20/07/23

SNS

* classification of signals
* Diff. b/w signals & systems

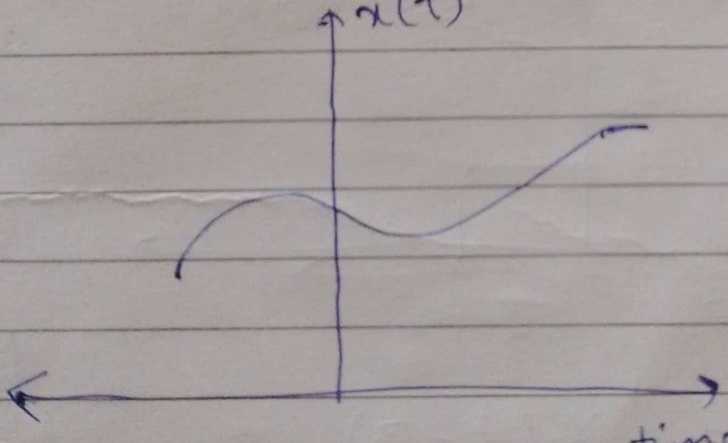
Unit-3 Imp.

$\boxed{\text{Book :- Oppenheim}}$

Signal :- Any physical quantity that contains information in a pattern of variation of some functions is known as signal.

Classification of signal :-

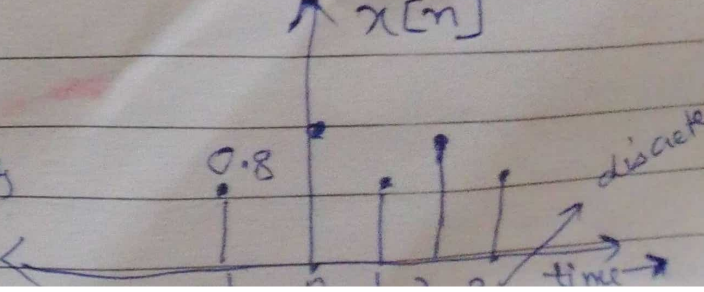
1. Continuous Time Signal :-



eg:- $\sin\omega_0 t$, $\cos\omega_0 t$

2. Discrete Time Signal :-

eg:- crime rates, budget, no. of objects



Page :
Date : / /

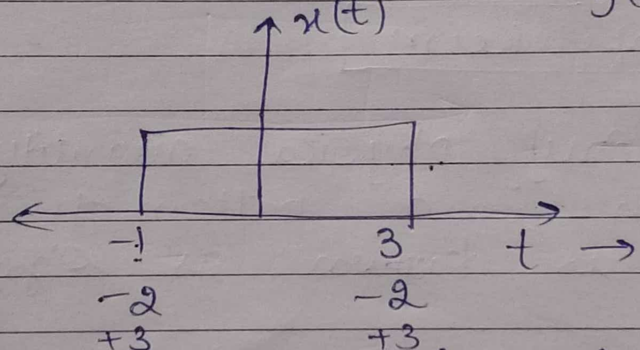
① Time is discrete in Discrete Time signal
② Digital signal :→ Quantized & coded
↳ levels me divide
↳ values either (0 , 1)

Basic Operations on Continuous Time signals :→
① Time Shifting :→ Time ke respect me signal ko shift krte h.

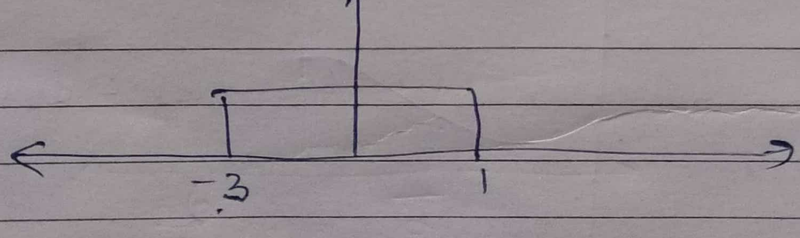
② Time reversing :- mirror image
③ Time Scaling :- Compress or expand signals

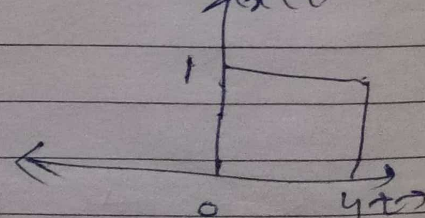
Graphical Represtation' Mathematical Rep.

$y(t) = x(t + \alpha)$
↓
Time shifting
+ve → advance
-ve → delay

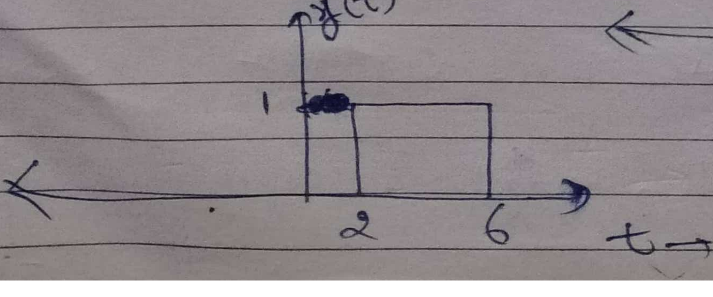


Time reversing
↳ $y(t) = x(t+2)$
$y(t) = x(t-2)$





$y(t) = x(t-3)$



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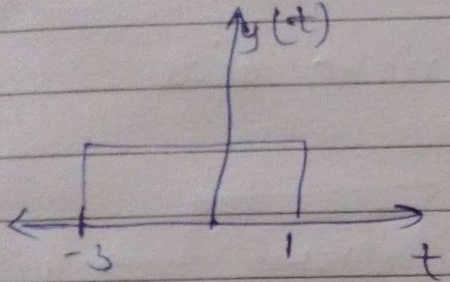
* Bandwidth will not be impacted by operations

② Time Reversal :- $y(-t)$ Along y-axis mirror image

$$y(t) = x(-t)$$



(not Time reversal)

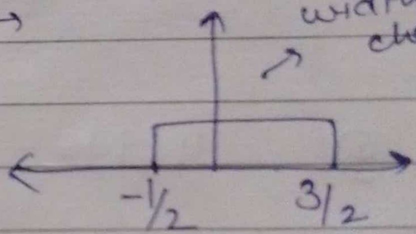
$-x(t) \neq x(-t)$

(mirror image along x-axis)

③ Time Scaling :- $y(t) = x(\alpha t)$

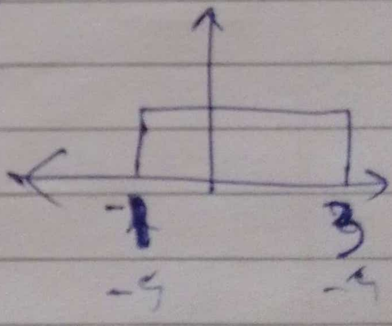
① ~~$\alpha \to +ve$~~ if $\alpha > 1$, compression

$y(t) = x(2t) \rightarrow$

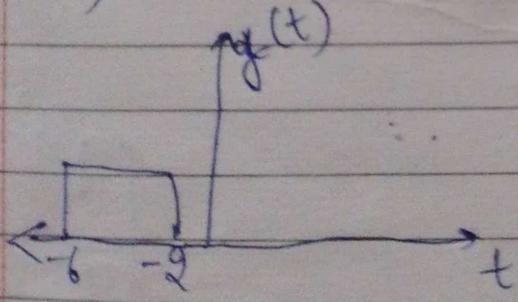


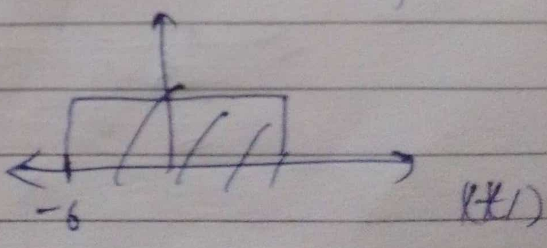
② $0 < \alpha < 1$, expand

$y(t) = x\left(\frac{t}{2}\right)$

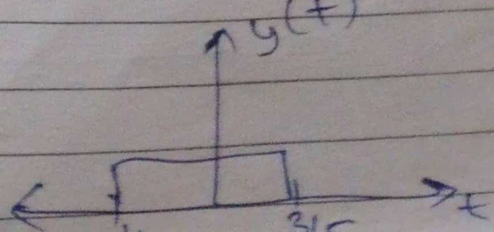


① $y(t) = x(t+5)$





② $y(t) = x(5t)$



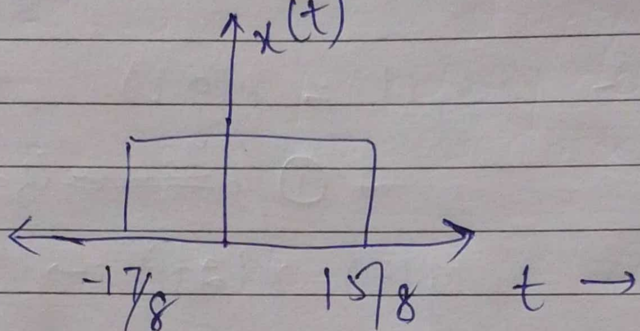
③ $y(t) = x\left(\frac{7}{8} - t\right)$

① convert it into std. form

$$y(t) = x\left(-t + \frac{7}{8}\right)$$

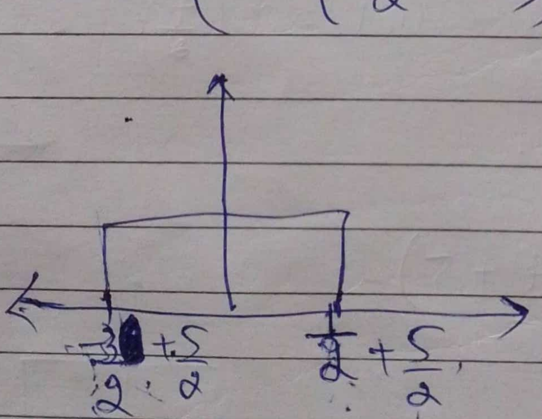
② start solving from right side

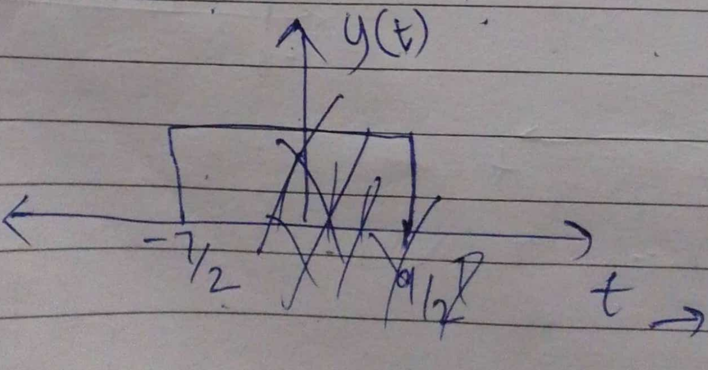
$x(t) \rightarrow$ [time adv. $7/8$] $\longrightarrow$ [Time reversal] $\longrightarrow y(t)$

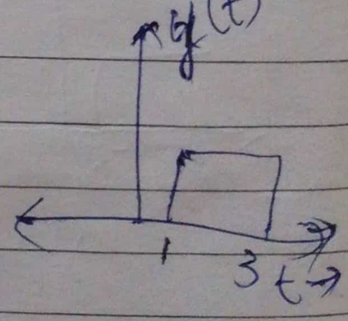


④ $y(t) = x(5 - 2t)$

$$y(t) = x\left(-2\left(-\frac{5}{2} + t\right)\right)$$







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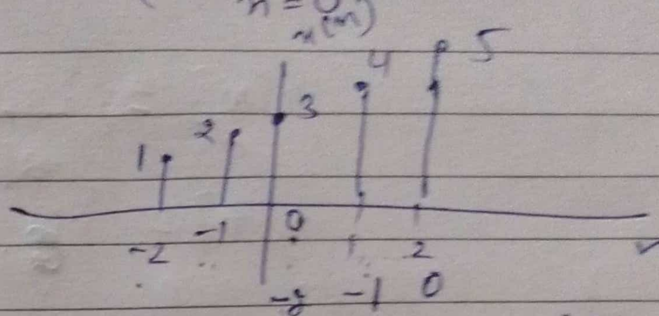
Discrete signals:- $x[n] = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$n = 0$



