Department of Mathematics and Natural Sciences
MAT215 : Complex Variables and Laplace Transformations
Assignment 1
Date: June 28, 2025

BRAC UNIVERSITY
Inspiring Excellence

Deadline : July 6,2024 Summer 2025 Total Marks: 90

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Use this page as the cover page of your assignment. No late submission will be graded.

(1) Express each of the following complex numbers in polar form. (any 1)

(a) $2+2\sqrt{3}i$, (b) $-\sqrt{6}-\sqrt{2}i$, (c) $-3i$. [5]

(2) Find each of the indicated roots and locate them graphically. (any 1)

(a) $(-1+i)^{1/3}$, (b) $(-2\sqrt{3}-2i)^{1/4}$ [5]

(3) Describe graphically the region represented by each of the following: (any 2)

(a) $1<|z+i|\leq 2$, (b) $\operatorname{Re}\{z^2\}>1$, (c) $|z+3i|>4$, (d) $|z+2-3i|+|z-2+3i|<10$, (e) $\operatorname{Re}(1/z)\leq 1/2$

(f) $\pi/2<\arg z<3\pi/2, |z|>2$ [6]

(4) Show that, $\ln(1-i)=\frac{1}{2}\ln 2+\left(2n+\frac{7}{4}\right)\pi i$ [5]

(5) Prove that (any 1)

(a) $\cos^{-1}z=\frac{1}{i}\ln\left(z+\sqrt{z^2-1}\right)$, (b) $\cot^{-1}z=\frac{1}{2i}\ln\left(\frac{z+i}{z-i}\right)$ indicating any restrictions. [5]

(6) Prove that $\lim_{z\to 1+i}\frac{z^2-z+1-i}{z^2-2z+2}=1-\frac{1}{2}i$. [5]

(7) Let $f(z)=\frac{2z-1}{3z+2}$. Prove that $\lim_{h\to 0}\frac{f(z_0+h)-f(z_0)}{h}=\frac{7}{(3z_0+2)^2}$ provided $z_0\neq -2/3$. [5]

(8) If $\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)^{50}=3^{25}(x+iy)$, where x and y are reals then find the values of x and y. [5]

(9) Evaluate using theorems on limits. (any 2)

(a) $\lim_{z\to 2i}\left(iz^4+3z^2-10i\right)$, (b) $\lim_{z\to i/2}\frac{(2z-3)(4z+i)}{(iz-1)^2}$,

(c) $\lim_{z \to e^{\pi/4}} \frac{z^2}{z^4+z+1}$, (d) $\lim_{z \to i} \frac{z^2+1}{z^6+1}$, (e) $\lim_{z \to 1+i} \left\{ \frac{z-1-i}{z^2-2z+2} \right\}^2$

(10) Use exponential form to compute (any 1)

i. $(1+\sqrt{3}i)^{2011}$; ii. $(1+\sqrt{3}i)^{-2011}$. [5]

(11) Sketch the region in xy plane represented by the following set of points: (any 2) [4]

(a) $\mathrm{Re}(\bar{z}-1) = 2$ (b) $\mathrm{Im}\left(z^2\right) = 4$ (c) $\left|\frac{2z-3}{2z+3}\right| = 1$ (d) $\mathrm{Re}(z)+\mathrm{Im}(z) = 0$

(12) Solve the following equations: (any 1)

(i) $\cosh z = \frac{1}{2}$ (ii) $\sinh z = i$. [5]

(13) Find all the roots of the following equations. (any 1)

(i) $z^6 = 64$ (ii) $z^4+z^2+1 = 0$. [5]

(14) Let $f(z) = \frac{z^2+4}{z-2i}$ if $z \neq 2i$, while $f(2i) = 3+4i$. [5]

(a) Prove that $\lim_{z \to i} f(z)$ exists and determine its value (b) Is $f(z)$ continuous at $z = 2i$? Explain. (c) Is $f(z)$ continuous at points $z \neq 2i$? Explain.

(15) Find all values of z such that: (any 1)

(i) $e^z = 1+\sqrt{3}i$ (ii) $e^{2z-1} = 1$ [5]

(16) Evaluate the following Limits using L'Hospital's rule (any 2) [5]

(i) $\lim_{z \to 2i} \frac{z^2+4}{2z^2+(3-4i)z-6i}$ (ii) $\lim_{z \to 0} \frac{z-\sin z}{z^3}$ (iii) $\lim_{z \to 0} \left(\frac{\sin z}{z}\right)^{\frac{1}{z^2}}$

(17) Find the modulus and argument of the following complex numbers: (any 1) [5]

(i) $\frac{\sqrt{3}+i}{\sqrt{3}-i}$ (ii) $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^2$

(18) If $f(z) = \begin{cases} \frac{z^2-4}{z^2-3z+2}, z \neq 2 \\ kz^2+6, z = 2 \end{cases}$, find **k** such that $f(z)$ becomes continuous at $z = 2$. [5]

1. a) $2+2\sqrt{3}\,i$

modulus, $r = \sqrt{(2)^2+(2\sqrt{3})^2} = 4$

Argument, $\theta = \tan^{-1}\left|\frac{2\sqrt{3}}{2}\right| = \pi/3$

$\therefore$ in polar form, $z = 4e^{i\pi/3}$

2. a) $(-1+i)^{1/3}$

Now, $z = (-1+i)^{1/3}$ consider $(-1+i)$: $x=-1$, $y=1$

$\mathrm{mod}\,(z) = \sqrt{(-1)^2+(1)^2} = \sqrt{2}$

$\mathrm{Arg}\,(z) = \pi - \tan^{-1}\left|\frac{1}{-1}\right| + 2n\pi = \pi - \frac{\pi}{4} + 2n\pi$

$= \frac{3\pi}{4} + 2n\pi$

$(-1+i)^{1/3} = (\sqrt{2})^{1/3}\left(\cos\frac{1}{3}\left(\frac{3\pi}{4}+2n\pi\right) + i\sin\frac{1}{3}\left(\frac{3\pi}{4}+2n\pi\right)\right)$

$\left[\because (\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)\right]$

$= (\sqrt{2})^{1/3}\cos\left(\frac{\pi}{4}+\frac{2n\pi}{3}\right) + i\sin\left(\frac{\pi}{4}+\frac{2n\pi}{3}\right)$

