This is the solution to Problem statement 1

Problem Statement

The random variable X and Y have the following joint probability density

$$f_{X,Y}(x,y) = \begin{cases} e^{-x-y} & 0 < x < \infty, 0 < y < \infty \\ 0 \end{cases}$$

What is
$$P(X < Y)$$

Solution

$$P(X < Y) = \int_0^\infty \int_0^y e^{-x} e^{-y} dx \cdot dy$$

$$\int_0^\infty e^{-y} \cdot \left[e^{-x} \right]_0^y$$

$$\int_0^\infty \left(e^{-y} - e^{-2y} \right) dy$$

$$\left[\frac{e^{-2y}}{2} - e^{-y} \right]_0^\infty$$

$$\frac{1}{2}$$

Answer:

0.5