Assignment-2

Student ID :- 202151120 Name :- Priyansh Vaishnav

DELUXE PAGE No.: DATE:

(1) $f(t) = 2t+6$

$$= \int_0^\infty (2t+6)e^{-st}\,dt$$

$$= 2\int_0^\infty t.e^{-st}\,dt + 6\int_0^\infty e^{-st}\,dt$$

$$\mathcal{L}(2t+6) = 2\left[\frac{1}{s^2}\right] + 6\left[\frac{1}{s}\right] = \frac{2+6s}{s^2} = \frac{2(1+3s)}{s^2}$$

(2) $a+bt+ct^2$

$$= \int_0^\infty (a+bt+ct^2).e^{-st}\,dt$$

$$= a\int_0^\infty a e^{-st}\,dt + b\int_0^\infty te^{-st} + c\int_0^\infty t^2e^{-st}\,dt$$

$$= \frac{a}{s} + b\left[\frac{1}{s^2}\right] + c\left[\frac{4}{s^3}\right]$$

$$\mathcal{L}(a+bt+ct^2) = \frac{as^2+bs+4c}{s^3}$$

(3) $\sin\pi t$

$$= \int_0^\infty \sin\pi t.\ e^{-st}\,dt$$

$$= \left[-\frac{e^{-st}(s\sin(\pi t)+\pi\cos\pi t)}{s^2+\pi^2}\right]_0^\infty$$

$$\mathcal{L}(\sin\pi t) = \frac{\pi}{s^2+\pi^2}$$

(4) $\cos^2\omega t$

$$= \int_0^\infty \cos^2\omega t\ .\ e^{-st}\,dt$$

$$= \left[\frac{e^{-st}(s\cos(\omega t)\,(2\omega\sin(\omega t) - s\cos(\omega t)) - 2\omega^2}{s.(s^2+4\omega^2)}\right]_0^\infty$$

$$\mathcal{L}(\cos^2\omega t) = \frac{s}{s^2+4\omega^2} + \frac{2\omega^2}{s.(s^2+4\omega^2)}$$

(5) e^{a-bt}

$$= \int_0^\infty e^{a-bt} \cdot e^{-st}\, dt$$

$$= \int_0^\infty e^{-st-bt+a}\, dt$$

$$= \left[-\frac{e^{-st-bt+a}}{s+b} \right]_0^\infty$$

$$\mathcal{L}(e^{a-bt}) = \frac{e^{a}}{s+b}$$

(6) $\mathcal{L}(\cancel{\sin 2t \cdot \cos 2t})$

(6) $\mathcal{L}(e^{t}\cosh 3t)$

$$= \int_0^\infty e^{t}\cosh(3t) \cdot e^{-st}\, dt$$

Putting $\cosh 3t = \frac{e^{3t}+e^{-3t}}{2}$

$$= \left[-\frac{\left((s+2)e^{6t} + s-4\right) e^{-(s+2)t}}{2(s-4)(s+2)} \right]_0^\infty$$

Assuming $\boxed{s \neq 1}$

$s-7 \neq -3$

$\boxed{s \neq 4}$

$s-1 = -3$

$\boxed{s \neq -2}$

$$= \frac{1}{2}\left[\frac{1}{s-4} + \frac{1}{s+2} \right]$$

$$= \frac{(s-1)}{(s-4)(s+2)}$$

(7) $\mathcal{L}(\sin(\omega t + \delta))$

$\sin(\omega t + \delta) = \sin \omega t \cos \delta + \cos \omega t \sin \delta$

$\mathcal{L}(\sin(\omega t + \delta)) = \cos\delta\, \mathcal{L}(\sin \omega t) + \sin\delta\, \mathcal{L}(\cos \omega t)$

$$= \cos\delta \left(\frac{\omega}{s^2+\omega^2} \right) + \sin\delta \left(\frac{s}{s^2+\omega^2} \right)$$

$$= \frac{s \sin\delta + \omega \cos\delta}{s^2+\omega^2}$$

(8) $\mathcal{L}(\sin 2t \cos 2t)$

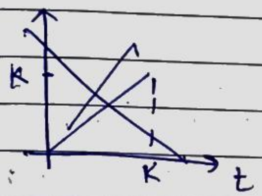
$$\mathcal{L}\left(\frac{\sin 4t}{2}\right) = \int \frac{e^{-st}\sin 4t}{2}\,dt$$

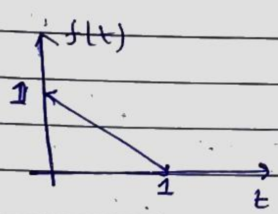
$$= \frac{1}{2}\left[\sin 4t \int e^{-st}\,dt - \int \left(4(-\cos 4t)\cdot \int e^{-st}\,dt\right)dt\right]$$

$$= \frac{1}{2}\left[-\frac{e^{-st}\sin 4t}{s} - \int \frac{4\cos 4t\, e^{-st}}{s}\,dt\right]_0^{\infty}$$

$$= \frac{2}{s^2+16}$$

(9)





$f(t) = -(t-1)$

$0 \le t \le 1$

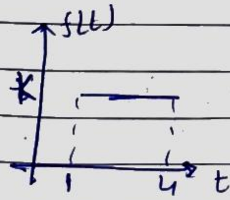
$$\mathcal{L}(f(t)) = \mathcal{L}(-(t-1)) = \int_0^1 -(t-1)e^{-st}\,dt$$

$$= \left[\frac{(s(t-1)+1)e^{-st}}{s^2}\right]_0^1 \Rightarrow -\left[\frac{(s(0-1)+1)}{s^2}\right]$$

$$\frac{-s+1}{s^2}$$

$$= \frac{s-1}{s^2}$$

(10)



