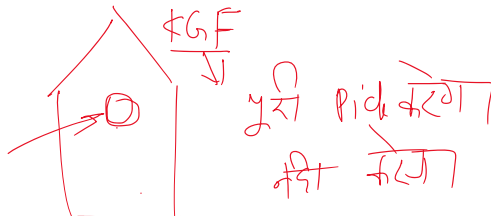
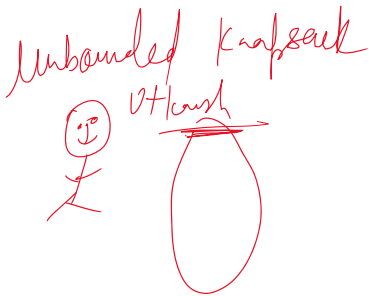
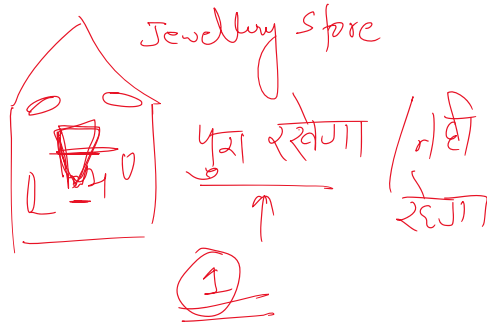
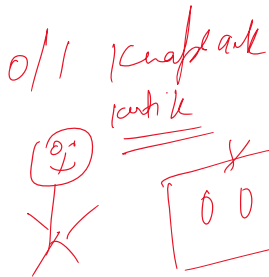
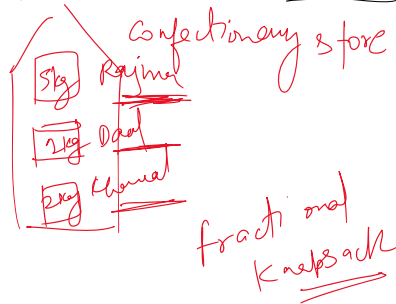
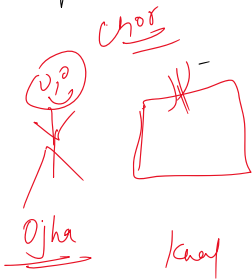
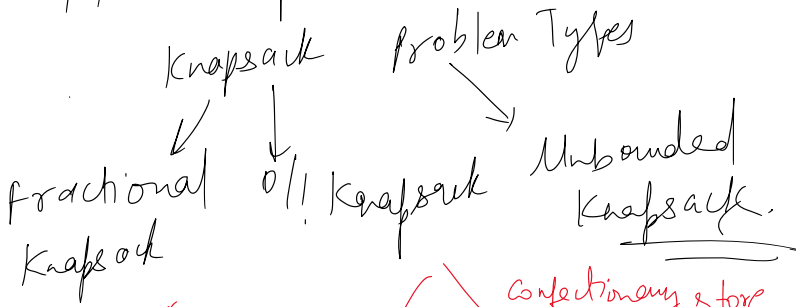