If arrow is not given then left most will be 0

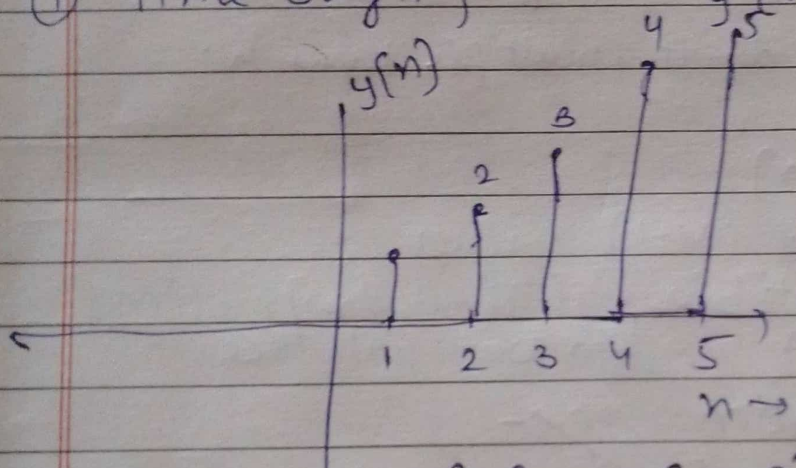
① Time shifting:- $y[n] = x[n + \alpha]$

α should be an integer

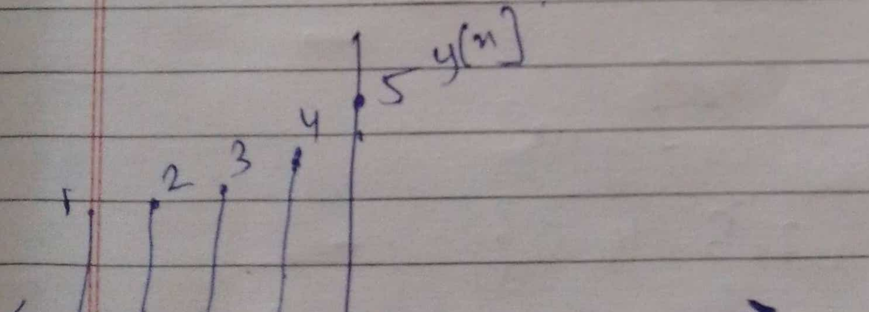
$\alpha < 0$ $\qquad$ $\alpha > 0$

$y[n] = x[n-3]$ $\qquad$ $y[n] = x[n+2]$

Time delay $\qquad$ Time Adv. (shift left)



$y[n] = x[n-3]$



② Time Reversal:- $y[n] = x[-n]$

$x[n] = \{1, 2, 3, 4, 5\} \rightarrow y[n] = \{5, 4, 3, 2, 1\}$

③ Time Scaling:- $y[n] = x[\alpha n]$ $\qquad$ α should be integer

$\alpha > 1$

(*) → Time Decimation :- $y[n] = x[2n]$

$$x[n] = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$$

$$\quad -2 \;\; -1 \;\; 0 \;\; 1 \;\; 2$$

$$y[n] = \underbrace{\{\cancel{-1}, -\tfrac{1}{2}, \cancel{0}, \tfrac{1}{2}, \cancel{1}\}}_{x \quad x \quad x \quad x}$$

$(-1, 0, 1)$ → discard these values (it's not defined in discrete)

n ki value pe divide, multiple karna h.

* $y[n] = x[Mn]$
  discard $(M-1)$ values
* It's not invertible (Time decimation) → Data will lose

(*) Time Interpolation o/p :-

$$y[n] = x\left[\frac{n}{\alpha}\right] \qquad \alpha > 1$$

$$y[n] = x[n/2]$$

$$x[n] = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$$

$$\{-2 \;\; -1 \;\; 0 \;\; 1 \;\; 2\}$$

$$\{-4 \;\; -2 \;\; 0 \;\; 2 \;\; 4\}$$

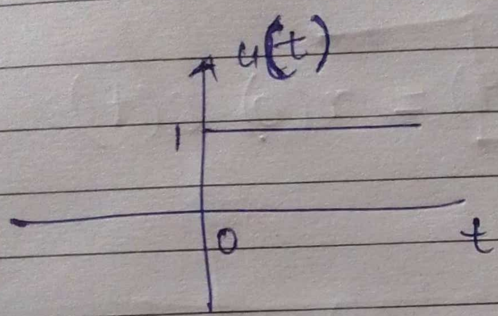
$$y(n) = [1, 0, 2, 0, 3, 0, 4, 0, 5]$$

$$y[n] = x\left[\frac{n}{M}\right]$$

Add (M-1) zeros in mid of two signals (every)

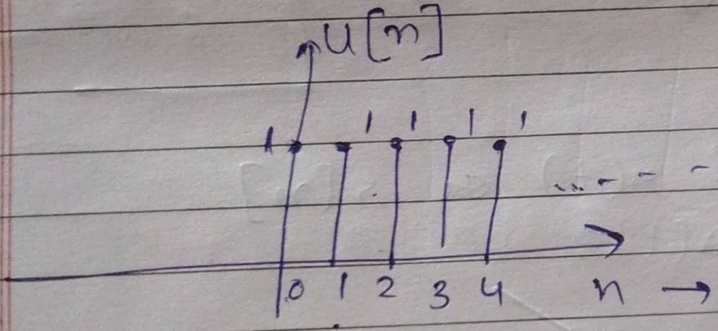
* Basic Elementary ~~Operation~~ Signals :-

① Unit step signal, $U(t)$, $U[n]$:→



$U(t) = 1 \; ; \; t \geq 0$
$\quad = 0 \; ; \;$ otherwise (or) $t < 0$

← (continuous)

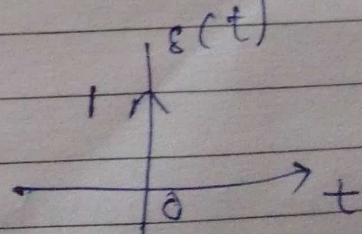


$u[n] = 1 \; ; \; n \geq 0$
$\quad = 0 \; ;$ otherwise

← discrete

Just take avg. $\; t = 0, \; u(t) = \frac{0+1}{2} = \frac{1}{2}$ (Practically)

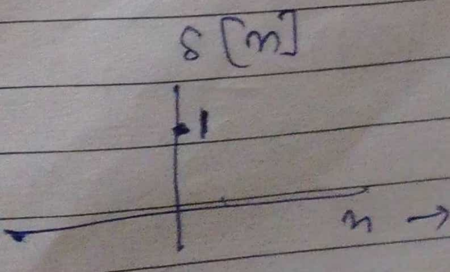
② Unit Impulse Signal :- $\delta(t)$ (or) $\delta(n)$



$\delta(t) = 1 \; ; \; t = 0$
$\quad = 0 \; ;$ otherwise

Dirac delta func.

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



$\delta[n] = 1 \; ; \; n = 0$
$\quad = 0 \; ;$ otherwise

* Properties of Impulse signals :-

① Time scaling

$$\delta(\alpha t) = \frac{1}{|\alpha|}\,\delta(t)$$

② Product Prop.

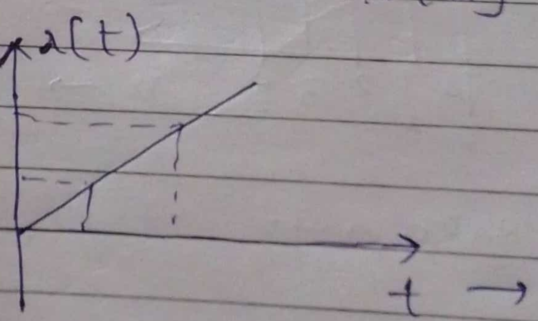
$$x(t)\,\delta(t) = x(0)\,\delta(t)$$

③ Sfting Prop :-

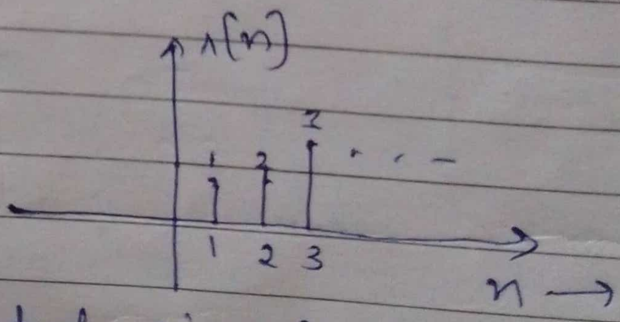
$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$$

3.) Unit Ramp signal $r(t)$ or $r[n]$

$r(t) = t, \quad t \geq 0$
$\quad\;\; = 0, \quad$ otherwise

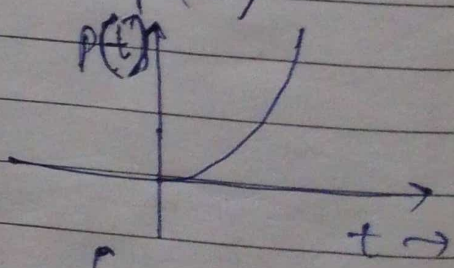


$r[n] = n \quad n \geq 0$
$\quad\;\; = 0 \quad$ otherwise



4.) Unit parabolic ~~symbol~~ signal :- $p(t)$

$p(t) = \frac{t^2}{2}, \quad t \geq 0$
$\quad\;\; = 0, \quad$ otherwise



$$\delta(t) \underset{d/dt}{\overset{\int}{\rightleftarrows}} u(t) \underset{d/dt}{\overset{\int}{\rightleftarrows}} r(t) \underset{d/dt}{\overset{\int}{\rightleftarrows}} p(t)$$

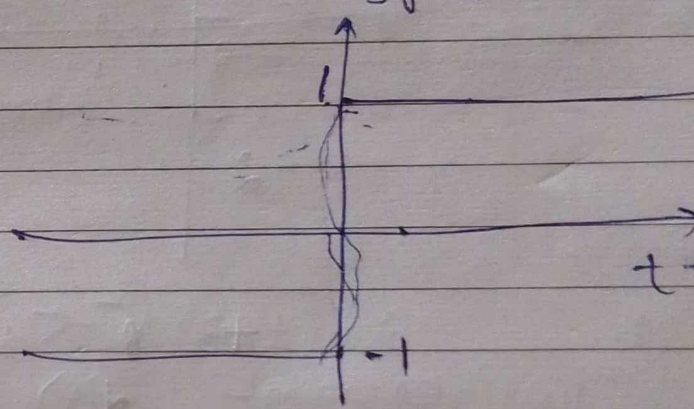
Relation b/w ramp signal & unit step signal :-

$$r(t) = t\,u(t)$$
$$r[n] = t\,u[n]$$

* Signum Signal :→

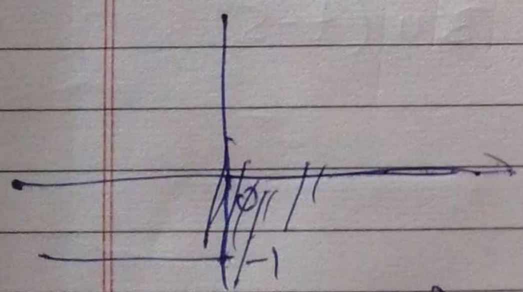
$$s(t) = 1 \; ; \quad t \geq 0$$
$$= -1 \; ; \quad t < 0$$
$$= 2 \; , \quad t = 0$$

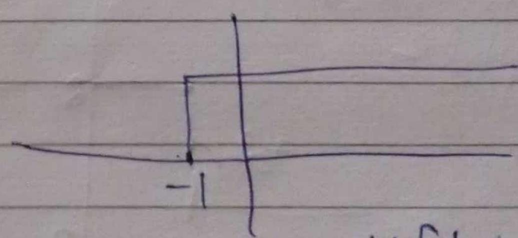
sgn(t)



$$\boxed{\begin{aligned} sgn(t) &= u(t) - u(-t) \\ sgn(t) &= 2u(t) - 1 \end{aligned}}$$

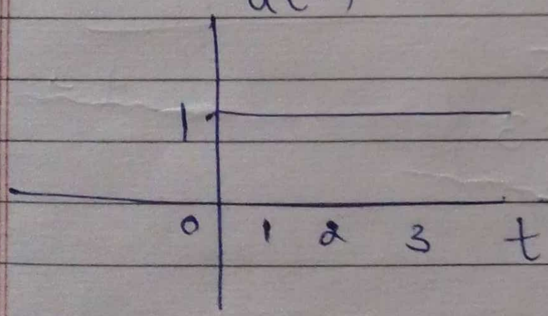
$-1u(t)$

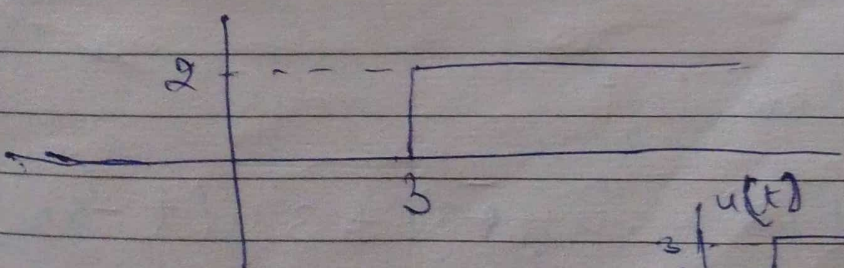




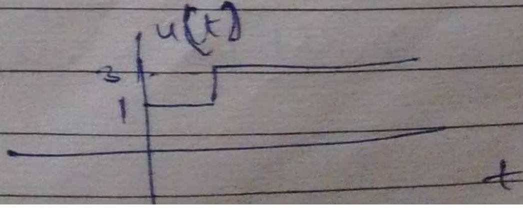
u[t+1]

u(t)