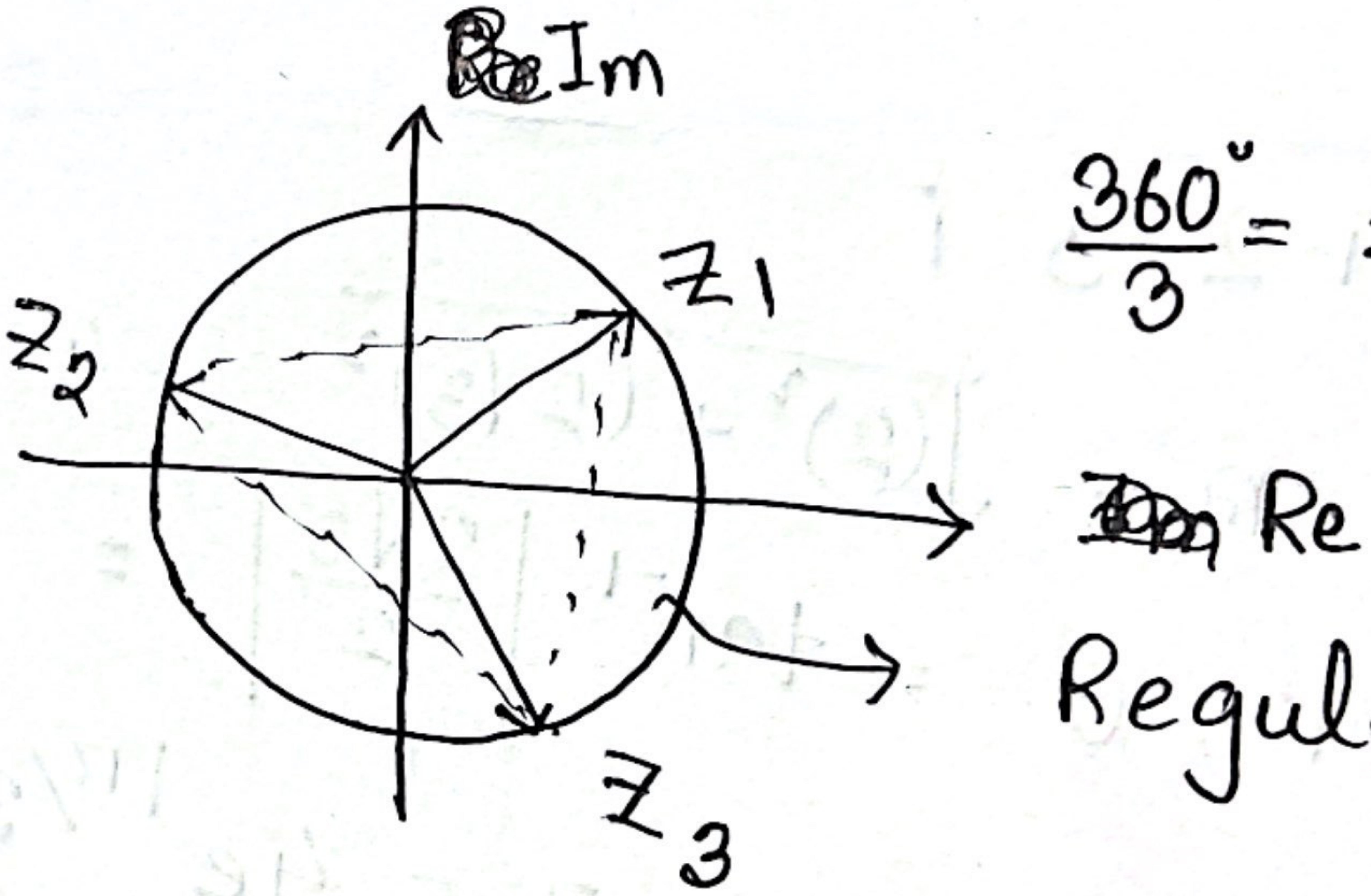
no. of roots = 3

$n = 0, 1, 2$

$n=0$, $z_0 = (\sqrt{2})^{1/3}\cdot\left(\cos(\pi/4) + i\sin(\pi/4)\right) = 0.793 + 0.793i$

$n=1$, $z_1 = (\sqrt{2})^{1/3}\cdot\left(\cos\left(\pi/4+\frac{2\pi}{3}\right) + i\sin\left(\pi/4+\frac{2\pi}{3}\right)\right) = -1.08 + 0.29i$

$n=2$, $z_2 = (\sqrt{2})^{1/3}\cdot\left(\cos\left(\pi/4+\frac{4\pi}{3}\right) + i\sin\left(\pi/4+\frac{4\pi}{3}\right)\right) = 0.29 - 1.08i$