0/1 Knapsack



Unbounded Knapsack

0-1 Knapsack

quantity of each item or

N items we pick one item only

values 

1	2	3
---	---	---

once

weights 

4	5	1
---	---	---

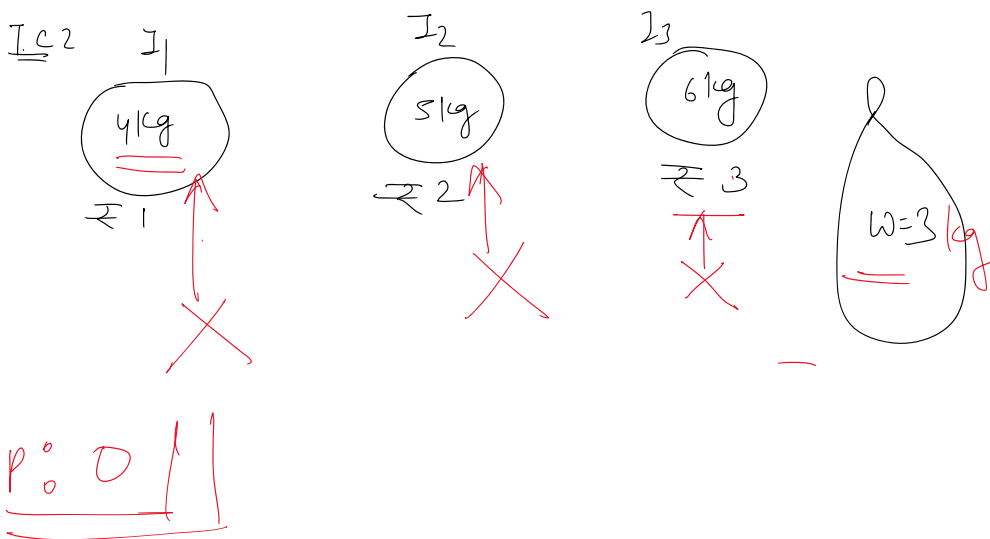
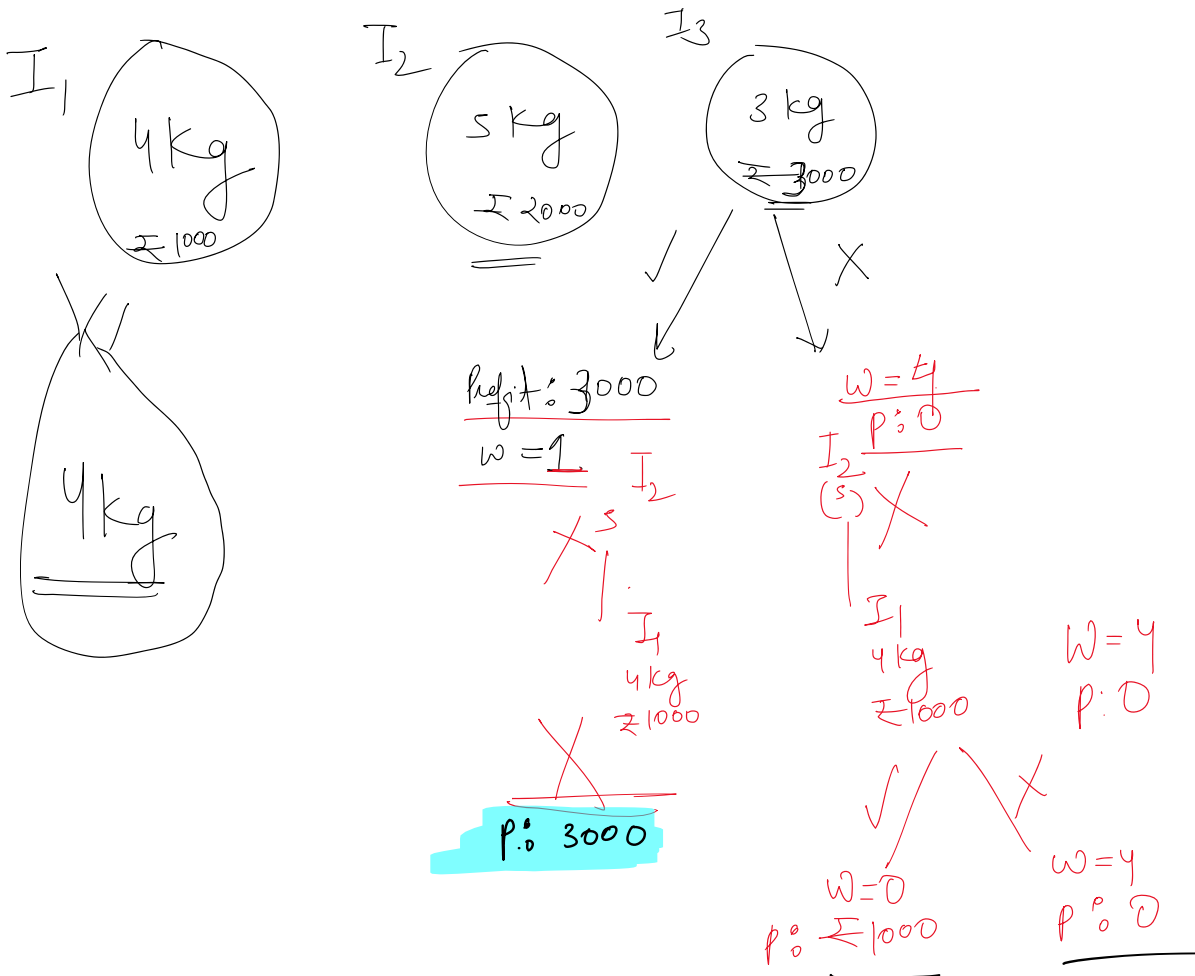
  
