Important Questions for Class 12

Physics

Chapter 6 - Electromagnetic Induction

Very Short Answer Questions **1 Mark**

1. A metallic wire coil is stationary in a non – uniform magnetic field. What is the emf induced in the coil?

Ans: No emf is induced in the coil because there is no change in the magnetic flux linked with the secondary coil.

2. Why does metallic piece become very hot when it is surrounded by a coil carrying high frequency (H.F) alternating current?

Ans: When a metallic piece is surrounded by a coil carrying high frequency (H.F) alternating current, it becomes hot because eddy currents are produced which in turn produces joule's heating effect.

3. An electrical element X when connected to an alternating voltage source has current through it leading the voltage by $\frac{\pi}{2}$ radian. Identify X and write expression for its reactance.

Ans: The X is a purely capacitive circuit.

The expression for capacitive reactance is $X_c = \frac{1}{\omega C} = \frac{1}{2\pi \upsilon c}$.

4. A transformer steps up $220V$ to $2200V$. What is the transformation ratio?

Ans: The transformation ratio, $K = \frac{N_S}{N_P} = \frac{E_S}{E_P} = \frac{2200}{220} = 10$

Therefore, the transformation ratio is 10.

5. The induced emf is also called back emf . Why?

Ans: It is because induced emf produced in a circuit always opposes the cause which produces it.

6. Why the oscillations of a copper disc in a magnetic field are lightly damped?

Ans: Copper disc oscillates because of the production of eddy currents which opposes its oscillating motion and as a result the motion gets damped.

7. A metallic wire coil is stationary in a non – uniform magnetic field. What is the emf. Induced in the coil?

Ans: No emf is induced in the coil as there is no change in the magnetic flux linked with the secondary coil.

Short Answer Questions 2 Marks

1. If the rate of change of current of $2A/s$ induces an emf of $10mV$ in a solenoid. What is the self-inductance of the solenoid?

Ans: Self-inductance is given by,

$$L=\frac{\varepsilon}{dI/dt}=\frac{10\times 10^{-3}}{2}=5\times 10^{-3}H$$

Therefore, self-inductance of the solenoid is $5\times 10^{-3}H$.

2. A circular copper disc. $10cm$ in radius rotates at a speed of $2\pi rad/s$ about an axis through its centre and perpendicular to the disc. A uniform magnetic field of $0.2T$ acts perpendicular to the disc.

a) Calculate the potential difference developed between the axis of the disc and the rim.

Ans: Given, radius $=10cm$, $B=0.2T$, $\omega=2\pi rad/s$

$$\varepsilon=\frac{1}{2}Bwr^2$$

$$\Rightarrow \varepsilon = \frac{1}{2} \times 0.2 \times 2\pi \times (0.1)^2$$

$$\therefore \varepsilon = 0.00628\text{volts}$$

Therefore, the potential difference developed is 0.00628volts.

b) What is the induced current if the resistance of the disc is 2Ω?

Ans: $I = \frac{\varepsilon}{R} = \frac{0.0628}{2}$

$$\therefore I = 0.0314\text{A}$$

If the resistance is 2Ω, the induced current is found to be 0.0314A.

3. An ideal inductor consumes no electric power in a.c. circuit. Explain.

Ans: Power consumed by inductor in a.c. circuit, $P = \text{ErmsIrms}\cos\phi$

But for an ideal inductor, $\phi = \frac{\pi}{2}$

$$\Rightarrow \cos\phi = \cos\frac{\pi}{2} = 0$$

$$\therefore P = 0$$

Therefore, an ideal inductor in a.c. circuit does not consume power.

4. Capacitor blocks d.c. why?

Ans: The capacitive reactance, $X_c = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon c}$

For d.c., $\upsilon = 0$

$$\therefore X_c = \propto$$

Since capacitor offers infinite resistance to d.c. flow, d.c. cannot pass through it.

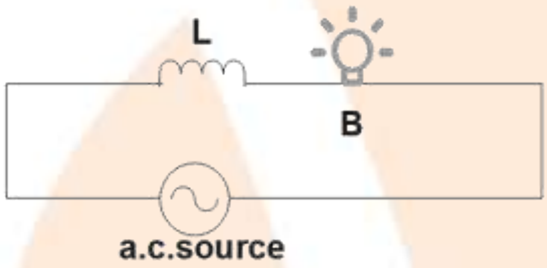
5. Why is the emf zero, when maximum number of magnetic lines of force pass through the coil?