2 u[t-3]

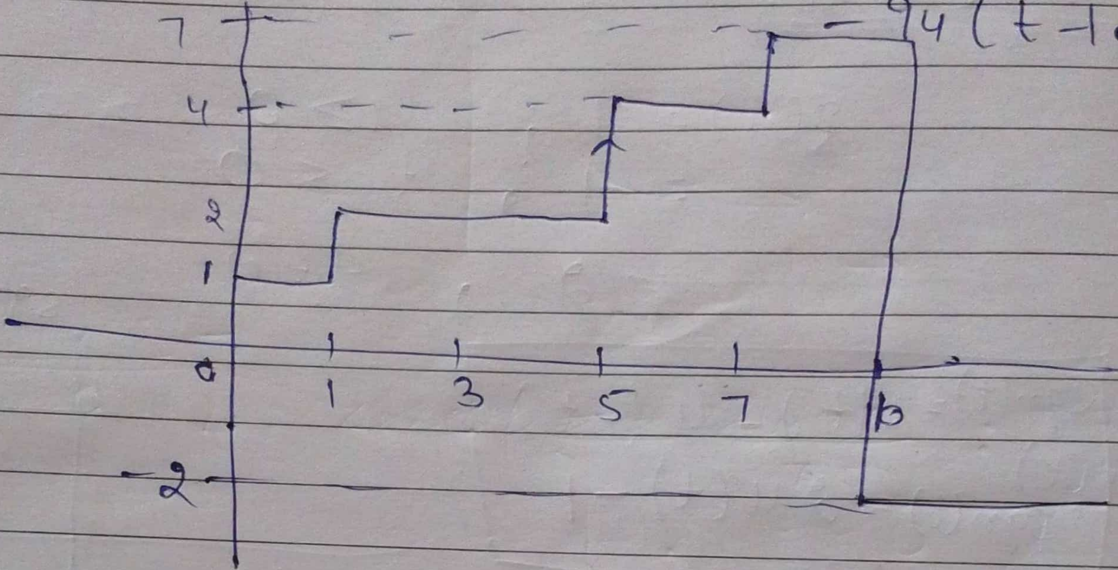


$\rightarrow$ amplitude

$$+1u(t)+2u(t-3)$$

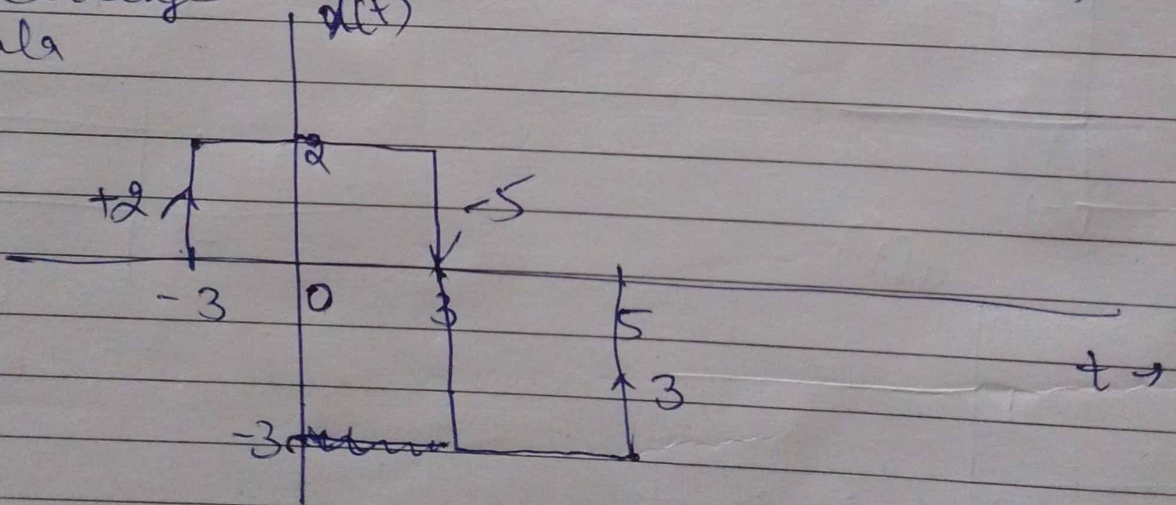
$\hookrightarrow t=0$ $\qquad\hookrightarrow t=3$

$$u(t) = u(t) + u(t-1) + 2u(t-5) + 3u(t-7) - 9u(t-10)$$



Q. $2u(t+3) - 5u(t-3) + 3u(t-5)$

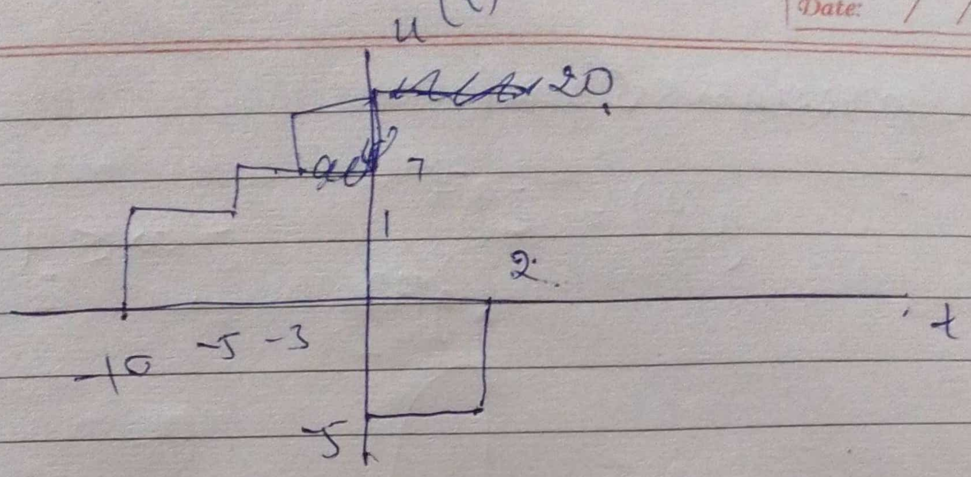
= first rearrange Adv. vala phle



Graph to eq :-

Start from left

$$2u(t+3) - 5u(t-3) + 3u(t-5)$$

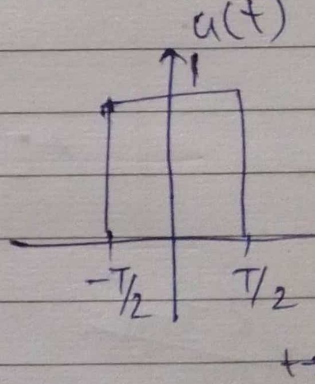


$$u(t+10) + 6u(t+5) + 13u(t+3) - 25u(t) + 4u(t-2)$$

* Rectangular pulse :-

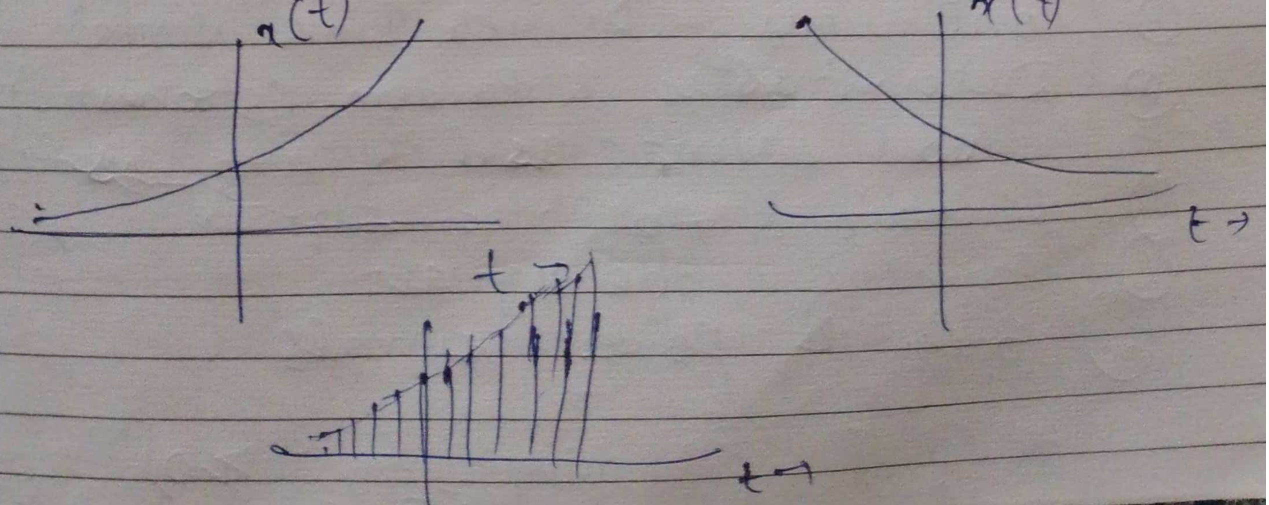
$\text{rect}\left(\frac{t}{T}\right)$ total duration of x

$$\text{rect}\left(\frac{t}{T}\right) = 1 \; ; \quad -\frac{T}{2} \le t \le \frac{T}{2}$$
$$= 0 \; ; \quad \text{otherwise}$$



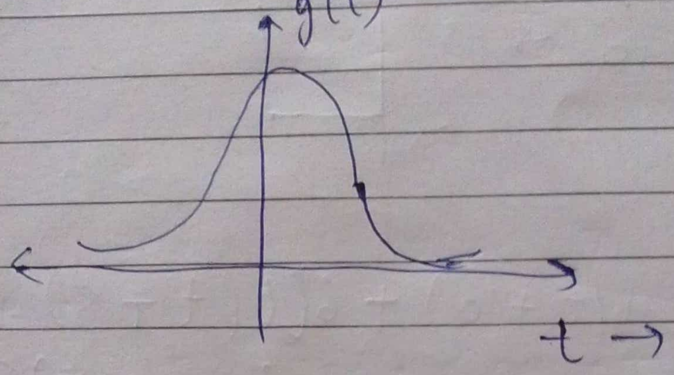
* Exponential Signal :-

$$x(t) = Ae^{Bt}$$



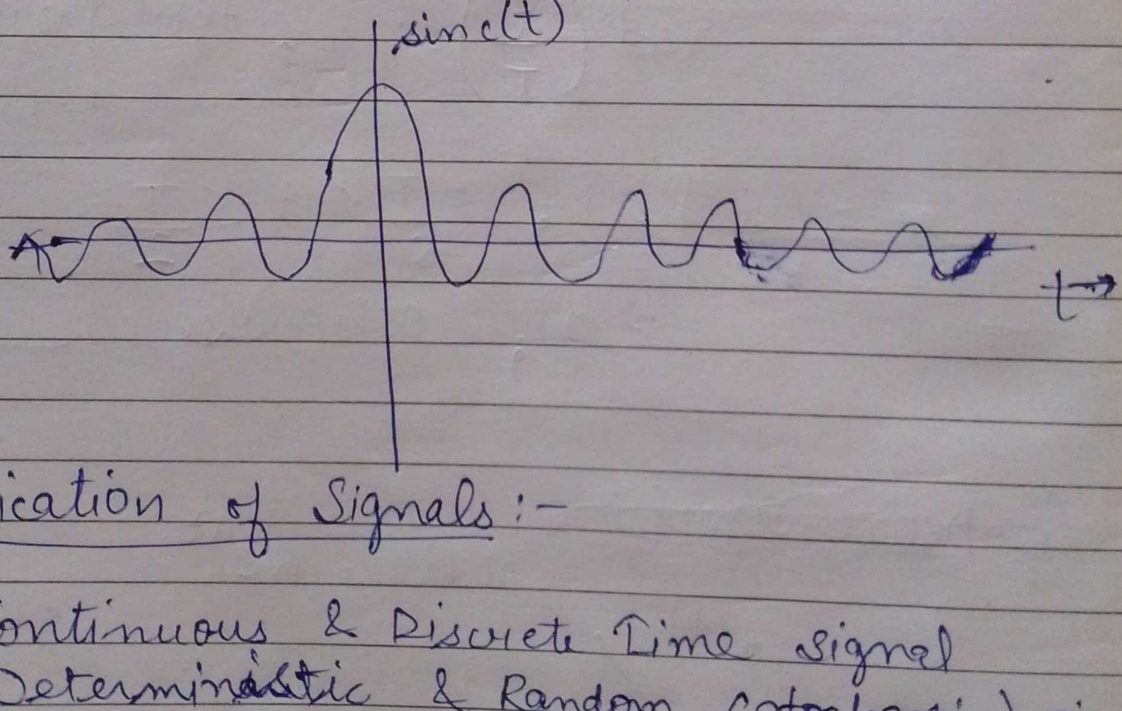
* Gaussian Signal :-

$$g(t) = e^{-a^2 t^2} \qquad -\infty \leq t \leq \infty$$



* Sinc. fx^n :-

$$\text{sinc}(t) = \frac{\sin \pi t}{\pi t} \; ; \; t \neq 0$$
$$= 1 \, , \quad t = 0$$



v. v. Imp.

