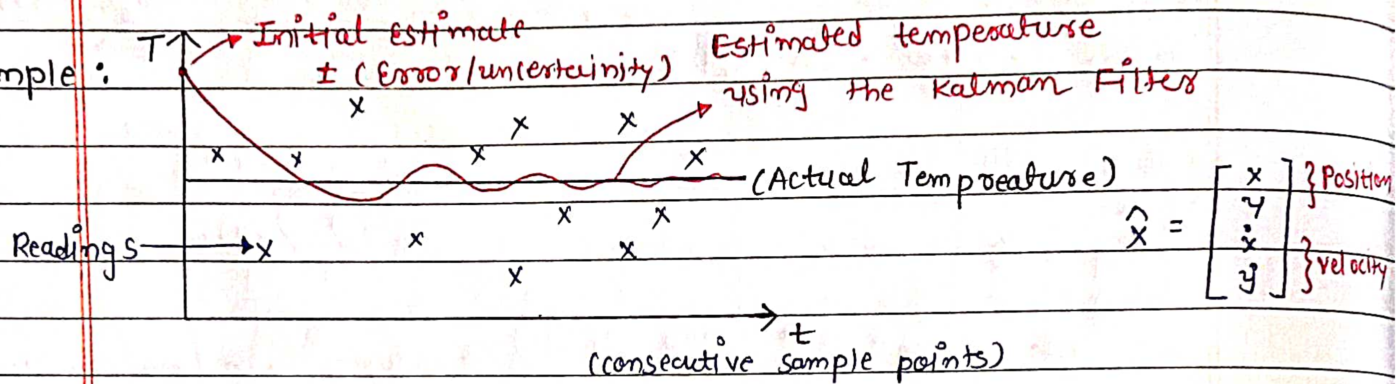
Kalman Filter

- It is a tool in order to estimate predicated values.

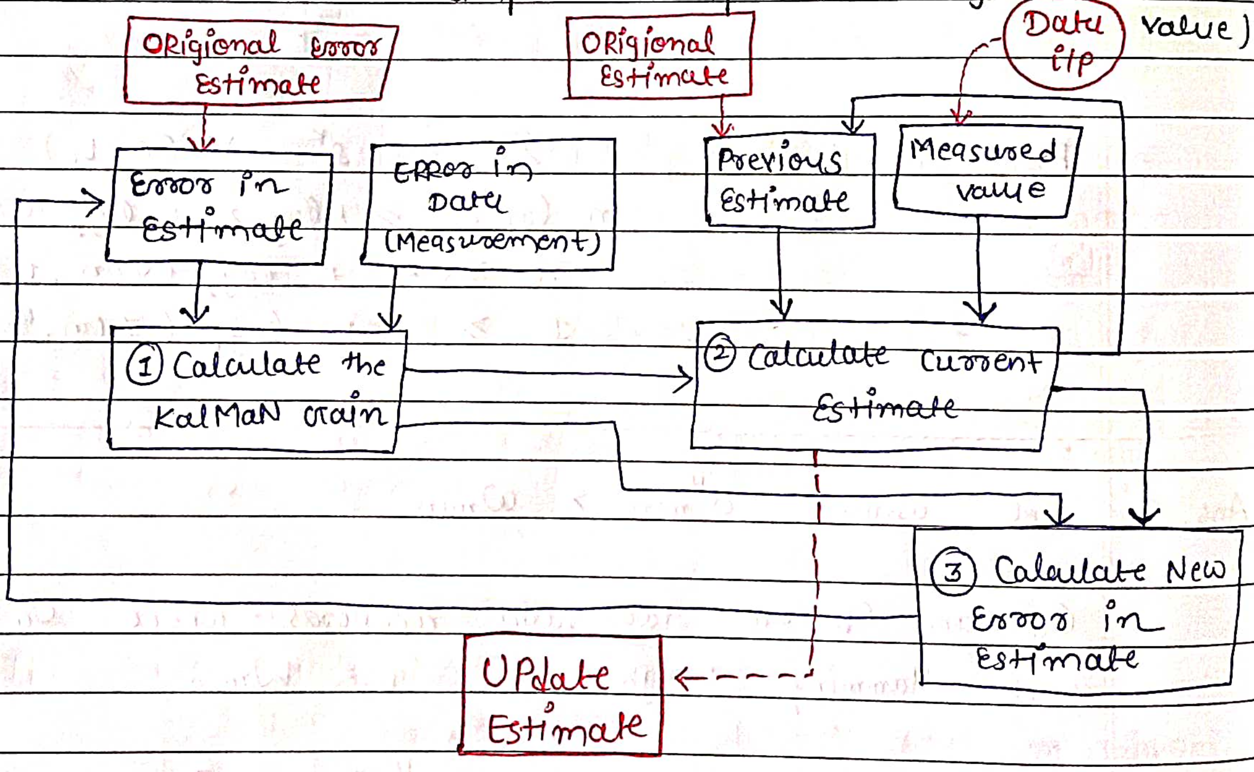
* It is an iterative mathematical process that uses a set of equations and consecutive data i/p s to quickly estimate the true value of the object being measured, when the measured values contain unpredicted or random error, uncertainty or Variation.

