Kalman Filter

"The Kalman filter is like a magic wand that can turn noisy measurements into accurate predictions."

Honor Code

We, Group 13, declare that

→ the work that we are presenting is our own work.

→ We have not copied the work (Matlab code, results, etc.) that someone else has done.

→ Concepts, understanding and insights we will be describing are our own.

→ Wherever we have relied on an existing work that is not our own, we have provided a proper reference citation

Introduction

- A Kalman filter is an optimal estimation algorithm.
- Developed by: Rudolf E. Kalman
- Essential Tool in various fields.
- Useful in situations where measurements are prone to errors
- Used in Apollo 11 Project.

[1]

Rudolf Kalman

- **IMU & ODOMETER SENSOR :**

  Updated frequently but measurement prone to error and give a relative distance

- **GPS :**

  Updated less frequently and noisy too but give absolute location

[2]

- Too high temperature can damage the engine.
- Closely monitor the internal temperature of the combustion chamber.
- Sensor could melt.
- Extract Information about what you can’t measure from what you can.
- Use Kalman Filter to get best estimate of internal temperature from external temperature

Kalman Filter Algorithm

Intialization

In this step, we determine some of the basic terms to start the filtering process

State Vector - X_0

X_0 contains terms of interest like position, velocity, etc.

Process Covariance Matrix - P_0

P_0 contains the covariance between terms of state vector

Prediction

In this step, we predict the current state of the object and process errors based on our knowledge of the previous observations

$$X_t^p = AX_{t-1} + Bu_t + w_t$$

- Predicting the current state using the previous state and control input.

$$P_t^p = AP_{t-1}A^T + Q_t$$

- Represents uncertainty in the state prediction.
- Calculated using the uncertainty in the previous cycle.

What are A and B Matrices?

We know the equations of position and velocity in terms of acceleration, A and B matrix helps us to perform these calculations in matrix form.

- For 1D, we consider position X and velocity V_x

$$A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \Delta t^2/2 \\ \Delta t \end{bmatrix}$$

- For 2D, we consider positions X, and Y and velocities V_x, V_y

$$A = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \Delta t^2/2 & 0 \\ 0 & \Delta t^2/2 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}$$

A simple calculation for 1D system,

Let's consider some initial values as follows,

$X_0 = 12$, $V_{x0} = 3$, $a = 1.2$

$\Delta X = 3$, $\Delta V_x = 0.6$, $\Delta PX = 2.5$, $\Delta PV_x = 0.2$, $\Delta t = 1$

$\mathbf{X_t = A*X_0 + B*u_t + w}$, consider w=0 for simplification

$$X_t = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 3 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} [1.2]$$

$$= \begin{bmatrix} 12 + 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 0.6 \\ 1.2 \end{bmatrix}$$

$X_t = \begin{bmatrix} 15.6 \\ 4.2 \end{bmatrix}$, this will be new predicted state

Calculation

In this part, we will calculate an important factor called Kalman Gain and also see how measured values are received

$$Y_t = CY_t^m + Z^m$$

- Measurement matrix Y
- Addition of original measurement and measurement noise

$$K = \frac{P_t^p H^T}{HP_t^p H^T + R}$$

- Kalman Gain Equation
- used for determining the weightage of measurement and prediction for the current state matrix.

Updation

Here, we will be updating the current state matrix and process co-variance matrix based on previous calculations

$$X_t = X_t^p + K[Y - HX_t^p]$$

- Updated state X_t
- Calculated from predicted state, measured state and Kalman Gain

$$P_t = (I - KH)P_t^p$$

- Updated process covariance matrix P_t
- Calculated from current process covariance matrix and Kalman Gain

This state and process co-variance matrix becomes the previous state and co-variance matrix for the next time cycle of the filtering process.

INITIAL STATE

$$X_0, P_0$$

X represents the State Matrix
P is the Process Covariance Matrix

PREVIOUS STATE

$$X_{t-1}, P_{t-1}$$

t is the iteration index
or time step

NEW PREDICTED STATE

$$X_t^p = AX_{t-1} + Bu_t + w_t$$

$$P_t^p = AP_{t-1}A^T + Q_t$$

small p represents the matrix has been updated with a new prediction

w and Q are error terms

Matrix A (State Transition) updates X and P based upon the time that has elapsed.