$$\frac{360^\circ}{3} = 120^\circ$$

Regular Triangle

3. e) $Re(1/z) \leq 1/2$

$\Rightarrow Re\left(\frac{1}{x+iy}\right) \leq 1/2$

$\Rightarrow Re\left(\frac{1(x-iy)}{x^2+y^2}\right) \leq 1/2$

$\Rightarrow \frac{x}{x^2+y^2} \leq 1/2$

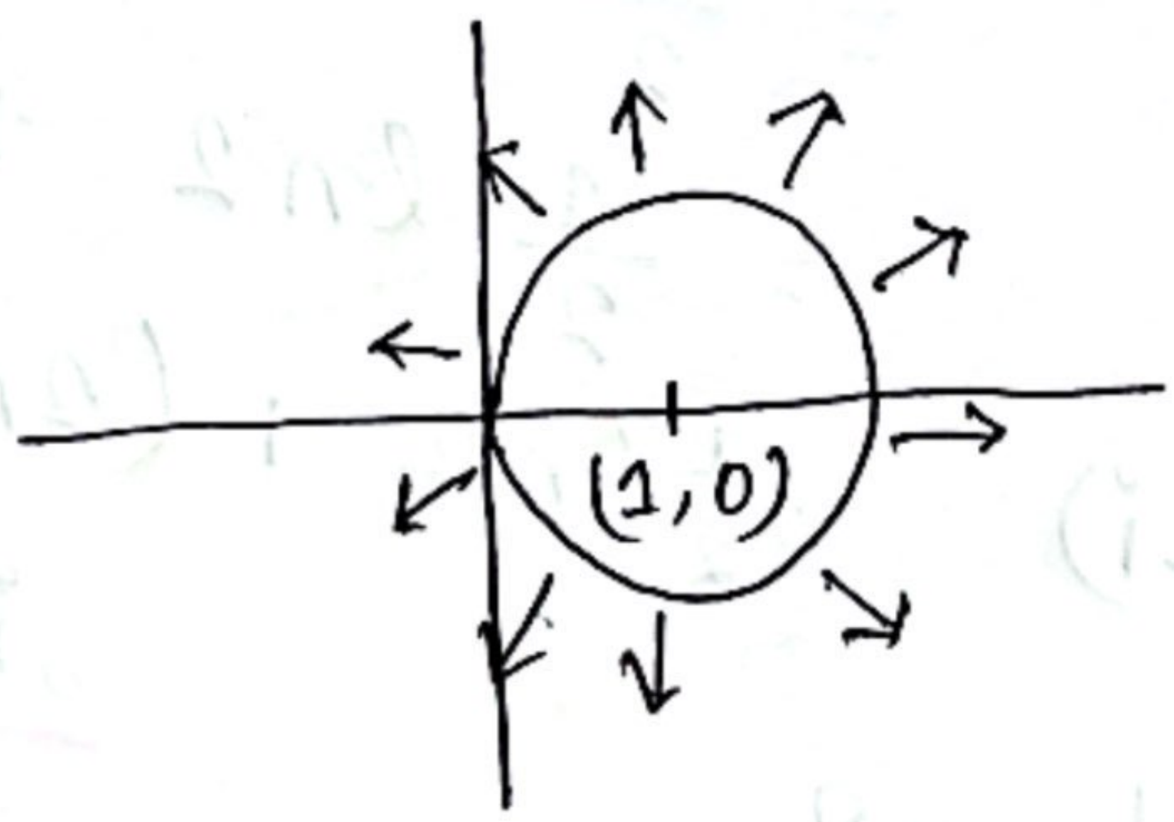
$\Rightarrow 2x \leq x^2+y^2$

$\Rightarrow x^2+y^2 \geq 2x$

$\Rightarrow x^2-2x+y^2 \geq 0$

$\Rightarrow x^2-2x+1+y^2 \geq 1$

$\Rightarrow (x-1)^2+(y-0)^2 \geq 1^2$



f) $\pi/2 < \underbrace{\arg z}_{\theta} < 3\pi/2$, $|z| > 2$

$\Rightarrow 90° < \arg z < 270°$

$\Rightarrow 90° < \theta < 270°$

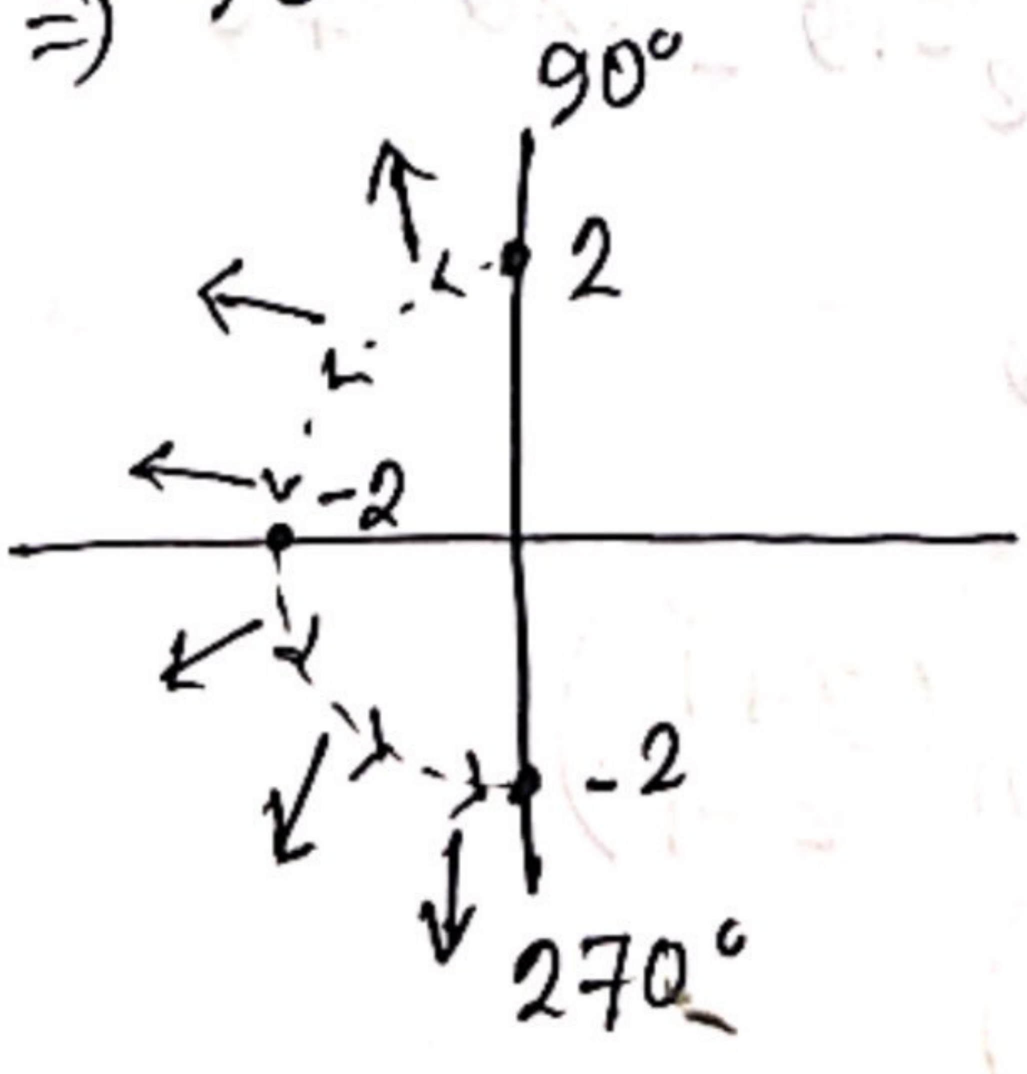
Given, $|z| > 2$

$\Rightarrow \sqrt{x^2+y^2} > 2$

$\Rightarrow x^2+y^2 > 2^2$

center $(0,0)$

$r = 2$



4. $1 - i \Rightarrow x = 1, y = -1$

$$r = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

$$\theta = - \tan^{-1}\left|\frac{-1}{1}\right| = - \pi/4 , 2\pi - \pi/4 = \frac{7\pi}{4}$$

Now, $re^{i(\theta + 2n\pi)}, n \geq 0$

$$1 - i = re^{i(\theta+2n\pi)}$$

$$= \sqrt{2}\, e^{i(7\pi/4 + 2n\pi)}$$

$$\ln(1-i) = \ln\left(\sqrt{2}\, e^{i(7\pi/4 + 2n\pi)}\right)$$

$$= \ln\sqrt{2} + \ln e^{(2n\pi + 7\pi/4)i}$$

$$= \ln 2^{1/2} + (2n\pi + 7\pi/4)i$$

$$= \frac{1}{2}\ln 2 + (2n\pi + 7\pi/4)i$$

$$= \frac{1}{2}\ln 2 + (2n + 7/4)\pi i$$

$$\therefore \ln(1-i) = \frac{1}{2}\ln 2 + (2n + 7/4)\pi i$$ [proved]

5. b) $\cot^{-1} z = \theta$

$$\Rightarrow z = \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{e^{i\theta} + e^{-i\theta}}{2}}{\frac{e^{i\theta} - e^{-i\theta}}{2i}}$$

$$\Rightarrow z = i\frac{e^{i\theta} + e^{-i\theta}}{e^{i\theta} - e^{-i\theta}}$$

$$\Rightarrow \frac{z+i}{z-i} = \frac{e^{i\theta} + \cancel{e^{-i\theta}} + e^{i\theta} - \cancel{e^{-i\theta}}}{\cancel{e^{i\theta}} + e^{-i\theta} - \cancel{e^{i\theta}} + e^{-i\theta}}$$

$$\Rightarrow \frac{z+i}{z-i} = \frac{e^{i\theta}}{e^{-i\theta}}$$

$$\Rightarrow e^{2i\theta} = \frac{z+i}{z-i}$$

$$\Rightarrow \ln\left(e^{2i\theta}\right) = \ln\left(\frac{z+i}{z-i}\right)$$

$$\Rightarrow 2i\theta = \ln\left(\frac{z+i}{z-i}\right)$$

$$\Rightarrow \theta = \frac{1}{2i} \ln\left(\frac{z+i}{z-i}\right)$$

$$\therefore \cot^{-1} z = \frac{1}{2i} \ln\left(\frac{z+i}{z-i}\right)$$ [Proved]

This expression is undefined at $z = \pm i$ because of division by zero.

