Department of Mathematics and Natural Sciences

Semester: Spring 2024

Final Examination

Course ID: MAT-215

Course Title: Complex Variables and Laplace Transformations (Mathematics III)

Total Marks: 35 Date: May 8, 2024

Time: 1 hour 30 minutes **Answer any five [5×7=35]**

1. [ 4+3]

   i. Use definition to find $\mathcal{L}\{f(t)\}$, where $f(t) = \begin{cases} 0, & 0 \le t < \frac{\pi}{2} \\ \cos t, & t \ge \frac{\pi}{2} \end{cases}$

   ii. Show that $\mathcal{L}\{y''(t)\} = s^2Y(s) - s\,y(0) - y'(0)$

2. Evaluate $\frac{1}{2\pi i}\oint_C \frac{e^{zt}}{(z^2+1)^2}dz$ if $t > 0$ and C is the circle $|z| = 3$. [7]

3. [4+3]

   i. Find the Laplace transform of $f(t) = te^{3t}\cos 2t$

   ii. Evaluate $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}e^{-\pi s}\right\}$

4. Evaluate $\int_i^{2-i}(3xy + iy^2)dz$ along the curve $x = 2t - 2, y = 1 + t - t^2$. [7]

5. Solve the following differential equation using Laplace transform: [7]

$$2y''' + 3y'' - 3y' - 2y = e^{-t}, y(0) = 0, y'(0) = 0, y''(0) = 1$$

6. Consider following: [7]

   - If $F(s) = \mathcal{L}\{f(t)\}$ and $a > 0$, then $\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}F(s)$,
   - If $f(t) = \mathcal{L}^{-1}\{F(s)\}$, then $\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\mathcal{U}(t-a)$,
   - Also $\mathcal{L}\{g(t)\mathcal{U}(t-a)\} = e^{-as}\,\mathcal{L}\{g(t+a)\}$

   Now, solve $y' + y = f(t)$, $y(0) = 5$ where $f(t) = \begin{cases} 0, & 0 \le t < \pi \\ 2\cos t, & t \ge \pi \end{cases}$

7. Expand $f(z) = \frac{1}{z^2+4z+3}$ in a Laurent Series valid for $1 < |z| < 3$. [7]