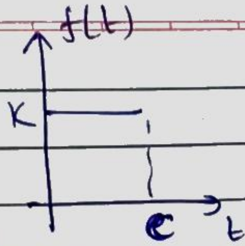
$1 \le t \le 4$

$$\mathcal{L}(f(t)) = \int_1^4 K e^{-st}\,dt = \frac{K}{s}\left[e^{-4s} - e^{-s}\right]$$

202151120

DELUXE
PAGE NO.:
DATE:

(11)

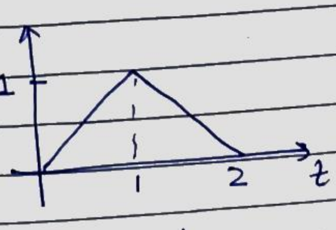


$$f(t) = K \qquad 0 \le t \le c$$

$$\mathcal{L}(f(t)) = \int_0^c K e^{-st}\, dt$$

$$\mathcal{L}(f(t)) = -\frac{K}{s}\left[e^{-sc} - 1\right]$$

(12)



$$f(t) = \begin{cases} t & 0 \le t \le 1 \\ -(t-2) & 1 < t \le 2 \end{cases}$$

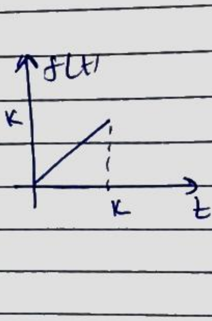
$$\mathcal{L}(f(t)) = \int_0^1 t e^{-st}\, dt + \int_1^2 -(t-2) e^{-st}\, dt$$

$$\Rightarrow \left\{ -\frac{(st+1)e^{-st}}{s^2} \right]_0^1 \cancel{\left[\frac{(s(t+1)+1)e^{-st}}{s^2}\right]_0^1} + \left\{ \frac{(s(t-2)+1)e^{-st}}{s^2} \right]_1^2$$

$$\Rightarrow \frac{e^{-s}(e^{s} - s - 1)}{s^2} + \frac{e^{-2s}((s-1)e^{s} + 1)}{s^2}$$

$$\mathcal{L}(f(t)) = \frac{e^{-s}(e^{s} - s - 1) + e^{-2s}((s-1)e^{s} + 1)}{s^2}$$

(13)



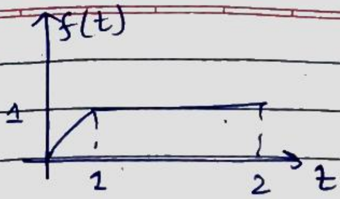
$$f(t) = t \qquad 0 \le t \le K$$

$$\mathcal{L}(f(t)) = \int_0^K t e^{-st}\, dt$$

$$= \cancel{\left[\frac{1}{s^2} - \frac{(s+1)e^{-s}}{s^2}\right]_0^K}$$

$$= \left[-\frac{(st+1)e^{-st}}{s^2}\right]_0^K = \frac{1}{s^2} - \frac{(Ks+1)e^{-Ks}}{s^2}$$

(14)



$$f(t) = \begin{cases} t & 0 \le t \le 1 \\ 1 & 1 \le t \le 2 \end{cases}$$

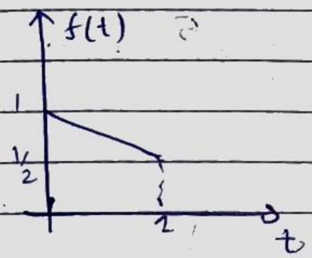
$$\mathcal{L}(f(t)) = \int_0^1 t e^{-st}\,dt + \int_1^2 e^{-st}\,dt$$

$$= \frac{1}{s^2} - \frac{e^{-s}(s+1)}{s^2} + \frac{e^{-s}}{s} - \frac{e^{-2s}}{s}$$

$$= \frac{1 - se^{-s} - e^{-s} + se^{-s} - se^{-2s}}{s^2}$$

$$= \frac{1 - e^{-s} - se^{-2s}}{s^2}$$

(15)

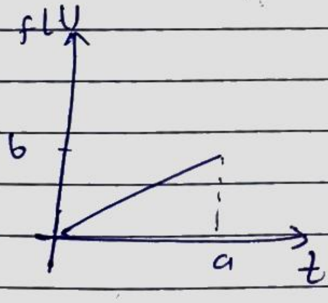


$$f(t) = \frac{t}{2} \qquad 0 \le t \le 1$$

$$\mathcal{L}(f(t)) = \int_0^1 \frac{t}{2} e^{-st}\,dt$$

$$= \frac{1}{2}\left[\frac{1 - se^{-s} - e^{-s}}{s^2}\right]$$

(16)



$$f(t) = \frac{bt}{a} \qquad 0 \le t \le a$$

$$\mathcal{L}(f(t)) = \int_0^a \frac{bt}{a} e^{-st}\,dt$$

$$= \frac{b}{a}\left[\frac{1 - e^{-as}\cdot as - e^{-as}}{s^2}\right]$$

20215120

DELUXE
PAGE NO.:
DATE:

(17) $\mathcal{L}^{-1}\left(\frac{0.1s + 0.9}{s^2 + 3.24}\right) = \mathcal{L}^{-1}\left(\frac{s+9}{10(s^2+3.24)}\right)$

$= \frac{1}{10}\mathcal{L}^{-1}\left(\frac{s}{s^2+3.24}\right) + \frac{9}{10}\mathcal{L}^{-1}\left(\frac{1}{s^2+3.24}\right)$

$= \frac{1}{10}\cos\frac{9t}{5} + \frac{9}{10}\times\frac{5}{9}\sin\left(\frac{9t}{5}\right)$

$= \frac{1}{10}\cos\frac{9t}{5} + \frac{1}{2}\sin\frac{9t}{5}$

(18) $\mathcal{L}^{-1}\left(\frac{5s}{s^2-25}\right)$ $\quad \frac{5s}{(s+5)(s+5)} = \frac{A}{s-5} + \frac{B}{s+5}$

$A = 5/2 = B$

$\frac{5}{2}\mathcal{L}^{-1}\left(\frac{1}{s-5}\right) + \frac{5}{2}\mathcal{L}^{-1}\left(\frac{1}{s+5}\right)$

$= \frac{5}{2}e^{-5t} + \frac{5}{2}e^{5t} = \frac{5}{2}\left(e^{5t} + e^{-5t}\right)$

(19) $\mathcal{L}^{-1}\left(\frac{-(s+10)}{s^2-s-2}\right) = \mathcal{L}^{-1}\left(\frac{-(s+10)}{(s-2)(s+1)}\right)$

$\frac{-(s+10)}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1}$

$A = -4, \quad B = 3$

$= -4\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) + 3\mathcal{L}^{-1}\left(\frac{1}{s+1}\right)$

$= -4e^{2t} + 3e^{-t}$

(20) $\mathcal{L}^{-1}\left(\frac{s-4}{(s+2)(s-2)}\right) = \mathcal{L}^{-1}\left(\frac{-1}{2(s-2)} + \frac{3}{2(s+2)}\right)$

$= \frac{-1}{2}\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) + \frac{3}{2}\mathcal{L}^{-1}\left(\frac{1}{s+2}\right)$

$= \frac{-e^{2t}}{2} + \frac{3}{2}e^{-2t}$

20215112.0

DELUXE PAGE NO.: DATE:

(21) $\mathcal{L}^{-1}\left(\frac{2.4}{s^4} - \frac{228}{s^6}\right)$

$\mathcal{L}^{-1}\left(\frac{2.4}{s^4}\right) - 228\mathcal{L}^{-1}\left(\frac{1}{s^6}\right)$

$= 2.4\frac{t^3}{3!} - 228\frac{t^5}{5!} = \frac{2t^3}{5} - \frac{57}{30}t^5$

(22) $\mathcal{L}^{-1}\left(\frac{60+6s^2+s^4}{s^7}\right)$

$= 60\mathcal{L}^{-1}\left(\frac{1}{s^7}\right) + 6\mathcal{L}^{-1}\left(\frac{1}{s^5}\right) + \mathcal{L}^{-1}\left(\frac{1}{s^3}\right)$

$= 60\frac{t^6}{6!} + 6\frac{t^4}{4!} + \frac{t^2}{2!}$

$= \frac{t^6}{12} + \frac{t^4}{4} + \frac{t^2}{2}$

(23) $\mathcal{L}^{-1}\left(\frac{s}{L^2s^2+n^2\pi^2}\right) = \frac{1}{L^2}\mathcal{L}^{-1}\left(\frac{s}{s^2+\left(\frac{n\pi}{L}\right)^2}\right)$

$= \frac{1}{L^2}\frac{\sin\left(\frac{\pi n}{L}t\right)}{\frac{\pi n}{L}}$

$= \frac{\sin\left(\frac{\pi n}{L}t\right)}{\pi n L}$

(24) $\mathcal{L}^{-1}\left(\frac{1-7s}{(s-3)(s-1)(s+2)}\right) = \mathcal{L}^{-1}\left(\frac{-2}{s-3} + \frac{1}{s-1} + \frac{1}{s+2}\right)$

$= -2e^{3t} + e^{t} + e^{-2t}$

202151120

DELUXE
PAGE NO.:
DATE:

(25) $\mathcal{L}^{-1}\left(\sum_{k=1}^{5} \frac{a_k}{s+k^2}\right)$

$= \mathcal{L}^{-1}\left(\frac{a_1}{s+1} + \frac{a_2}{s+4} + \frac{a_3}{s+9} + \frac{a_4}{s+16} + \frac{a_5}{s+25}\right)$

$= a_1 e^{-t} + a_2 e^{-4t} + a_3 e^{-9t} + a_4 e^{-16t} + a_5 e^{-25t}$

$\mathcal{L}^{-1}\left(\sum_{k=1}^{5} \frac{a_k}{s+k^2}\right) \Rightarrow \sum_{k=1}^{5} a_k e^{-k^2 t}$

(26) $\mathcal{L}^{-1}\left(\frac{s^4+6s-18}{s^5-3s^4}\right) = \mathcal{L}^{-1}\left(\frac{s^4+6s-18}{s^4(s-3)}\right)$

$= \mathcal{L}^{-1}\left(\frac{1}{s-3}\right) + \mathcal{L}^{-1}\left(\frac{6s-18}{s^4(s-3)}\right)$