Example :



$$\hat{x} = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$$

* Flowchart of a simple example : (single measurd value)



* The Kalman Gain :

- it is used to determine how much of the new measurements to use to update the new estimate.

Date ______

Page ______

Kalman Gain = KG

Error in Estimate = E_{EST}

Error in Measurement = E_{MEA}

$$\boxed{KG = \frac{E_{EST}}{E_{EST} + E_{MEA}}}$$

Error in Estimate → ① Calculate the Kalman Gain ← Error in Data (Measurement)

$0 \leq KG \leq 1$

Measurement are accurate	KG	Estimates are unstable
↑	1	↑
	0.9	
	0.8	
	0.7	
	0.6	
	0.5	
	0.4	
	0.3	
	0.2	
↓	0.1	↓
Measurement are inaccurate	0	Estimates are Stable (Small Errors)

Current Estimate = EST_t

Previous Estimate = EST_{t-1}

Measurement = MEA

$$EST_t = EST_{t-1} + KG\,[MEA - EST_{t-1}]$$

* The 3 Calculations :

Calculate Kalman Gain → Calculating Current Estimate → Calculating new estimate Error

① $$KG = \frac{E_{EST}}{E_{EST} + E_{MEA}}$$

② $$EST_t = EST_{t-1} + KG\,[MEA - EST_{t-1}]$$

③ $$E_{EST_t} = \frac{(E_{MEA})(E_{EST\,t-1})}{E_{MEA} + E_{EST\,t-1}}$$

$$\Rightarrow E_{EST_t} = [1 - KG]\,(E_{EST_{t-1}})$$

* Multi - Dimension Model :

Position, Velocity

$X_0 \longrightarrow$ State Matrix $(x_i, \dot{x}_i)$

$P_0 \longrightarrow$ Process Covariance Matrix

(Represents errors in Estimate / Process)

ΔT = Time for 1 Cycle

* New State = Privious State + Controlling Para. for change in State + Noise

Date

Page

→ Which is More trustable, the theoritical (Pridecated future) or the practical (i/p measurement)?
 - Kalman filter is all about this.

Initial State

X_0
P_0

(K-1)
Previous State

X_{K-1}
P_{K-1}

K_p

New State (Predicted)

$X_{Kp} = A X_{K-1} + B u_K + \omega_K$

$P_{Kp} = A P_{K-1} A^T + Q_K$

Predicated Stated based on pysical Model and previous State

u = control Variable Matrix

ω = Predicted State noice matrix

Q = Process noise covariance matrix

K → K-1

Current becomes previous

Update with New measurement and Kalman Gain

$K = \dfrac{P_{Kp} H}{H P_{Kp} H^T + R}$

$X_K = X_{Kp} + K[Y - H X_{Kp}]$

$P_K = (I - KH) P_{Kp}$

X_p

Output of updated State

X_K
P_K

Identity Matrix

Measurement Input

$Y_K = C X_{K_M} + Z_K$

Measurement of the State

Measurement Noise (Uncertainty)

K = Kalman gain

R = Sensor noise covariance Matrix (Measurement Error)

* The State Matrix :

