Traffic Control

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1 INTRODUCTION

In this project, we worked on a method for shortening the time that cars must wait at traffic lights. We are employing the idea of Graph Theory in Discrete Mathematics and solving some equations with multiple variables while working on this.
In order to give you a fast understanding of the criteria at the outset, we have included some terminology that you may not be familiar with. The amount of traffic or the width of the road are not factors in this project. This is accomplished by making the simple assumption that the signal point has ideal roads and ideal traffic in every direction.

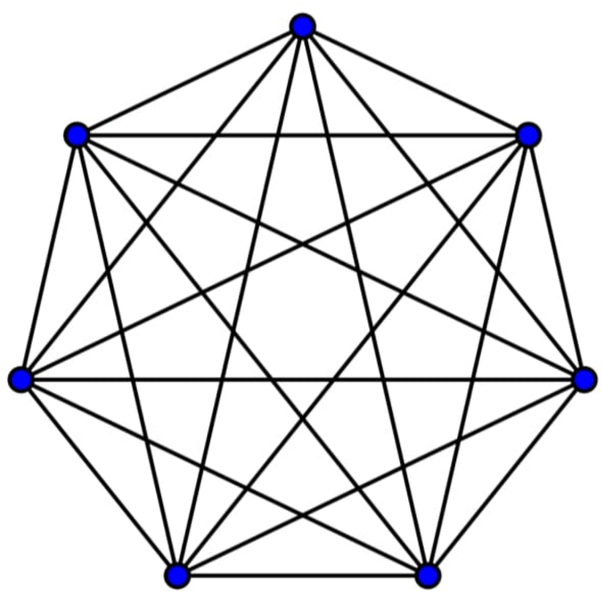
2 PROBLEM STATEMENT

Create a way to reduce the red light period implicitly for all vehicle streams approaching a junction at a traffic signal using the appropriate mathematical theories. It's possible to overlook the volume of roads and their size in relation to the issue. By considering the ideal situation, it may be resolved.

3 PRE- REQUISITE KNOWLEDGE

3.1 COMPLETE GRAPHIC:

An undirected graph in which every pair of distinct vertices is connected by a unique edge.
- The graph should just be symmetrical, and that's all we need to worry about.



3.2 Graph Notation:

• G(V,E) , in this term 'V' denotes the number of vertices and 'E' denotes the number of edges.

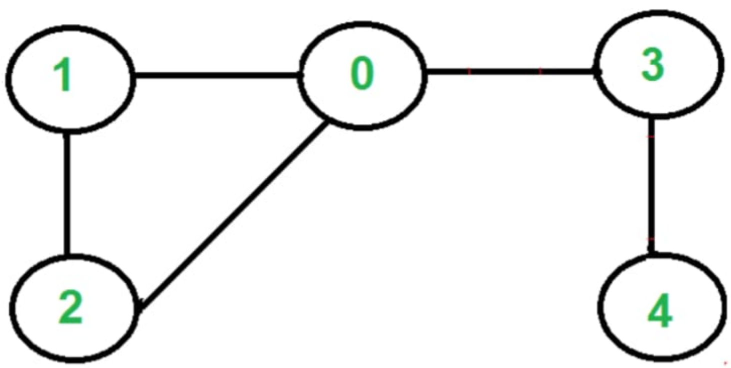
•Now, we are going to discuss about 'H' which is a sub-graph of 'G'.

3.3 Subgraph

It consists of a selection of edges and vertices from the main graph.

3.4 Clique

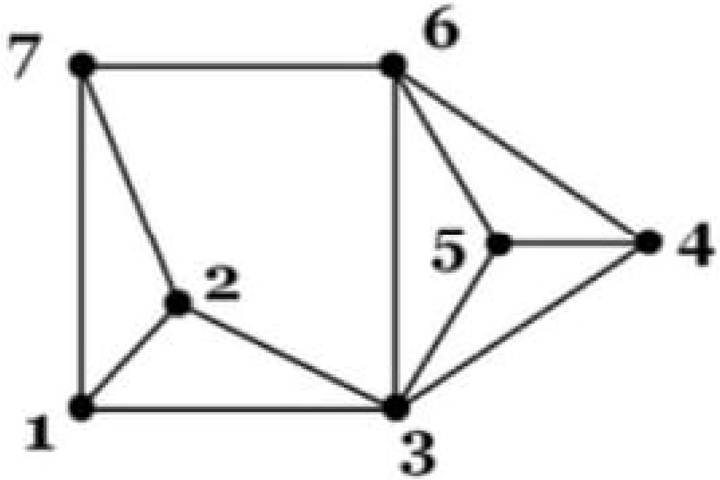
A clique is a subset of undirected graph vertices where each pair of different vertices is connected to the vertex next to it.A complete subgraph of a graph is called a clique.