Classification of Signals :-

1. Continuous & Discrete Time signal
2. Deterministic & Random (stochastic) signal
3. Causal, Non-causal & anti causal signal
4. Periodic & Aperiodic signals
5. Even & odd signals
6. Energy & power signals.

* Causal (only defined for $t \geq 0$)
  Anti causal ($t < 0$ only) $u(-t)$

Page :
Date: / /

Multiply with u(t) to make causal signals.

$\text{sinc}\left(\frac{\pi t}{\pi t}\right)$
cost, rect $\left(\frac{t}{T}\right)$
t^2

* Even & odd Signal :-

* if $x(-t) = x(t)$; Even ; sym. abt y-axis

$x(-t) = -x(t)$; odd ; Anti symm.
sin (t), signum

* $E + E = E$
$O + E = NENO$ ← sint + cost
$O + O = O$
$E \times E = E$
$E \times O = O$
$O \times O = E$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

* Periodic & Aperiodic signals :-

$x(t+T) = x(t)$ → Periodic
↳ period

Eg:- sint, cost

u(t) → Aperiodic

Periodic

$x(t) = 5 \sin 7\pi t$
$\omega = 7\pi$
$\omega = \frac{2\pi}{T} = 7\pi$
$T = \frac{2}{7}$

$x(t) = 5\sin 2\pi t + 3\cos 8\pi t$

Method 1 :-
find ω_1 & ω_2 ...
$\omega_0 = HCF(\omega_1, \omega_2 ...)$
$T = \frac{2\pi}{\omega_0}$

(2) find T_1, T_2, T_3 ...
= LCM (T_1, T_2, ...)

Page :
Date: / /

How to check periodic

$= 5 \sin 2\pi t + 3 \cos 8\pi t$

$\frac{\omega_1}{\omega_2}$ or $\frac{T_1}{T_2} = \frac{2}{8}$ = rational then periodic

* Either all terms contains π otherwise no one.

$5 \sin 2\pi t + 3 \cos 8\pi t$

$HCF(2\pi, 8\pi)$
$\Rightarrow 2\pi$

$T = \frac{2\pi}{\omega} = 1$

$N = \frac{2\pi}{\omega_0} k$

$x[n] = \cos 5\pi x$

$\cos x = 5\pi$

$N = \frac{2\pi}{\omega_0} k$

~~Put~~ $x = \frac{2\pi}{5\pi} k$

$\boxed{k = 5}$ $\boxed{N = 2}$

Q. $x[n] = \cos 5n$

$\omega_0 = 5$

$N = \frac{2\pi}{5} k$

k must be an integer so, $k = \frac{5}{\pi}$

it's not integer.

Aperiodic

NMIT SNS

03/08/23

(1) $x[n] = e^{j\frac{4\pi}{7}n} + e^{j\frac{2\pi}{5}n}$

$$\omega_0 = \frac{4\pi}{7}, \quad \omega_1 = \frac{2\pi}{5}$$

$$\omega_0 = HCF\left(\frac{4\pi}{7}, \frac{2\pi}{5}\right)$$

$$\omega_0 = \frac{2\pi}{35}$$

$$N = \frac{2\pi}{\omega_0} k$$

$$N = \frac{\cancel{2\pi}}{\cancel{2\pi}} \times 35\, k$$

$$N = 35 \text{ sec}$$

(2) $x[n] = \cos 7\pi n + \cos 5\pi n$

$$\omega_0 = HCF(7\pi, 5\pi)$$

$$\omega_0 = \pi$$

$$N = \frac{2\pi}{\pi} k$$

$$\boxed{N = 2 \text{ sec}}$$

(3) $x[n] = \cos 7\pi n + \cos 5n \rightarrow$ Aperiodic.

$E \rightarrow$ if E = finite & $P = 0$

Power if P = finite & $E = \infty$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad \text{or} \quad \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \quad \text{or} \quad \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2$$

Just to verify.

* If $t \to \infty$, signal value = 0; Energy
* if $t \to \infty$, ______ = finite, Power
* If ______ $= \infty$, Neither energy nor Power

$x[t] = A \to$ Power

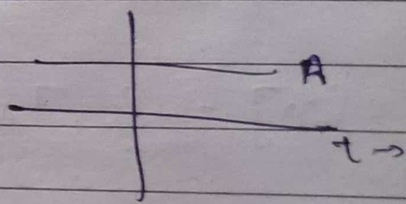
Q: $x(t) = e^{-2t} u(t)$

$t \to \infty$

$x(t) = 0$

↳ energy

Q: $x[t] = A \to$ Power



$x[t] = e^{-2t}(u(t))$

$$\int_{-\infty}^{\infty} |x[t]|^2$$

$$= \int_{-\infty}^{\infty} \left(e^{-2t}(u(t))\right)^2 dt$$

$$\Rightarrow \int_{-\infty}^{0} 0 + \int_{0}^{\infty} e^{-4t} dt = \left[\frac{e^{-4t}}{-4}\right]_0^{\infty}$$

~~Energy~~ $= \frac{1}{4}$ Joules

$P = 0$ W

$$P = \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$$

$x[t] = A$

Power $= A^2$

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04/08/23.

Page:
Date: / /

* Classification of System :-

$$\xrightarrow[\text{I/P}]{x(t)} \boxed{\text{S/S}} \xrightarrow[\text{O/P}]{} y(t) = x(t-2)$$

$\tau \rightarrow$ response of S/S

$$y(t) = \tau[x(t)]$$

$y(t) = x(t^2)$
$y(t) = 2x(t)$
$y(t) = x(-t)$
$y(t) = \cos t \, x(t)$

$\Bigg\} \rightarrow$ All these are systems

V.V Imp:→ Qs
Classification of system :-
Type of system to identify.
One numerical
Graph → eqn. or eqn. to graph

① Static & Dynamic (Memory or Memoryless) S/S :-
(Static → Memoryless; Dynamic → memory)

② Causal & Non causal S/S (Reliazable & Non-reliazab

Static :- A system is said to be static if the response at any instance of time depends on i/p only at that instant of time (at present instant of time) Also known as memory-less system.

$t \rightarrow 0$, $t \rightarrow +ve$, $t \rightarrow -ve$
(To check)

$y(t) = 2x(t)$
$y(0) = 2x(0)$
$y(2) = 2x(2)$
$y(-3) = 2x(-3)$

$\Bigg\} \rightarrow$ static

Page :
Date: / /

(response)

* $y(t) = \cos t \; x(t) \rightarrow$ static

$y(t) = x^2(t) \rightarrow$ static

$y(t) = x(2t) \rightarrow$ Dynamic

Q: $y(t) = x^2(t)$ check whether static, dyna L/NL, V/IV, - -

~~Dynamic~~ Static (Memoryless)

② Causal & Non-Causal :-

A system is said to be a causal system if the response at any instant of time depends on past ~~or~~ present i/p only.

If it depends upon (past or present) & future i/p. non-causal.

If it depends upon future i/p only is called Anti-Causal.

$y(t) = x^2(t)$ Reliazable

$y(0) = x^2(0)$

$y(2) = x^2(2)$

$y(-3) = x^2(-3)$

$\rightarrow$ Causal

$y(t) = x(2t) \rightarrow$ Non-causal

Non-reliazable

$y(t) = x\left(\frac{t}{2}\right) =$ N.C.

$y(t) = \cos t \; x(t) \rightarrow$ C

$y(t) = x(t-2) =$ C

$y(t) = x(t+5) =$ NC

* ③ Linear & Non-Linear S/S :-

write all steps in numerical:-

① $y(t) = x^2(t)$

② $x_1(t) \xrightarrow{\tau} y_1(t) = x_1^2(t)$

③ $x_2(t) \xrightarrow{\tau} y_2(t) = x_2^2(t)$

④ if $y_1(t) + y_2(t) = \tau[x_1(t) + x_2(t)]$

(Response)

Also, $a x_1(t) \xrightarrow{\tau} a y_1(t) = a x_1^2(t)$

(homogenity prop.)

$$a x_1(t) + b x_2(t) \xrightarrow{\tau} a y_1(t) + b y_2(t)$$

if $x_1^2(t) + x_2^2(t) = [x_1(t) + x_2(t)]^2$

$\Rightarrow$ false $\Rightarrow$ N.L.

If equal then Linear

$$y_1(t) + y_2(t) = \tau[x_1(t) + x_2(t)]$$

Q: $y(t) = \frac{d}{dt} x(t)$

$$\frac{d}{dt} x_1(t) + \frac{d}{dt} x_2(t) = \frac{d}{dt}[x_1(t) + x_2(t)]$$

$\Rightarrow$ linear

Q: $y(t) = \frac{d}{dt} x(t) + c$

$$\frac{d}{dt} x_1(t) + c + \frac{d}{dt} x_2(t) + c = \frac{d}{dt}[x_1(t) + x_2(t)] + c$$

$\Rightarrow$ N. Linear

Page :
Date: / /

(4) Time Variant & Invariant system:-
If response of system is changing with time it is variant system.

If delayed response = response of s/s to delayed i/p

$$y(t-t_0) = \tau[x(t-t_0)]$$

→ Then time Invariant (shift Invariant)

Q: $y(t) = 2t\, x(t)$

~~$y_1(t)$~~ $x_1(t) = y_1(t) = 2t\, x_1(t)$

$x_1(t-t_0) = y_1(t-t_0) = 2(t-t_0)\, x_1(t-t_0)$

$2(t-t_0)\, x_1(t-t_0) = 2t[x(t-t_0)]$

→ variant

Q: $y(t) = 3x^2(t)$

$y(t-t_0) = 3x^2(t-t_0)$

$x(t) \xrightarrow{t_0} x(t-t_0) \xrightarrow{\tau} 3x^2(t-t_0)$

$3x^2(t-t_0) = 3(x(t-t_0))^2$
$= 3x^2(t-t_0)$

→ Invariant

Q: $y(t) = 3x(t^2)$

$y(t-t_0) = 3x((t-t_0)^2)$

$\Rightarrow 3x((t-t_0)^2)$

$$x(t) \xrightarrow{t_0} x(t-t_0) \xrightarrow{T} 3x((t^2-t_0))$$

$\Rightarrow$ variant

Q: $y(t) = x(-t)$

$y(t-t_0) = x(-t+t_0)$

$$x(t) \xrightarrow{t_0} x(t-t_0) \xrightarrow{T} x(-t_0-t_0)$$

$\Rightarrow$ variant

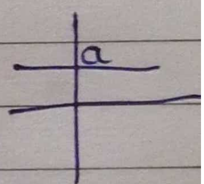
Shortcut!- t pr Operation no sha h $\rightarrow$ variant
Agr $x(t)$ ——————— $\rightarrow$ Invariant

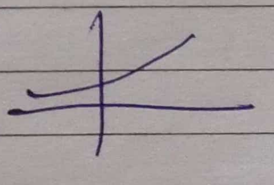
* $y(t) = x(-t) \longrightarrow$ variant

(5) Stable & Unstable system :- A system is said to be a stable system, if bounded i/p results in bounded o/p.

