BRAC
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Department of Mathematics and Natural Sciences

Semester: Fall 2022

Midterm Examination

Course Title: Mathematics for Machine Learning & Signal Processing

Course Code: MAT 215, Section: 05

Total marks: 40   Date: November 7, 2022

Times: 1 hour (11.5 am -12.5 pm)   Room: UB70303

Answer any FOUR including question 1:

Q1.   [3+4+3]

   a. Find all the values of z for which $z^5 = -32$.

   b. Locate these value in the complex plane.

   c. State the necessary and sufficient conditions for being a function analytic. Hence show that the function $\sin z$ is analytic.

Q2.   [3+4+3]

   a. Solve the equation $z^2 + (2i - 3)z + 5 - i = 0$.

   b. If $\left(\frac{3}{2} + \frac{i\sqrt{3}}{2}\right)^{50} = 3^{25}(x + iy)$, where x and y are reals then find the values of x and y.

   c. Evaluate $\lim\limits_{z \to 2e^{\frac{\pi i}{3}}} \dfrac{z^3}{z^4 + 4z^2 + 16}$.

Q3.   [3+4+3]

   a. Show that $\sin^{-1}z = -i\,log[iz + (1 - z^2)^{\frac{1}{2}}]$.

   b. Find $\lim\limits_{z \to 0} \left(\dfrac{\sin z}{z}\right)^{\frac{1}{z^2}}$.

   c. Determine whether the following function u is harmonic. For harmonic function, find the conjugate harmonic function v and express $u + iv$ as an analytic function of z.

$$e^{-2xy}\sin(x^2 - y^2)$$

Q4. [3+4+3]

   a. Show that $\log(-1+i)^2 \neq 2\log(-1+i)$.

   b. Sketch the set of point determine by the given condition: $|z+i| \leq 3$.

   c. Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic.

Q5. [3+4+3]

   a. Suppose $f(z) = \begin{cases} z^2 + 2z \text{ if } z \neq i \\ 3 + 2i \text{ if } z = i \end{cases}$; Find $\lim\limits_{z \to i} f(z)$ and justify your answer.

   b. Let $f(z) = \frac{2z-1}{3z+2}$. Prove that $\lim\limits_{h \to 0} \frac{f(z_0+h)-f(z_0)}{h} = \frac{7}{(3z_0+2)^2}$ provide $z_0 \neq -\frac{2}{3}$.

   c. If the equation $\frac{|z-i|}{|z+i|} = 3$ represent a circle in the complex plane, then find the radius and the center of the circle.