The logarithm is multivalued and undefined when the argument is zero or negative real.

So, we avoid:

$$z = i \Rightarrow \ln(0)$$
$$z = -i \Rightarrow \ln(\infty)$$

6. $$\lim_{z \to 1+i} \frac{z^2 - z + 1 - i}{z^2 - 2z + 2}$$

$$= \lim_{z \to 1+i} \frac{2z - 1}{2z - 2}$$ [$\because$ Apply L' Hospital's Rule]

$$= \frac{2(1+i) - 1}{2(1+i) - 2}$$

$$= \frac{2 + 2i - 1}{2 + 2i - 2}$$

$$= \frac{1 + 2i}{2i}$$

$$= \frac{(1+2i)}{2i} \cdot \frac{(-2i)}{(-2i)}$$

$$= \frac{-4i^2 - 2i}{-4i^2} = \frac{4 - 2i}{4} = 1 - \frac{1}{2} i$$

$$\therefore \lim_{z \to 1+i} \frac{z^2 - z + 1 - i}{z^2 - 2z + 2} = 1 - 1/2\, i$$ [Proved]

7. $$f(z) = \frac{2z-1}{3z+2}$$

$$f(z_0+h) = \frac{2(z_0+h)-1}{3(z_0+h)+2} = \frac{2z_0+2h-1}{3z_0+3h+2}$$

$$f(z_0) = \frac{2z_0-1}{3z_0+2}$$

$$f(z_0+h) - f(z_0) = \frac{2z_0+2h-1}{3z_0+3h+2} - \frac{2z_0-1}{3z_0+2}$$

$$= \frac{(2z_0+2h-1)(3z_0+2) - (2z_0-1)(3z_0+3h+2)}{(3z_0+3h+2)(3z_0+2)}$$

$$= \frac{\cancel{6z_0^2} + \cancel{6z_0h} - \cancel{3z_0} + \cancel{4z_0} + 4h - \cancel{2} - \cancel{6z_0^2} + \cancel{3z_0} - \cancel{6z_0h} + 3h - \cancel{4z_0} + \cancel{2}}{(3z_0+3h+2)(3z_0+2)}$$

$$= \frac{7h}{(3z_0+3h+2)(3z_0+2)}$$

$$L.H.S = \lim_{h\to 0} \frac{f(z_0+h) - f(z_0)}{h}$$

$$= \lim_{h\to 0} \frac{7\cancel{h}}{(3z_0+3h+2)(3z_0+2)} \times \frac{1}{\cancel{h}}$$

$$= \lim_{h\to 0} \frac{7}{(3z_0+3h+2)(3z_0+2)}$$

$$= \frac{7}{(3z_0+2)(3z_0+2)}$$

$$= \frac{7}{(3z_0+2)^2} = R.H.S$$

[Proved]

8. $z = \frac{3}{2} + \frac{i\sqrt{3}}{2}$

$r = \sqrt{(3/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}/2}{3/2}\right) = \pi/6$

z in polar form: $z = \sqrt{3}\,(\cos \pi/6 + i \sin \pi/6)$

Now,

$$\left(\frac{3}{2} + \frac{i\sqrt{3}}{2}\right)^{50} = 3^{25}(x+iy)$$

$$\Rightarrow (\sqrt{3})^{50}\left(\cos\frac{50\pi}{6} + i\sin\frac{50\pi}{6}\right) = 3^{25}(x+iy)$$

$$\Rightarrow 3^{25}\left(\cos\frac{50\pi}{6} + i\sin\frac{50\pi}{6}\right) = 3^{25}(x+iy)$$

$$x = \cos\left(\frac{50\pi}{6}\right) = 1/2$$

$$y = \sin\left(\frac{50\pi}{6}\right) = \sqrt{3}/2$$

9. a) $\lim_{z \to 2i} (iz^4 + 3z^2 - 10i)$

$= i(2i)^4 + 3(2i)^2 - 10i$

$= i(16i^4) + 3(4i^2) - 10i$

$= 16i^5 + 12i^2 - 10i$ $\qquad i^5 = i^2 \cdot i^2 \cdot i = i$

$= 16i - 12 - 10i$

$= 6i - 12$

b) $\lim_{z \to i/2} \frac{(2z-3)(4z+i)}{(iz-1)^2}$

$= \frac{(i-3)\ (2i+i)}{i^2 \cdot i^2/4 - 2i \cdot i/2 + 1}$

$= \frac{(i-3) \cdot 3i}{1/4 + 1 + 1}$

$= \frac{3i^2 - 9i}{1/4 + 2}$

$= \frac{-3 - 9i}{9/4}$

$= (-3 - 9i) \times \frac{4}{9}$

$= -\frac{12}{9} - \frac{36i}{9}$

$= -4/3 - 4i$

10. i) $(1+\sqrt{3}\,i)^{2011}$

$z = (1+\sqrt{3}\,i)$

$r = \sqrt{(1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right| = \pi/3 + 2n\pi$

z in exponential form $= 2e^{\pi/3\, i + 2n\pi}$

Now, $(1+\sqrt{3}\,i)^{2011} = 2^{2011} \cdot e^{2011 \cdot (\pi/3\, i + 2n\pi)}$