I<sub>1</sub> I<sub>2</sub> I<sub>3</sub>



Maximum Profit

You cannot break an item

You cannot break an item  
Fractional Knapsack X



Choice + Decision = Recursion

Recursion (Top to Bottom)

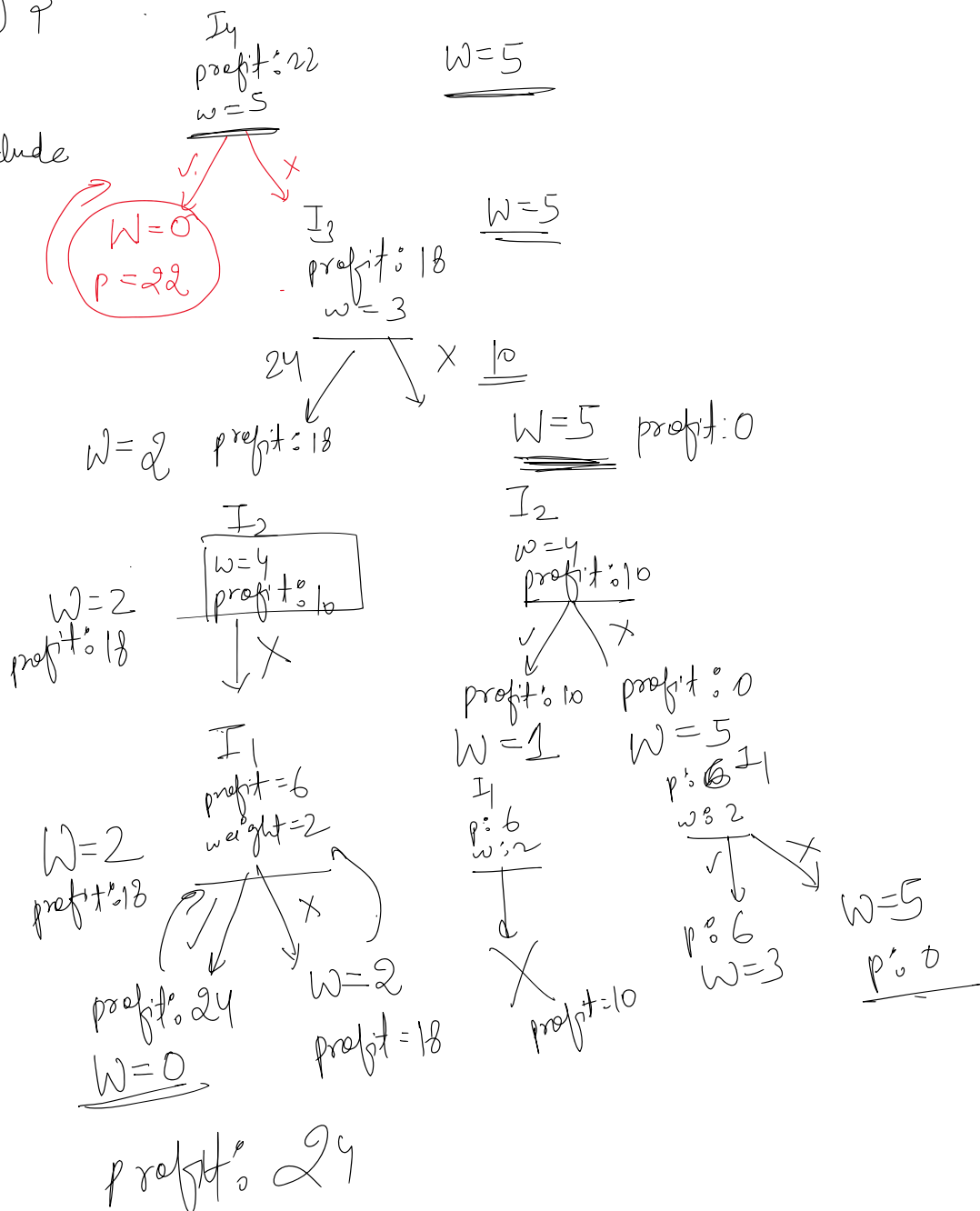
T.C3

	$I_1$	$I_2$	$I_3$	$I_4$
values	6	10	18	22
weight	2	4	3	5

$W = 5$

Recursive (Top to Bottom)

