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|----------|----------------------------|-------------------|
| v | Ascent Rate | m/s |
| F | Free Lift | N |
| F_g | Gross Lift | N |
| C_d | Drag Coefficient | |
| ρ_a | Air Density | kg/m ³ |
| ρ_g | Gas Density | kg/m ³ |
| r | Launch Radius | m |
| A | Launch Area | m ² |
| V_L | Launch Volume | m ³ |
| m_p | Payload Mass | kg |
| m_b | Balloon Mass | kg |
| g | Gravitational Acceleration | m/s ² |

$$v = \sqrt{\frac{F}{\frac{1}{2}C_d\rho_a A}} \quad (1)$$

$$A = \pi r^2 \quad (2)$$

$$F = F_g - (m_p + m_b)(g) \quad (3)$$

$$F_g = (\rho_a - \rho_g) \times V_L \quad (4)$$

$$V_L = \frac{4}{3}\pi r^3 \quad (5)$$

$$F_g = (\rho_a - \rho_g)\left(\frac{4}{3}\pi r^3\right) \quad (6)$$

$$F = (\rho_a - \rho_g)\left(\frac{4}{3}\pi r^3\right) - (m_p + m_b)(g) \quad (7)$$

$$v = \sqrt{\frac{(\rho_a - \rho_g)\left(\frac{4}{3}\pi r^3\right) - (m_p + m_b)(g)}{\left(\frac{1}{2}C_d\rho_a\right)(\pi r^2)}} \quad (8)$$

$$v^2\left(\frac{1}{2}C_d\rho_a\pi r^2\right) = \left((\rho_a - \rho_g)\frac{4}{3}\pi r^3\right) - (m_p + m_b)(g) \quad (9)$$

$$\left[(\rho_a - \rho_g)\frac{4}{3}\pi\right]r^3 - \left[\frac{1}{2}v^2C_d\rho_a\pi\right]r^2 - [(m_p + m_b)g] = 0 \quad (10)$$

yuck! a cubic!