423 Assignment

Q 1. a) Perspective projection is considered more realistic because it mimics how the human eye proceives depth and distance. In perspective projection parallel lines converge at a vanishing point, creating a sense of depth. For example, when looking down a straight road, the sides appear to converge in the distance, which is accurrately represented in perspective projection. This realism is crucial in 3D graphics for applications like video games and simulations, where immersion is important. In contrast, parallel projection maintains the size and the shape of objects regardless of their distance from the viewer. This is useful in engineering and architectural drawings, where accurate measurements and proportions are essential. For instance, in architectual plans, an orthographic projection allows architects to see the exact dimensions of a building without the distorction caused by perspective, making it easier to convey percise information.

b) $\alpha = 63.4^\circ$ $\quad \lambda = 0.5$

$\beta = 30^\circ$ $\quad x = 11$

$\Delta x = x - 11$

$y' = y + \lambda \cos\beta \times \Delta x$

$z' = z + \lambda \sin\beta \times \Delta x$

$0.5 \cos 30^\circ = 0.433$

$0.5 \sin 30^\circ = 0.25$

$$\begin{bmatrix} 0 & 0 & 0 & 11 \\ 0.43 & 1 & 0 & -4.76 \\ 0.25 & 0 & 1 & -2.75 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) $\alpha = 45^\circ$; $\beta = 30^\circ$ $\lambda = 1$

$x' = x + \lambda \cos\beta \, 4z$ $\quad p(10, 20, -40)$

$y' = y + \lambda \sin\beta \, 4z$

$$P' = \begin{bmatrix} 1 & 0 & \cos 30^\circ & 0 \\ 0 & 1 & \sin 30^\circ & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 20 \\ -40 \\ 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} -24.4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$p'(-24.4, 0)$ on the XY plane.

d) $P(40, 75, -250)$

PP is at $y = 0$ (xz plane)

COP is at $(0, -150, 0)$

$d = -150$

$x' = \dfrac{x}{(1 - \frac{y}{d})} = \omega x$

$z' = \dfrac{z}{(1 - \frac{y}{d})} = \omega z'$

$\omega = (\frac{-1}{d}) y + 1$

$$P' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1/150 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 40 \\ 75 \\ -250 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 40 \\ 0 \\ -250 \\ 1.495 \end{bmatrix} = \begin{bmatrix} 26.75 \\ 0 \\ -167.22 \\ 1 \end{bmatrix}$$

$p'(26.75, 0, -167.22)$

Q2. a) The projection technique applied in this illustration is Cavalier Projection which is a type of oblique parallel projection.

Justification

1. True Shape of the front face: In Caveller projection the plane that is parallel to the projection plane is drawn without any distortion. In this case, the front wall of the wooden cabin is shown in its actual proportions, which is a defining characteristic of Cavalier projection.

2. Depth represented at 45° without scale reduction: Another key property of cavalier is that the receding (length or depth) axis is drawn at an angle of 45° to the horizontal, and the full length of that axis is preserved (scalling factor $\lambda = 1$). This matches the given description when the cabin's length is drawn along lines inclined at 45° with no reduction.

Because of these two properties, the projection clearly corresponds to Cavalier projection.

Effect of reducing the length axis to half:

If the length axis were reduced to half of its actual length ($\lambda = 0.5$), the projection would become a cabinet projection. Their reduction would make the cabin appear less elogated and visually less distorted, resulting in a more realistic appearance while still maintaining parallel projection properties.