$= 2^{2011} \cdot \left\{ \cos\left(\frac{2011\pi}{3} + 4022n\pi\right) + i \sin\left(\frac{2011\pi}{3} + 4022n\pi\right) \right\}$

$$= 2^{2011} \cdot \left\{ \cos\left(\frac{2011\pi}{3}\right) + i \sin\left(\frac{2011\pi}{3}\right) \right\} \quad [n=0]$$

$$= 2^{2011} \cdot \left\{ \frac{1}{2} + i \cancel{\sin} \frac{\sqrt{3}}{2} \right\}$$

$$= \frac{2^{2011}}{2} + \frac{\sqrt{3} \cdot 2^{2011}}{2} i$$

$$= 2^{2010} + \sqrt{3} \cdot 2^{2010} i$$

11. a) $Re(\bar{z} - 1) = 2$

We know,

$$z = x + iy$$

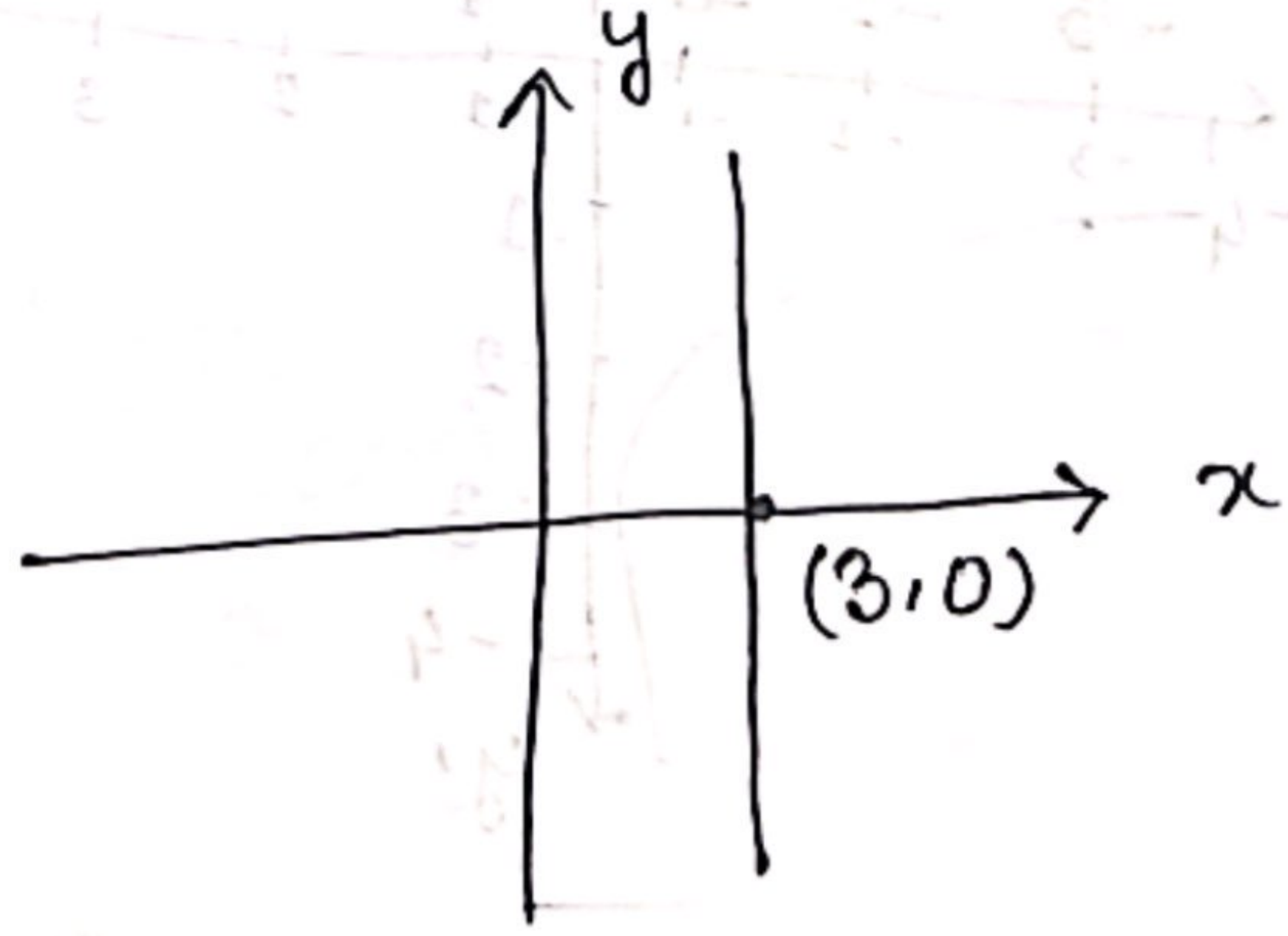
$$\bar{z} = x - i \cdot y$$

$$Re(x - iy - 1) = 2$$

$$\Rightarrow x - 1 = 2$$

$$\Rightarrow x = 2 + 1$$

$$\therefore x = 3$$



b) $Im(z^2) = 4$

We know,

$$z = x + iy$$

Now,

$$Im(z^2) = 4$$

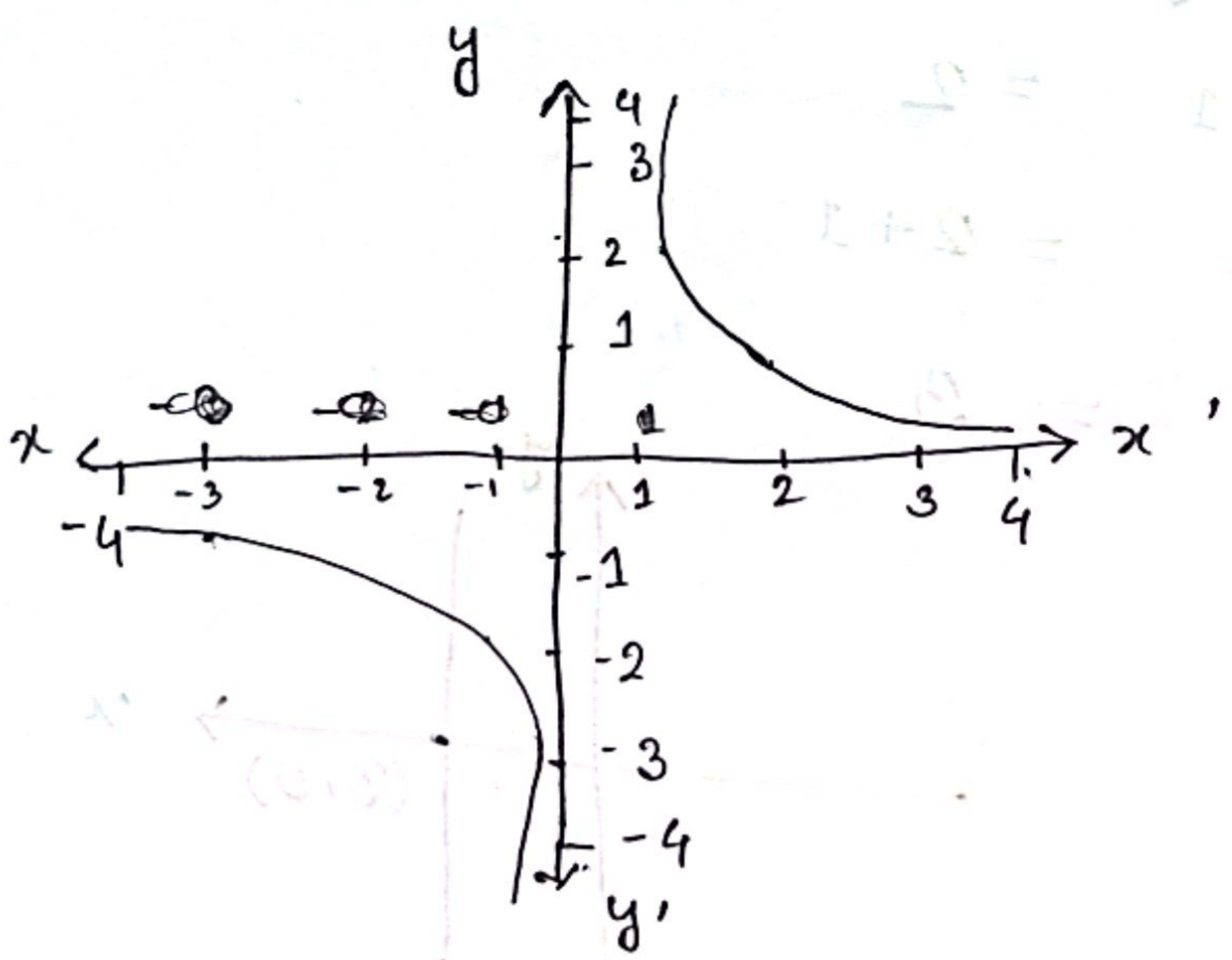
$$\Rightarrow Im\{(x+iy)^2\} = 4$$

$$\Rightarrow Im(x^2 + 2xyi - y^2) = 4$$

$$\Rightarrow 2xy = 4$$

$$\Rightarrow y = 2/x$$

x	-1	1	-2	2	-3	3	-4	4
y	-2	2	-1	1	$-2/3$	$2/3$	$-\frac{1}{2}$	$\frac{1}{2}$



12. 1) $\cosh z = 1/2$

$$\Rightarrow \frac{1}{2}\left(e^{z} + e^{-z}\right) = 1/2$$

$$\Rightarrow e^{z} + e^{-z} = 1$$

$$\Rightarrow (e^{z})^{2} + 1 = e^{z}$$ [multiply by e^{z} to avoid the -ve power]