Ans: The vertical position of the coil offers maximum magnetic flux.

But as the coil rotates, $\frac{d\phi}{dt} = 0$

Therefore, produced emf, $\varepsilon = \frac{d\phi}{dt} = 0$.

6. An inductor L of reactance X_L is connected in series with a bulb B to an a.c. source as shown in the figure.



Briefly explain how does the brightness of the bulb change when

(a) Number of turns of the inductor is reduced.

Ans: We know, $Z = \sqrt{R^2 + {X_L}^2}$

When number of turns of the inductor gets reduced X_L and Z decreases and in turn current increases. Because of this the bulb will grow more brightly.

(b) A capacitor of reactance $X_C = X_L$ is included in series in the same circuit.

Ans: When capacitor is included in the circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

But, given that, $X_C = X_L$

That implies at minimum, $Z = R$

Therefore, the brightness of the bulb will become maximum.

7. A jet plane is travelling towards west at a speed of $1800 km/hr$. What is the voltage difference developed between the ends of the wing having a span of $25m$, if the Earth's magnetic field at the location has a magnitude of $5 \times 10^{-4} T$ and the dip angle is 30°?

Ans: Given that, speed of the jet plane, $v = 1800 km/h = 500 m/s$

Wingspan of jet plane, $l = 25\text{m}$

Earth's magnetic field strength, $B = 5 \times 10^{-4}\text{T}$

Angle of dip, $\delta = 30^{\circ}$

Vertical component of Earth's magnetic field,

$B_v = B\sin\delta$

$\Rightarrow B_v = 5 \times 10^{-4}\sin 30^{\circ}$

$\Rightarrow 2.5 \times 10^{-4}\text{T}$

So, voltage difference between the ends of the wing,

$e = (B_v) \times l \times v$

$\Rightarrow e = 2.5 \times 10^{-4} \times 25 \times 500$

$\therefore 3.125\text{V}$

Therefore, the voltage difference developed between the ends of the wings is 3.125V.

8. A pair of adjacent coils has a mutual inductance of 1.5H. If the current in one coil changes from 0 to 20A in 0.5s, what is the change of flux linkage with the other coil?

Ans: Given that, mutual inductance of a pair of coils, $\mu = 1.5\text{H}$

Initial current, $I_1 = 0\text{A}$

Final current, $I_2 = 20\text{A}$

Change in current, $dI = I_2 - I_1 = 20 - 0 = 20\text{A}$

Time taken for the change, $dt = 0.5\text{s}$

Induced emf, $e = \frac{d\phi}{dt}$ .....(1)

Where, $d\phi$ is the change in the flux linkages with the coil.

Emf is related with mutual inductance as, $e = \mu\frac{dI}{dt}$ .....(2)

Equating equations (1) and (2),

$$\Rightarrow \frac{d\phi}{dt} = \mu \frac{dl}{dt}$$

$$\Rightarrow d\phi = 1.5 \times (20)$$

$$\therefore d\phi = 30\text{Wb}$$

Therefore, the change in the flux linkage is 30Wb.

9. A horizontal straight wire 10m long extending from east to west is falling with a speed of 5.0ms^{-1}, at right angles to the horizontal component of the earth's magnetic field, $0.30 \times 10^{-4}\text{Wbm}^{-2}$.

(a) What is the instantaneous value of the emf induced in the wire?

Ans: Given that, length of the wire, $l = 10\text{m}$

Falling speed of the wire, $v = 5.0\text{m/s}$

Magnetic field strength, $B = 0.3 \times 10^{-4}\text{Wbm}^{-2}$

Emf induced in the wire, $e = Blv$

$$\Rightarrow e = 0.3 \times 10^{-4} \times 5 \times 10$$

$$\therefore e = 1.5 \times 10^{-3}\text{V}$$

Therefore, instantaneous emf induced is $1.5 \times 10^{-3}\text{V}$.

(b) What is the direction of the emf?

Ans: The direction of the induced emf is from West to East by Fleming's right-hand rule.

(c) Which end of the wire is at the higher electrical potential?

Ans: The eastern end of the wire is at higher potential.