$= \mathcal{L}^{-1}\left(\frac{1}{s-3}\right) + 6\,\mathcal{L}^{-1}\left(\frac{1}{s^4}\right)$

$= e^{3t} + \frac{6 \cdot t^3}{3!} = t^3 + e^{3t}$

(27) $\mathcal{L}^{-1}\left(\frac{1}{(s+\sqrt{2})(s-\sqrt{3})}\right) =$

$= \frac{-1}{(\sqrt{2}+\sqrt{3})}\mathcal{L}^{-1}\left(\frac{1}{s+\sqrt{2}}\right) + \frac{1}{(\sqrt{2}+\sqrt{3})}\mathcal{L}^{-1}\left(\frac{1}{s-\sqrt{3}}\right)$

$= \frac{-e^{-\sqrt{2}t} + e^{\sqrt{3}t}}{\sqrt{2}+\sqrt{3}}$

$= \frac{1}{\sqrt{2}+\sqrt{3}}\left(e^{\sqrt{3}t} - e^{-\sqrt{2}t}\right)$

202151120

DELUXE
PAGE NO. :
DATE:

(28) $$L^{-1}\left(\frac{2s^3}{s^4-1}\right) = L^{-1}\underbrace{\left(\frac{s}{s^2+1} + \frac{1}{2(s+1)} + \frac{1}{2(s-1)}\right)}_{\text{by partial fraction}}$$

$$= L^{-1}\left(\frac{s}{s^2+1}\right) + \frac{1}{2}L^{-1}\left(\frac{1}{s+1}\right) + \frac{1}{2}L^{-1}\left(\frac{1}{s-1}\right)$$

$$= \cos t + \frac{1}{2}e^{-t} + \frac{1}{2}e^{t}$$

$$= \cos t + \frac{1}{2}\left[e^{t} + e^{-t}\right)$$

20215|120

(29) $f(t) = t^2 e^{-3t}$.

if $L(f(t)) = F(s)$

$L(e^{-at} \cdot f(t)) = F(s+a)$

$\cancel{f(t) = t^2}$ We know

$$L(t^n) = \frac{n!}{s^{n+1}}$$

$n = 2$ & $a = -3$

$$L(t^2 e^{-3t}) = \cancel{\frac{2!}{s^3}} \quad \frac{2!}{(s+3)^3}$$

(30) $f(t) = e^{-\alpha t} \cos\beta t$

$$L(\cos\beta t) = \frac{s}{s^2+\beta^2}$$

$$L(e^{-\alpha t} \cdot f(t)) = F(s+a)$$

$$L(e^{-at} \cos\beta t) = \frac{(s+\alpha)}{(s+\alpha)^2+\beta 2}$$

(31) fxn :- $5e^{2t} \sinh 2t$

$$L(5e^{2t} \sinh(2t)) = (-1)^1 \frac{d}{ds} \underbrace{L(e^{-2t} \sin h(2t))}_{= 1/s+2}$$

$$\frac{d}{ds}\left(\frac{1}{s+2}\right) = -\frac{1}{(s+2)^2}$$

$$L(5e^{2t} \sinh 2t) = \frac{-1}{(s+2)^2}$$

(32) $2e^{-t} \cos^2(t/2)$

$$L[2e^{-t} \cos^2(t/2)] = (-1)^1 \frac{d}{ds} L(\cos^2(t/2) \cdot e^{-t})$$

$$L(\cos^2(t/2) \cdot e^{-t}) = \frac{s+1}{(s+1)^2 + (1/2)^2}$$

DELUXE
PAGE NO. :
DATE:

202151120

$$\frac{d}{ds}\left(\frac{s+1}{(s+1)^2+(\frac{1}{2})^2}\right) = \frac{-2(s+1)}{\left((s+1)^2+(\frac{1}{2})^2\right)^2}$$

(33) $L(\sinh t \cos t) = L(\sinh t)$ $\quad a=0$, $f(t) = \sinh t$

$L(\sinh t) = 1/2$

$L(\sinh t . \cos t) = 1/s$

(34) $L((t+1)^2 . e^t) = (-1)^2 \frac{d^2}{ds^2} L(e^t)$

$= (-1)^2 \frac{d^2}{ds^2}\left(\frac{1}{s+1}\right) = \frac{2}{(s+1)^3}$

$L((t+1)^2 . e^t) = \frac{2}{(s+1)^3}$

(35) $L^{-1}\left(\frac{1}{(s+1)^2}\right)$

$L^{-1}(F(s+a)) = e^{-at} f(t)$

where $L^{-1}(F(s)) = f(t)$

in $L^{-1}\left(\frac{1}{(s+1)^2}\right)$ $\quad a=1$, $F(s) = \frac{1}{s^2}$

$\frac{1}{s+1} = \frac{1}{s-3}$

$L^{-1}\left(\frac{1}{s^2}\right) = f(t) = t$

$L^{-1}\left(\frac{1}{(s+1)^2}\right) = e^{-t} . t$

202151120

(36) $\mathcal{L}^{-1}\left(\cancel{e^{-a}}\frac{12}{(s-3)^4}\right)$

$a = -3$

$F(s) = \frac{12}{s^4}$

$\mathcal{L}^{-1}\left(\frac{12}{s^4}\right) = f(t)$

$= 12\,\mathcal{L}^{-1}\left(\frac{1}{s^4}\right) = \frac{t^3}{3!}$

$\mathcal{L}^{-1}\left(\frac{12}{(s-3)^4}\right) = \frac{t^3}{3!}e^{3t}$

(37) $\mathcal{L}^{-1}\left(\frac{3}{s^2+6s+18}\right) = \mathcal{L}^{-1}\left(\frac{3}{(s+3)^2+9}\right)$ $\boxed{a = 3}$