$$\Rightarrow (e^{z})^{2} - e^{z} + 1 = 0$$

$$\Rightarrow e^{z} = \frac{1 \pm \sqrt{1-4}}{2}$$

$$\Rightarrow e^{z} = \frac{1 \pm i\sqrt{3}}{2}$$

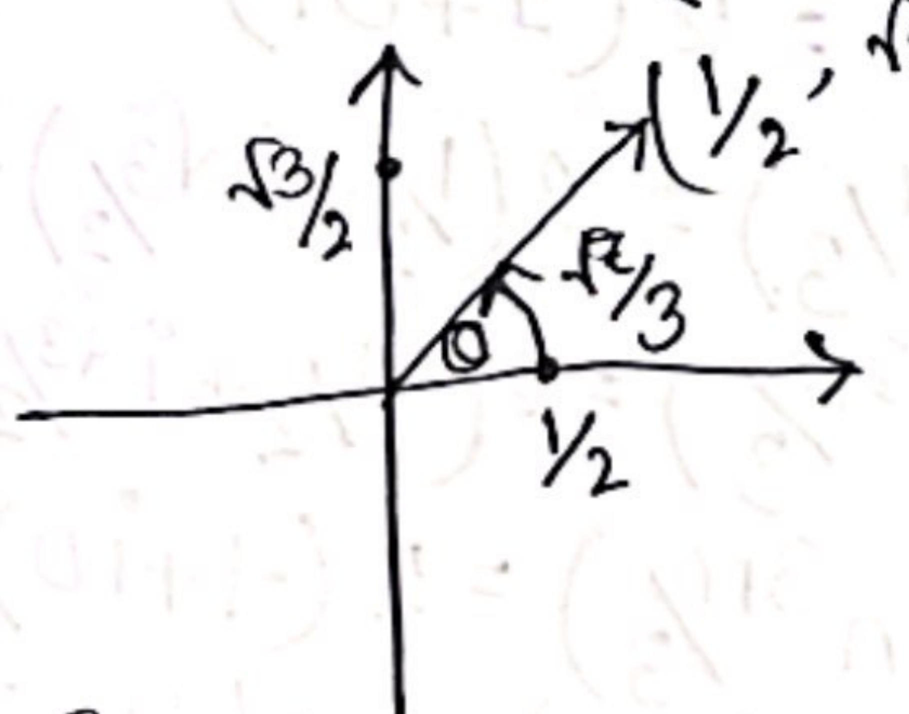
$$\Rightarrow e^{z} = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

Consider the complex number $\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

for $e^{z} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$$r = \sqrt{\left(\frac{1}{2}\right)^{2} + \left(\frac{\sqrt{3}}{2}\right)^{2}} = 1$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \pi/3$$

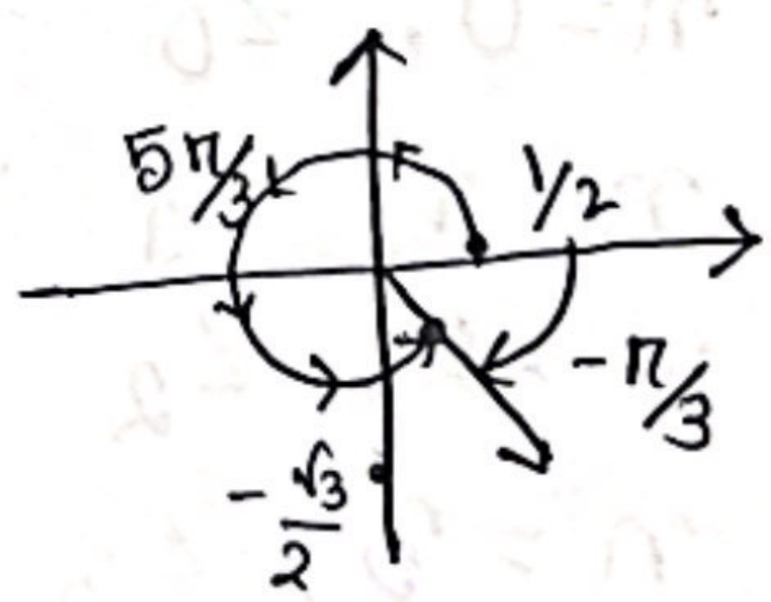


for $e^{z} = 1/2 - i\frac{\sqrt{3}}{2}$

$$r = \sqrt{(1/2)^{2} + (-\sqrt{3}/2)^{2}} = 1$$

$$\theta = -\tan^{-1}\left(-\frac{\sqrt{3}/2}{1/2}\right) = -\pi/3$$

$$= 2\pi - |-\pi/3| = \frac{5\pi}{3}$$



$\theta = \pi/3$

$\theta_a = 2\pi - |\theta|$

$= 2\pi - \theta$

For,

$$e^{z} = \frac{1}{2} + \frac{i\sqrt{3}}{2} = re^{i(\theta + 2n\pi)}, \; n \geq 0$$

$$= 1e^{i(\pi/3 + 2n\pi)}$$

$$e^{z} = e^{i(\pi/3 + 2n\pi)}$$

$$\Rightarrow z = i(\pi/3 + 2n\pi) = \pi i(1/3 + 2n); \; n \geq 0$$

for, $e^z = \frac{1}{2} - \frac{i\sqrt{3}}{2} = re^{i(\theta+2n\pi)}$, $n \geq 0$