10. A 1.0ms long metallic rod is rotated with an angular frequency of 400rads^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Ans: From the given data, Length of the rod, $l = 1m$

Angular frequency, $\omega = 400 rad/s$

Magnetic field strength, $B = 0.5T$

One end of the rod has zero linear velocity, while the other end has a linear velocity of $l_{(t)}$.

Average linear velocity of the rod, $v = \frac{l_{(t)} + 0}{2} = \frac{l_{(t)}}{2}$

Emf developed between the centre and the ring,

$$e = Blv = Bl\left(\frac{l_{(t)}}{2}\right) = \frac{Bl^2(i)}{2}$$

$$\Rightarrow e = \frac{0.5 \times (l)^2 \times 400}{2} = 100V$$

Therefore, the emf developed between the centre and the ring is $100V$.

11. Use Lenz's law to determine the direction of induced current in the situations described by Figure:

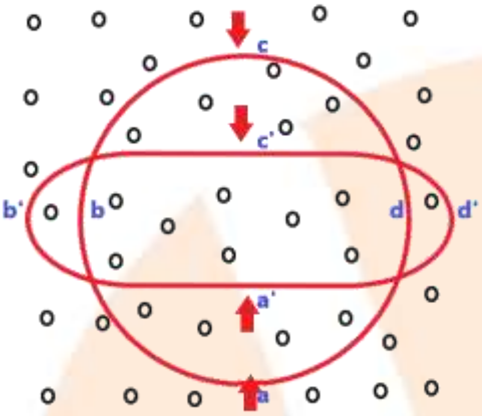
a) A wire of irregular shape turning into a circular shape:

Ans: According to Lenz's law, the direction of the induced emf is such that it tends to produce a current that would oppose the change in the magnetic flux that produced it.

The wire here is expanding to form a circle, which means that force would be acting outwards on each part of wire because of the magnetic field (acting in the ownwards direction). Now, the direction of induced current should be such that it will produce magnetic field in the upward direction (towards the reader).

Therefore, the force on wire will be towards inward direction, i.e., induced current would be flowing in anticlockwise direction in the loop from cbad.

b) A circular loop being deformed into a narrow straight wire.



Ans: On deforming the shape of a circular loop into a narrow straight wire, the flux piercing the surface decreases. Therefore, the induced current flows along abcd according to Lenz's law.

Long Answer Questions **3 Marks**

1. How is the mutual inductance of a pair of coils affected when

a) Separation between the coils is increased.

Ans: When the Separation between the coils is increased, the flux linked with the secondary coils decrease. Therefore, the mutual induction decreases.

b) The number of turns of each coil is increased.

Ans: As mutual inductance, $M = \frac{\mu_0 N_1 N_2 A}{l}$;

Therefore, when N_1 and N_2 increases, the mutual induction also increases.

c) A thin iron sheet is placed between two coils, other factors remaining the same. Explain the answer in each case.

Ans: As mutual inductance, $M \propto \mu_r$ (Relative permeability of material).

Therefore, mutual induction will increase.

2. Distinguish between resistances, reactance and impedance of an a.c. circuit?

Ans: They can be differentiated as:

Resistance	Reactance	Impedance
Opposition offered by the resistor to the flow of current.	Opposition offered by the inductor or capacitor to the flow of current.	Opposition offered by the combination of resistor, inductor or capacitor.
It is independent of the frequency of the source.	It depends on the frequency of the source.	It depends on the frequency of the source.

3. A sinusoidal voltage, $V = 200\sin 314t$ is applied to a resistor of 10Ω resistance. Calculate:

a) rms value of the voltage

Ans: Given that, $V = 200\sin 314t$

$V = V_0 \sin wt$

$V_0 = 200V$

$w = 314 rad/s$

$R = 10\Omega$

We have,

$V_{rms} = \sqrt{2}V_0$

$\therefore V_{rms} = \sqrt{2} \times 200 = 282.8V$

Therefore, the rms value of voltage is $282.8V$.

b) rms value of the current

Ans: Rms value of current, $I_{rms} = \frac{V_{rms}}{R} = \frac{282.8}{10}$

$\therefore I_{rms} = 28.28A$

Therefore, the rms value of current is $28.28A$.

c) Power dissipated as heat in watt.

Ans: Because of purely resistive circuit, flux, $\phi = 0$

We have,

$P = E_v I_v \cos\phi = E_V I_V \cos 0$

$\Rightarrow P = E_v I_v = 282.8 \times 28.28$

$\therefore P = 7.998 \text{watt}$

Therefore, the power dissipated is 7.998watt.

