

- ① Prime Number
- ② LCM
- ③ GCD

Topics

Prime Number  
Worst T.C  $O(N)$

# Prime Number

n

↓  
 itself or divisible

$n=2$

1, 2

$n=3$

1, 3

$n=4$

1, 2, 4

$n=5$

1, 5

Print

n

1, n

2 to n-1

C++ code      Worst       $n=5$        $i=2$   
 $5 \% 2 \rightarrow \neq 0$  ✓  
 (X)

```

int n;
cin >> n;

for(int i=2; i<=n-1; i++){
    if(n%i==0){
        print("False"); break;
    }
}
print("True");
  
```

T.C —  $O(N)$

$S_1$  (16GB, 256 SSD)

→ 2ms  
 → 3ms

$S_2$  (16GB, 256 SSD)

→ 3ms

Asymptotic Notations

Worst Case T.C

↓

Big Oh       $O(n)$

T.C -  $O(1)$

$O(5)$

$n = 23$

$1 \times 23 = 23$



☆☆☆☆

Divisor of a  
Number always  
occur in pair.

$n = 24$

$1 \times 24 = 24$

$2 \times 12 = 24$

$3 \times 8 = 24$

$4 \times 6 = 24$

$6 \times 4 = 24$

$8 \times 3 = 24$

$12 \times 2 = 24$

$24 \times 1 = 24$

\* Point to  $\sqrt{n}$  value repeat  $\sqrt{n}$   
don't  $\frac{n}{2} //$

$n = 24$

$\sqrt{24}$  =  $\left| \begin{array}{c} 4.6 \\ \hline \end{array} \right|$  -

$\text{ceil}(4.2) \Rightarrow 5$

$\text{ceil}(4.6) \Rightarrow 5$

(-5)

$i = 2$  to  $\sqrt{n}$

$$i = 2 \text{ to } \sqrt{n}$$

$$n \% i$$

$$24 \% 2 \rightarrow$$

$$n = 18$$

$$\sqrt{n} \Rightarrow 4.2$$

$$\textcircled{5}$$

$$2 \text{ to } 5$$

False

$$n = 23$$

$$\sqrt{23} = 4.5$$

$$\textcircled{5}$$

$$2 \text{ to } 5$$

$$23 \% 2 \times$$

$$23 \% 3 \times$$

$$23 \% 4 \times$$

$$23 \% 5 \times$$

Prime

Algo

- ① Divisors always occur in pairs
- ②  $i = 2 \text{ to } \text{ceil}(\sqrt{n});$
- ③  $\text{if } (n \% i == 0) \{$

③ if  $(n \% i == 0)$  {  
 print("Not a Prime"); break;  
 }

④ At the end of the loop  
~~print~~ ("Prime");

T.C -  $O(N)$   
 $n = 200000$   
            
 (200000)

T.C -  $O(\sqrt{n})$   
 $n = 200000$   
            
 ④  $\sqrt{n}$   
 (450)

Prime Number

2 or 3    6n+1 or 6n-1

$6n+1$ , or  $6n-1$      $n=1$  (

2 (X)

3 (X)

5  $n=1$

7  $n=2$

$6n+1$ ,  $6n-1$

7

5

13

11

7	n=2	13 ✓	11 ✓
11	n=3	19 ✓	17 ✓
13			
17	n=4	25 ✗	23 ✓

Prime Number

T.C -  $O(\sqrt{n})$

bool primeNumber(int n) {

✓ if (n == 2 || n == 3) return true;

if (n % 2 == 0 || n % 3 == 0) return false;

★  $\left( \frac{n+1}{6}, \frac{n-1}{6} \right)$   $(i) \leq (\sqrt{n})^2$  i = 2 to sqrt(n)

★ Divisors always occur in pairs

for (int i = 5;  $i \times i \leq n$ ;  $i = i + 6$ ) {

or  $i \leq \sqrt{n}$

if (n % i == 0 ✓  
||  
✓ n % (i + 2) == 0) {

return false;

}

return true;