If BIBO $\rightarrow$ stable

bounded $\rightarrow$ signal ka amplitude finite





$\rightarrow$ unbounded

Example :→

$y(t) = \cos(x[t])$ — Put $x[t] = u[t]$
$\quad$ -1 to 1 — If bounded o/p $\rightarrow$ stable
$\Rightarrow$ bounded $\rightarrow$ stable — unbounded $\rightarrow$ unstable

$y(t) = \cos t \; x[t]$
$\quad$ ↳ bounded $\rightarrow$ stable

$y(t) = \dfrac{x[t]}{\cos t}$ = unbounded $\rightarrow$ Unstable

$y(t) = t(x(t)) = t(u(t)) = r(t) =$ Unstable

$y(t) = x^2(t) =$ Stable

⑥ Invertible or Non - Invertible :-

If ~~x~~ ~~Inverse~~ distinct i/ps results in distinct o/p then the system is said to be invertible.
And if distinct i/p result in ~~dist~~ same o/p then the system is non-invertible

* $y(t) = x^2(t)$
Non- Invertible

$$\underbrace{x(t) \longrightarrow \boxed{s/s} \xrightarrow{y(t)} \boxed{s/s}}_{\text{(Invertible)}} \longrightarrow x(t)$$

* Response of unit Impulse system is $h(t)$ Impulse response

* Response of step Impulse System is $s(t)$ step response.

07/08/23

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Def Convolution Integral :- is a mathematical tool for finding the response of any LTI system for any arbitrary i/p $x(t)$ provided the impulse response is known.

$$x(t) \longrightarrow \boxed{} \longrightarrow y(t)$$

$$y(t) = \tau[\{x(t)\}]$$

$$\delta(t) \longrightarrow \boxed{S/S} \longrightarrow h(t)$$

$$u(t) \longrightarrow \boxed{S/S} \longrightarrow s(t)$$

$y(t) = x(t) * h(t)$ (convolution)

$$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau}$$

Proof :-

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

(Prop. of imp. signal)

$$y(t) = \tau[x(t)] = \tau\left[\int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau\right]$$

$$\Rightarrow \int_{-\infty}^{\infty} \tau[x(\tau)\, \delta(t-\tau)\, d\tau]$$

$$= \int_{-\infty}^{\infty} x(\tau)\, \tau[\delta(t-\tau)]\, d\tau$$

$$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = x(t) * h(t)}$$

If

$$\boxed{y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)}$$

* Properties of Convolution :-

① Commulative prop :→ $x(t) * h(t) = h(t) * x$

L.h.s $\quad x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

let $t-\tau = p \quad d\tau = -$

$$= \int_{-\infty}^{\infty} x(t-p)\, h(p)\, dp$$

$$= \int_{-\infty}^{\infty} h(p)\, x(t-p)\, dp$$

$$= h(t) * x(t)$$

② Associative Prop :- * connecting in cascade

Cascade Interconnection

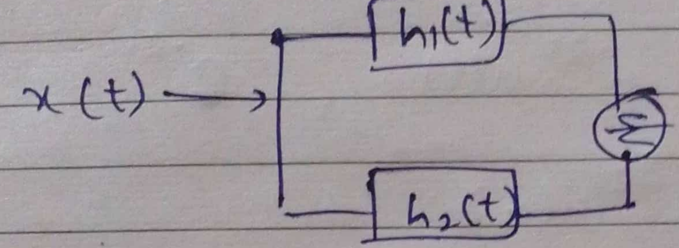
$$[x_1(t) * x_2(t)] * h(t) = x_1(t) * [x_2(t) * h($$

③ Distibutive Prop :- Parallel Inconnection.

$$x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2$$

① $x(t) \rightarrow \boxed{h_1(t)} \rightarrow \boxed{h_2(t)} \rightarrow y(t)$

$\equiv \quad x(t) \rightarrow \boxed{h_1(t) * h_2(t)} \rightarrow y(t)$

② 

$y(t) \equiv x(t) \rightarrow \boxed{h_1(t) + h_2(t)}$

Steps to solve numericals :-

Step 1 :→ Change of time index

① $t \rightarrow \tau$ $\quad x(\tau)$, $h(\tau)$

② $h(\tau) \rightarrow h(-\tau)$

③ shifting → $h(t-\tau)$

④ Multiplication $x(\tau)\, h(t-\tau)$

⑤ Integration

Q: $x(t) = 2u(t)$ $\quad$ find response?

$h(t) = u(t)$ $\quad$ " convolution?

$u(t) \rightarrow$ Causal

If both causal, then it changes to $\int_0^t$

$x(t) = 2u(t) = 2, \; t \geq 0$

$h(t) = u(t) = 1, \; t \geq 0$

$y(t) = \int_0^t x(\tau)\, h(t-\tau)\, d\tau = \int_0^t 2.1\, d\tau = \int_0^t 2 d\tau$

Page:
Date: / /

$= 2t, \; t \geq 0 \quad = 2t\,u(t)$

Q: $x(t) = e^{-2t} u(t)$
$h(t) = e^{-5t} u(t)$

$\cdot\; \cancel{x(t)} = e^{-2t} u(t) \quad , \quad e^{-2t}$

$x(\tau) = e^{-2\tau} u(\tau)$
$\cancel{x(t)} =$
$h(t-\tau) = e^{-5(t-\tau)} u(t-\tau)$

$\Rightarrow \quad x(t) = e^{-2t} \quad , \; t \geq 0$
$h(t) = e^{-5t} \quad , \; t \geq 0$

$\Rightarrow y(t) = \int_0^t x(\tau)\, h(t-\tau)\, d\tau$

$\Rightarrow y(t) = \int_0^t e^{-2\tau} e^{-5(t-\tau)}\, d\tau$

$\Rightarrow y(t) = \int_0^t e^{-2\tau} e^{-5t} e^{5\tau}\, d\tau$

$= e^{-5t} \int_0^t e^{3\tau}\, d\tau$

$\Rightarrow e^{-5t} \left[\frac{e^{3\tau}}{3} \right]_0^t$

$\Rightarrow \frac{e^{-5t} \cdot (e^{3t} - 1]}{3}$

$= \frac{1}{3} [e^{-2t} - e^{-5t}]\, u($

* Std. form answer is must

11/08/23 Graphical Method → (Always when not mentioned)

$x(t) = u(t)$

$h(t) = u(t)$

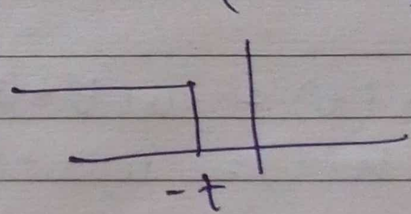
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

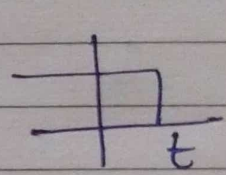
$x(\tau) = u(\tau)$

$h(\tau) = u(\tau)$, $h(t-\tau) = u(t-\tau)$

$$= \int_{0}^{\infty} u(t-\tau)\, d\tau = \int_{0}^{\infty} u(-\tau - t)\, d\tau$$

$t < 0$ → $u(-\tau - t)$; $t > 0$ → $u(-\tau + t)$





$$y(t) = \int_{0}^{\infty} d\tau , \ t \geq 0$$

$= 0$

$y(t) = t , \ t \geq 0$

$= 0 , \ t < 0$

$y(t) = t\,u(t)$

$= r(t)$

★ **Note:-** If u convolve unit step signal with unit step signal the resultant signal will be ramp signal.

Q.★ $x(t) = u(t+3)$

$h(t) = u(t+3)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

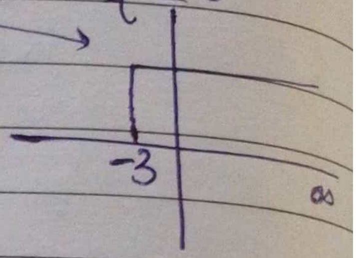
$x(\tau) = u(\tau+3)$, $h(\tau) = u(\tau+3)$

$\times \quad h(t-\tau) = ~~u~~ (t-\tau+3)$

$$\int_0^{\infty} u(\tau+3)\, u(t-\tau+3)\, d\tau$$

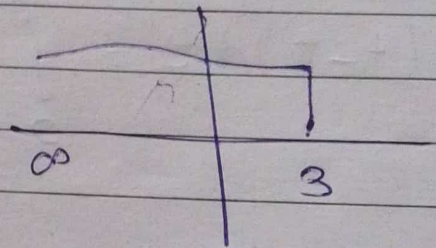
$$\times \quad \int_{-3}^{\infty} u(t-\tau+3)\, d\tau$$

$t<0$



$t>0$

$\times$



$y(t) =$

Q.

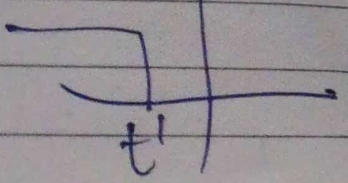
$x(t) = u(t+3)$
$h(t) = u(t-5)$

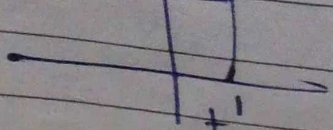
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} u(\tau-5)\, u(-\tau+\underbrace{t+3}_{t'})\, d\tau$$

$$\int_5^{\infty} 1 \cdot u(t'-\tau)\, d\tau$$

$t'<5 \qquad t' \geq 5$





$y(t) = \int_5^{t'} 1 \cdot d\tau,$

$= 0,$

$y(t) = t'-5 \qquad t' \geq 5$
$= 0 \qquad$ otherwise

$y(t) = t'-5 \;, \qquad t+3 \geq 5$
$= t+3-5 \;, \qquad t \geq 2$
$= t-2$

$y(t) = t-2 \qquad$ for $t \geq 2$
$= 0 \qquad$ otherwise

NOTE
$$\boxed{\begin{gathered} u(t) * u(t) = r(t) \\ u(t \pm \alpha) * u(t \pm \beta) = r(t \pm \alpha \pm \beta) \end{gathered}}$$

Q. $x(t) = u(t+7)$
$h(t) = u(t+2)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\int_{-\infty}^{\infty} u(\tau+2)\, u(\underbrace{-\tau+t+7}_{t'})\, d\tau$$

In discrete :-> $\quad x[n] = u[n+2]$
$h[n] = u[n-3]$
$r[n] = r[n-1]$

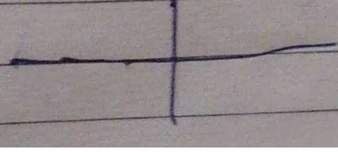
* $\boxed{x[n]\,\delta[n] = x[n]}$

* $\boxed{x[n+2]\,\delta[n-1] = x[n+1]}$

$$\Rightarrow \int_{-2}^{\infty} 1.\, u(t'-\tau)\, d\tau$$

$t' < -2 \qquad t' \geq -2$

$\Rightarrow$ graphical representation



Page :
Date : / /

$$y(t) = \int_{-2}^{t'} 1 \, d\tau \quad , \quad t' \geq -2$$
$$0 \quad , \quad \text{otherwise}$$

$$y(t) = t' + 2 \quad , \quad t+7 \geq -2$$
$$t+7+2 \qquad t \geq -9$$
$$t+9$$

$$y(t) = t+9 \quad \text{for } t \geq -9$$
$$0 \quad \text{otherwise}$$

$$\sum_{-1}^{1} 1 = 3 \quad (1+1+1)$$

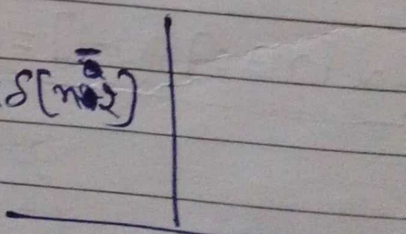
[upper lt - lower lt +

* Convolution Proof } → 2
* " Numerical
* Discrete value (****)

Q. * $x[n] = u[n+2]$
$h[n] \quad u[n-3]$

**** $x[n] = [1, 2, 3, 2]$ (Arrow is given otherwise left most)
$h[n] , [1, 0, 1, 2]$
$y[n] = ?$

$$x[n] = \delta[n+1] + 2\delta[n] + 3\delta[n-1] + 2\delta[n-2]$$
$$h[n] = \delta[n+2] + \delta[n] + 2\delta[n-1]$$



$$y[n] = x[n] * y[n]$$

$$y[n] = \delta[n+1] + 2\delta[n] + 3\delta[n-1] + 2\delta[n-2] *$$
$$\delta[n+2] + \delta[n] + 2\delta[n-1]$$

$x[n] * \delta[n] = x[n]$

$x[n+\alpha] * \delta[n] = x[n+\alpha]$

$x[n+\alpha] * \delta[n+\beta] = x[n+\alpha \pm \beta]$

Page:
Date: / /

$$y(n) = \delta[n+3] + \delta[n+1] + 2\delta[n] + 2\delta[n+2] + 2\delta[n] + 4\delta[n-1] + 3\delta[n+1] + 3\delta[n-1] + 6\delta[n-2] + 2\delta[n] + 2\delta[n-2] + 4\delta[n-3]$$

$$\Rightarrow \delta[n+3] + 4\delta[n+1] + 6\delta[n] \dots$$

$$y[n] = [1, 2, 4, 6, 7, 8, 4]$$

* Tabular method :-

* $x[n] = [1, 2, 3, 2]$ = m terms

$h[n] = [1, 0, 1, 2]$ = n terms

	1	2	3	2
1	1	2	3	2
0	0	0	0	0
1	1	2	3	2
2	2	4	6	4

$y(n) = [1, 2, 4, 6, 7, 8, 4]$

No. of terms in y(n) = $m - 1 + n$

~~4+1-4~~ $4 + 4 - 1 = 7$

Q. $x[n] = [1, 2, 4, 3, 2]$

$h[n] = [1, 0, 1]$

Tabular method :-

	1	2	4	3	2
1	1	2	4	3	2
0	0	0	0	0	0
1	1	2	4	3	2

$y[n] = [1, 2, 5, 5, 6, 3, 2]$

No. of terms: $M + N - 1$

$= 5 + 3 - 1 = 7$

1.5 marks)
(conditions
③ cond.