4. Obtain an expression for the self-inductance of a long solenoid. Hence define one henry.

Ans: Consider current I flowing through a long solenoid of area A,

Let N be the total number of turns in the solenoid,

Total flux, $\phi = NBA$

Here, $B = \mu_0 nI$

Where, n is no. of turns per unit length of the solenoid

$N = nl$

$\Rightarrow \phi = nl \times \mu_0 nIA$

$\Rightarrow \phi = \mu_0 n^2 AlI$ ....(1)

Also, $\phi = LI$ ....(2)

From equation (1) & (2)

$$\mu_0 n^2 A l \cancel{I} = L \cancel{I}$$

$$\Rightarrow L = \mu_0 n^2 A l$$

$$\Rightarrow L = \frac{\mu_0 N^2 A}{l} \quad \text{where,} [n = N / l]$$

One henry(1H) can be defined as: If current is changing at a rate of $1A/s$ in a coil inducing an emf of 1volt in it then the inductance of the coil is one henry.

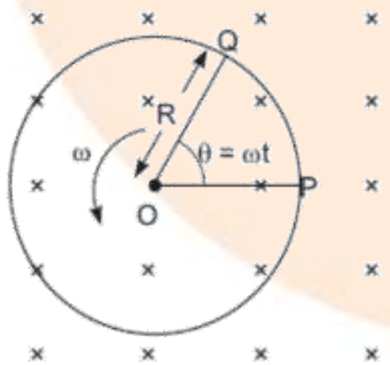
5. A conducting rod rotates with angular speed w with one end at the centre and other end at circumference of a circular metallic ring of radius R, about an axis passing through the centre of the coil perpendicular to the plane of the coil A constant magnetic field B parallel to the axis is present everywhere. Show that the emf. between the centre and the metallic ring is $\frac{1}{2} B w R^2$.

Ans: If a circular loop connects the centre with point P with a resistor,

The potential difference across the resistor $=$ induced emf.

$\varepsilon = B \times$ Rate of change of area of loop

If the resistor QP is rotated with angular velocity ω and turns by an angle θ in time t then,



Area swept,

$$A = \frac{1}{2} \times R \times R\theta$$

$$\Rightarrow A=\frac{1}{2}R^2\theta$$

Now, we have,

$$\phi = BA\cos\theta^\circ = BA$$

$$\Rightarrow \phi = B\times\frac{1}{2}R^2\theta$$

So,

$$\varepsilon=\frac{d\phi}{dt}=\frac{d}{dt}\left(\frac{1}{2}BR^2\theta\right)=\frac{1}{2}BR^2\left(\frac{d\theta}{dt}\right)$$

$$\therefore \varepsilon=\frac{1}{2}B\omega R^2$$

Therefore, it is proved that the emf. between the centre and the metallic ring is $\frac{1}{2}B\omega R^2$.

6.

(a) At a very high frequency of a.c., capacitor behaves as a conductor. Why?

Ans: As capacitive Reactance, $X_C=\frac{1}{2\pi\upsilon c}$

For a.c. when $\upsilon\to\infty X_C=0$

Therefore, at a very high frequency of a.c. capacitor behaves as a conductor.

(b) Draw the graph showing the variation of reactance of

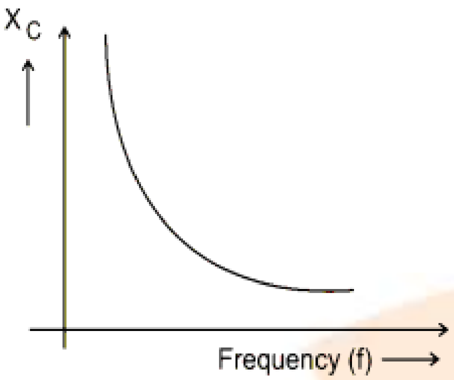
(i) A capacitor

(ii) An inductor with a frequency of an a.c. circuit.