Matrix B applies acceleration(u) to provide values to update the position and velocity of AX

CALCULATE KALMAN GAIN (K) AND MEASURED DATA (Y)

$$K = \frac{P_t^p H^T}{HP_t^p H^T + R}$$

H matrix helps transform the matrix format of P into the format desired for the K matrix

$$Y_t = CY_t^m + Z^m$$

Y is a matrix containing measurement data (m)
C is a matrix transform to allow it be summed with Z
Z is the error term of the measurement

UPDATE PROCESS AND STATE MATRIX

$$P_t = (I - KH)P_t^p$$

$$X_t = X_t^p + K[Y - HX_t^p]$$

CURRENT STATE BECOMES PREVIOUS STATE

$$X_t = X_{t-1}$$

$$P_t = P_{t-1}$$

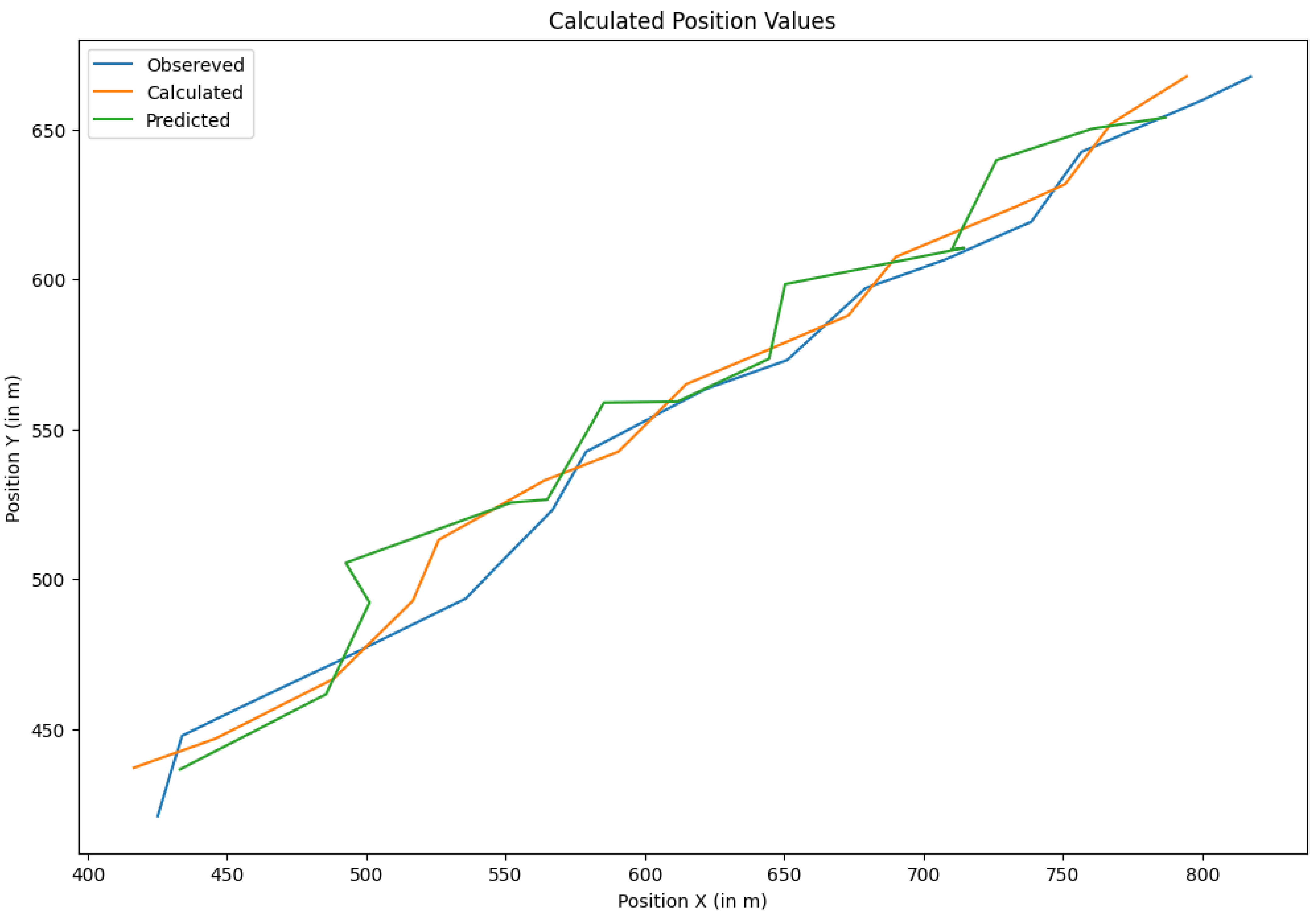
REPEAT

[3]

