

SET-A

①

$$2p + 2k + 3s = 350$$

$$2p + 3k + 7s = 560$$

$$6p + 5k + s = 430$$

$$45p + 15k + 27s = 2000$$

②

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 7 \\ 6 & 5 & 1 \\ 45 & 15 & 27 \end{bmatrix} \quad x = \begin{bmatrix} p \\ k \\ s \end{bmatrix} \quad b = \begin{bmatrix} 350 \\ 560 \\ 430 \\ 2000 \end{bmatrix}$$

$$Ax = b$$

$$\Rightarrow \begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 7 \\ 6 & 5 & 1 \\ 45 & 15 & 27 \end{bmatrix} \begin{bmatrix} p \\ k \\ s \end{bmatrix} = \begin{bmatrix} 350 \\ 560 \\ 430 \\ 2000 \end{bmatrix}$$

③

$$\begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 7 \\ 6 & 5 & 1 \\ 45 & 15 & 27 \end{bmatrix}$$

\downarrow
 u_1

\downarrow
 u_2

\downarrow
 u_3

$$\text{Step -1 : } k=1$$

$$P_1 = u_1 = \begin{pmatrix} 2 \\ 2 \\ 6 \\ 45 \end{pmatrix} \quad |P_1| = 45 \cdot 48626$$

$$\therefore a_1 = \frac{P_1}{|P_1|} = \frac{1}{45 \cdot 48626} \begin{pmatrix} 2 \\ 2 \\ 6 \\ 45 \end{pmatrix} = \begin{pmatrix} 0.04397 \\ 0.04397 \\ 0.13191 \\ 0.98931 \end{pmatrix}$$

Step -2

$$\begin{aligned}
 P_2 &= u_2 - (u_2^T q_1) q_1 \\
 &= \begin{pmatrix} 2 \\ 3 \\ 5 \\ 15 \end{pmatrix} - \begin{pmatrix} 2 & 3 & 5 & 15 \end{pmatrix} \begin{pmatrix} 0.04397 \\ 0.04397 \\ 0.13191 \\ 0.98931 \end{pmatrix} \cdot \begin{pmatrix} 0.04397 \\ 0.04397 \\ 0.13191 \\ 0.98931 \end{pmatrix} \\
 &= \begin{pmatrix} 1.309 \\ 2.309 \\ 2.927 \\ -0.551 \end{pmatrix} \\
 q_2 &= \frac{P_2}{|P_2|} = \begin{pmatrix} 0.32811 \\ 0.57877 \\ 0.73368 \\ -0.13811 \end{pmatrix}
 \end{aligned}$$

$|P_2| = 3.98947$

Step 3 $k = 3$

$$\begin{aligned}
 P_3 &= u_3 - [(u_3^T q_1) q_1 + (u_3^T q_2) q_2] \\
 u_3^T q_1 &= (3 \ 7 \ 1 \ 27) \begin{pmatrix} 0.04397 \\ 0.04397 \\ 0.13191 \\ 0.98931 \end{pmatrix} = 27.28298 \\
 u_3^T q_2 &= (3 \ 7 \ 1 \ 27) \begin{pmatrix} 0.32811 \\ 0.57877 \\ 0.73368 \\ -0.13811 \end{pmatrix} = 2.04043 \\
 P_3 &= \begin{pmatrix} 3 \\ 7 \\ 1 \\ 27 \end{pmatrix} - \begin{pmatrix} 27.28298 \end{pmatrix} \begin{pmatrix} 0.04397 \\ 0.04397 \\ 0.13191 \\ 0.98931 \end{pmatrix} + 2.04043 \begin{pmatrix} 0.32811 \\ 0.57877 \\ 0.73368 \\ -0.13811 \end{pmatrix} = \begin{pmatrix} 1.13088 \\ 4.61943 \\ -4.6959 \\ 0.29048 \end{pmatrix}
 \end{aligned}$$

$$|P_3| = 6.2832 \quad \begin{pmatrix} 0.18 \\ 0.7352 \\ -0.6519 \\ 0.0436 \end{pmatrix} \\
 q_3 = \frac{P_3}{|P_3|} =$$

$$\Phi = \begin{bmatrix} 0.04397 & 0.32811 & 0.18 \\ 0.04397 & 0.57877 & 0.7352 \\ 0.13191 & 0.73368 & -0.6519 \\ 0.98931 & -0.13811 & 0.436 \end{bmatrix}$$

$$R = \begin{bmatrix} u_1^T q_1 & u_2^T q_1 & u_3^T q_1 \\ 0 & u_2^T q_2 & u_3^T q_2 \\ 0 & 0 & u_3^T q_3 \end{bmatrix} = \begin{bmatrix} 95.4863 & 15.719 & 27.283 \\ 0 & 3.989 & 2.0402 \\ 0 & 0 & 6.2831 \end{bmatrix}$$