Ans: As capacitive Reactance, $X_C=\frac{1}{2\pi\upsilon c}$

$$\Rightarrow X_C\propto\frac{1}{\upsilon}$$

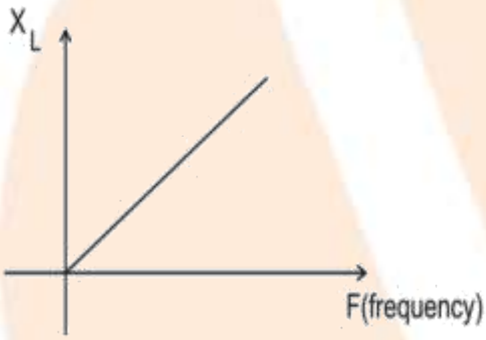
So, the corresponding graph would be:



And, inductive reactance, $X_L = \omega L = 2\pi \upsilon L$

$\Rightarrow X_L \propto \upsilon$

Therefore, the corresponding graph here will be,



7. Calculate the current drawn by the primary of a transformer which steps down $200V$ to $20V$ to operate a device of resistance 20Ω. Assume the efficiency of the transformer to be 80%.

Ans: Given that, $\eta = 80\%$

$E_p = 200V$

$E_s = 20V$

$Z = 20\Omega$

$$I_s = \frac{E_s}{Z} = \frac{20}{20} = 1A$$

Now, we have,

$$\eta = \frac{E_s I_s}{E_p I_p}$$

Substituting the given values,

$$\eta = \frac{80}{100} = \frac{20 \times 1}{200 \times I_p}$$

$$\Rightarrow I_p = \frac{2000}{80 \times 200}$$

$$\therefore I_p = 0.125\text{A}$$

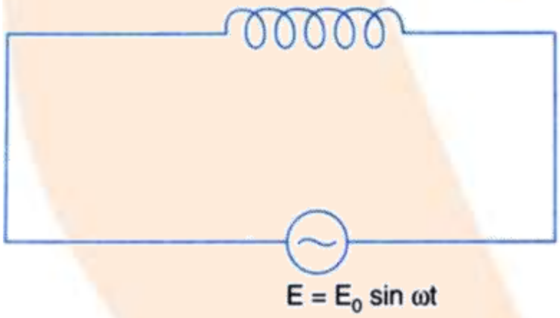
Therefore, the current drawn by primary of transformer is found to be 0.125A.

8. An a.c. voltage $E = E_0 \sin\omega t$ is applied across an inductance L. Obtain the expression for current I.

Ans: Given that, $E = E_0 \sin\omega t$

Emf produced across $L = \frac{-LdI}{dt}$

Total emf of the circuit $= E + \left(\frac{-LdI}{dt}\right)$



As there is no circuit element across whose potential drop may occur,

$$\Rightarrow E + \left(\frac{-LdI}{dt}\right) = 0$$

$$\Rightarrow E = \frac{LdI}{dt}$$

$$\Rightarrow dI = \frac{E}{L}dt$$

$$\Rightarrow dI = \frac{E_0}{L}\sin\omega t dt$$

On Integrating,

$$I = \frac{E_0}{L}\int \sin\omega t dt$$

$$\Rightarrow I = \frac{E_0}{L}\left(\frac{-\cos\omega t}{\omega}\right)$$

$$\Rightarrow I = \frac{-E_0}{wL}\cos\omega t$$

$$\Rightarrow I = \frac{E_0}{wL}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\Rightarrow I = \frac{E_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

Where, $\frac{E_0}{X_L} = I_0$ (maximum value of current)

$$\therefore I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

Therefore, the expression for current is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$.

9. A series circuit with $L = 0.12H$, $C = 0.48mF$ and $R = 25\Omega$ is connected to a $220V$ variable frequency supply. At what frequency is the circuit current maximum?

Ans: Given that, $L = 0.12H$

$C = 0.48mF = 0.48 \times 10^{-3} F$

$E_v = 220V$

$$I_v = \frac{E_v}{\sqrt{R^2 + (X_L - X_C)^2}}$$

In the circuit when I is maximum, R will be minimum

$$\Rightarrow X_L = X_C$$

$$\Rightarrow I = \frac{E_v}{R}$$

Now, we have,

$$f=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\times3.14\sqrt{0.12\times0.48\times10^{-3}}}$$

$\therefore f=21Hz$

Therefore, the frequency with maximum circuit current is $21Hz$.