$X_K = A X_{K-1} + B u_K + \omega_K$

X = State matrix

u = control variable matrix

ω = Noise in process

Δt = Time for 1 cycle

1D : $\begin{bmatrix} x \\ \dot{x} \end{bmatrix}$ or $\begin{bmatrix} y \\ \dot{y} \end{bmatrix} = X$

2D : $\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$ or $\begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} = X$

3D : $\begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$ or $\begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix} = X$ → Position & Velocity

Date ____
Page ____

→ Example : $\underbrace{X}_{\text{updated}} = \underbrace{X_0}_{\text{Privious}} + \underbrace{\dot{x}t + \frac{1}{2}\ddot{x}t^2}_{\text{controll (?)}} + \underset{\text{(Error)}}{\text{Noise}}$

→ Example in 1D : (1) Rising Fluid in a Tank
(2) Falling Object
(3) Moving Vehicle in 1 Dimension.

Calculating "A" Matrix (How previous position effects the current one) :

Rising Fluid: $X = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$ — Falling Object: $X = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$ — Moving Object: $X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$

$A = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix}$ ← 1 time period

Same for all without carring about dimension

So that, $AX = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$

$\therefore AX = \begin{bmatrix} y + \dot{y}\Delta T \\ \dot{y} \end{bmatrix}$

→ Here, we are only dealing with velocity & previous part. Controlling i/p parameter acclaration will be taken care by "Bu_K" Parameter.

→ So, $\underbrace{AX_{K-1}}_{\text{without acelaration}} + \underbrace{Bu_K}_{\text{only acelaration (?)}}$

Calculating "B" Matrix (Controlling Part) :

→ Example as above (X is same)

Rising Fluid

$B = \begin{bmatrix} \frac{1}{2}\Delta T^2 \\ \Delta T \end{bmatrix}$

$u = [0]$

↳ no controll in the case (no acelaration)

$Bu = \begin{bmatrix} \frac{1}{2}\Delta T^2 \\ \Delta T \end{bmatrix} [0]$

Falling Object

$B = \begin{bmatrix} \frac{1}{2}\Delta T^2 \\ \Delta T \end{bmatrix}$

$u = [g]$

↳ gravity provides acceleration

$Bu = \begin{bmatrix} \frac{1}{2}\Delta T^2 \\ \Delta T \end{bmatrix} [g]$

Moving object

$B = \begin{bmatrix} \frac{1}{2}\Delta T^2 \\ \Delta T \end{bmatrix}$

$u = [a]$

↳ It has it own.

$Bu = \begin{bmatrix} \frac{1}{2}\Delta T^2 \\ \Delta T \end{bmatrix} [a]$

← Same for any dimention move (x or y or z)

Only 1-D

$Bu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ → no acceleration → no update from there

$Bu = \begin{bmatrix} g\frac{1}{2}\Delta T^2 \\ g\Delta T \end{bmatrix}$ → change in position → change in velocity

$Bu = \begin{bmatrix} \frac{1}{2}a\Delta T^2 \\ a\Delta T \end{bmatrix}$

Update in observation :

$$X_K = AX_{K-1} + Bu_K + \omega_K$$
$$Y_K = CX_K + Z_K$$

Y = Observation

Z = Measurement Noise

→ If $x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$ then,

Only Position : Y_K

$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ $\quad x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$

$\Rightarrow CX = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{1\times 2} \underbrace{\begin{bmatrix} y \\ \dot{y} \end{bmatrix}}_{2\times 1} = \underbrace{\begin{bmatrix} y \end{bmatrix}}_{1\times 1}$ Measured

Both : $CX = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} y \\ \dot{y} \end{bmatrix}}_{2\times 1} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}^{2\times 1}$ Measured

* The State Matrix Calculation for 2-D :

$$X_K = AX_{K-1} + Bu_K + \omega_K$$
$$[4\times 4] \quad [4\times 1]$$

X = State Matrix

u = control Variable Matrix

ω = Noise in process

ΔT = Time for 1 cycle

AX_{K-1}

$X = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$ $\quad A = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow AX = \begin{bmatrix} x + \Delta T\dot{x} \\ y + \Delta T\dot{y} \\ \dot{x} \\ \dot{y} \end{bmatrix}$