(d) $R_{\lambda} = \Phi^T b$

$$\begin{bmatrix} 95.4863 & 15.719 & 27.283 \\ 0 & 3.989 & 2.0402 \\ 0 & 0 & 6.2831 \end{bmatrix} \begin{bmatrix} p \\ k \\ s \end{bmatrix} = \begin{bmatrix} 0.04397 & 0.04397 & 0.13191 & 0.98931 \\ 0.32811 & 0.57877 & 0.73368 & -0.13811 \\ 0.18 & 0.7352 & -0.6519 & 0.436 \end{bmatrix} \cdot b$$

$$\begin{bmatrix} 95.4863 & 15.719 & 27.283 \\ 0 & 3.989 & 2.0402 \\ 0 & 0 & 6.2831 \end{bmatrix} \begin{bmatrix} p \\ k \\ s \end{bmatrix} = \begin{bmatrix} 2074.354 \\ 478.2121 \\ 281.595 \end{bmatrix}$$

$$s = \frac{281.595}{6.2831} = 44.818$$

$$3.989k + 2.0402 * 44.818 = 478.2121$$

$$k = \frac{478.2121 - 2.0402 * 44.818}{3.989} = 96.96$$

$$45.4863p + 15.719 * 96.96 + 27.283 * 44.81 = 2074.354$$

$$p = -14.785$$

$$\begin{bmatrix} p \\ k \\ s \end{bmatrix} = \begin{bmatrix} -14.785 \\ 96.96 \\ 44.818 \end{bmatrix}$$

(Ans)

SET - B

(a)

$$3T + 2M + 4B = 450$$

$$4T + 3M + 2B = 500$$

$$5T + 2M + 3B = 550$$

$$50T + 20M + 30B = 2500$$

(b) $A = \begin{bmatrix} 3 & 2 & 9 \\ 4 & 3 & 2 \\ 5 & 2 & 3 \\ 50 & 20 & 30 \end{bmatrix} \begin{bmatrix} T \\ M \\ B \end{bmatrix} = \begin{bmatrix} 450 \\ 500 \\ 550 \\ 2500 \end{bmatrix}$

(c) $P_1 = U_1 = \begin{pmatrix} 3 \\ 4 \\ 5 \\ 50 \end{pmatrix} \quad |P_1| = 5\sqrt{102}$

$$q_1 = \frac{1}{5\sqrt{102}} \begin{pmatrix} 3 \\ 4 \\ 5 \\ 50 \end{pmatrix} = \begin{pmatrix} 0.0594 \\ 0.0794 \\ 0.099 \\ 0.9901 \end{pmatrix}$$

S-2 : $P_2 = U_2 - (U_2^T q_1) q_1$

$$= \begin{pmatrix} 2 \\ 3 \\ 2 \\ 20 \end{pmatrix} - (2 \ 3 \ 2 \ 20) \begin{pmatrix} 0.0594 \\ 0.0794 \\ 0.099 \\ 0.9901 \end{pmatrix} \begin{pmatrix} 0.0594 \\ 0.0794 \\ 0.099 \\ 0.9901 \end{pmatrix}$$

$$= \begin{pmatrix} 0.7906 \\ 1.3875 \\ -0.0157 \\ -0.1589 \end{pmatrix} \quad |P_2| = 1.6047$$

$$q_2 = \frac{1}{1.6047} \begin{pmatrix} 0.7906 \\ 1.3875 \\ -0.0157 \\ -0.1589 \end{pmatrix} = \begin{pmatrix} 0.4927 \\ 0.8646 \\ -0.0098 \\ -0.0978 \end{pmatrix}$$