10. A rectangular wire loop of sides $8cm$ and $2cm$ with a small cut is moving out of a region of uniform magnetic field of magnitude $0.3T$ directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is $1cms^{-1}$ in a direction normal to the

(a) longer side,

Ans: Given that, length of the rectangular wire, $l=8cm=0.08m$

Width of the rectangular wire, $b=2cm=0.02m$

So, area of the rectangular loop,

$A=lb=0.09\times0.02$

$\Rightarrow16\times10^{-4}m^2$

Magnetic field strength, $B=0.3T$

Velocity of the loop, $v=1cm/s=0.01m/s$

Emf developed in the loop is given as:

$e=Blv$

$\Rightarrow0.3\times0.08\times0.01=2.4\times10^{-4}V$

Time taken to travel along the width is,

$$\therefore t=\frac{\text{Distance travelled}}{\text{Velocity}}=\frac{b}{v}=\frac{0.02}{0.01}=2s$$

Therefore, the induced voltage is $2.4\times10^{-4}V$ which lasts for $2s$.

(b) shorter side of the loop? For how long does the induced voltage last in each case?

Ans: Emf developed, $e=Bbv$

$\Rightarrow0.3\times0.02\times0.01=0.6\times10^{-4}V$

Time taken to travel along the length,

$$\therefore t = \frac{\text{Distance travelled}}{\text{Velocity}} = \frac{1}{v} = \frac{0.08}{0.01} = 8s$$

Therefore, the induced voltage is $0.6\times10^{-4}V$ which lasts for $8s$.

11. Current in a circuit falls from $5.0A$ to $0.0A$ in $0.1s$. If an average emf of $200V$ induced, give an estimate of the self-inductance of the circuit.

Ans: Given that, initial current, $I_1 = 5.0A$

Final current, $I_2 = 0.0A$

Change in current, $dl = l_1 - l_2 = 5A$

Time taken for the change, $t = 0.1s$

Average emf, $e = 200V$

For self-inductance (L) of the coil, we have the relation for average emf as,

$$e = L\frac{di}{dt}s$$

$$\Rightarrow L = \frac{r}{\left(\frac{di}{dt}\right)}$$

$$\therefore \frac{200}{\frac{5}{0.1}} = 4H$$

Therefore, the self-induction of the coil is $4H$.

12. Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of $0.3T$ at the rate of $0.02Ts^{-1}$. If the cut is joined and the loop has a resistance of 1.6Ω how much power is dissipated by the loop as heat? What is the source of this power?

Ans: Given that, sides of the rectangular loop are $8cm$ and $2cm$.

So, area of the rectangular wire loop, $A = \text{length}\times\text{width}$

$\Rightarrow A = 8 \times 2 = 16cm^2$

$\Rightarrow A = 16 \times 10^{-4} m^2$

Initial value of the magnetic field, $B' = 0.3T$

Rate of decrease of the magnetic field, $\frac{dB}{dt} = 0.02T/s$

Emf developed in the loop is given as:

$$e = \frac{d\phi}{dt}$$

$d\phi =$ Change in flux through the loop area

$$\Rightarrow d\phi = d(AB)$$

$$\Rightarrow e = \frac{d(AB)}{dt} = \frac{AdB}{dt}$$

$$\Rightarrow e = 16 \times 10^{-4} \times 0.02 = 0.32 \times 10^{-4} V$$

Resistance of the loop, $R = 1.6\Omega$

The current induced in the loop is given as:

$$i = \frac{r}{R}$$

$$\Rightarrow i = \frac{0.32 \times 10^{-4}}{1.6} = 2 \times 10^{-5} A$$

Power dissipated in the loop in the form of heat is given as:

$$P = i^2 R$$

$$\Rightarrow \left(2 \times 10^{-5}\right)^2 \times 1.6$$

$$\therefore P = 6.4 \times 10^{-10} W$$

Therefore, power dissipated in loop as heat is $6.4 \times 10^{-10} W$. The source of this heat loss is an external agent, changing the magnetic field with time.