if ( $Iw \leq Kc$ ) {  
 // choice  
 ① Include  
 ② Not Include  
}

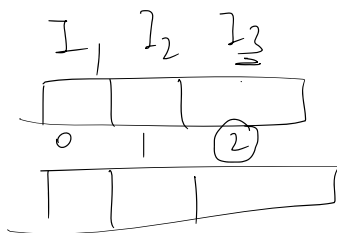


choice Diagram  
 if ( $I_w \leq K_c$ ) {

// Include

// Not Include

} else {  
 // Not Include  
 }



Recursion TLE

D.P.

memoization

Tabulation

~~Optimize~~  
 Space Optimize

But Important

(1) changing variable

2-D → 1-D → 1D-changing  
 → 2-D → (2-changing)

Recursive call  
 $f(2, 1)$

```
class Solution
{
public:
//Function to return max value that can be put in knapsack of capacity W.
int solve(int Knapsack_Capacity, int Item_Weights[], int Item_Profit[], int No_of_Item){
// base case
if(No_of_Item == 0 || Knapsack_Capacity == 0) return 0;
// choice diagram
int maxi = 0;
if(Item_Weights[No_of_Item-1] <= Knapsack_Capacity){
// include
maxi = max(maxi, solve(Knapsack_Capacity-Item_Weights[No_of_Item-1], Item_Weights, Item_Profit, No_of_Item-1) + Item_Profit[No_of_Item-1]);
// not include
maxi = max(maxi, solve(Knapsack_Capacity, Item_Weights, Item_Profit, No_of_Item-1));
}
else{
// not include
}
```

```

// not include
maxi = max(maxi, solve(Knapsack_Capacity, Item_Weights, Item_Profit, No_of_Item-1));
}
return maxi;

}
int knapSack(int Knapsack_Capacity, int Item_Weights[], int Item_Profit[], int No_of_Items)
{
    return solve(Knapsack_Capacity, Item_Weights, Item_Profit, No_of_Items);
}
};

```

$T.C - O(2^N)$   $S.C - \underline{O(N)}$   
 $\uparrow$   
No. of Items

```

int solve(int Knapsack_Capacity, int Item_Weights[], int Item_Profit[], int No_of_Items vector<vector<int>> dp){
    // base case
    if(No_of_Item == 0 || Knapsack_Capacity == 0) return 0;
    // check for recomputation
    if(dp[Knapsack_Capacity][No_of_Item] != -1) return dp[Knapsack_Capacity][No_of_Item];
    // choice diagram

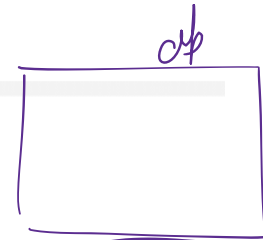
    int maxi = 0;
    if(Item_Weights[No_of_Item-1] <= Knapsack_Capacity){
        // Include
        maxi = max(maxi, solve(Knapsack_Capacity - Item_Weights[No_of_Item-1], Item_Weights, Item_Profit, No_of_Item-1, dp) + Item_Profit[No_of_Item-1]);

        // not include
        maxi = max(maxi, solve(Knapsack_Capacity, Item_Weights, Item_Profit, No_of_Item-1, dp));
    } else {
        // not include
        maxi = max(maxi, solve(Knapsack_Capacity, Item_Weights, Item_Profit, No_of_Item-1, dp));
    }
    return dp[Knapsack_Capacity][No_of_Item] = maxi;
}

int knapSack(int Knapsack_Capacity, int Item_Weights[], int Item_Profit[], int No_of_Items)
{
    // knapsack capacity
    // NO. of items
    vector<vector<int>> dp(Knapsack_Capacity + 1, vector<int>(No_of_Items + 1, -1));
    return solve(Knapsack_Capacity, Item_Weights, Item_Profit, No_of_Items, dp);
}

