Tutorial - 4

① $T(n) = 3T(\frac{n}{2}) + n^2$

$T(n) = aT(\frac{n}{b}) + f(n)$

$a \geq 1, b > 1$

comparing

$a = 3 \quad b = 2, \quad f(n) = n^2$

Now $c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$f(n) > n^c$

② $T(n) = 4T(n/2) + n^2$

$a > 1 \quad b > 1$

$a = 4, b = 2, f(n) = n^2$

$T(n) = \Theta(n^2 \log_2 n)$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$T(n) = \Theta(n^2 \log_2 n)$

③ $T(n) = T(n/2) + 2^n$

Sol $a = 1 \quad b = 2$

$F(n) = 2^n$

$c = \log_b a = \log_2 c = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

④ $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$

$b = 2 \quad F(n) = n^n$

$c = \log_b a = \log_2 2^n$

Q5 $T(n) = 16T(n/4) + n$

$a = 16 \quad b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$T(n) = \Theta(n^2)$

Q6 $T(n) = 2T(n/2) + n \log n$

$a = 2, b = 2$

$F(n) = n \log n$

$c = \log 2 = 1$

$n^c = n^1 = n$

Since, $n \log n > n$

$f(n) > n^c$

$T(n) = \Theta(n \log n)$

7 $T(n) = 2T(\frac{n}{2}) + n/\log n$

$a=2 \quad b=2, \quad f(n) = n/\log n$

$c = \log_2^2 = 1$

$n^c = n = n$

Since $\frac{n}{\log n} < n$

$f(n) < n^c$

$T(n) = \Theta(n)$

(9) $T(n) = 0.5T(n/2) + 1/n$

$\therefore a = 0.5 \quad b = 2$

Since are to master theorem $a \geq 1$ but here a is 0.5 so we cannot apply master theorem.

(11) $4T(n/2) + \log n$

$a=4, \quad b=2, \quad f(n) = \log(n)$

$c = \log_b a = \log_2 4 = 2.$

$n^c = n^2$

$f(n) = \log n.$

$\log n < n^2$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$\Theta(n^2)$

(13) $T(n) = 3T(n/2) + n$

$a = 3 \quad b = 2 \quad f(n) = n.$

$c = \log_b a = \log_2 3 = 1.53$

$n^c = n^{1.5849}$

$n < n^{1.5849}$

$f(n) < n^c$

$T(n) = O(n^{(1.58)})$

(15) $T(n) = 4T(n/2) + n$

$a = 4, \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n$

$n < n^2$ (for any constant)

$f(n) < n^c$

$T(n) = O(n^2)$

(17) $T(n) = 3T(n/3) + n/2$

$a = 3 \quad b = 3$

$c = \log_b a = \log 3^3 = 1$

$f(n) = n/2.$

$n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$

$T(n) = O(n)$

19) $T(n) = 4T(n/2) + n\log n$

$a = 4, b = 2, f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \Theta(n^2)$

21) $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7$

$n^c = n^{1.771}$

$n^{1.77} < n^2$

$T(n) = \Theta(n^2)$

22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1 \quad b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$T(n) = \Theta(n(2 - \cos n))$