Page:
Date / /

Static / Dynamic LTI system :- A discrete sys. is said to be static if $h[n]=0$ for $n \neq 0$ (memoryless)
such a system has impulse response $h[n] = k\delta[n]$ where $k = h[0]$ is a constant & $y[n] = kx[n]$ where $k = h[0]$ is a const. & $y[n] = kx[n]$.

(*) Causality :-

$$y(t) = x(t) * h(t) = h[t] * x[t]$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$\Rightarrow h(\infty)\, x(t-\infty) + \ldots h(1)\, x(t-1) + h(0)\, x(t) + \underbrace{h(-1)\, x(t+1) \ldots}_{\text{future values}}$$

(Responsible for non-causality)

if $h[n] = 0$ for $n < 0$
if $h(t) = 0$ " $t < 0$
then our system is called causal
we can't make 0 to i/p, we can make 0 response.

A sys. is said to be causal if it's impulse response is 0 for -ve values of t or n.

(*) Stability for LTI :-

A System is said to be stable if it's impulse response is absolutely integrable. Then sys tem is stable.

* LTI hoga hi hoga.

18/08/23

* $x[n] = [1, 2, 1, 1]$, $\uparrow$ at 1
$h[n] = [1, 2, 1, -1]$
$y(n) =$
−1, −3 / 0 / −2 / −1
−4 / −3

By Tabular method,

	1	2	1	1
1	1	2	1	1
2	2	4	2	2
1	1	2	1	1
−1	−1	−2	−1	−1

$$y[n] = \{1 \quad \underset{\uparrow}{4} \quad 6 \quad 4 \quad 1 \quad 0 \quad -1\}$$
−1 0 1 2 3 4 5

$$y(n) = \sum_{n=-\infty}^{\infty} x[n]\, h[n-k]$$

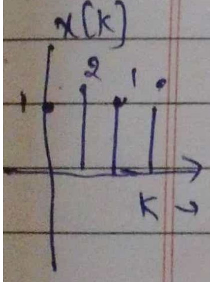
$$y(n) = \sum_{n=-1}^{5} x[k]\, h[n-k]$$

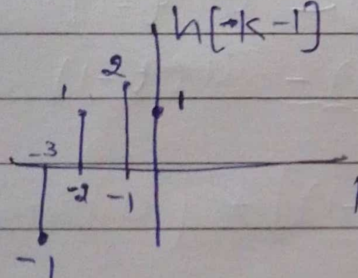
Put $n = -1$

$y(-1) = x[k]\, h[-1-k]$
$= x[k]\, h[-k-1]$

$h[k-1] = [\underset{\uparrow}{1}, 2, 1, -1]$

$h[-k-1] = [-1, 1, 2, \underset{\uparrow}{1}]$
(Swap)





= 1

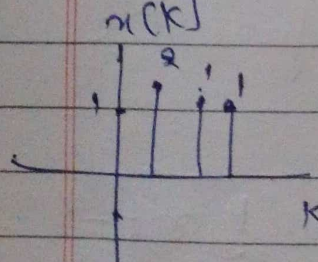
Put $n = 1$

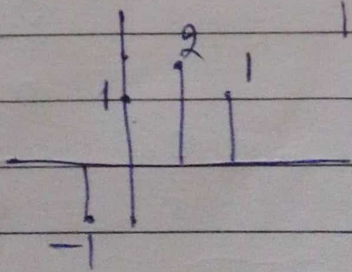
$y[1] = x[k]\, h[1-k] = x[k]\, h[-k+1] =$

$1+4+1$
$= \boxed{6}$

$h[k+1]$
$= [1\ 2\ 1\ -1]$

$h[-k+1] = [-1\ 1\ 2\ 1]$

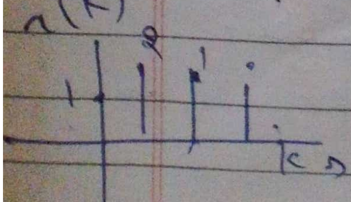


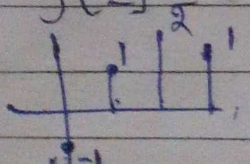


Put $n = 2$

$y[2] = x[k]\, h[2-k]$

$h[k+2]$
$= [1\ 2\ 1\ -1]$





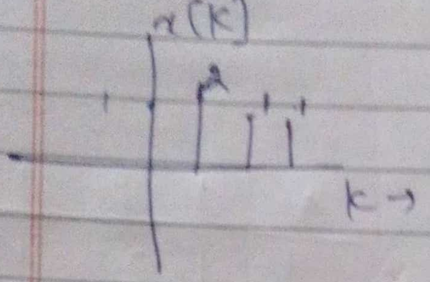
$\Rightarrow -1 + 2 + 2 + 1$

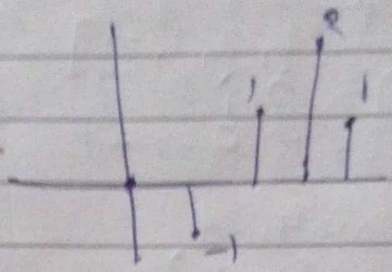
$= [-1\ 1\ 2\ 1]$

Page :
Date : / /

$y(3) = h[3-k]$

$n=3$

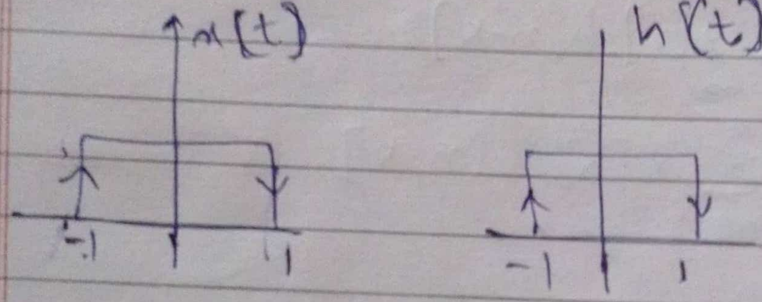




$-2+1+2 = 1$

verify by $y(n)$

Q.

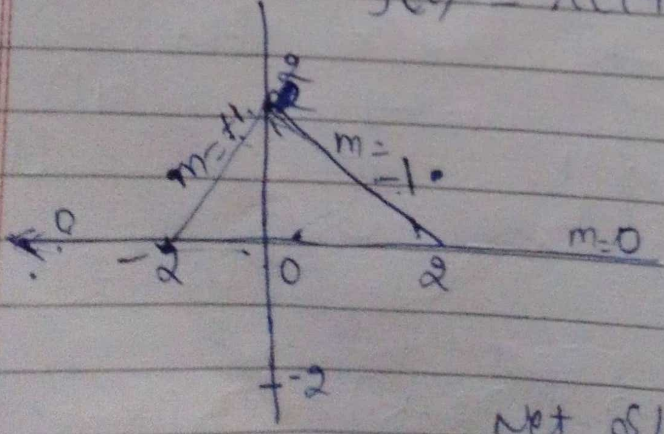


$y(t) = ?$

$x(t) = u(t+1) - u(t-1)$
$h(t) = u(t+1) - u(t-1)$

$y(t) = [u(t+1) - u(t-1)] * [u(t+1) - u(t-1)$

$r(t+2) - r(t) - r(t) + r(t-2)$
$y(t) = r(t+2) - 2r(t) + r(t-2)$



Ramp func me slo
mention.

$y(t) = \underbrace{1}_{\text{net slope}} r(t+2)$

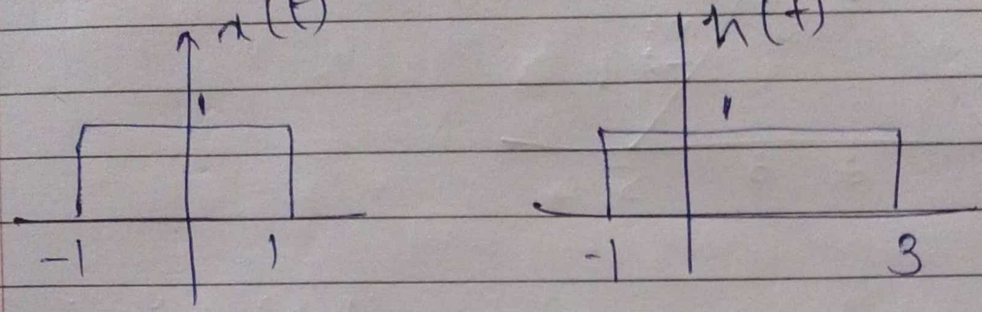
Net slope = final slope - initial slope

Page :
Date: / /

$$y(t) = r(t+2) - 2r(t) + r(t-2)$$

* If two rec. pulses of duration t are convolved then the resultant signal is a triangular pulse of duration (2t)

Q:

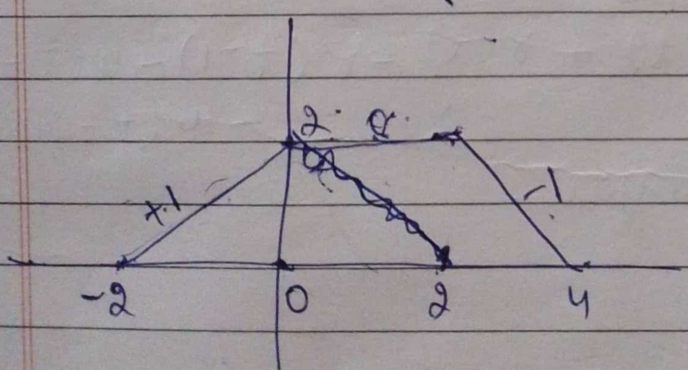


$$x(t) = u(t+1) - u(t-1)$$
$$h(t) = u(t+1) - u(t-3)$$

$$y(t) = x(t) * h(t)$$

$$y(t) = [u(t+1) - u(t-1)] * [u(t+1) - u(t-3)]$$

$$y(t) = r(t+2) - r(t-2) - r(t) + r(t-4)$$

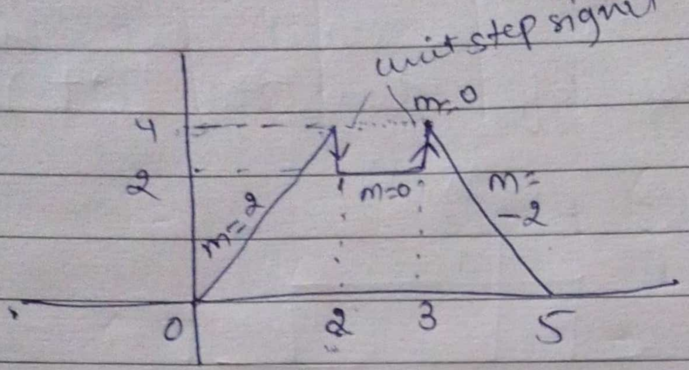


* If we convolve 2 rec. pulses of duration t_1 & t_2 then, the resultant signal will be a trapizodi pulse of duration $(t_1 + t_2)$

* The starting pt. of resultant signal y(t) is equal to the sum of starting pt. of x(t) & h(t).

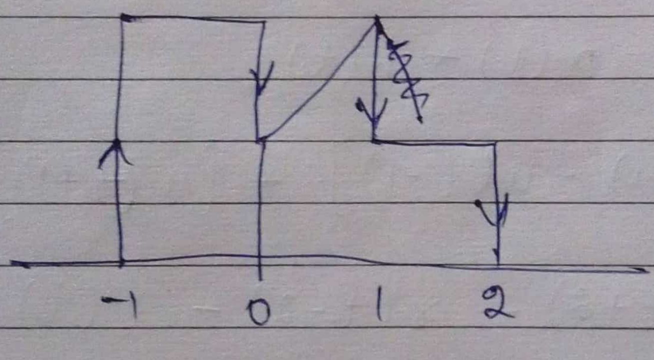
* The ending of $y(t)$ is equal to the sum of ~~resultant~~ ending pt. of $x(t)$ & $h(t)$.