$$P_3 = u_3 - (u_3^T q_1) q_1 + (u_3^T q_2) q_2$$

$$(u_3^T q_1) q_1 = \begin{pmatrix} 9 & 2 & 3 & 30 \end{pmatrix} \begin{pmatrix} 0.0594 \\ 0.0794 \\ 0.099 \\ 0.9901 \end{pmatrix} = 30.3975$$

$$(u_3^T q_2) q_2 = \begin{pmatrix} 9 & 2 & 3 & 30 \end{pmatrix} \begin{pmatrix} 0.4927 \\ 0.8646 \\ -0.0098 \\ -0.0978 \end{pmatrix} = 0.07381$$

$$P_3 = \begin{pmatrix} 9 \\ 2 \\ 3 \\ 30 \end{pmatrix} - 30.3975 \begin{pmatrix} 0.0594 \\ 0.0794 \\ 0.099 \\ 0.9901 \end{pmatrix} - 0.07381 \begin{pmatrix} 0.4927 \\ 0.8646 \\ -0.0098 \\ -0.0978 \end{pmatrix} = \begin{pmatrix} 1.8305 \\ -1.046 \\ -0.0026 \\ 0.0259 \end{pmatrix}$$

$$c_{l_3} = \frac{P_3}{|P_3|} = \frac{1}{2.1084} \begin{pmatrix} 1.8305 \\ -1.046 \\ -0.0026 \\ 0.0259 \end{pmatrix} = \begin{pmatrix} 0.8682 \\ -0.4961 \\ -0.0012 \\ -0.0123 \end{pmatrix} \quad |P_3| = 2.1084$$

$$\varrho = \begin{bmatrix} 0.0594 & 0.4927 & 0.8682 \\ 0.0792 & 0.8646 & -0.4961 \\ 0.099 & -0.0098 & -0.0012 \\ 0.9901 & -0.0978 & -0.0123 \end{bmatrix} \quad R = \begin{bmatrix} u_1^T q_1 & u_2^T q_1 & u_3^T q_1 \\ 0 & u_2^T q_2 & u_3^T q_2 \\ 0 & 0 & u_3^T q_3 \end{bmatrix}$$

$$R = \begin{bmatrix} 50.4975 & 20.3574 & 30.3975 \\ 0 & 1.6046 & 0.073805 \\ 0 & 0 & 2.10843 \end{bmatrix}$$

$$\textcircled{d} \quad R_x = Q^T b$$

$$Q^T b = \begin{bmatrix} 0.0594 & 0.0792 & 0.099 & 0.9901 \\ 0.4927 & 0.8646 & -0.0098 & -0.0978 \\ 0.8682 & -0.4961 & -0.0012 & -0.0123 \end{bmatrix} \begin{bmatrix} 450 \\ 500 \\ 550 \\ 2500 \end{bmatrix} = \begin{bmatrix} 2596.03 \\ 404.125 \\ 111.23 \end{bmatrix}$$

$$R_x = Q^T b$$

$$\begin{bmatrix} 50.4975 & 20.3574 & 30.3975 \\ 0 & 1.6046 & 0.73805 \\ 0 & 0 & 2.10843 \end{bmatrix} \begin{bmatrix} T \\ M \\ B \end{bmatrix} = \begin{bmatrix} 2596.03 \\ 404.125 \\ 111.23 \end{bmatrix}$$

$$2.10843 B = 111.23$$

$$\Rightarrow B = \frac{111.23}{2.10843} = 52.755$$

$$1.6046 M + 0.73805 * 52.755 = 404.125$$

$$\Rightarrow M = \frac{404.125 - (0.73805 * 52.755)}{1.6046} = 227.5889$$

$$50.4975 T + 20.3574 * 227.5889 + 30.3975 * 52.755 = 2596.03$$

$$50.4975 T + 20.3574 * 227.5889 + 30.3975 * 52.755 = 2596.03$$

$$\Rightarrow T = \frac{2596.03 - (20.3574 * 227.5889 + 30.3975 * 52.755)}{50.4975}$$

$$\Rightarrow T = -72.0968$$

$$\begin{bmatrix} T \\ M \\ B \end{bmatrix} = \begin{bmatrix} -72.0968 \\ 227.5889 \\ 52.755 \end{bmatrix} \quad (\text{Ans})$$