

Practical Assignment 1 (NM)

1. The function $f(x) = 27x^4 + 162x^3 - 180x^2 + 62x - 7$ has a zero at $x = 1/3$. Perform ten iterations of Newton's method on this function, starting with $p_0 = 0$. What is the apparent order of convergence of the sequence of approximations? What is the multiplicity of the zero at $x = 1/3$? Would the sequence generated by the bisection method converge faster?
2. Commissioner Gordon had been found dead in his office. At 8:00 PM, the county coroner determined the core temperature of the corpse to be 90°F . One hour later, the core temperature had dropped to 85°F . Captain Furillo believed that the infamous Doc B had killed the commissioner. Doc B, however, claimed to have an alibi. Lois Lane was interviewing him at the Daily Planet Building, just across the street from the commissioner's office. The receptionist at the Daily Planet Building checked Doc B into the building at 6:35 PM, and the interview tapes confirmed that Doc B was occupied from 6:40 PM until 7:15 PM. Use Newton's Law of Cooling to model the temperature of the corpse as a function of time and determine the time of death. (Use secant method to solve nonlinear equation with initial guess $p_0 = 0.1$ and $p_1 = 1$ for finding constant of cooling and $p_0 = -2$ and $p_1 = 0$ for time of death. Iterate until $|p_n - p_{n-1}| < 5 \times 10^{-7}$).
3. Suppose it was discovered that Commissioner Gordon had the flu when he died, and his core temperature at the time of his death was 103°F , solve the equation
$$72 + t_d - \frac{1}{k} + (18 + \frac{1}{k})e^{-kt_d} = 103$$
determine the time of death based on this new information.
4. The function $f(x) = e^x + x^2 - x - 4$ has a unique zero on the interval $(1, 2)$. Create three different iteration functions corresponding to this function, and compare their convergence properties for approximating the zero on $(1, 2)$. Use the same starting approximation, p_0 , for each iteration function.
5. (a) Verify that the equation $x^4 - 18x^2 + 45 = 0$ has a root on the interval $(1, 2)$. Next, perform six iterations of the method of false position and bisection method. Given that the exact value of the root is $x = \sqrt{3}$, compute the absolute error in the six approximations just obtained. What is the apparent order of convergence? What explanation can you provide for this behavior?
(b) Verify that the equation $x^4 - 18x^2 + 45 = 0$ also has a root on the interval $(3, 4)$. Perform five iterations of the method of false position and bisection method, and compute the absolute error in

each approximation. The exact value of the root is $x = \sqrt{15}$. What is the apparent order of convergence in this case?

(c) What explanation can you provide for the different convergence behavior between parts (a) and (b).

6. Suppose that the equation

$$\frac{3}{2}x - 6 - \frac{1}{2}\sin(2x) = 0$$

has a unique real root.

(a) Find an interval on which this unique real root is guaranteed to exist.

(b) Using the interval found in part (a) and the bisection method, approximate the root to within an absolute tolerance of 10^{-5} .