Q:



$$\Rightarrow 2r(t) - 2r(t-2) - 2u(t-2) + 2u(t-3) - 2r(t-3) + 2r(t-5)$$

Q:



$$\Rightarrow 2u(t+1) - u(t) + r(t) - r(t-1) - u(t-1) - u(t-2)$$

Book:- Offanman, A. Nagur.

SNS

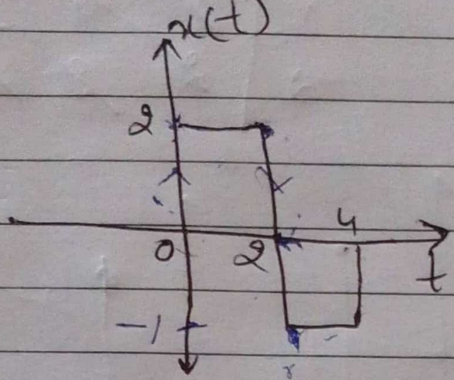
21/08/23

$$s(t) = \int h(t)\,dt$$
$$u(t) = \int \delta(t)$$

v.v.v. Imp

Q: Laplace transform of $\delta(t)$, $u(t)$, $r(t)$.

Q:



$\Rightarrow$ $2u(t) - 3u(t-2) + 1u(t-4)$

Q: $x(t) = 5\sin 2\pi t + 8\cos 7\pi t$ is periodic

It's periodic :- $T_1 = \frac{2\pi}{\omega}$ $\quad T_2 = \frac{2\pi}{7\pi}$

$T_1 = \frac{2\pi}{2\pi} = 1$, $\quad T_2 = \frac{2}{7}$

$T_0 = LCM(T_1, T_2)$

$LCM(1, \frac{2}{7}) = \frac{2}{7}$

Q: $y(t) = \cos t\, x(t)$ = ~~Non~~ causal

$\Rightarrow$ Static

$\Rightarrow$ ~~Non linear~~ ~~Non~~ linear.

$\Rightarrow$ Stable

$\Rightarrow$ TIV (t pr operation ho to variant)

linear :- $y_1(t) = \cos t\, x_1(t)$ $\quad y_2(t) = \cos t\, x_2(t)$

~~$\cos t\, x_1(t) + \cos t\, x_2(t) = \cos(t_1+t_2)\, x_1(t_1+t_2)$~~

$[x_1 + x_2] = y_1(t) + y_2(t)$

$\cos t[x_1(t) + x_2(t)] = \cos t\, x_1(t) + \cos t\, x_2(t)$ (linear)

Page :
Date : / /

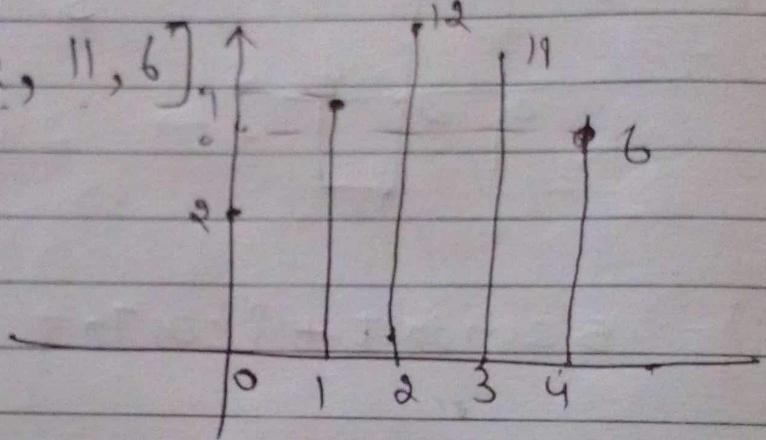
Q. $x[n] = [1, 4, 3]$

$h[n] = [\underset{\uparrow}{2}, 1, 2]$

(By Tabular)

	1	4	3
2	2	8	6
1	1	4	3
2	2	8	6

$y[n] = [\underset{\uparrow}{2}, 9, 12, 11, 6]$



Q.

Q. $y(t) = \frac{d}{dt} x(t)$

Q. $y(t) = e^{x(t)}$

Q. $y(t) = \log x(t)$

Q. $y(t) = \int x(t)$

Q. $y(t) = x(-t)$

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Page :
Date: / /

25/08/23

Laplace Transform :- is used to transform a time domain signal into complex frequency domain.

Continuous Time signal $\xrightarrow[\text{Transform}]{\text{Laplace}}$ Complex freq. domain

Discrete Time signal $\xrightarrow{\text{z-transform}}$ freq.

$$S = \sigma + j\omega$$

$j\omega \rightarrow$ freq. in rad/sec

$\sigma \rightarrow$ ~~Laplace~~ Neper freq. in neper/sec.

$$\boxed{X(S) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt} \rightarrow \text{V.V.V.V.V. Imp}$$

$X(S) \rightarrow$ Laplace $[x(t)]$

If this integration has finite value then it exists.

Q. 1 $x(t) = e^{-at} u(t)$

Q. 2 $x(t) = -e^{-at} u(-t)$

①. $$X(S) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-st}\,dt$$

$$\Rightarrow \int_{-\infty}^{0} e^{-at} u(t) e^{-st}\,dt + \int_{0}^{\infty} e^{-at} u(t) e^{-st}\,dt$$

$$0 + \int_{0}^{\infty} e^{-at} e^{-st}\,dt$$

Unit 1 + Unit 2
+ laplace
+ last unit

$$\Rightarrow \int_0^{\infty} e^{-(a+s)t}\,dt$$

$$\Rightarrow \left[\frac{e^{-(a+s)t}}{-(a+s)}\right]_0^{\infty}$$

$$\Rightarrow \frac{1}{s+a}$$

Q:

$$x(t) = -e^{-at}\,u(-t)$$

$$X(s) = \int_{-\infty}^{0} -e^{-at}\,u(-t)\,e^{-st}\,dt + 0$$

$$\Rightarrow \int_{-\infty}^{0} -e^{-at}\,e^{-st}\,dt$$

$$= -\int_{-\infty}^{0} e^{-(a+s)t}\,dt$$

$$\Rightarrow -\left[\frac{e^{-(a+s)t}}{-(a+s)}\right]_{-\infty}^{0}$$

$$\Rightarrow -\left[\frac{1}{-s-a} + 0\right] = \frac{1}{s+a}$$

laplace ~~exist~~ exist.

ROC:→ Region of convergence :- It includes those values of sigma Re[s] for which in laplace transform converges.
↳ (finite value)

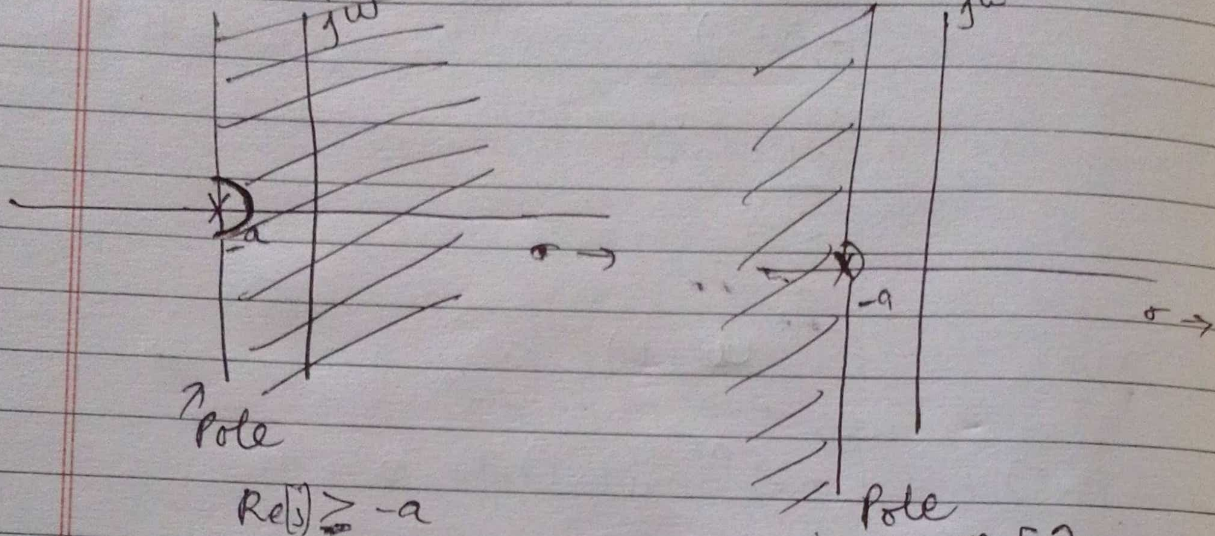
If $u(t)$ h to put den ≥ 0 $\quad s+a<0$

If $u(-t)$ h to put den ≤ 0

Page
Date / /

* LT of $e^{-at}u(t) = \frac{1}{s+a}$, $Re[s] \geq -a$

* LT of $-e^{-at}u(-t) = \frac{1}{s+a}$, $Re[s] \leq -a$



$Re[s] \geq -a$ $\quad\quad$ $Re[s] \leq -a$

$X(s) = \frac{p}{q}$ $\quad p = 0 \rightarrow$ zero $\quad q = 0 \rightarrow$ pole

