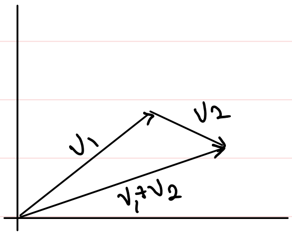
Linear Algebra

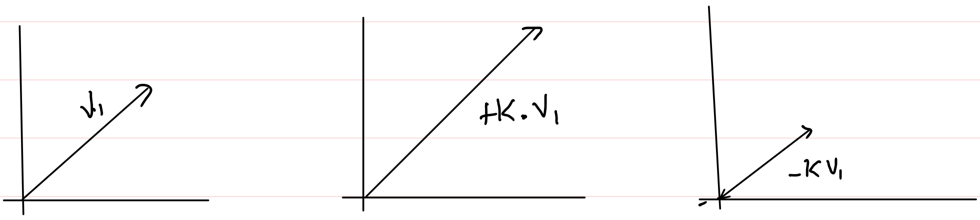
16 April 2024 22:00

- Vectors

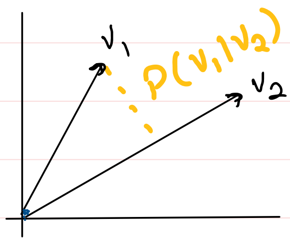
→ Vector Addition (Dot Product)



→ Scalar Multiplication



→ Projection



If we are able to project v_1 on v_2 then we are able to find unknown features of v_1.

- Matrices

Suppose we have 2 equations:

$$2x + 2y = 10$$
$$4x + y = 18$$

This in Matrix Form is:

$$\begin{bmatrix} 2 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \end{bmatrix}$$

→ Matrix Addition

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} a+A & b+B \\ c+C & d+D \end{bmatrix}$$

Same order ⇒ must

→ Matrix Multiplication

$$\begin{bmatrix} 2 & 2 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2*2+2*1 & 2*3+2*4 \\ 4*2+1*1 & 4*3+1*4 \end{bmatrix} \rightarrow \begin{bmatrix} 4+2 & 6+8 \\ 8+1 & 12+4 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 6 & 14 \\ 9 & 16 \end{bmatrix}$$

→ Transpose (Row ⇄ Col)

$$x = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad x^T = \begin{bmatrix} A & C \\ B & D \end{bmatrix}$$

→ Determinant of Matrix

it gives the scalar value of the Matrix

$$x = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$Det(x) = a * \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b * \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

→ Inverse of Matrix

- matrix when multiplied with inverse gives identity Matrix.
- it helps to apply transformations easily.
- Some matrix do not have an inverse that mean they Contain very noisy & un-reliable data.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$Inv(A) = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For order 3 and above

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$Inv(A) = \frac{1}{det(A)} \begin{bmatrix} \begin{vmatrix} e & f \\ h & i \end{vmatrix} & \begin{vmatrix} d & f \\ g & i \end{vmatrix} & \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ \begin{vmatrix} b & c \\ h & i \end{vmatrix} & \begin{vmatrix} a & c \\ g & i \end{vmatrix} & \begin{vmatrix} a & b \\ g & h \end{vmatrix} \\ \begin{vmatrix} b & c \\ e & f \end{vmatrix} & \begin{vmatrix} a & c \\ d & f \end{vmatrix} & \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{bmatrix}$$

→ Scaling

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

→ Shearing

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ m & 1 \end{bmatrix}$$

→ Rotation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

→ Row Echelon method

$$2x + y - z = 2$$
$$x + 3y + 2z = 1$$
$$x + y + z = 2$$

matrix

$$\begin{bmatrix} 2 & 1 & -1 & 2 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 3 & 2 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

ans

$x = 2$

$y = -1$

$z = 1$

Step 1 $\quad R_1 = R_1 / 2$

$$A = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1/2 & -1/2 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

Step 2 $\quad R_2 = R_2 - R_1$

$$\begin{bmatrix} 1 & 1/2 & -1/2 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1/2 & -1/2 & 1 \\ 0 & 5/2 & 5/2 & 0 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

Step 3 $\quad R_3 = R_3 - R_1$

$$\begin{bmatrix} 1 & 1/2 & -1/2 & 1 \\ 0 & 5/2 & 5/2 & 0 \\ 1 & 1 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1/2 & -1/2 & 1 \\ 0 & 5/2 & 5/2 & 0 \\ 0 & 1/2 & 3/2 & 1 \end{bmatrix}$$

