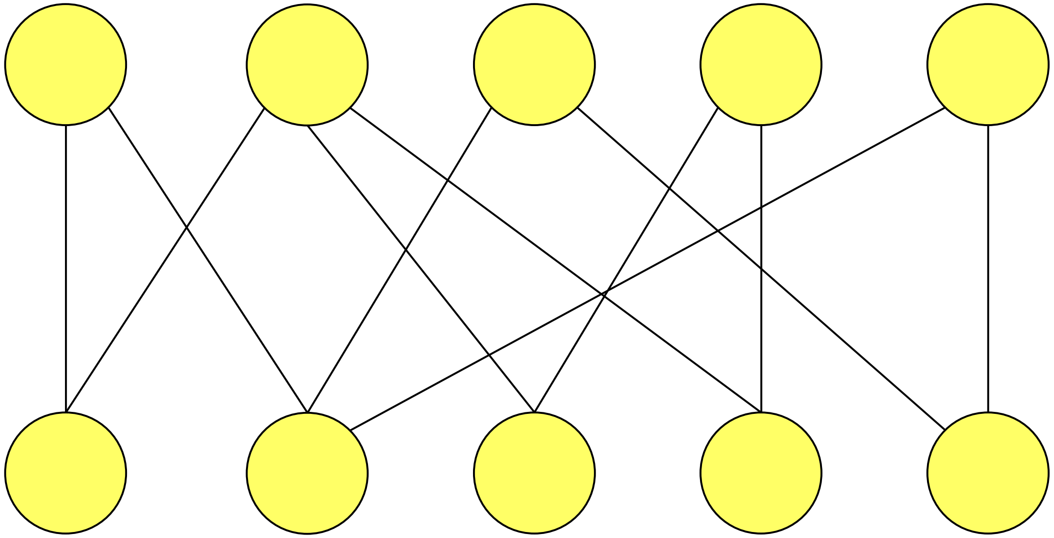
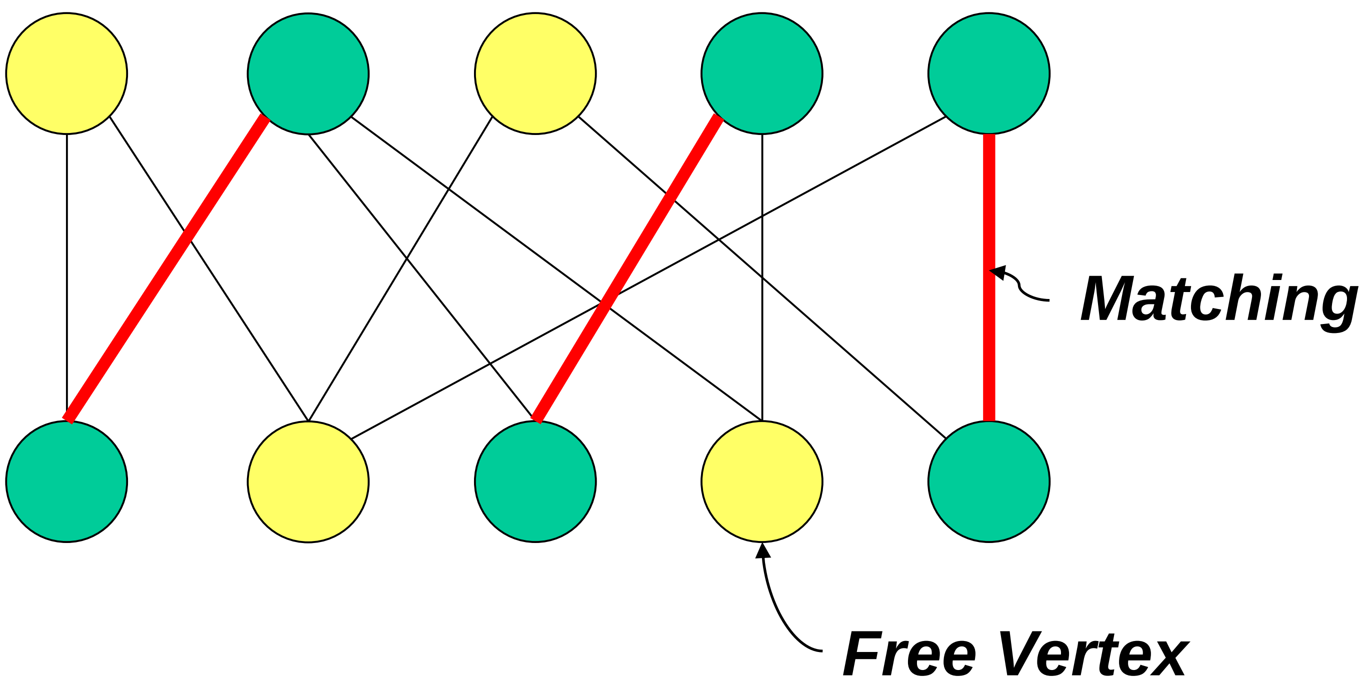
Maximum Bipartite Matching