$\underbrace{F(s) = \frac{3}{(s+3)^2+9} = \frac{A}{s+3} + \frac{B(s+3)}{(s+3)^2+9}}_{\text{partial fraction}}$

~~30s~~

$\underbrace{3 = A\left((s+3)^2+9\right) + B(s+3)}$

$\boxed{A=3}$ $\boxed{B=0}$

$F(s) = \frac{3}{s+3}$

$\mathcal{L}^{-1}(F(s)) = f(t) = \mathcal{L}^{-1}\left(\frac{3}{s+3}\right) = 3e^{-3t}$

$\mathcal{L}^{-1}\left(\frac{3}{s^2+6s+18}\right) = 3e^{-3t}$

(5)

202151120

(4)

DELUXE
PAGE NO.:
DATE:

$\cancel{L^{-1}\left(\frac{4}{s^2-2s-3}\right) = e^{3t} \cdot L^{-1}\left(\frac{1}{s+1}\right)}$

(38) $L^{-1}\left(\frac{4}{s^2-2s-3}\right) = L^{-1}\left(\frac{4}{(s-3)(s+1)}\right) = L^{-1}\left(\frac{1}{s-3} - \frac{1}{s+1}\right)$

$= L^{-1}\left(\frac{1}{s-3}\right) - L^{-1}\left(\frac{1}{s+1}\right)$

$a_1 = -3$ $\qquad$ $a_2 = 1$

$= e^{-3t} \cdot t - e^{t} \cdot t \qquad = t\left(\frac{1}{e^{3t}} - e^{t}\right)$

$= \cancel{-t\left(\frac{1}{e^{t}} + \frac{1}{e^{3t}}\right)} \qquad = t \cdot e^{-3t}\left(1 - e^{4t}\right)$

(39) $L^{-1}\left(\frac{s}{(s+1/2)^2+1}\right) \qquad a = -1/2$

$= e^{-t/2} L^{-1}\left(\frac{s}{s^2+1^2}\right) = e^{-t/2} \cos t$

(40) $L^{-1}\left(\frac{2}{s^2+s+1/2}\right) = L^{-1}\left(\frac{2}{s^2+s+1/4+1/4}\right) = L^{-1}\left(\frac{2}{(s+1/2)^2+}\right.$

$a = -1/2$

$= e^{-t/2} L^{-1}\left(\frac{2}{s^2+1/2^2}\right)$

$= 2 \cdot e^{-t/2} \cos \frac{t}{2}$

(2) $\mathcal{L}(t\,u(t-1)) = \mathcal{L}((t-1+1)\,u(t-1))$

$= \mathcal{L}((t-1)u(t-1)) + \mathcal{L}\,u(t-1)$

$= \dfrac{e^{-s}}{s^2} + \dfrac{e^{-s}}{s}$

$= \dfrac{e^{-s} + s e^{-s}}{s^2} = \dfrac{e^{-s}(s+1)}{s^2}$

(3) Using second shift theorem

$\mathcal{L}((t-1)\,u(t-1))$

unit step fn
$\begin{cases} 0 & t<1 \\ t-1 & t\geq 1 \end{cases}$

$\mathcal{L}(f(t-a)\,u(t-a)) = e^{-as}F(s)$

$\mathcal{L}((t-1)u(t-1)) = \mathcal{L}(t-1).\,e^{-s}$

$= \dfrac{e^{-s}}{s^2}$

(4) $\mathcal{L}((t-1)^2 u(t-1))$

$\mathcal{L}[f(t).\,u(t-c)] = e^{-cs}\mathcal{L}[f(t+c)]$

$f(t) = (t-1)^2 \;,\; c=1$

$\mathcal{L}\{[(t+1)-1]^2\} = \dfrac{2}{s^3}$

$\mathcal{L}((t-1)^2 u(t-1)) = \dfrac{e^{-s}.2}{s^3} = \dfrac{2e^{-s}}{s^3}$

202151120

(5) $\mathcal{L}(t^2 u(t-1))$

$\therefore \mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\left(\mathcal{L}(f(t))\right)$

$f(t) = u(t-1) \quad , \quad K=2$

$\mathcal{L}(t^2 u(t-1)) = (-1)^2 \frac{d^2}{ds^2} \mathcal{L}(u(t-1))$

$\Rightarrow \frac{d^2}{ds^2}\left(\frac{e^{-s}}{s}\right)$

$= \frac{e^{-s}(s^2+2(s+1))}{s^3}$

(6) $\mathcal{L}(e^{-2t} u(t-3))$

$\cancel{f(t) = u(t-3), \quad a=-2}$

$\cancel{= \mathcal{L}(u(t-3)) \, (s+2)}$

$\cancel{= \frac{e^{-3s}}{s}(s+2)}$

$\mathcal{L}(f(t)) = F(s) \longrightarrow \mathcal{L}(e^{at} f(t)) = F(s-a)$

in $e^{-2t} u(t-3) \Rightarrow f(t) = u(t-3), \quad a=-2$

$= \mathcal{L}(H(t-3)) (s+2)$

$= \frac{e^{-3(s+2)}}{(s+2)}$

20215 1120

(7) $\mathcal{L}\left(4u(t-\pi)\cos t\right) = 4\mathcal{L}\left(\cos t\, u(t-\pi)\right)$