Step 4 $\quad R_2 = \frac{2}{5}(R_2)$

$$\begin{bmatrix} 1 & 1/2 & -1/2 & 1 \\ 0 & 5/2 & 5/2 & 0 \\ 0 & 1/2 & 3/2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1/2 & -1/2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1/2 & 3/2 & 1 \end{bmatrix}$$

Step 5 $\quad R_1 = R_1 - \frac{1}{2}(R_2)$

$$\begin{bmatrix} 1 & 1/2 & -1/2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1/2 & 3/2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1/2 & 3/2 & 1 \end{bmatrix}$$

Step 6 $\quad R_3 = R_3 - \frac{1}{2}(R_2)$

$$\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1/2 & 3/2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Step 7 $\quad R_1 = R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

step 8 $\quad R_2 = R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$x = 2$
$y = -1$
$z = 1$

only they should have values (1)
rest all should be zero.

∴ Transform matrix such that only diagonals have 1's rest all other should contain 0's.

→ Inverse

$A A^{-1} = I$

lets suppose $AB = C$

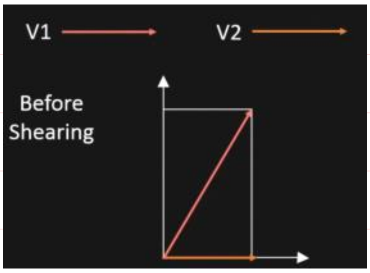
$\Rightarrow A A^{-1} B = C A^{-1}$

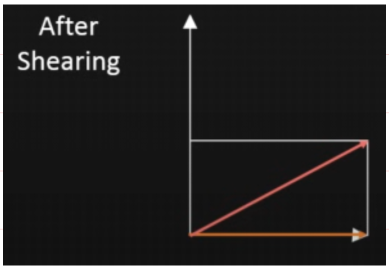
$\Rightarrow B = C A^{-1}$

$$A^{-1} = \frac{1}{ab - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

→ Eigen Vector

- doesnt change direction even after transformation is applied.





this is the reason even after applying Shearing operation vectors remains the same

→ Uses of Linear algebra

a) PCA
b) Pictures
c) Dataset Encoding
d) Singular Value decomposing
e) latent Semantic Analysis for NLP

Multivariate Calculus

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→ Differentiation :-

Constant Rule	$\frac{d}{dx}[C] = 0$
Power Rule	$\frac{d}{dx}x^n = nx^{n-1}$
Product Rule	$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
Quotient Rule	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x))\,g'(x)$

$$x^n = n x^{n-1}$$
$$x^a + x^b = a x^{a-1} + b x^{b-1}$$
$$x^a \cdot x^b = a x^{a-1} x^b + b x^{b-1} x^a$$

→ Partial Differentiation :-

Partial Differentiations

$f^1(x, y, z) =$

With respect to x

$(y, z).f^1(x)$

With respect to y

$(x, z).f^1(y)$

With respect to z

$(x, y).f^1(z)$

Complete Differentiation

$f^1(x, y, z) = (y, z).f^1(x) + (x, z).f^1(y) + (x, y).f^1(z)$

Eg: $f = x^2 + 3y + 4z^2 x$

$x \to 2x + 4z^2$
$y \to 3$
$z \to 8zx$

$$f(x,y,z) = 2x + 4z^2 + 3 + 8zx$$

→ Applications -

a) Jacobian helps find global maxima of dataset.
Also helps linearizing a non-linear function to linear at a point.

b) Hessian helps minimize error.

c) Deep learning

d) Gradient Descent method for optimizing weights.

Probability

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→ it is the measure of how likely an event with occur.

$P = \text{desired} / \text{total}$

Events → Joint (Common Outcome)
→ Dis-Joint (un-Common OutCome)

Distribution → Probability Density function
⇒ it describes a Continious distribution
→ Normal Distribution
⇒ it depends on mean & std deviation.

→ Central Limit Theorm
⇒ Sampling distribution of mean of any independent random variable will be normal, if Sample Size is big enough.

Probability → Marginal
⇒ probability of occurence of Single event.

→ Joint Probability
⇒ prop. of two events happing at Same time

→ Conditional Probability

⇒ Prob. of event B is that the event will occur given that event A has already occured.

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$P(A)$

-> Bayes Theorm

Shows relation b/w one Conditional probability & its inverse.

$$P(B|A) = \frac{P(A|B)\ P(B)}{P(B)} \longrightarrow \{Prior\}$$

Prob of occurence of B
given A
{Posterior}

-> Application

a) optimize Modal
b) classification by algo requires Probability
c) Loss can be Calculated by probability