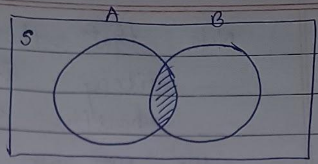
Assignment - 1

Asn. 1)

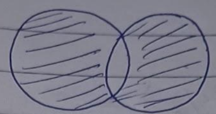
$P(A) = 0.3$

$P(B) = 0.4$

$P(A \cap B) = 0.2$



For exactly one to occur:
venn diagram will be $\longrightarrow$



a) P (exactly one occurs either A or B) $= P(A \cup B) - P(A \cap B)$

we know $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.3 + 0.4 - 2 \times 0.2$

$= \boxed{0.3}$ Ans

b) P (atleast on of the event A or B occur) $= P(A \cup B)$

$= 0.3 + 0.4 - 0.2$

$= \boxed{0.5}$ Ans

c) P (none of A or B occur) $= 1 - P(A \cup B)$

$= \boxed{0.5}$ Ans

Asn. 3) different possibilities:

3R Ball in bag	4R ball in bag	5R ball in bag	6R ball in bag
E_1	E_2	E_3	E_4

A = 3 Balls drawn without replacement is Red.

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Note that the outcomes E_1 E_2 E_3 & E_4 are equally likely & they are mutually exclusive & exhaustive too.

We can use Bay's theorem:

$$P\left(\frac{E_4}{A}\right) = \frac{P(E_4).\ P\left(\frac{A}{E_4}\right)}{P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right) + P(E_3).P\left(\frac{A}{E_3}\right) + P(E_4).P\left(\frac{A}{E_4}\right)}$$

$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$ [Equally likely event]

$P\left(\frac{A}{E_1}\right) = \frac{{}^{3}C_3}{{}^{6}C_3} = \frac{1}{20}$ $\quad P\left(\frac{A}{E_2}\right) = \frac{{}^{4}C_3}{{}^{6}C_3} = \frac{4}{20}$

$P\left(\frac{A}{E_3}\right) = \frac{{}^{5}C_3}{{}^{6}C_3} = \frac{10}{20}$ $\quad P\left(\frac{A}{E_4}\right) = \frac{{}^{6}C_3}{{}^{6}C_3} = \frac{20}{20}$

Substituting the values

$$P\left(E_4/A\right) = \frac{\cancel{\frac{1}{4}} \cdot 1}{\cancel{\frac{1}{4}} \cdot \frac{1}{20} + \cancel{\frac{1}{4}} \cdot \frac{4}{20} + \cancel{\frac{1}{4}} \cdot \frac{10}{20} + \cancel{\frac{1}{4}} \cdot \frac{20}{20}}$$

$$= \frac{20}{35} = \boxed{\frac{4}{7}}$$

Ans

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equally
&

Qsn. 4) $P(x < 0.5) = \underset{x=0.2}{0.1} + \underset{x=0.4}{0.2} + \underset{\text{otherwise}}{0} = \boxed{0.3}$ Ans

$P(0.25 < X < 0.75) = \underset{(x=0.4)}{0.2} + \underset{(x=0.5)}{0.2} + \underset{\text{otherwise}}{0} = \boxed{0.4}$ Ans

). $P\left(A/E_3\right)$

$\cancel{P(X = 0.2 \mid X \leq 0.6)} = \cancel{P(A) + P(B)} - \cancel{P(A \cap B)}$

$= \cancel{0.2 + (0.1 + 0.2 + 0.5)}$

$P\left(A/E_4\right)$

$P(X = 0.2 \mid X < 0.6) = \dfrac{0.1 + \cancel{0.2} + 0.2}{0.1 + 0.2 + 0.2} = \boxed{0.2}$ Ans

ly event

~~Qsn. 5)~~ ~~by property of distribution function.~~

$\frac{4}{20}$

$\cancel{0} + \cancel{\frac{c}{3}} + \cancel{\frac{c+6c}{6}} + \cancel{\frac{4c^2 - 3c + 8}{4}} + \cancel{\frac{1}{2}} < \cancel{1}$

$\frac{20}{20}$

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Qsn. 6)

$$p(x) = \begin{cases} 1 & x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