$$= 4\left\{-\frac{e^{-\pi s}s}{s^2+1}\right\}$$

$$= \frac{-4s\,e^{-\pi s}}{s^2+1}$$

(8) $t^2 \quad (0<t<1)$

$\mathcal{L}(t^2) = \dfrac{2}{s^3}$

(9) $\sin\omega t \quad 0<t<\pi/\omega$

$\mathcal{L}(\sin\omega t) = \dfrac{\omega}{s^2+\omega^2}$

(10) $1-e^{-t} \quad (0<t<2)$

$\mathcal{L}(1-e^{-t}) = \mathcal{L}(1) - \mathcal{L}(e^{-t})$

$= \dfrac{1}{s} - \dfrac{1}{s+1}$

(12) $\sin t \quad 2\pi<t<4\pi$

$\mathcal{L}(\sin t) = \dfrac{1}{s^2+1}$

(11) $\mathcal{L}(e^t) \quad 0<t\leq 1$

$\mathcal{L}(e^t) = \dfrac{1}{s-a}$

(13) $10\cos\pi t \quad 1<t<2$

$\mathcal{L}(\cos\pi t) = \dfrac{s}{s^2+\pi^2}$

$\mathcal{L}(10\cos\pi t) = \dfrac{10s}{s^2+\pi^2}$

20215/20

DELUXE
PAGE NO. :
DATE:

(14) $L^{-1}\left(\frac{4(e^{-2s}-2e^{-5s})}{s}\right)$

$= 4\left(\frac{1}{s+2} - \frac{2}{s+5}\right)$

$= \frac{4}{s+2} - \frac{8}{s+5}$

$L^{-1}\left(\frac{4}{s+2}\right) = 4e^{-2t} - 8e^{-5t}$

(15) $L^{-1}\left(\frac{e^{-3s}}{s^3}\right)$

$F(s) = \frac{e^{-3s}}{s^3}$

$F'(s) = \frac{d}{ds}\left(\frac{e^{-3s}}{s^3}\right)$

$F''(s) = \frac{9e^{-3s}}{s^3} + \frac{18e^{-3s}}{s^4} + \frac{9e^{-3s}}{s^5}$

$F'''(s) = \frac{d}{ds}(F''(s))$

$= \frac{-27}{s^3}e^{-3s} - \frac{162e^{-3s}}{s^4} - \frac{405e^{-3s}}{s^5} + \frac{243e^{-3s}}{s^6}$

$L^{-1}\left(\frac{e^{-3s}}{s^3}\right) = 27u(t-3)t^2 + 162u(t-3)t + 405u(t-3)$
$+ 243u(t-3)t$

(16) $\frac{e^{-3s}}{(s-1)^3} \Rightarrow \frac{e^{-3(s-1)}}{s^3}$

$F'(s) = \frac{d}{ds}\left(\frac{e^{-3(s-1)}}{s^3}\right) = \frac{-3e^{-3(s-1)}}{s^3} - \frac{3e^{-3(s-1)}}{s^4}$

20215I120

DELUXE
PAGE NO.:
DATE:

$$F''(s) = \frac{d}{ds}(F'(s)) = \frac{9e^{-3(s-1)}}{s^3} + \frac{18e^{-3(s-1)}}{s^4} + \frac{9e^{-3(s-1)}}{s^5}$$

$$F'''(s) = \frac{d}{ds}(F''(s)) = \frac{-27e^{-3(s-1)}}{s^3} - \frac{162e^{-3(s-1)}}{s^4}$$
$$- \frac{405e^{-3(s-1)}}{s^5} - \frac{243e^{-3(s-1)}}{s^6}$$

$$\mathcal{L}(f(t)) = 254\,u(t-1)\,t^2e^{t-1} + 162\,u(t-1)\,te^{t-1} + 405\,u(t-1)\,e^{t-1}$$
$$+ 243\,u(t-1)\,t \cdot e^{t-1}$$

(17) $$\mathcal{L}^{-1}\left(\frac{3(1-e^{-\pi s})}{s^2+9}\right) = \frac{3}{s^2+9} - \frac{3e^{-\pi s}}{s^2+9}$$

$$\mathcal{L}\left(\frac{3}{s^2+9}\right) = 3\sin 3t \quad , \quad \mathcal{L}\left(\frac{\cancel{3}e^{-\pi s}}{s^2+9}\right) = \sin 3t$$

$$\mathcal{L}^{-1}\left(\frac{3(1-e^{-\pi s})}{s^2+9}\right) = 3\sin 3t - 3(\sin 3t)$$
$$= 0$$

(18) $$\mathcal{L}^{-1}\left(\frac{e^{-2\pi s}}{s^2+2s+2}\right)$$

$$\mathcal{L}^{-1}(e^{-2\pi s}) \rightarrow \delta(t-2\pi)$$

$$\delta(t-2\pi) \cdot e^{-t}\cos t = \mathcal{L}^{-1}\left(\frac{e^{-2\pi s}}{s^2+2s+2}\right)$$

$$\mathcal{L}^{-1}\left(\frac{e^{-2\pi s}}{s^2+2s+2}\right) = e^{-t}\cos(t-2\pi)$$

$$\mathcal{L}^{-1}\left(e^{-t}\cos(t-2\pi)\right) = \frac{s+1}{(s+1)^2+1}$$

20215120

DELUXE
PAGE NO. :
DATE:

(19) $\mathcal{L}^{-1}\left(\frac{se^{-2s}}{s^2+\pi^2}\right)$ $\frac{se^{-2s}}{s^2+\pi^2} = \frac{s}{s^2+\pi^2} \cdot e^{-2s}$

$\delta(t-2)\cdot\cos(\pi t) = \mathcal{L}^{-1}\left(\frac{se^{-2s}}{s^2+\pi^2}\right)$

$\mathcal{L}^{-1}\left(\frac{se^{-2s}}{s^2+\pi^2}\right) = \delta(t-2)\cos\pi t$

$= e^{-2s}\,\frac{s}{s^2+\pi^2}$

$= e^{-(t-1)}\cdot u(t-1)$

20215 1120

(20) $4y'' - 4y' + 37y = 0 \qquad y(0) = 3, \quad y'(0) = 10.5$

$\mathcal{L}(4y'' - 4y' + 37y) = \mathcal{L}(0)$

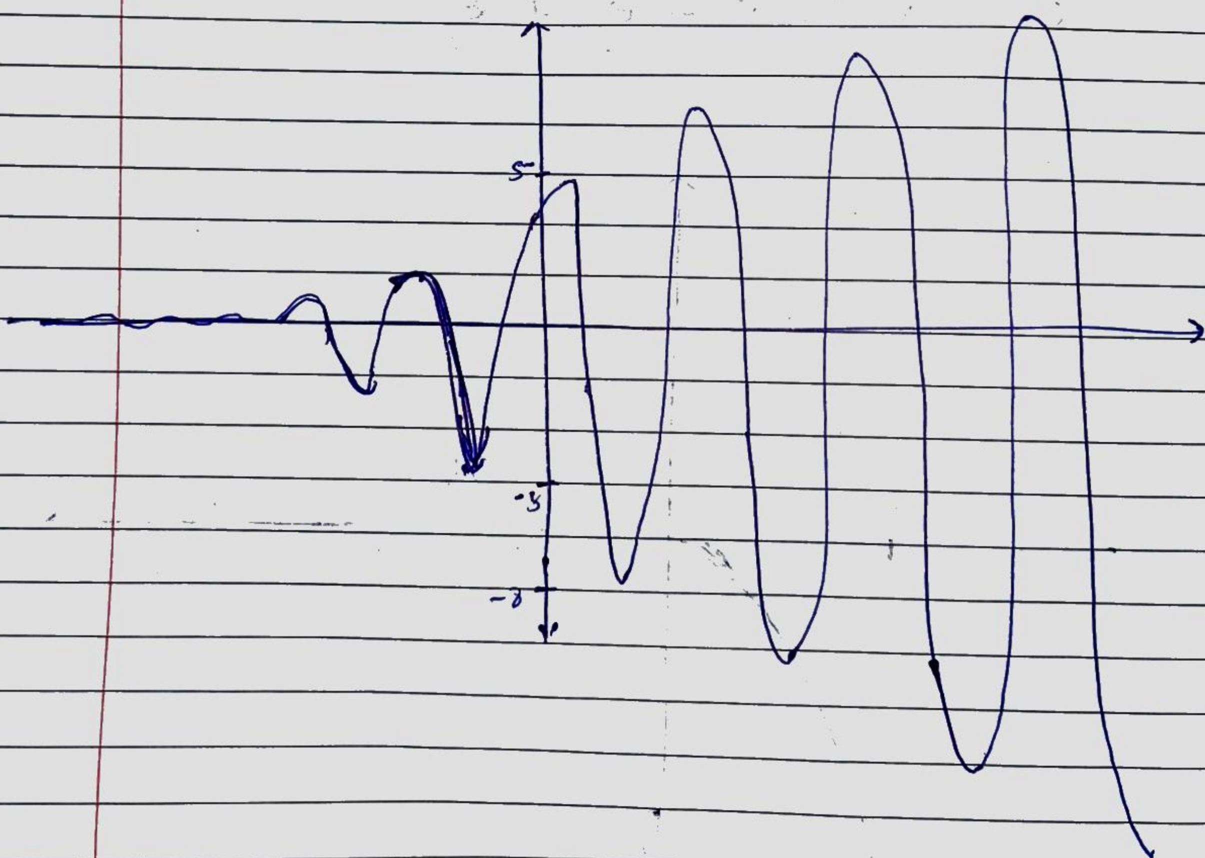
$4\mathcal{L}(4y'') - 4\mathcal{L}(y') + 37\mathcal{L}(y) = 0$

$$\left.\begin{aligned} \mathcal{L}(y') &= s\mathcal{L}(y) - y(0) \\ \mathcal{L}(y'') &= s^2\mathcal{L}(y) - y'(0) - sy(0) \end{aligned}\right\}$$