$= 1e^{i\left(\frac{5\pi}{3}+2n\pi\right)}$

$\Rightarrow e^z = e^{i\left(\frac{5\pi}{3}+2n\pi\right)}$

$\Rightarrow z = i\left(\frac{5\pi}{3}+2n\pi\right)$

$= \pi i\left(5/3+2n\right)$; $n \geq 0$

13. 1) $z^6 = 64$

$\Rightarrow \cancel{8}z = (64)^{1/6} = (64+0i)^{1/6}$

$x = 64$, $y = 0$ $r = \sqrt{(64)^2} = 64$

$\tan^{-1}\left(\frac{0}{64}\right) = 0°$

$z = r^{1/6}\left(\cos\frac{1}{6}(\theta+2n\pi) + i\sin\frac{1}{6}(\theta+2n\pi)\right)$

$= (64)^{1/6}\left(\cos\frac{1}{6}(0+2n\pi) + i\sin 1/6(0+2n\pi)\right)$

$= 2\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)$

6 roots means $n = 0, 1, 2, 3, 4, 5$

$n=0$, $z_0 = 2(\cos 0° + i\sin 0°) = 2(1+0i) = 2$

$n=1$, $z_1 = 2(\cos \pi/3 + i\sin \pi/3) = 2(1/2 + i\sqrt{3}/2) = 1+i\sqrt{3}$

$n=2$, $z_2 = 2(\cos 2\pi/3 + i\sin 2\pi/3) = 2(-1/2 + i\sqrt{3}/2) = -1+i\sqrt{3}$

$n=3$, $z_3 = 2(\cos 3\pi/3 + i\sin 3\pi/3) = 2(-1+i\cdot 0) = -2$

$n=4$, $z_4 = 2(\cos 4\pi/3 + i\sin 4\pi/3) = 2(-1/2 - \sqrt{3}/2\, i) = -1-i\sqrt{3}$

$n=5$, $z_5 = 2(\cos 5\pi/3 + i\sin 5\pi/3) = 2(1/2 - \sqrt{3}/2\, i) = 1-i\sqrt{3}$

14. $f(z) = \begin{cases} \dfrac{z^2+4}{z-2i}, & z \neq 2i \\ 3+4i, & z = 2i \end{cases}$

a) $\lim\limits_{z \to i} f(z)$

$= \lim\limits_{z \to i} \dfrac{z^2+4}{z-2i}$

$= \dfrac{i^2+4}{i-2i} = \dfrac{3}{-i} = -\dfrac{3}{i} \times \dfrac{i}{i} = 3i$

$\therefore \lim\limits_{z \to i} f(z)$ exists and value is $3i$.

b) $\lim\limits_{z \to 2i} f(z) = \lim\limits_{z \to 2i} \dfrac{z^2+4}{z-2i}$

$= \cancel{\dfrac{4i^2+4}{2i}} \quad \dfrac{2z}{1} = \dfrac{2 \cdot 2i}{1} = 4i$

but function value $f(2i) = 3+4i$

limit $\neq$ function

$\therefore$ Not continuous at $z = 2i$.

c) $f(z) = \begin{cases} \dfrac{z^2+4}{z-2i}, & z \neq 2i \\ 3+4i, & z = 2i \end{cases}$

continuous at all points ~~Other than~~

~~2i because~~ where the denominator is non zero.

We considering only $z \neq 2i$, the denominator $z-2i \neq 0$. so, function is continuous.

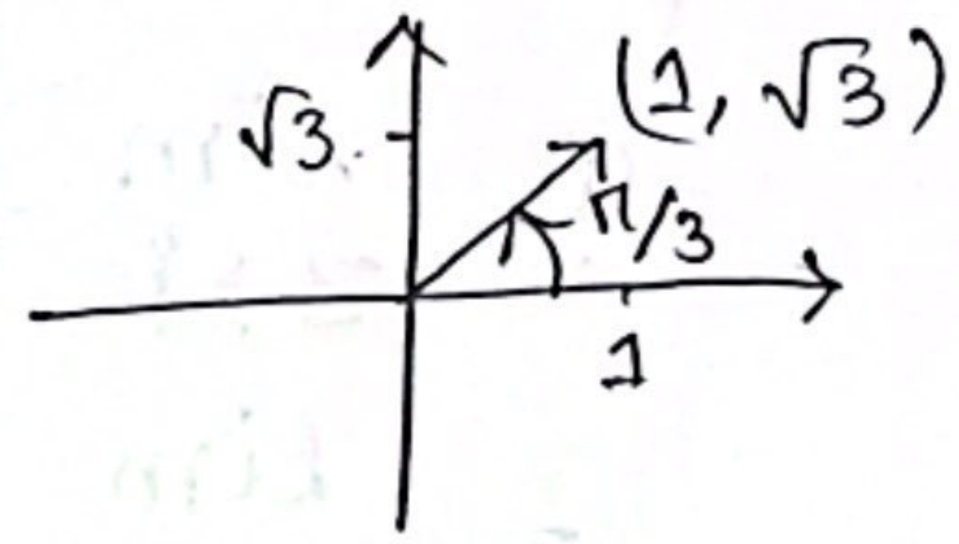
15.1) $e^z = 1+\sqrt{3}\,i$

consider $e^z = 1+\sqrt{3}\,i$ (Given)

$1+\sqrt{3}\,i$ is a complex number

$r = \sqrt{1^2+(\sqrt{3})^2} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \pi/3$



$1+\sqrt{3}\,i = re^{i(\theta+2n\pi)}$ $\quad n \geq 0$

$e^z = 2e^{(\pi/3+2n\pi)i}$

$\Rightarrow e^z = 2e^{(2n+1/3)\pi i}$

$\Rightarrow \ln e^z = \ln\left[2e^{(2n+1/3)\pi i}\right]$ [$\because$ take ln on both sides]