Q: $X(s) = \frac{(s+2)(s+3)}{s(s^2+6s+13)}$

* (Put num = 0 for zeroes) $= (s+2)(s+3) = 0$

$s = -2, -3$

* (Put den. = 0 for pole) $= s(s^2+6s+13) = 0$

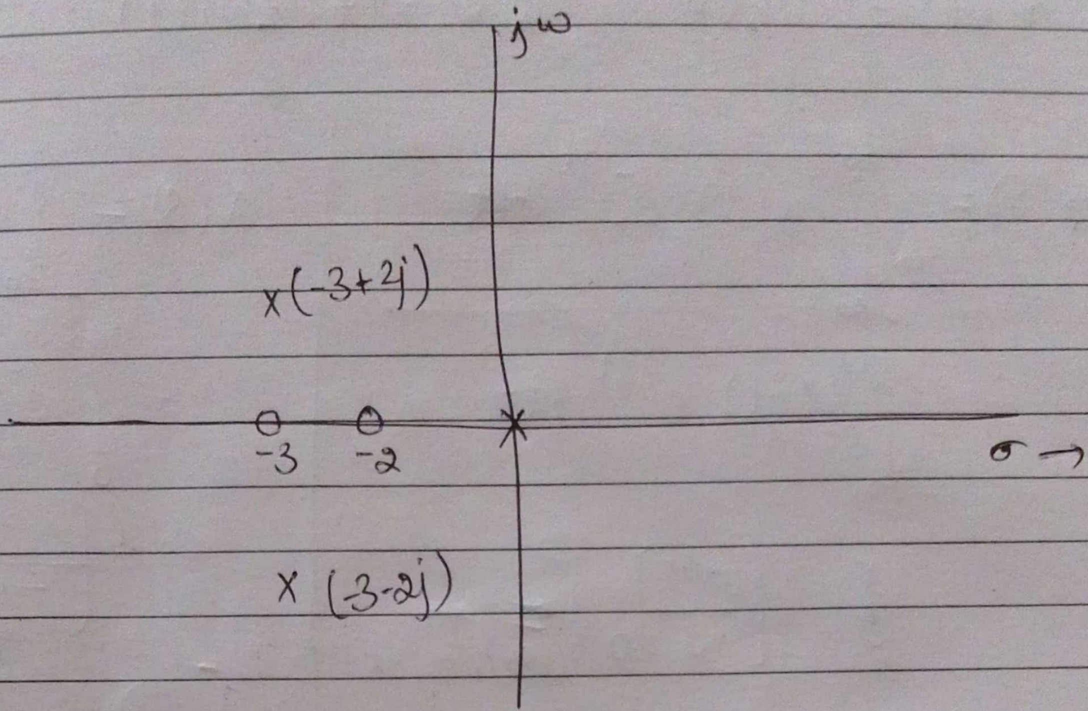
$\frac{-6 \pm \sqrt{36-52}}{2}$

$= \frac{-6 \pm 4i}{2}$

$= -3+2i, -3-2i, 0$

Page :
Date: / /

zero → O pole → X



Q_2: $X(s) = \dfrac{s^2 - 3s + 2}{s(s^2 + 4s + 8)}$

Num ⇒ $s^2 - 2s - s + 2$

$s(s-2) - 1(s-2)$

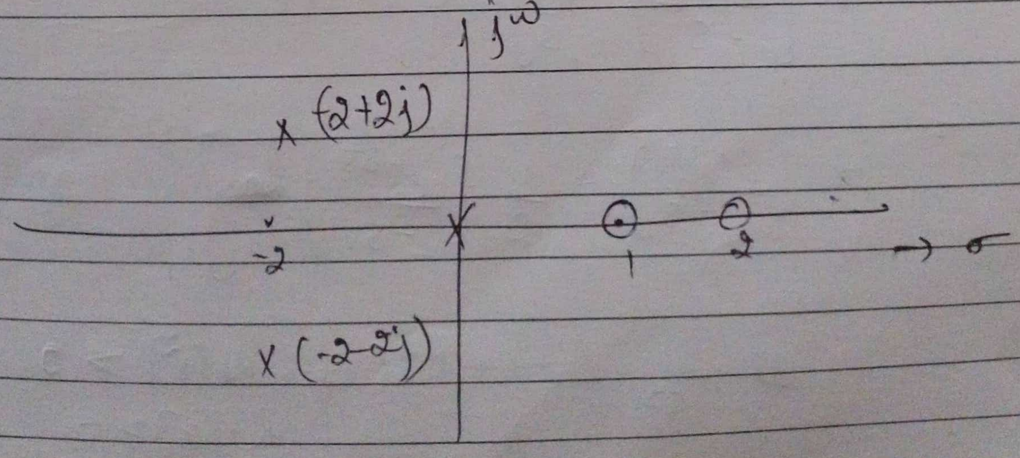
zeros $= 1, 2$

Den ⇒ $s(s^2 + 4s + 8)$

$\dfrac{-4 \pm \sqrt{16-32}}{2} = \dfrac{-4 \pm 4j}{2}$

$= -2 \pm 2j$

Poles → $\boxed{0, -2 \pm 2j}$



Page :
Date: / /

All transform = 1

Q.1 $x(t) = \delta(t)$ Q.2 $x(t) = u(t)$ Q3 $x(t) = r(t)$

$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$

$\int_{-\infty}^{\infty} \delta(t) e^{-st} dt$

$\left[\frac{e^{-st}}{-s}\right]_0 = 1$

ROC = Entire s plane

$X(s) = \int_{-\infty}^{\infty} u(t) e^{-st} dt$

$= \int_0^{\infty} e^{-st} dt$

$\Rightarrow \left[\frac{e^{-st}}{-s}\right]_0^{\infty}$

$\Rightarrow \frac{1}{s}$

ROC :→ $s > 0$

Q.3. $x(t) = r(t)$

$\int_{-\infty}^{\infty} t\, u(t)\, e^{-st} dt$

$\int_0^{\infty} t\, e^{-st} dt$

$= \left[t \int e^{-st} dt - \int \int e^{-st} dt\right]_0^{\infty}$

$= t\left[\frac{e^{-st}}{-s}\right] + \frac{1}{s}\int e^{-st} dt$

$= \left(t\left[\frac{e^{-st}}{-s}\right] + \frac{1}{s}\left[\frac{e^{-st}}{-s}\right]\right]_0^{\infty}$

$= \frac{1}{s^2}$ ROC = $Re[s^2] > 0$

$Re[s] > 0$

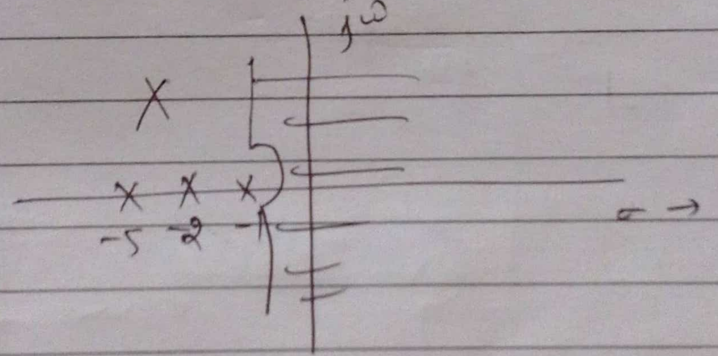
* Right sided signal :- Ek pt. se phle uski saari value 0 ho.
signal for which $x(t) = 0$ before some finite time.

Properties :- (ROC)

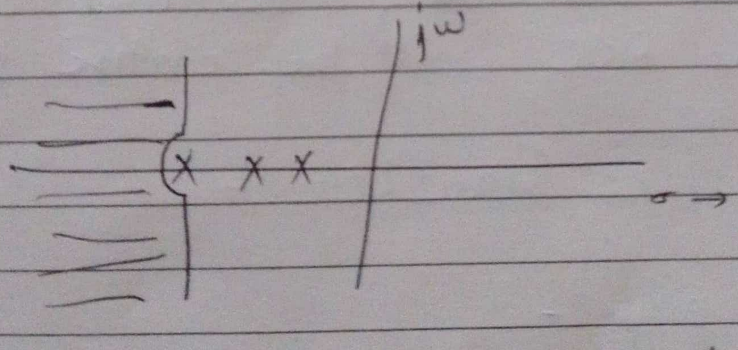
1. ROC never contain any pole.
2. ROC of laplace transform $X(S)$ consists of lines parallel to $j\omega$ axis :: ROC is only dependent upon real part of s.
3. For a right sided signal the ROC will be right to the right most pole.

Ex :- $S = -2, -5, -1$

$$x(t) = u(t)$$



4. for a left sided signal, ROC will be left to the left most pole.



5. If $x(t)$ is a two sided signal then ROC will be b/w the poles but never contains any pole.

$$x(t) = e^{-2|t|}$$

$$X(s) = \int_{-\infty}^{\infty} e^{-2|t|} e^{-st}\, dt$$

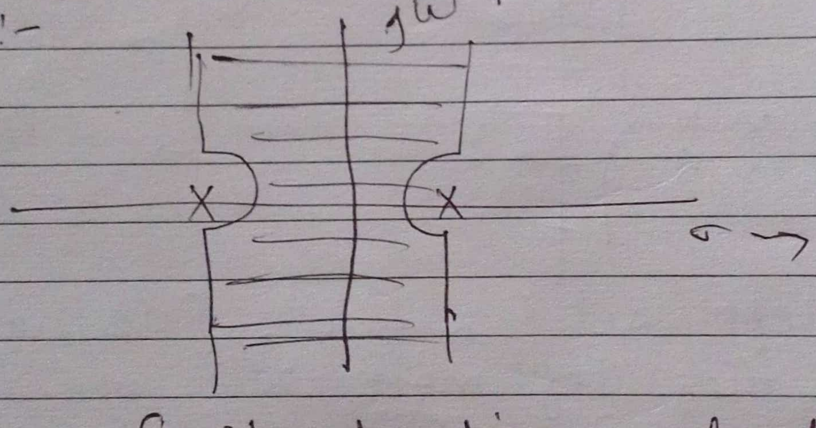
$$x(t) = e^{-2t}, \; t \geq 0$$
$$= e^{2t}, \; t < 0$$

XX $\boxed{L\left[t^n u(t)\right] = \frac{n!}{s^{n+1}}}$

$$\cancel{X(s) = \int_{0}^{\infty} e^{-(2+s)t}\, dt}$$

$$e^{-2t}u(t), \; t \geq 0 \rightarrow \frac{1}{s+2} \quad = s > -2$$
$$e^{2t}u(-t), \; t < 0 \rightarrow \left(\frac{-1}{s-2}\right) \quad s < 2$$

ROC:-



6. For a finite duration, absolutely integrable signal, the ROC will be entire s-plane.

Properties of laplace Transform:-

1. Linearity prop:-

If $x_1(t) \rightarrow X_1(s)$, ROC $= R_1$
$x_2(t) \rightarrow X_2(s)$, ROC $= R_2$

Acc to Linearity prop:-

$$a x_1(t) + b x_2(t) \longrightarrow a X_1(s) + b X_2(s),$$
ROC containing $R_1 \cap R_2$

Page :
Date: / /

Q. $x_1(t) = e^{-2t} u(t)$
$x_2(t) = e^{-4t} u(t)$

$X_1(s) = \frac{1}{s+2}$, $s > -2$

$X_2(s) = \frac{1}{s+4}$, $s > -4$

$X(s) = \frac{1}{s+2} + \frac{1}{s+4}$ $\boxed{s > -2}$

Q. $X_1(s) = \frac{1}{s+1}$ $s > -1$

$X_2(s) = \frac{1}{(s+1)(s+2)}$ $s > -1$

$\frac{1}{s+1} - \frac{1}{(s+1)(s+2)}$

$\frac{\cancel{s+1}}{\cancel{(s+1)}(s+2)} = \left(\frac{1}{s+2}\right)$

ROC:-
$\boxed{s > -2}$

2. Time shifting prop:-
If $x(t) \rightarrow X(s)$, ROC = R
then $x(t \pm t_0) \rightarrow e^{\pm st_0}$
↳ ROC = R

v. Imp
Proof 1→

$L(x(t+t_0)) = X(s) = \int_{-\infty}^{\infty} \underbrace{x(t+t_0)}_{\text{let } t+t_0 = u} e^{-st} dt$

$= \int_{-\infty}^{\infty} x(u) e^{-s(u-t_0)} du$

$$= \int_{-\infty}^{\infty} x(u)\, e^{-su}\, e^{st_0}\, du$$

$$= e^{st_0}\, X(s)$$

Q: $x(t) = u(t)$

$L[x(\underbrace{t-t_0}_{u})] = ?$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-s(u+t_0)}\, du$$

$$\Rightarrow e^{-st_0}\, X(s)$$

04/09/23

SNS

* If laplace transform of $x(t)$ is $X(S)$ then according to time reversal property laplace transform of $x(-t)$ is $X(-S)$

Q: Time reversal property

If $x(t) \longrightarrow X(S)$; ROC = R
then $x(-t) \longrightarrow X(-S)$, ROC = R

* freq. shifting prop.
$e^{\pm S_0 t} x(t) \longrightarrow X(S \mp S_0)$; ROC $= R + R_0(S_0)$

Q: $x(t) = e^{-2t} u(t) \longleftrightarrow \frac{1}{S+2}$

* Time Scaling prop:-

$x(t) \longrightarrow X(S)$, ROC = R

$x(\alpha t) \rightarrow \frac{1}{|\alpha|} X\left(\frac{S}{\alpha}\right)$, ROC = αR

Q: $x(t) = e^{-2t} u(t)$

find L.T. of $x(3t)$

$= \frac{1}{3} \left(\frac{1}{S/3+2}\right)$

Page :
Date : / /

Q: If $x(t) = r(t)$

$x(-2t) \rightarrow ?$ $= \frac{1}{2}\left[\frac{1}{\left(\frac{s^2}{-2}\right)}\right]$

$\Rightarrow \frac{2}{s^2}$

convolution prop:-

convolution prop is ~~one~~ multiplicati in time domain.

If $x_1(t) \rightarrow X_1(s)$; $ROC = R_1$

$x_2(t) \rightarrow X_2(s)$; $ROC = R_2$

then $x_1(t) * x_2(t) \rightarrow X_1(s) \cdot X_2(s)$, $ROC =$ cond. $R_1 \cap R_2$

$$X_1(s) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau\right] e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} x_1(\tau) \left[\int_{-\infty}^{\infty} x_2(t-\tau)\, e^{-st}\, dt\right] d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, e^{-s\tau}\, X_2(s)\, d\tau$$

$$= X_2(s) \cdot X_1(s)$$

Q: If $x_1(t) = e^{-2t} u(t)$ & $x_2(t) = e^{-4t} u(t)$

find the L.T. of $x_1(t) * x_2(t)$.

$\Rightarrow \frac{1}{s+2} \times \frac{1}{s+4}$ (If graphical method mention then use that, otherwise this)

convert it into t form.

$$\frac{1}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4}$$

$$= Ae^{-2t}u(t) + Be^{-4t}u(t)$$

* **Multiplication Prop:-**

If $x_1(t) \to X_1(s)$; $ROC = R_1$

$x_2(t) \to X_2(s)$; $ROC = R_2$

then $x_1(t) \cdot x_2(t) \to \frac{1}{2\pi}\left[X_1(s) * X_2(s)\right]$

[Unit 1 ([illegible]) → friday Test]

* Differentiation in Time Domain:-

If $x(t) \to X(s)$; $ROC = R$

Proof:- $\frac{d}{dt}x(t) = s(X(s))$, ROC = containing R

$$\frac{d^2}{dt^2}x(t) = s^2 X(s) - s\,x(0) - x'(0)$$

$$\frac{d^3}{dt^3}x(t) = s^3 X(s) - s^2 x(0) - s\,X(0) - x'(0)$$

* Multiplication by 't' in time domain (or) Differentiation in S-domain.

if $x(t) \to X(s)$

then $t\,x(t) \to -\frac{d}{ds}X(s)$