

## Practice Problems: Chapter 1

1. The floating point representation can be expressed in any of the following forms:

Standard Normalized Form	:	$F = (\pm 0.d_1d_2d_3 \cdots d_m)_\beta \times \beta^e; \quad d_1 \neq 0.$
IEEE Normalized Form	:	$F = (\pm 0.1d_1d_2d_3 \cdots d_m)_\beta \times \beta^e.$
IEEE Denormalized Form	:	$F = (\pm 1.d_1d_2d_3 \cdots d_m)_\beta \times \beta^e.$

- a) Consider a system with  $\beta = 2$ ,  $m = 4$ , and  $-3 \leq e \leq 4$ . Find out the **maximum** and **minimum** numbers this system can store with and without **negative support**. Express the numbers both in binary and decimal digits for all three forms.
- b) How many numbers can this system represent or store in all these forms?
- c) Using **Standard Form**, find all the decimal numbers without **negative support**, plot them on a real line, and show if the number line is **equally spaced** or not.
- d) For the **IEEE standard** for double-precision (64-bit) arithmetic, find the **smallest positive number** and the **largest number** representable by a system that follows this standard. Do not find their decimal values, but simply represent the numbers in the following format:

$$(\pm 0.1d_1d_2d_3 \cdots d_m)_\beta \times \beta^{e-\text{exponentBias}}.$$

Be mindful of the conditions for representing  $\pm\infty$  and  $\pm 0$  in this IEEE standard.

- e) In the above IEEE standard, if the exponent bias were to be altered to  $\text{exponentBias} = 500$ , what would the **smallest positive number** and the **largest number** be? Write your answers in the same format as in part (d). Note that the conditions for representing  $\pm\infty$  and  $\pm 0$  are still maintained as before.
2. If  $x = 3/8$  and  $y = 5/8$ , find  $fl(x \times y)$  where  $m = 4$ . Also check whether  $x \times y = fl(x \times y)$ . If not, find the **rounding error** of the product of these two numbers.
3. Consider the quadratic equation,  $x^2 - 60x + 1 = 0$ . Working to **6 significant figures**, compute the **roots** of the quadratic equation and check that there is a **loss of significance**. Find the **correct roots** such that loss of significance does not occur.
4. Given  $\beta = 2$ ,  $m = 5$ ,  $-100 \leq e \leq 100$ . Using the IEEE **Normalized form**, answer the following:
- a) Compute the Machine Epsilon ( $\epsilon_M$ ).
- b) Compute the minimum of  $|x|$ .
- c) How many non-negative numbers can you represent using this system?

5. Consider the quadratic equation  $x^2 - 16x + 3 = 0$ . Explain how the loss of significance occurs in finding the roots of the quadratic equation if we restrict to **4 significant figures**. Discuss how to avoid this and find the roots.

6. Given a system parameterized by  $\beta = 2$ ,  $m = 3$ , and  $e_{\min} = -1 \leq e \leq e_{\max} = 2$ , where  $e \in \mathbb{Z}$ . For this system answer the following:

- (a) Find the floating-point representation of the numbers  $(6.25)_{10}$  and  $(6.875)_{10}$  in the Normalized Form. That is, find  $fl(6.25)_{10}$  and  $fl(6.875)_{10}$ .
- (b) What are the rounding errors  $\delta_1, \delta_2$  in part (a)?
- (c) Can the values  $(6.25)_{10}$  and  $(6.875)_{10}$  be represented in the Denormalized Form? If so, find the floating-point representations. If not, then concisely explain why?
- (d) Find the rounding error for Standard Form, Normalized and Denormalized Form.

7. Consider the **real number**  $x = (8.235)_{10}$

- (a) First convert the decimal number  $x$  in binary format at least up to 8 binary places.
- (b) What will be the binary value of  $x$  [Find  $fl(x)$ ] if you store it in a system with  **$m = 6$**  using the **Denormalized** form of floating point representation.
- (c) Now convert back to decimal form the stored values you obtained in the previous part, and calculate the **rounding error of both numbers**.

8. Consider the quadratic equation:

$$x^2 - 12x + 5 = 0$$

- a) Compute the roots of the quadratic equation while keeping to **four significant figures**.
- b) Explain how **loss of significance** occurs in this case due to the subtraction of nearly equal numbers.
- c) Discuss an alternative approach to computing the roots to **avoid loss of significance**, and use this method to determine the correct roots.

9. Consider a computing system with base  $\beta = 2$ ,  $m = 3$ , and  $e_{\min} = -3 \leq e \leq e_{\max} = 2$

- a) In the Standard form of this system, determine the total number of representable values including support for negative numbers. Also, compute the maximum value of delta.
- b) Express the floating-point representations (binary format) for the numbers  $x = 4/8$  and  $y = 7/8$  in this system.
- c) Compute  $fl(x \times y)$  and determine whether this value can be stored within the given floating-point system.