TOTORIAL - 1

Ques 1) What do you understand by Asymptotic notation. Define different Asymptotic notation with examples.

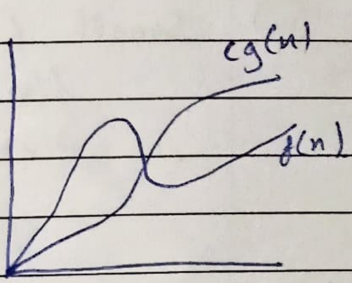
Ans Asymptotic notation means towards infinity. They are used to tell the complexity of an algorithm having input ~~sto~~ size very large.
It is priority analysis.
Different types of asymptotic notation are:

(I) Big oh Notations:

$$f(n) = O(g(n)) \quad , \text{ if } 0 \leq f(n) \leq c(g(n)) \; \forall \; n \geq n_0$$

g(n) is tight upper bound of f(n).

Example:
```
for (int i=0; i<n; i++) {
        cout << i << endl;
}
```

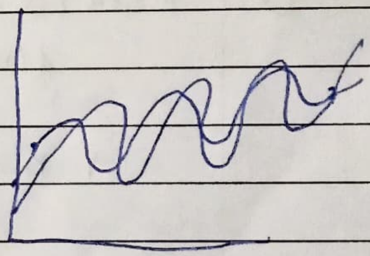


$T(n) = O(n)$

(II) Small oh Notation:

$$f(n) = o(g(n)), \text{ if } f(n) < c(g(n)) \; \forall \; n > n_0 \; \& \; \forall \; c > 0$$

g(n) is upper bound of f(n).



(III) Big Omega (Ω)

$$f(n) = \Omega g(n) \quad \text{if } f(n) = c(g(n)) \cdot \forall \; n \geq n_0.$$

Some constant $c > 0$

g(n) is tight lower bound of f(n).

Example. $f(n) = 6n^2+n+1$

$0 \le g(n) \le f(n)$

$0 \le c.g(n) \le f(n)$

$c \le 6 + \frac{1}{n} + \frac{1}{n^2}$ on putting $n=\infty$, $\frac{1}{n} = \frac{1}{\infty}$

$c \le 6$ $\cdot 6 \le 6+1+1 \le 6+0$ True.

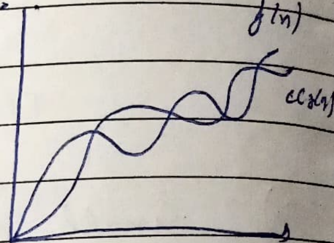
$c > 0$ and $n > n_0$ $(n=1)$

$f(n) = \Omega(n^2)$

(iv) Small Omega (ω)

$f(n) = \omega(g(n))$, if $f(n) > c(g(n))\ \forall\ n > n_0$.

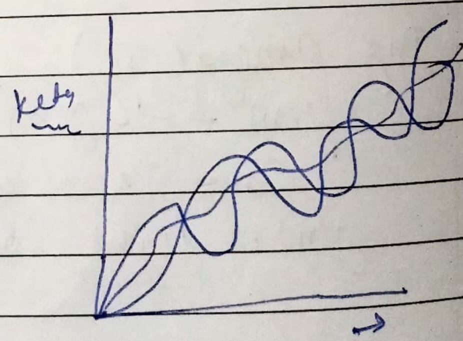
g(n) is the lower bound of f(n).



(v) Theta (θ)

$f(n) = \theta(g(n))$, if $c_1(g(n)) \le f(n) \le c_2(g(n))$

$\forall\ n \ge \max(n_1, n_2)$ and some constant c_1 & $c_2 > 0$



Q2 What should be time complexity of
for (i=1 to n) (i=i*2)

Sol^n · i would have 1, 2, 4, 8, 16, --- n.

let say these are K terms.
It is a G.P with $a=1$, $r=2$
K^{th} term $= t_k = ar^{K-1}$
$n = 1(2)^{K-1}$
$= 2^{K-1}$

taking $\log_2$ on the both sides
$\log_2 n = \log_2 (2^{K-1})$
$\log_2 n = (K-1) \log_2 2$
$\log n = (K-1) \Rightarrow K = 1 + \log_2 n$

$T(n) = O(K) = O(1 + \log n) = O(\log n)$

Q. 3 $T(n) = [3T(n-1)$ if $n>0$, otherwise

$T(n) = 3T(n-1)$ — ①
by backward situation
$T(n) = 3T(n-1)$
$T(n-1) = 3T(n-1-1)$.
$T(n-1) = 3T(n-2)$ — ②

Put ② in ①
$T(n) = 3\{3T(n-2)\} \Rightarrow T(n) = 9T(n-2)$ — ③
$T(n-2) = 3T(n-3)$
$T(n) = 27T(n-3)$
Continue for K times
$T(n) = 3^K T(n-K)$
assume $n-K=0 \Rightarrow n=K$.

$T(n) = 3^k T(0)$

$T(n) = 3^k$

$T(n) = O(3^n)$

Ques 4 $T(n) = \{2T(n-1) - 1$ if $n > 0$, otherwise $1\}$

$T(n) = 2T(n-1) - 1$ — (1)

by using Backward substitution.