$E(x^2 + y^2) = 1$ $\qquad$ $Var(y) = \frac{5}{9}$

$$E[X] = \int_{-\infty}^{\infty} x\, p(x)\, dx = \int_0^1 x\, p(x)\, dx = \int_0^1 x\, dx = \boxed{\frac{1}{2}}$$

Ans

$$E[X^2] = \int_{-\infty}^{\infty} x^2 p(x)\, dx = \int_0^1 x^2\, dx = \boxed{\frac{1}{3}}$$

$$Var[X] = E(X^2) - (E(X))^2$$

$$= \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$$

Ans

$$E[X^2 + Y^2] = 1$$

$$E[X^2] + E[Y^2] = 1$$

$$E[Y^2] = 1 - \frac{1}{3} \Rightarrow \frac{2}{3}$$

$$Var(Y) = \frac{5}{9}$$

$$E[Y^2] - (E[Y])^2 = \frac{5}{9}$$

$$\frac{2}{3} - (E[Y])^2 = \frac{5}{9} \qquad E[Y] = \sqrt{\frac{2}{3} - \frac{5}{9}} = \boxed{\frac{1}{3}}$$

Ans

$$E[X+Y] = E[X] + E[Y] = \frac{1}{2} + \frac{1}{3} = \boxed{\frac{5}{6}}$$

Ans

$Var(x) = \exp(x^2) - [\exp(x)]^2$

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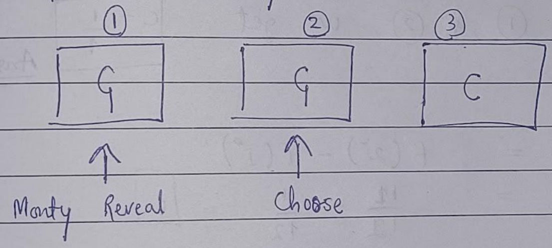
$\boxed{E[x] = \frac{1}{2} \quad Var[x] = \frac{1}{12} \quad E[y] = \frac{1}{3} \quad E[x+y] = \frac{5}{6}}$ Ans

Q: 2) **Case 1:** Stick with the original decision :
the probability of wining in this case will be $\frac{1}{3}$ and it will not change as initially there were 3 doors & 1 door has car behind it.

Case 2: If we change our decision :
Initially choosing a door will have a probability of wining of $\frac{1}{3}$ but after Monty reveal one of the door which has goat behind, the probability of car behind the door will increase.
Suppose initially car was behind door 3, &



you have choosen intially door 2, after monty reveal probability of car behind door 3 become $\frac{1}{3} + \frac{1}{3}$

as you have got to know that car was not behind 1.
Hence switching our decision will increase the probability of winining, which is $\frac{2}{3}$.

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Ans 5) Using RHL continuity:

$$f(3) = f(3^+) \Rightarrow \frac{4c^2 - 9c + 6}{4} = 1$$

$$\Rightarrow 4c^2 - 9c + 2 = 0$$

$$\Rightarrow (4c-1)(c-2) = 0$$

$$\underbrace{c = \frac{1}{4}, 2}$$ — 1

& for the LHL at $x = 1$

$$f(1^-) \leq f(1) \Rightarrow \frac{2}{3} \leq \frac{7-6c}{6}$$

$$\underbrace{c \leq \frac{1}{2}}$$ — 2

Property:

1. $P(x_1 < x < x_2) = F(x_2^-) - F(x_1)$
2. $P(x_1 \leq x < x_2) = F(x_2^-) - F(x_1^-)$
3. $P(x_1 < x \leq x_2) = F(x_2) - F(x_1)$
4. $P(x_1 \leq x \leq x_2) = F(x_2) - F(x_1^-)$

Combining 1 & 2 we get $\boxed{c = \frac{1}{4}}$ Ans

$$P(1 < X < 2) = f(2^-) - f(1^+)$$
$$= \frac{11}{12} - \frac{11}{12} = \boxed{0}$$

$$P(2 \leq X < 3) = f(3^-) - f(2^-)$$
$$= 1 - \frac{11}{12} = \boxed{\frac{1}{12}}$$

$$P(0 < X \leq 1) = f(1) - f(0)$$
$$= \frac{11}{12} - \frac{2}{3} = \boxed{\frac{1}{4}}$$

$$P(1 \leq X \leq 2) = f(2) - f(1^-)$$
$$= 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

$$P(X \geq 3) = f(3^+) - f(3^-) = 1 - 1 = 0$$

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