$\Rightarrow z = \ln 2 + \ln e^{(2n+1/3)\pi i}$

$\therefore z = \ln 2 + (2n+1/3)\pi i$

16. 1) $\lim\limits_{z\to 2i} \dfrac{z^2+4}{2z^2+3z-4iz-6i}$

$= \lim\limits_{z\to 2i} \dfrac{2z}{4z+3-4i}$

$= \dfrac{4i}{8i+3-4i}$

$= \dfrac{4i}{3+4i}$

$= \dfrac{4i(3-4i)}{9-16i^2} = \dfrac{12i-16i^2}{9+16} = \dfrac{16+12i}{25}$

$= \dfrac{16}{25} + \dfrac{12}{25}i$

11) $\lim\limits_{z \to 0} \dfrac{z - \sin z}{z^3}$

$= \lim\limits_{z \to 0} \dfrac{1 - \cos z}{3z^2}$

$= \lim\limits_{z \to 0} \dfrac{\sin z}{6z}$

$\cancel{= \lim\limits_{z \to 0} \dfrac{\cos z}{6}}$ $= \dfrac{1}{6} \lim\limits_{z \to 0} \dfrac{\sin z}{z}$ $\left[\because \lim\limits_{z \to 0} \dfrac{\sin z}{z} = 1\right]$

$\cancel{= \dfrac{\cos 0^\circ}{6}}$ $= \dfrac{1}{6}$

$\cancel{= 1/6}$.

17. 11) $\left(\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^2$

$= \dfrac{1 + 2\sqrt{3}i + 3i^2}{1 - 2\sqrt{3}i + 3i^2}$

$= \dfrac{-2 + 2\sqrt{3}i}{-2 - 2\sqrt{3}i}$

$= \dfrac{-(1-\sqrt{3}i)}{-(1+\sqrt{3}i)}$

$= \dfrac{(1-\sqrt{3}i)(1-\sqrt{3}i)}{1 - 3i^2}$

$= \dfrac{1 + 2\sqrt{3}i + 3i^2}{4}$

$= \dfrac{-2 - 2\sqrt{3}i}{4} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$z = -1/2 - \sqrt{3}/2\, i$

modulus:

$$|z| = \sqrt{(-1/2)^2 + (-\sqrt{3}/2)^2}$$

$$= \sqrt{1/4 + 3/4}$$

$$= 1$$

Argument:

$$\theta = -\pi + \tan^{-1}\left|\frac{-\sqrt{3}/2}{-1/2}\right|$$

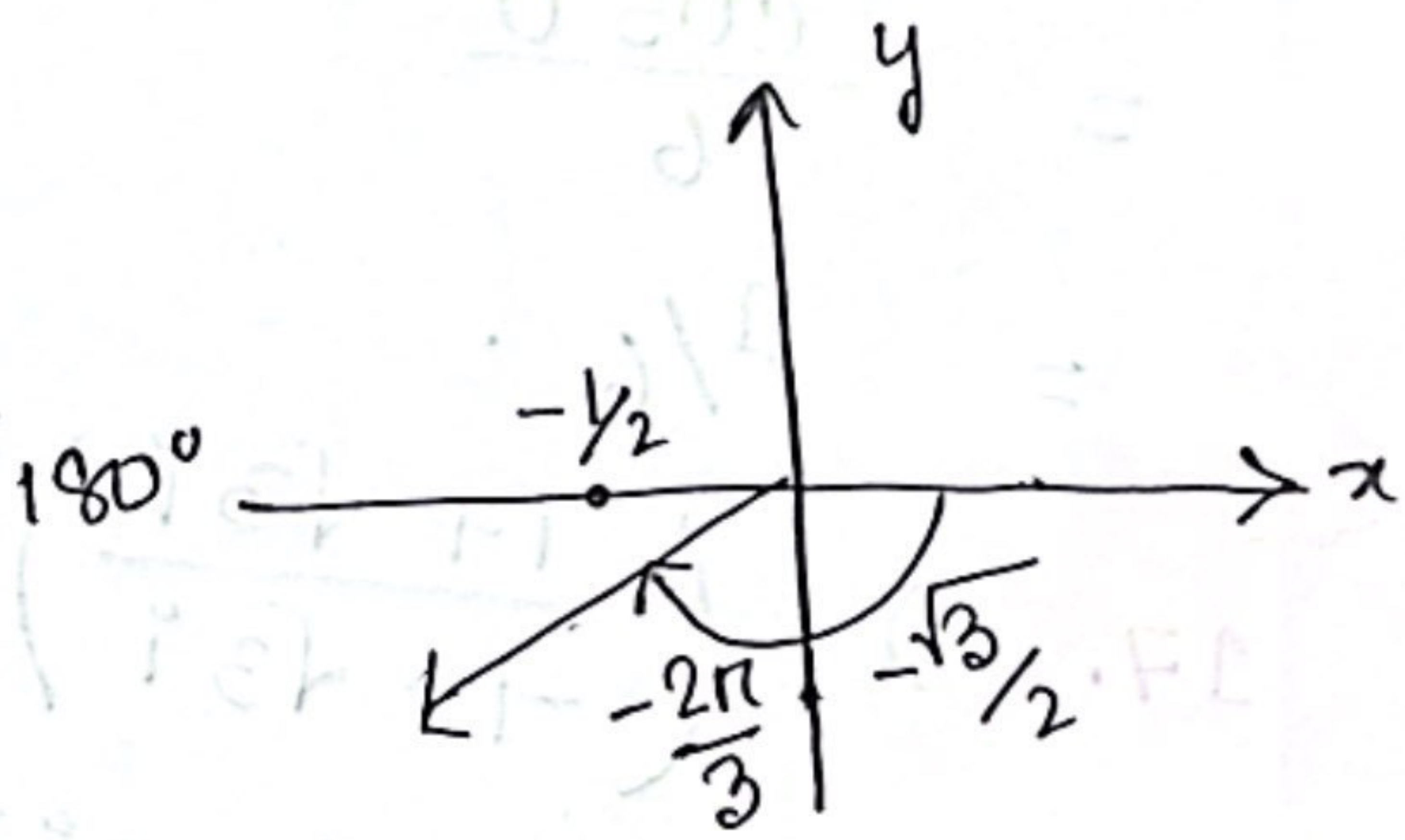
$$= -\pi + \pi/3$$

$$\neq \cancel{2\pi - \pi/3}$$

$$\neq \cancel{5\pi/3}$$

$$= \frac{-3\pi + \pi}{3}$$

$$= \frac{-2\pi}{3}$$



18. $f(2) = K \cdot 2^2 + 6 = 4K + 6$

$$\lim_{z \to 2} f(z) = \lim_{z \to 2} \frac{z^2 - 4}{z^2 - 3z + 2}$$

$$= \lim_{z \to 2} \frac{2z}{2z - 3}$$

$$= \frac{2 \cdot 2}{2 \cdot 2 - 3} = 4$$

For continity,

Function value = limit

$\Rightarrow 4K + 6 = 4$

$\Rightarrow 4K = -2$

$\Rightarrow K = -1/2$