$X = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$ $\quad A = \begin{bmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow AX = \begin{bmatrix} x + \Delta T\dot{x} \\ \dot{x} \\ y + \Delta T\dot{y} \\ \dot{y} \end{bmatrix}$

Date
Page

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$$\begin{bmatrix} \frac{1}{2}\Delta T^2 & 0 \\ 0 & \frac{1}{2}\Delta T^2 \\ \Delta T & 0 \\ 0 & \Delta T \end{bmatrix}_{4\times 2} \begin{bmatrix} a_x \\ a_y \end{bmatrix}_{2\times 1} = \begin{bmatrix} \frac{1}{2} a_x \Delta T^2 \\ \frac{1}{2} a_y \Delta T^2 \\ a_x \Delta T \\ a_y \Delta T \end{bmatrix}$$

where $X_K = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$

Final

Without Noise

$$X_K = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{K-1} \\ y_{K-1} \\ \dot{x}_{K-1} \\ \dot{y}_{K-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta T^2 & 0 \\ 0 & \frac{1}{2}\Delta T^2 \\ \Delta T & 0 \\ 0 & \Delta T \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix}$$

$$\therefore X_K = \begin{bmatrix} x_{K-1} + \dot{x}_{K-1}\Delta T + a_x \frac{1}{2}\Delta T^2 \\ y_{K-1} + \dot{y}_{K-1}\Delta T + a_y \frac{1}{2}\Delta T^2 \\ \dot{x}_{K-1} + a_x \Delta T \\ \dot{y}_{K-1} + a_y \Delta T \end{bmatrix}$$

Applications Of Kalman Filter :

① Aerospace & Defense : for guidance, navigation & control system. Estimation of position, velocity & orientation of a vehicle or aircraft & to track targets.

② Robotics : to estimate the position & velocity of Robot, & to improve its motion control. Sensor fusion, where multiple sensors are used to gather data about the robot's environment.

③ Finance : to estimate the value of assets and to predict market trends. Portfolio optimization, where they help to allocate assets based on risk & return.

④ Signal Processing : to estimate & remove noise and to detect & track signals in noisy environments. Also used in image processing to improve the quality of images & videos.

⑤ Biomedical engineering : to track the movements of organs. (BP & Oxygen saturation)

Date
Page

⑥ Control System : to estimate the state of System, and to predict its future behavior. Such as autonomous vehicles, where they help to control the vehicle's speed, direction & trajectory.

Overall, it is a versatile tool that can be applied to a wide range of fields and applications where real-time filtering and prediction of data is required.

* Controlling Units :

① Aerospace :

- aircraft nevigation system → aircraft's velocity, accelaration, attitude angles, external i/ps : GPS measurement and other sensor data
- Spacecraft guidence & Control → thruster firing commands, attitude control system commands, sensors

→ estimate the state of system & adjust control i/ps accordingly for optimal performance.

Thruster firing Command → instructions sent to trusters of spacecraft to change its velocity / attitude

- Satellite tracking → Satellite ehemeris (position & Velocity), Sensor (Radar, optimal cameras), Control inputs (like antenna)

② Robotics :

- Odometry → The process of estimating the motion of Vehicle based on sensor measurement

Wheel encoder readings : sensor measure rotational motion of wheel & calculate distance

IMU data : accelaration & Rotation motion

Control i/p : Steering angles, throttle & braking Commands, etc.

Application ↳ navigation, autonomous driving & robotics

Date ____________
Page ____________

- Robot vision : Camera measurement → includes position, size and orientation of objects in robot's environment
  Robot odometry
  Control i/ps → steering, acceleration, deceleration commads
  Applications → Object tracking, object recognition, navigation
- Simultaneous Localization & Mapping (SLAM)
  ↳ computational problem in robo. that involves building a map of unknown environment while simultaneously tracking the robot's location within environment
  (Sensor : Cameras, LIDAR, sonar, radar)
  Applications → mobile robotics, autonomous vehicles, augmented reality, virtual reality
  Sensor Measurement
  ↳ LIDAR, Cameras, Sonar
  Robot odometry
  Control i/ps

* Extra Use : Apollo navigation Computer → Neil Armstrong journey to moon & back home
  Satellite navigation device, Smart phone, computer games