

## **Practice Problems: Chapter 2**

1. Evaluate the interpolation polynomial of a function  $f(x) = \sin(x)$  at the nodes  $\{-\pi/2, 0, \pi/2\}$  by using the Vandermonde method. Note you need to show a detailed calculation to get full credit.
2. Consider the function  $f(x) = xe^x$ . Using Taylor Expansion a polynomial of degree 3 is calculated.
  - a. Using Taylor Expansion, write  $f(x)$  as an infinite series.
  - b. Find the coefficients  $a_0, a_1, a_2$  and  $a_3$  for the polynomial  $P_3(X)$ .
  - c. Compute  $f(0.1)$  and  $P(0.1)$  up to seven significant figures.
  - d. Find the percentage of error using the answers got from Question (c)
3. Consider the function  $f(x) = \ln(1+x)$ . Using Taylor Expansion a polynomial of degree 3 is calculated.
  - a. Using Taylor Expansion, write  $f(x)$  as an infinite series.
  - b. Find the coefficients  $a_0, a_1$  and  $a_2$  for the polynomial  $P_2(X)$ .
  - c. Compute  $f(0.2)$  and  $P(0.2)$  up to seven significant figures.
  - d. Find the percentage of error using the answers got from Question (c)
4. A function,  $f(x) = x \sin(x)$ , is to be interpolated at three nodes:  $x_0 = -\pi/2, x_1 = 0$  and  $x_2 = \pi/2$ . Answer the following:
  - a. Draw a sketch of the function  $f(x)$  and identify the nodes. Briefly explain what kind of interpolation polynomial is expected and discuss the properties of the polynomial.
  - b. Compute Lagrange bases.
  - c. Write the interpolation polynomial and compare the properties of the interpolation polynomial with the expectation in Part-(a).
5. Read the following and answer accordingly:
  - a. Find an interpolating polynomial of appropriate degree using Newton's divided-difference method for  $f(x) = x \cos(x)$ . Consider the nodes  $[-\pi/2, 0, \pi/2]$ .
  - b. Use the interpolated polynomial to find an approximate value at  $\pi/4$ , and compute the relative error at  $\pi/4$ .
  - c. Add a new node  $\pi$  to the above nodes, and find the interpolating polynomial.
6. Given,  $f(x) = \sin(x)$  and Consider the nodes  $[0, \pi/2, \pi]$ . Use Newton's Divided-Difference method for the following questions-
  - a. Find the values of  $a_0, a_1$  and  $a_2$ .
  - b. Write down the interpolating polynomial.
  - c. Add a new node  $3\pi/2$  to the existing nodes and find a new interpolating polynomial.
  - d. Estimate the upper bound of the interpolating error for the polynomial  $P_3(x)$ .

7. Consider the following table of data points/nodal points:

Time (sec) $t$	Velocity ( $\text{ms}^{-1}$ ) $v(t)$
2	10
4	20
6	25

- Find an interpolating polynomial of velocity that goes through the above data points by using Vandermonde Matrix method. Also compute an approximate value of acceleration at Time,  $t = 7$  sec.
  - Find an interpolating polynomial of velocity that goes through the above data points by using Lagrange method.
  - If a new data point is added in the above scenario, which method should you use in finding a new interpolating polynomial? Also what will be the degree of that new polynomial?
8. Use a Hermite polynomial that agrees with the data listed below to find the approximation of  $f(1.5)$ .

$k$	$x_k$	$f(x_k)$	$f'(x_k)$
0	1.3	0.6200860	-0.5220232
1	1.6	0.4554022	-0.5698959
2	1.9	0.2818186	-0.5811571

9. Find the value of  $y$  for  $x = 2.1$  using 2<sup>nd</sup> order Lagrange polynomials with the appropriate data.

SI.	$x$	$y$
1	-1	2.2
2	0	10.6
3	1	17.0
4	2	22.4
5	3	25.8

10. Let  $f(x) = \cos(x)$  and the nodes are  $(-\pi/4, 0, \pi/4)$ . Use Cauchy's theorem to find the upper bound of the error.

11. a. Compute the upper bound of error using Cauchy's Theorem for  $f(x) = \sin^2(x/2)$  with nodes  $\{-\pi/3, 0, \pi/3\}$  within the interval  $[-1.2, 1.2]$ .  
 b. Why are Chebyshev nodes an optimal choice in Interpolation?

12. The following nodes come from the function  $f(x) = \ln(5x+9)$  :

x	f(x)
-0.5	1.87
0	2.20
0.5	2.44

- a. Using Newton's divided difference method, find the equation of a second degree polynomial which fits the above data points.  
 b. Expand the function  $f(x) = \ln(5x + 9)$  using Taylor Series, centered at 0. Include till the  $x^2$  term of the Taylor series.  
 c. Should the equation which you found in part (a) and part (b) match? Comment on why, or why not.

13. Consider the following nodes:

x	f(x)
0	5
3	9.5
6	5

- a. If an equation of a polynomial which fits through the above nodes is found using both the Vandermonde Matrix approach and the Lagrange approach, will both the equations match?  
 b. Find the equation of a polynomial which fits through the above nodes using the Vandermonde matrix approach.  
 c. Find the equation of a polynomial which fits through the above nodes using the Lagrange approach.

14. Given a Runge Function,  $f(x) = \frac{8}{5+16x^2}$  between the interval  $[-6,6]$  where  $n=3$

- a. Calculate the equal angled points of  $\theta$ 's.  
 b. Calculate the values of the Chebychev's nodes.  
 c. Find the Lagrange basis  $l_2(x)$ .

15. Consider the following dataset:

$x$	$f(x)$	$f'(x)$
0.1	-0.62050	3.58502
0.2	-0.28340	3.14033

Answer the following based on the above data:

- Compute the Hermite bases:  $h_0(x)$ ,  $h_1(x)$ ,  $\hat{h}_0(x)$  and  $\hat{h}_1(x)$ .
- Write the Hermite polynomial and find the value at  $x = 0.15$