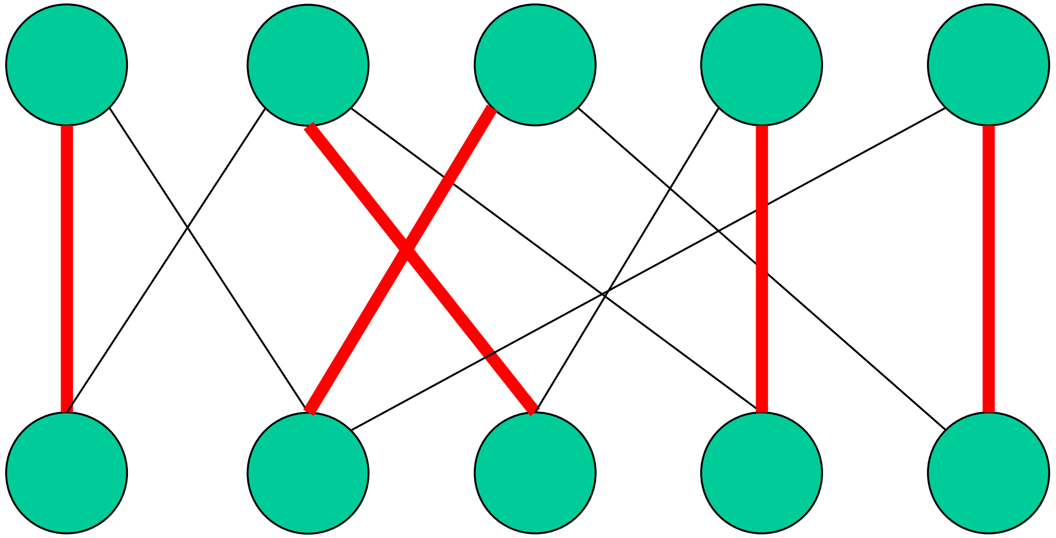
Bipartite Matching



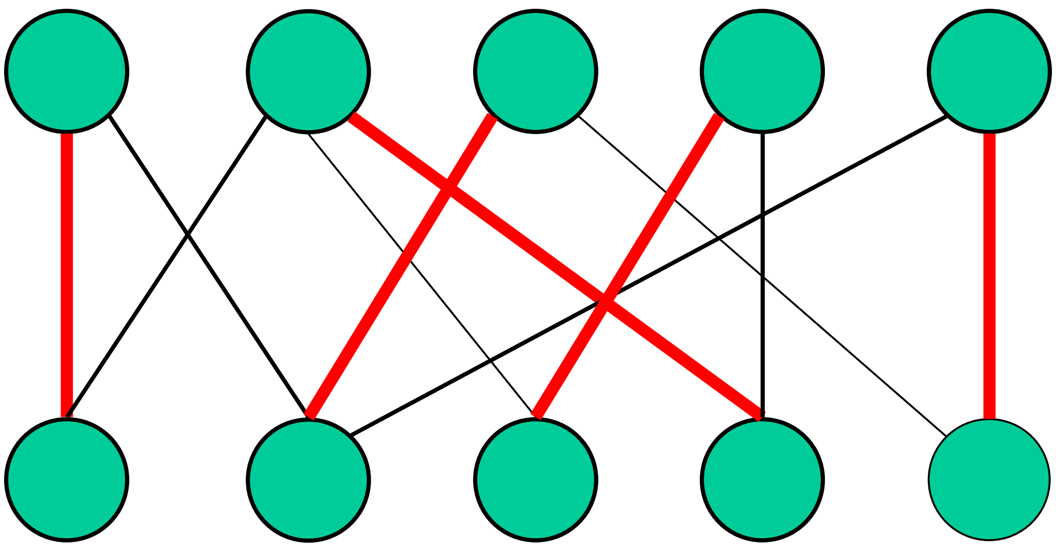
Matching



- Maximum Matching: matching with the largest number of edges



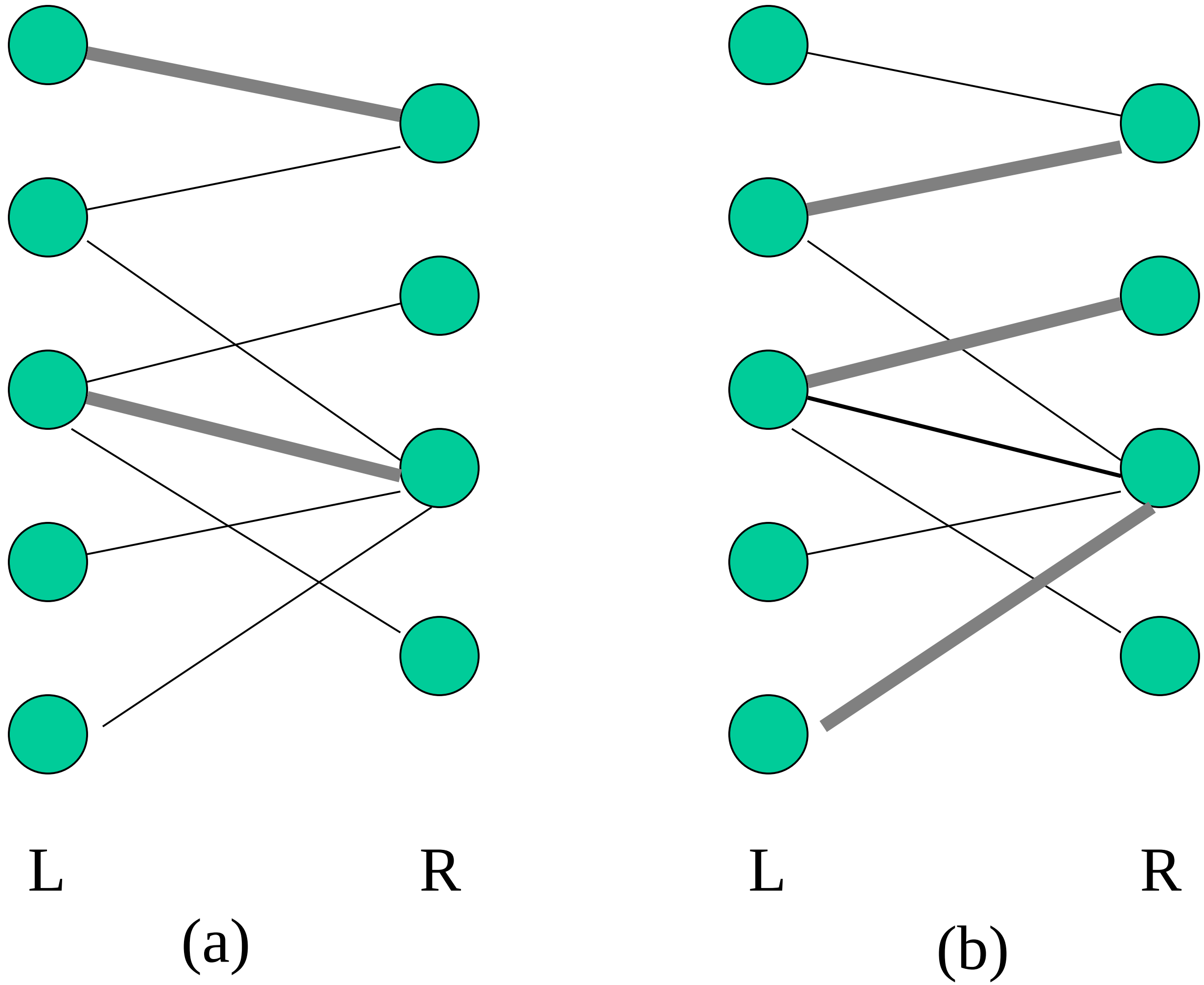
- Note that maximum matching is not unique.



Maximum bipartite matching:

- Bipartite graph: a graph (V, E), where $V=L\cup R$, $L\cap R$=empty, and for every $(u, v)\in E$, $u \in L$ and $v \in R$.
- Given an undirected graph G=(V,E), a matching is a subset of edges $M\subseteq E$ such that for all vertices $v\in V$,at most one edge of M is incident on v.We say that a vertex $v \in V$ is matched by matching M if some edge in M is incident on v;otherwise, v is unmatched. A maximum matching is a matching of maximum cardinality,that is, a matching M such that for any matching M', we have .