↳

$$n = \underline{\underline{67}}$$

$$n = 73$$

①  ~~$i \times i \leq n$~~

②  $i = i + 6$

$$i = 5 \quad 67 \% 5 \quad \times \quad 25 \leq 67$$

$$i = 7 \quad 67 \% 7 \quad \times \quad 49 \leq 67$$

$$i = 11$$

$$121 \leq 67$$

$$n = 73$$

$$i = 5 \quad 73 \% 5 \quad \times$$

$$25 \leq 73 \quad \checkmark$$

$$49 \leq 73$$

$$73 \% 7 \quad \times$$

~~$i = 11$~~

~~$(121 < 73)$~~

return true;

①  $i \times i \leq n$

②  $i = i + 6$

③  $n \% i == 0$

$$n \% i + 2 == 0$$

return false

i = 2 to 8

★★

$$T.C - \underline{\underline{O(\sqrt{n})}}$$

GCD

a=4    b=6

$\frac{2 \times 2}{2 \times 3} \quad \frac{(2)}{\text{Euclid's Algorithm}}$

4  $\overline{) 6}$  2  
4  
2  
4  
0  
Ans  $\rightarrow$  2

n=12  
m=36

12     $\frac{12 \times 1}{12 \times 3}$

12  $\rightarrow$  {2  $\times$  2  $\times$  3}

36  $\rightarrow$  {2  $\times$  2  $\times$  3  $\times$  3}

$\Downarrow$   
{2, 2, 3}



$$2 + 2 + 3 = \underline{\underline{12}}$$

Euclid's Algorithm

$$a = 4 \quad b = 6$$

$$\begin{array}{r} 4 \overline{) 6} \\ \underline{4} \phantom{0} \\ 2 \end{array}$$

$\text{GCD} = 2$   
 $\text{LCF} = 12$

$$a = 12 \quad b = 36$$

$$\begin{array}{r} 12 \overline{) 36} \\ \underline{36} \\ 0 \end{array}$$

$\text{GCD} = 12$

Recursion

```

int gcd(int divisor, int dividend) {
    if (dividend % divisor == 0)
        return divisor;
    return gcd(dividend % divisor, divisor);
}