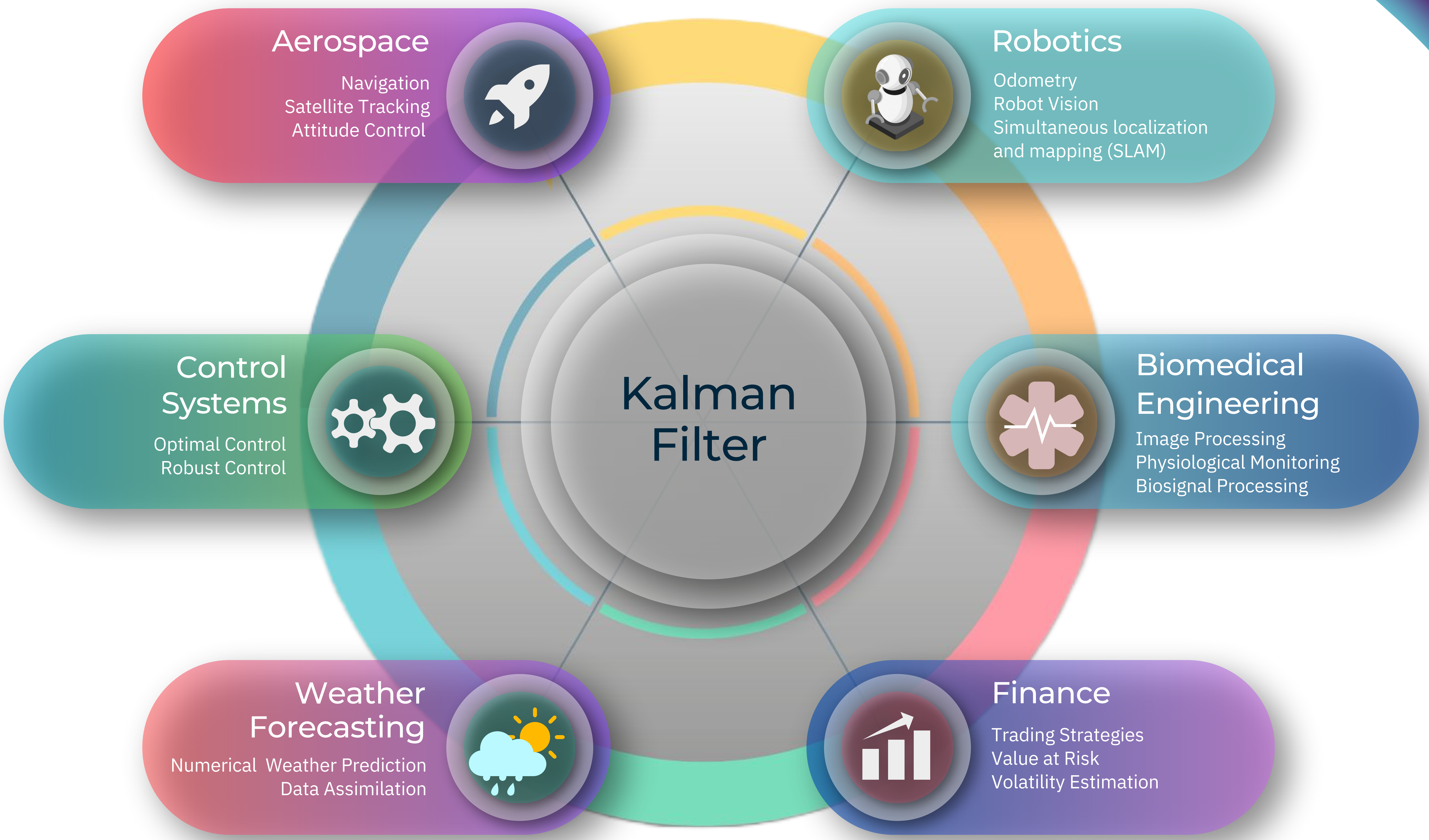
Let's explore the process by an example...

Case of tracking an air plane

Say, we are tracking ar air plane which is moving with a constant acceleration and we are trying to estimate it's state at different time.



Applications


Kalman Filter
Aerospace
Navigation
Satellite Tracking
Attitude Control
Robotics
Odometry
Robot Vision
Simultaneous localization and mapping (SLAM)
Control Systems
Optimal Control
Robust Control
Biomedical Engineering
Image Processing
Physiological Monitoring
Biosignal Processing
Weather Forecasting
Numerical Weather Prediction
Data Assimilation
Finance
Trading Strategies
Value at Risk
Volatility Estimation

Drawbacks and Alternatives

Drawbacks

While the Kalman filter is a powerful and widely used estimation technique, it is important to carefully consider its limitations.

- **Assumption of linearity**
- **Lack of handling constraints**
- **Sensitivity to initial conditions**
- **Difficulty in handling high-dimensional systems**

Alternatives

As powerful as the Kalman filter is, it is not always the most suitable choice for every estimation problem. In such cases, exploring alternative approaches can lead to better accuracy and more efficient computations.

- **Particle Filter**
- **Extended Kalman filter**
- **Unscented Kalman Filter**

Conclusion

References

[1]http://www.edubilla.com/award/national-medal-of-science/rudolf-e-kalman/

[2] Referred from: Matlab Youtube CHANNEL
https://www.youtube.com/watch?v=mwn8xhgNpFY&list=PLn8PRpmsu08pzi6EMiYnR-076Mh-q3tWr&index=1

[3] Flowchart of Kalman Filter Matrix Process by James Teow, inspired by Prof. Biezen's flowchart

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Thank you!

Appendix