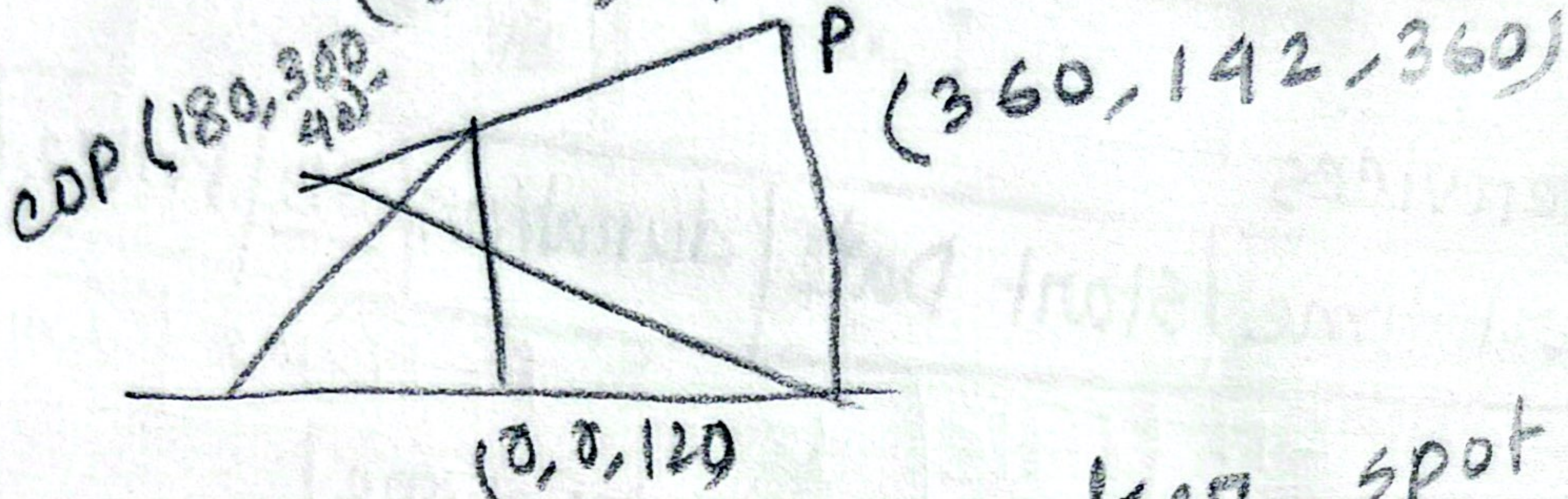
b) $a = (200, 150, 360)$

$b = (600, 130, 360)$

$b - a = (400, -20, 0)$

marker spot $= (200, 150, 360) + 0.4\,(400, -200)$

$= (360, 142, 360)$



distance from panel to marker spot of z-axis $360 - 120 = 240$

eye to panel distence is one third of that distance $\therefore \frac{240}{3} = 80$

$\therefore$ z coordinate of eye $120 - 80 = 40$

$\therefore$ COP $(180, 300, 40)$

$q_x = 180 - 0 = 180$

$q_y = 300 - 0 = 300$

$q_z = 40 - 120 = -80$

$z_p = 120$

$-\frac{q_x}{q_z} = \frac{-180}{-80} = \frac{9}{4}$

$-\frac{q_y}{q_z} = \frac{-300}{-80} = \frac{15}{4}$

$-\frac{z_p}{q_z} = \frac{-120}{-80} = 1.5$

$z_p \frac{q_x}{q_z} = 120 \times \frac{180}{-80} = -270$

$z_p \frac{q_y}{q_z} = 120 \times \frac{300}{-80} = -450$

$z_p + \frac{z_p^2}{q_z} = 120 + \frac{120^2}{-80} = -60$

$1 + \frac{z_p}{q_z} = 1 + \frac{120}{-80} = -0.5$

$-\frac{1}{q_z} = \frac{-1}{-80} = \frac{1}{80}$

so, the generalised formula of perspective matrix

$$\begin{bmatrix} x'w \\ y'w \\ z'w \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{q_x}{q_z} & z_p \frac{q_x}{q_z} \\ 0 & 1 & -\frac{q_y}{q_z} & z_p \frac{q_y}{q_z} \\ 0 & 0 & -\frac{z_p}{q_z} & z_p + \frac{z_p^2}{q_z} \\ 0 & 0 & -1/q_z & 1 + \frac{z_p}{q_z} \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 9/4 & -270 \\ 0 & 1 & 15/4 & -450 \\ 0 & 0 & 1.5 & -60 \\ 0 & 0 & 1/80 & -0.5 \end{bmatrix} \times \begin{bmatrix} 360 \\ 142 \\ 360 \\ 1 \end{bmatrix} = \begin{bmatrix} 900 \\ 1042 \\ 480 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 225 \\ 260.5 \\ 120 \\ 1 \end{bmatrix}$$

$\therefore$ (225, 260.5, 120)