$T(n) = 2[2T(n-2) - 1] - 1$

$2^2 T(n-2) - 2 - 1$

$T(n) = 2T(n-1) - 1$

$T(n-1) = 2T(n-2) - 1$

$T(n-2) = 2T(n-3) - 1$

$2^2[2T(n-3) - 1] - 2 - 1$

$2^3 T(n-3) - 4 - 2 - 1$

Continue for K times.

$T(n) = 2^k T(n-K) - 2^{k-1} - 2^{k-2} \text{----} 1$

Assume $n - K = 0 \Rightarrow n - K$

$2^n T(0) - 2^{n-1} - 2^{n-2} \text{----} 1$

$2^n - 2^{n-1} - 2^{n-2} \text{----} 1$

$2^n - [2^{n-1} + 2^{n-2} \text{---} + 1]$

GP K terms.

$a = 2^{n-1};\ r = 2^{-1} = \frac{1}{2}$

Sum of G.P

$\frac{a(1-r^{n-1})}{1-r} = \frac{2^{n-1}(1-(1/2)^{n-1}/(1/2))}{1/2}$

$= 2^n(1 - 2(1/2)^n)$

$= \frac{2^n(2^n - 2)}{2^n} = 2^n - 2$

$T(n) = O(1)$ Ans

Page No.:
Date:

Ques 5 what should be the complexity of.

```
int i=1, s=1
while (s<=n) {
    i++
    s=s+i;
    printf("#");
}
```

i	s
1	1
2	3
3	6
4	10
5	15
n	K times

$S = 1, 3, 6, 10, 15 \ldots n$

let say K terms

$\frac{K(K+1)}{2} = n$

$K = 2n$ $\quad K = \sqrt{n}$ $\quad O\sqrt{n}$

$+1$ would be constant

Ques 6 Time complexity of

```
Void function(int n) {
    int i, count=0.
    for(i=1; i*i<n; i++)
        count++; }
```

$1^2, 2^2, 3^2, \ldots n$

let say k terms.

$t_k = k^2$

$n = k^2 \Rightarrow k = \sqrt{n}$

$T(n) = O(\sqrt{n})$

Ques 7 Time complexity of

```
Void function (int n) {
    int i, j, k, count=0
    for(int i=n/2, i<=n; i++)
        for (int j=1; j<=n, j=j+2)
            for( k=1, k<=n ; k=k*2)
                count++;
}
```

$i = \frac{n}{2}, \frac{n}{2}+1, \frac{n}{2}+2 \dots n$

$= \frac{n}{2}, \frac{n+2}{2}, \frac{n+4}{2} \dots n$

given of form $= \frac{n+0*2}{2} + \frac{n+1*2}{2} + \frac{n+2*2}{2}, \dots n$

$\frac{n+k*2}{2}$ $(k=0,1,2,n)$.

Total terms $= k+1$

$t_{k+1} = n$

$\frac{n+(k+1)*2}{2} = n \Rightarrow 2n = n + (k+1)*2$

$n - 2 = 2k$

$k = \frac{n}{2} - 1$

i	j	k
$n/2$	log n times	$(\log n)^2$
$\frac{n+2}{2}$	log n times	$(\log n)^2$
n	log n times	$(\log n)^2$

$= \left(\frac{n}{2}-1\right)(\log n)^2$

$\frac{n}{2}\log^2 n - \log^2 n$

$T(n) = O(n \log^2 n)$

Ques 8 Time complexity of

```
function (int n){
    if (n==1) return;
    for(i=1 to n){
        for (j=1 to n)
            printf("*");
    }}
    function(n-3);
```

Function call would be $n, n-3, n-6, n-9 \ldots$

it Saw K terms

AP, $a = n, d = -3,$

$a_n = a + (n-1)d.$

$1 = n + (k-1)(-3)$

$1 = n - 3k + 3.$

$3k = n + 2$

$k = \frac{n+2}{2}$

Function have recurssive call $\frac{n+2}{2}$ times.

Time complexity for two inner loop.

$\left(\frac{n+2}{2}\right) n^2 \Rightarrow n^3$

$T(n) = O(n^3)$

Q.7 Time complexity of Void function(int n) {
for (i=1 to n)
for (j=1, i≤n ; j=j+i)
Print('*');
}
}

i(outer loop)

when $i=1 \Rightarrow j = 1, 2, 3 \ldots n = n$

when $i=2 \Rightarrow j = 1, 2, 3 \ldots n/2.$

$\sum_{j=n}^{1} n + \frac{n}{2} + \frac{n}{3} + \ldots 1$

$\sum_{j=n} n\left(1 + \frac{1}{2} + \frac{1}{3} \ldots + \frac{1}{n}\right)$

$O(n \log n)$ Ans.

Ques 10. For the function, n^k and c^n what is the asymptotic relationship b/w these functions. Assume that $k >= 1$ and $c > 1$ are constants. Find out the value of c and n_0 for which relation hold.

as given n^k and c^n
relation b/w n^k and c^n is $n^k = O(c^n)$

as $n^k \le ac^n \ \forall \ n \ge n_0$ for d constant $a > 0$.
for $n = 1$
$c = 2$

$1^k \le a2^1$
$n_0 = 1$ and $c = 2$.