



Inspiring Excellence

Department of Mathematics and Natural Sciences

Semester: Spring 2024

Midterm Examination

Course Title: MATHMATICS III: COMPLEX VARIABLES & LAPLACE TRANSFORMATIONS

Course Code: MAT 215

Total marks: 40

Date: March 9, 2024

Times: 1 hour

Answer any FOUR:

Q1. [3+3+4]

- Find the modulus and argument of $\frac{2-i}{2+i}$
- Sketch the region in xy -plane represented by $\text{Im}(z^2) = 4$
- Evaluate $\lim_{z \rightarrow 0} \left(\frac{\sin z}{z} \right)^{1/z^2}$

Q2. [5+5]

- Show that the function $\sinh z$ is analytic.
- Show that $\sin^{-1} z = -i \ln[iz \pm (1 - z^2)^{\frac{1}{2}}]$.

Q3. [10]

Determine whether the following function u is harmonic. For harmonic function, find the conjugate harmonic function v and express $u + iv$ as an analytic function of z .

$$u = e^{-x}(x \sin y - y \cos y)$$

Q4. [5+5]

- Find all the values of z for which $z^5 = (-4 + 4i)$.
- If $f(z) = \begin{cases} \frac{z^2-4}{z^2-3z+2}, & z \neq 2 \\ kz^2 + 6, & z = 2 \end{cases}$, find k such that $f(z)$ becomes continuous at $z = 2$.

Q5.

[5+5]

- a. Let $f(z) = \frac{2z-1}{3z+2}$. Prove that $\lim_{h \rightarrow 0} \frac{f(z_0+h)-f(z_0)}{h} = \frac{7}{(3z_0+2)^2}$ provide $z_0 \neq -\frac{2}{3}$.
- b. If $(1+i)^{2024} = x + iy$, then find (x, y) .