This graph can be divided into two cliques : One clique contains 0,1,2 and other clique contains 3,4

3.5 Maximal Clique

Clique that can never be expanded by adding another nearby vertex. A maximal clique is also one that is not contained in another clique of the graph.

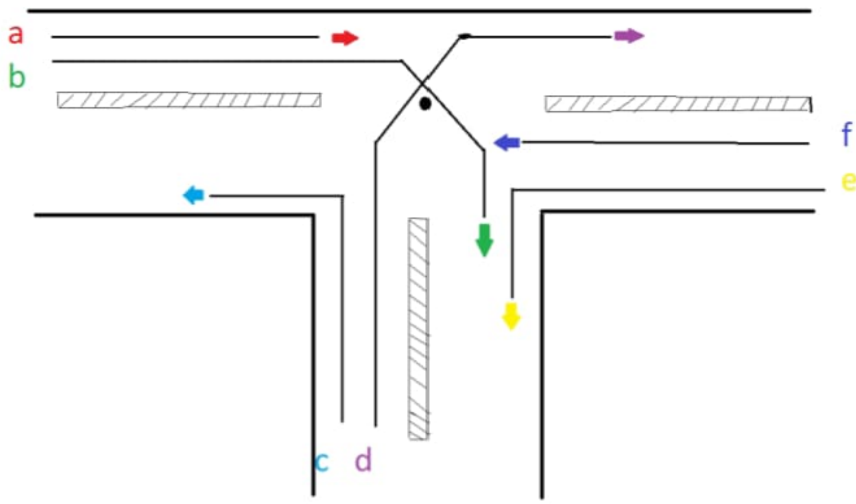


The maximal cliques in the above graph are 6, 3, 4, 5, 7, 2, 1, 2, 1, 3, These flower brackets indicate every maximal clique. Here, 7, 2, 1 is a maximal clique because if we try to include an adjacent vertex like 6, then it would not become a clique.

4 THE MATHEMATICS FORMULATION

1. The issue can be resolved using a specific approach that works for all issues of this nature.
2. After looking at the vehicle streams, we create a compatibility graph out of it.
3. Next, we must segment the graph into phases, also known as cliques.
4. There will be a specific window of time for each step to receive the all-clear.
5. The phase includes specific streams.
6. Based on our requirements, we set various limits, such as the length of a cycle overall and the minimum amount of time for each phase to receive a green light.
7. After that, we follow the mathematics that results in the least amount of red light time across all streams by supplying variable names for phases.

5 THE MATHEMATICAL SOLUTION

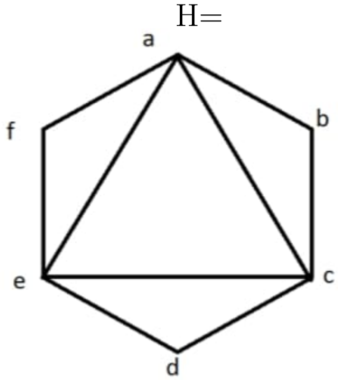


Now we are going to make compatible graph.

5.1 Compatible graph

In the graph, each stream is represented by a vertex. The streams that can coexist with one another are combined. Here, compatibility refers to streams that run concurrently without interfering with one another.

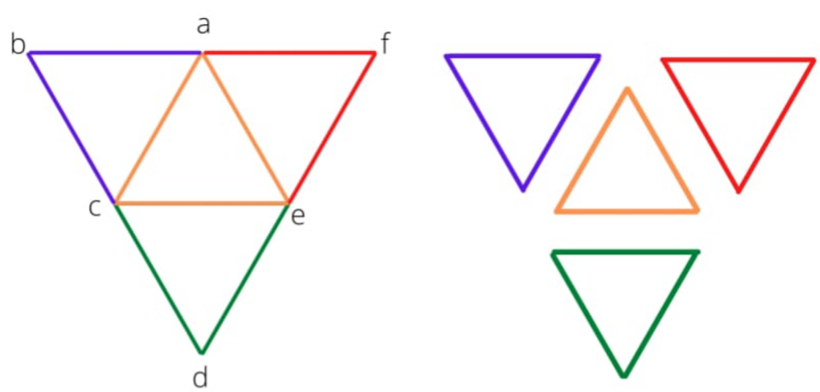
H=



a is compatible with - b, c, f, e
b is compatible with - c, a
c is compatible with - d, e, a, b
d is compatible with - e, c
e is compatible with - f, a, d, c
f is compatible with - a, e