```

$T.C - O(\text{Knap Capacity} \times \text{No. of Item})$   
 $S.C - O(\text{No. of Item} \times \text{Knap Capacity})$



Knap Capacity x No. of Item

① DP (-1) // 0

② Base Case (Return on)  $\Rightarrow$  Initialization

③ 1st find value set start ~~from~~ loop TTTTT

④ Recursive  $\Rightarrow$   $DP[i][j]$

```

int knapSack(int Knapsack_Capacity, int Item_Weights[], int Item_Profit[], int No_of_Items)
{
    // knapsack capacity
    // NO_of_items

    vector<vector<int>> dp(No_of_Items + 1, vector<int>(Knapsack_Capacity + 1));
    // dp[i][j]
    // i--> NO_of_Items
    // j --> knapsack_capacity
    // initialization
    for(int i = 0; i < Knapsack_Capacity + 1; i++){
        // no_of_item == 0
        dp[0][i] = 0;
    }
    for(int i = 0; i < No_of_Items + 1; i++){
        // knapsack == 0
        dp[i][0] = 0;
    }

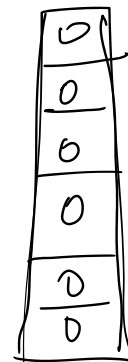
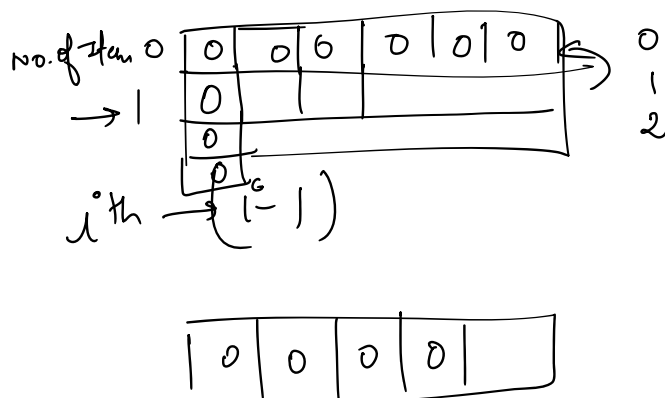
    for(int i = 1; i < No_of_Items + 1; i++){
        for(int j = 1; j < Knapsack_Capacity + 1; j++){
            int maxi = 0;
            if(Item_Weights[i-1] <= j){
                // include
                maxi = max(maxi, dp[i-1][j-Item_Weights[i-1]] + Item_Profit[i-1]);

                // not include
                maxi = max(maxi, dp[i-1][j]);
            } else {
                // not include
                maxi = max(maxi, dp[i-1][j]);
            }
            dp[i][j] = maxi;
        }
    }

    // return solve( Knapsack_Capacity, Item_Weights, Item_Profit, No_of_Items, dp);
    return dp[No_of_Items][Knapsack_Capacity];
}

```

$T.C - O(\text{Knap-Capacity} \times \text{No. of Items})$   
 $S.C - O(\text{Knap-Capacity} \times \text{No. of Items})$



$1 \rightarrow \text{prev\_day}$   
 $1 \rightarrow \text{curr\_day}$   
 $dp[i-1]$   
 $dp[i]$

```

vector<int> curr_item(Knapsack_Capacity + 1,0);
for(int i = 1; i < No_of_Items + 1; i++){
    auto prev_item = curr_item;
    for(int j = 1; j < Knapsack_Capacity + 1; j++){
        int maxi = 0;
        if(Item_Weights[i-1] <= j){
            // include
            maxi = max(maxi, prev_item[j-Item_Weights[i-1]] + Item_Profit[i-1]);

            // not include
            maxi = max(maxi, prev_item[j]);
        } else {
            // not include
            maxi = max(maxi, prev_item[j]);
        }
        curr_item[j] = maxi;
    }
}

return solve(Knapsack_Capacity, Item_Weights, Item_Profit, No_of_Items, dp);
return curr_item[Knapsack_Capacity];

```

$T.C = O(N \times C)$   $\times$  No of Item)     $S.C = O(C)$