```

$$a = 4 \quad b = 6$$

$$\text{gcd}(4, 6)$$

$$6 \% 4 = 2$$

$$\begin{array}{r} 4 \overline{) 6} \\ \underline{4} \\ 2 \end{array}$$

$$\begin{array}{r} 2 \overline{) 4} \\ \underline{4} \\ 0 \end{array}$$

$$6 \% 4 = 2 \quad \hookrightarrow 1$$

$$\underline{\underline{\text{gcd}(2, 4)}}$$

$$4 \% 2 = 0 \quad \hookrightarrow 2$$

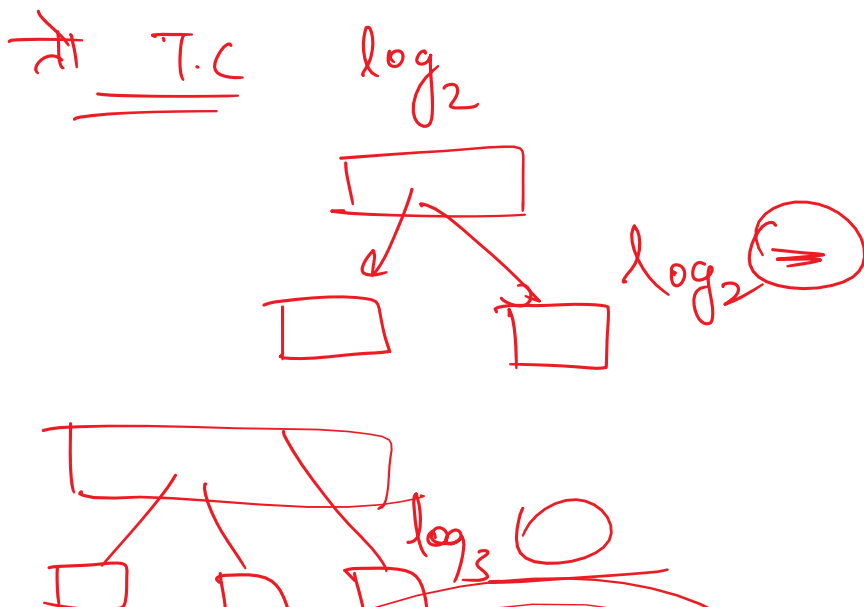
$$O(\log(\min(a, b)))$$

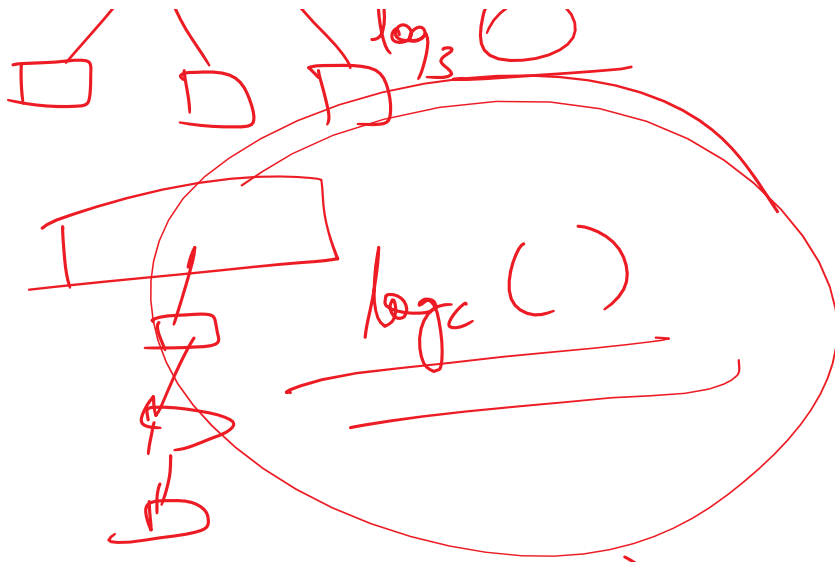
T.C — ~~min~~

$n = 5$ $\textcircled{1}$ $5 \overline{) 5}$ $\textcircled{1}$ $\text{gcd}$	$m = 6$ $\textcircled{1}$ $5 \overline{) 6}$ $\textcircled{1}$ $\text{gcd}$	$m = 100$ $n = 3$ $3 \overline{) 100}$ $99$ $\textcircled{1}$ $\text{gcd}$
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T.C —  $O(\log(\min(a, b)))$

જો બીજા મી ચીજ Divide. દોતો છે





$$T.C - O(\log(\min(a, b)))$$

LCM

$$a=4 \quad b=6$$

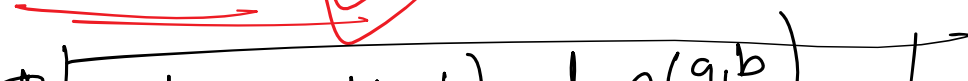
2	7, 6
2	2, 3
3	1, 3
1	1, 1

$$2 \times 2 \times 3$$

$$4 \times 3 = 12$$

~~LCM~~

GCD



$$a \times b = \text{gcd}(a, b) \times \text{lcm}(a, b)$$

$$a=4 \quad b=6$$

$$\begin{array}{r} 4 \overline{) 6} \\ \underline{4} \phantom{0} \\ 2 \end{array}$$

$$2 \times 6 = 12 = \text{lcm}(4, 6)$$

$$12 = \text{lcm}(4, 6)$$

$$T.C - O(\log_c(\min(a, b)))$$

```
int lcm(int a, int b) {
```

```
    int i = 2; int ans = 1;
```

```
    while(a != 1 && b != 1) {
```

```
        if(a % i == 0 && b % i == 0)
```

```
            a /= i; b /= i; ans *= i;
```

```
    }
```

```
    return ans;
```

$$ans = 2 \times 2 \times 3$$

$$(12)$$

$$a=4 \quad b=6$$

$$\begin{array}{r} 2 \overline{) 4, 6} \\ \underline{2} \phantom{0} \\ 2 \end{array}$$

```

if (a % i == 0)
{
    a = a / i;
}
if (b % i == 0)
{
    b = b / i;
}
ans *= i;

```

2	4, (6)
2	2, 3
3	1, 3
	1, 1

return ans; ✓

T.C  $\max(a, b)$

eg  $a=2$   $b=4$

2	2, 4
2	1, 2
	1, 1

T.C  $\rightarrow \max(a, b)$

T.C  $\log_c(\min(a, b))$

7.6

① GCD (2)