$$|M| \geq |M'|$$

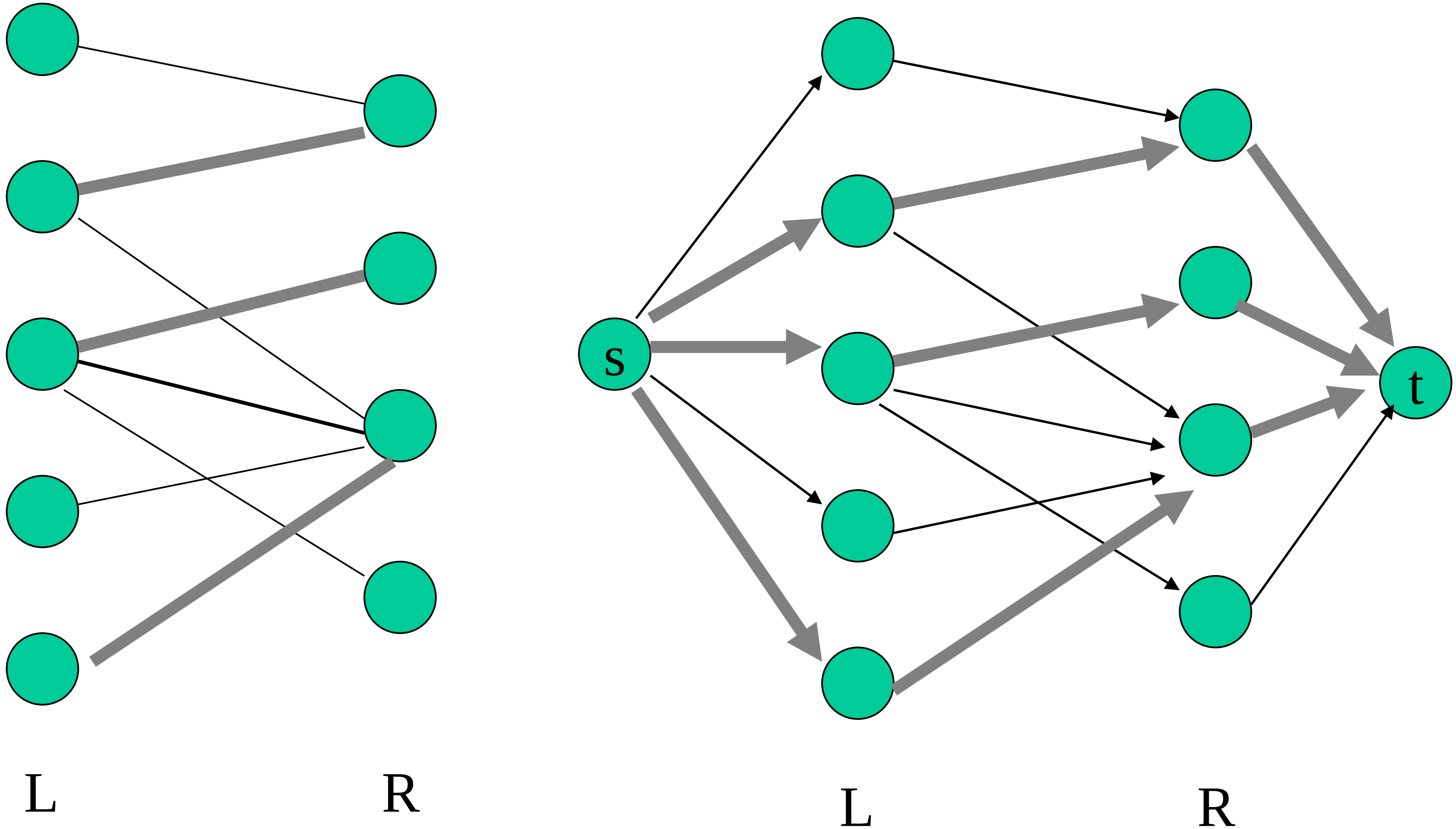


A bipartite graph G=(V,E) with vertex partition $V=L\cup R$.(a)A matching with cardinality 2.(b) A maximum matching with cardinality 3.

Finding a maximum bipartite matching:

- We define the corresponding flow network $G'=(V',E')$ for the bipartite graph G as follows. Let the source s and sink t be new vertices not in V, and let $V'=V\cup\{s,t\}$.If the vertex partition of G is $V=L \cup R$, the directed edges of G' are given by $E'=\{(s,u):u\in L\} \cup\{(u,v):u \in L,v \in R,\text{and } (u,v) \in E\} \cup\{(v,t):v \in R\}$.Finally, we assign unit capacity to each edge in E'.

- We will show that a matching in G corresponds directly to a flow in G's corresponding flow network G'. We say that a flow f on a flow network $G=(V,E)$ is integer-valued if $f(u,v)$ is an integer for all $(u,v) \in V*V$.



(a) (b)

(a)The bipartite graph G=(V,E) with vertex partition V=L∪R. A maximum matching is shown by shaded edges.(b) The corresponding flow network.Each edge has unit capacity.Shaded edges have a flow of 1,and all other edges carry no flow.

Continue:

- Lemma .
- Let G=(V,E) be a bipartite graph with vertex partition V=L∪R,and let G'=(V',E') be its corresponding flow network.If M is a matching in G, then there is an integer-valued flow f in G' with value $|f|=|M|$ .Conversely, if f is an integer-valued flow in G',then there is a matching M in G with cardinality $|M|=|f|$.
- Reason: The edges incident to s and t ensures this.
  - Each node in the first column has in-degree 1
  - Each node in the second column has out-degree 1.
  - So each node in the bipartite graph can be involved once in the flow.