$4s^2\mathcal{L}(y) - 42 - 12s - 4s\mathcal{L}(y) + 12 + 37\mathcal{L}(y) = 0$

$$\mathcal{L}(y) = \frac{12s + 30}{4s^2 - 4s + 37}$$

$$y = 3e^{t/2}(\sin 3t + \cos 3t)$$



202151120

(21) $y'' + 6y' + 8y = e^{-3t} - e^{-5t}$ $\quad y(0) = 0$

$y'(0) = 0$

$\mathcal{L}(y'' + 6y' + 8y) = \mathcal{L}(e^{-3t} - e^{-5t})$

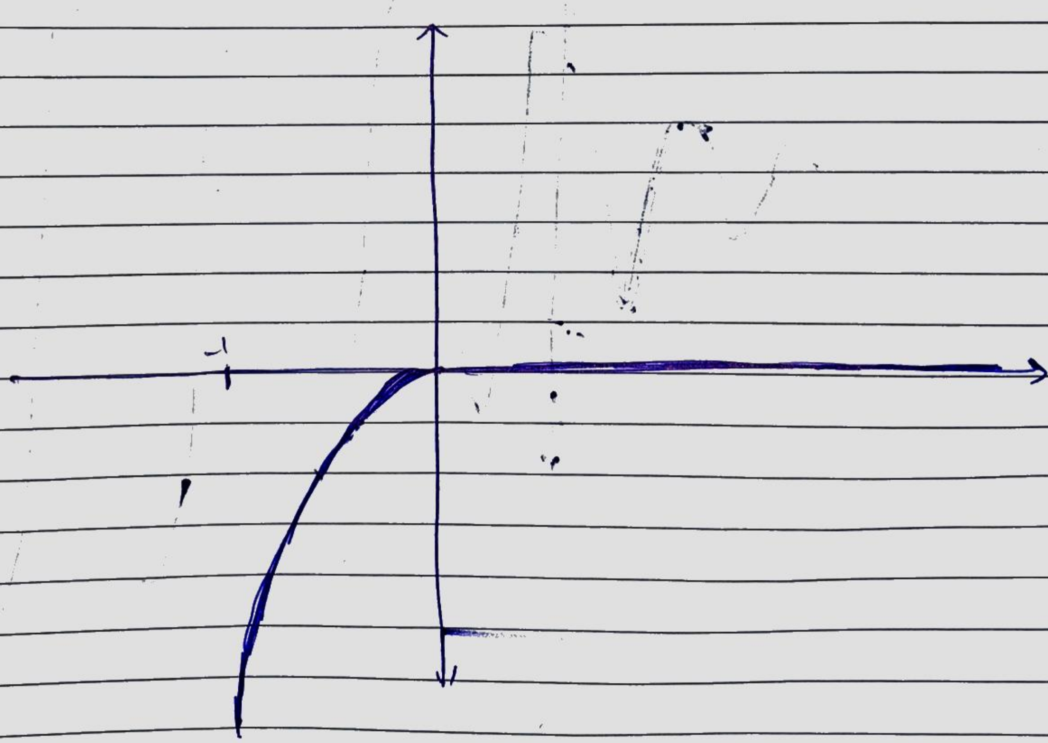
$\mathcal{L}(y'') + 6\mathcal{L}(y') + 8\mathcal{L}(y) = \frac{1}{s+3} - \frac{1}{s+5}$

$s^2\mathcal{L}(y) - y'(0) - sy(0) + 6s\mathcal{L}(y) - 6y(0) + 8\mathcal{L}(y)$

$\Rightarrow \frac{1}{s+3} - \frac{1}{s+5}$

$\mathcal{L}(y) = \frac{2}{(s+2)(s+3)(s+4)(s+5)}$

$y = \frac{e^{-5t}}{3} + e^{-4t} - e^{-3t} + \frac{e^{-2t}}{3}$



202151120

DELUXE
PAGE NO.:
DATE:

(22) $y'' + 3y' + 2y = 4t \qquad 0<t<1 \qquad y=0$ &
$y'' + 3y' + 2y = 8 \qquad t>1 \qquad y'(0)=0$

$s^2 \mathcal{L}(y) - s y(0) - y'(0)$
$+ 3\left(s \mathcal{L}(y) - y(0)\right) + 2\mathcal{L}(y) = \frac{4}{s^2}$

$\mathcal{L}(y) = \frac{4}{s^2(s^2+3s+2)} = \frac{-1}{s} + \frac{2}{s^2} + \frac{c}{s+1} + \frac{3-c}{s+2}$

$\mathcal{L}^{-1}$
$f(t) = -1 + 2t + c\,\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) + (3-c)\,\mathcal{L}^{-1}\left(\frac{1}{s+2}\right)$
$f(t) = -1 + 2t + c\,e^{-t} + (3-c)\,e^{-2t}$

(23) $y'' + 9y = 8\sin t \qquad 0<t<a \qquad y(0)=0$
$= 0 \qquad t>a \qquad y'(0)=4$

$\mathcal{L}(y''+9y) = \mathcal{L}(8\sin t)$

$s^2\mathcal{L}(y) - s y(0) - y'(0) + 9\mathcal{L}(y) = \frac{8}{s^2+1}$

$\mathcal{L}(y) = \frac{8}{(s^2+1)(s^2+9)} + \frac{4s}{s^2+9}$

20215I120

DELUXE
PAGE NO.:
DATE:

$$\mathcal{L}(y) = -\frac{s}{8(s^2+1)} + \frac{1}{8(s^2+1)} + \frac{1}{6}\cdot\frac{s}{s^2+9} - \frac{3}{6(s^2+9)} + \frac{4s}{s^2+9}$$

$$= -\frac{\cos t}{8} + \frac{\sin t}{8} + \frac{\cos 3t}{6} - \frac{\sin 3t}{2} + 2\cos 3t$$