• We must create a subgraph of the graph "H" that contains the most cliques possible.
• The reason for this is that if we can do it, we might be able to run more streams concurrently, which would save time.
(More streams get green line at the same time)
As a result, subgraph G, which turns out to be H itself, has the most maximal cliques. This is the subparagraph that follows a brief modulation.

The subgraph looks like



6 Cliques:

a, b, c , a, e, f , c, e, d , a, c, e

Let's now give them four variables: K1, K2, K3, and K4. During each of these four phases, each clique will receive a green light.

$$K_1 = \{a, b, c\} \qquad K_3 = \{c, e, d\}$$
$$K_2 = \{a, e, f\} \qquad K_4 = \{a, c, e\}$$

Now, let's choose a few times when each phase will be given the all-clear:

$$K_1 - d_1 \qquad K_3 - d_3$$
$$K_2 - d_2 \qquad K_4 - d_4$$

d1,d2,d3,d4- They are the respective time of green light for each phase.
The red light has now turned on for each stream:

$$a - d_3$$
$$b - d_2 + d_3 + d_4$$
$$c - d_2$$
$$d - d_1 + d_2 + d_4$$
$$e - d_1$$
$$f - d_1 + d_3 + d_4$$

Let's call the total time of red light across all streams as Z. So,

$$Z = (d_3) + (d_2 + d_3 + d_4) + (d_2) + (d_1 + d_2 + d_4) + (d_1) + (d_1 + d_3 + d_4)$$

$$Z = 3(d_1 + d_2 + d_3 + d_4)$$

Let us assume our traffic system has a minimum time of 20 seconds of green light for each stream. .... (a)

Now let's look at a cycle where all four phases have already displayed a green light.

So, we assume our cycle is of 120 seconds which means
d1 + d2 + d3 + d4 = 120 ....(1)
and from equation (a) ,

$$d_1 + d_3 + d_4 \geq 20 \qquad ....(2)$$

$$d_1 \geq 20 \qquad ....(3)$$

$$d_1 + d_2 + d_4 \geq 20 \qquad ....(4)$$

$$d_2 \geq 20 \qquad ....(5)$$

$$d_2 + d_3 + d_4 \geq 20 \qquad ....(6)$$

finally we have,

d1 + d2 + d3 + d4 = 120 ....(1)

- Finding d1, d2, d3, and d4 such that Z is minimum is our primary goal.Thus, the overall length of the red light is minimal.

- Therefore, we obtain Z = 360 seconds from equation (1), indicating that Z is constant for the selected graph. 360 seconds is both the minimum and maximum duration.

• To keep Zmin = 360, the values of d1, d2, d3, and d4 are as follows:

$$100 \geq d_1 \geq 20$$

$$100 \geq d_2 \geq 20$$

$$100 \geq d_3 \geq 20$$

$$100 \geq d_4$$

- In this case, we receive Z as a constant value, but with this approach, we can also obtain Zmin.

- Zmin is obtained by solving all the equations from equation (1) to equation (7).

7 Practical Interpretation of Application

The final solution to this puzzle illustrates that, while we can reduce even the busiest traffic on earth, not every circumstance can be resolved precisely. We may be able to significantly shorten the red light time in certain places while finding it difficult to do so in others. The stream clubbing and in-depth flow observations made possible by this use of graph theory were very helpful.
When the amount of traffic and the size of the road are roughly equal, this could really work in certain circumstances, but not always. The automobiles will now feel much better with this solution since if a systematic method or a disciplined style is followed, every task is completed with very little mistake.

8 CONCLUSION

The value of Z that we derived is the stream with the least amount of red light. This states that 360 seconds is the total amount of red light for each individual stream.
The solution of our chosen problem mentioned previously , is in itself an explanation for the implementation in real life. This solution gives us an implicit way to come out of this problem and as said before this problem excludes the volume of traffic and the size of roads into consideration.
As a result, this direct will not work as intended in the current context. However, it does offer a useful means of demonstrating both the elegance of mathematics and its application to actual

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