13. An air-cored solenoid with length $30cm$, area of cross-section $25cm^2$ and number of turns 500, carries a current of $2.5A$. The current is suddenly switched off in a brief time of $10^{-3}s$. How much is the average back emf

induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Ans: Given that, length of the solenoid, $l = 30cm = 0.3m$

Area of cross-section, $A = 25cm^2 = 25\times10^{-4}m^2$

Number of turns on the solenoid, $N = 500$

Current in the solenoid, $I = 2.5A$

Current flows for time, $t = 10^{-3}s$

Average back emf,

$$e = \frac{d\phi}{dt} \qquad ....(1)$$

Where,

$d\phi = \text{Change in flux}$

$$d\phi = NAB \qquad ....(2)$$

Where,

$B = \text{Magnetic field strength}$

$$B = \mu_0 \frac{NI}{l} \qquad ....(3)$$

Where,

$\mu_0 = \text{Permeablity of free space} = 4n\times10^{-7}sTmA^{-1}$

Using equations (2) and (3) in equation (1), we get,

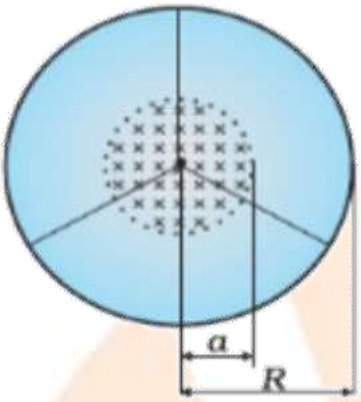
$$e = \frac{\mu_0 N^2 IA}{lt}$$

$$\therefore e = \frac{4\pi\times10^{-7}\times(500)^2\times2.5\times25\times10^{-4}}{0.3\times10^{-3}} = 6.5V$$

Therefore, the average back emf induced in the solenoid is $6.5V$.

14. A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig. 6.22). A uniform magnetic field extends over a circular region within the rim. It

is given by, $B = -B_0 k \ \left(r \le a; a < R\right) = 0$ (otherwise) What is the angular velocity of the wheel after the field is suddenly switched off?



Ans: Given that, line charge per unit length,

$$\Rightarrow \lambda = \frac{\text{Total charge}}{\text{Length}} = \frac{Q}{2\pi r}$$

Where,

$r =$ Distance of the point within the wheel

Mass of the wheel $= M$

Radius of the wheel $= R$

Magnetic field, $B = -\vec{B} = B_0 \hat{k}$

At distance r, the magnetic force is balanced by the centripetal force, that is,

$$BQv = \frac{Mv^2}{r}$$

Where

$v =$ linear velocity of the wheel

$$\Rightarrow B2\pi r\lambda r^2 = \frac{Mv}{r}$$

$$\Rightarrow v = \frac{B2\pi r\lambda r^2}{M}$$

$$\Rightarrow BQv = \frac{Mv^2}{r}$$

So Angular velocity,

$$\omega = \frac{v}{R} = \frac{B2\pi r\lambda r^2}{M}$$

For $r \leq$ and $a < R$ we get,

$$\therefore \omega = -\frac{2B_0a^2\lambda}{MR}k$$

Therefore, the angular velocity of the wheel is $-\frac{2B_0a^2\lambda}{MR}k$.

Long Answer Questions **5 Marks**

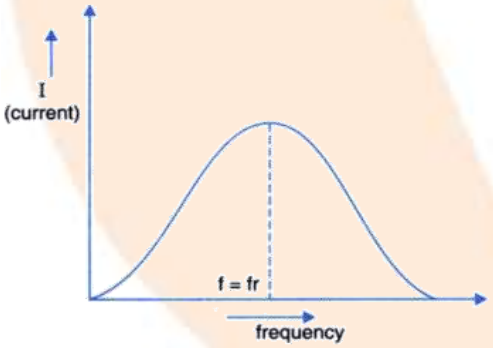
1.

(a) State the condition under which the phenomenon of resonance occurs in a series LCR circuit. Plot a graph showing the variation of current with frequency of a.c. sources in a series LCR circuit.

Ans: A series LCR circuit will have resonance when,

$$\mathbf{X_L = X_C}$$

The variation of current with frequency of a.c. source in series LCR circuit,



(b) Show that in a series LCR circuit connected to an a.c. source exhibits resonance at its natural frequency equal to $\frac{1}{\sqrt{LC}}$.

Ans: In a series LCR circuit Electrical resonance takes place when circuit allows maximum alternating current for which,

$$X_L = X_C$$

Impedance, $Z=\sqrt{R^2+(X_L-X_C)^2}$

$$I=\frac{E}{Z}-\frac{E}{\sqrt{R^2+(X_L-X_C)^2}}$$

For electrical resonance,

$$X_L=X_C$$

$$\omega L=\frac{1}{\omega C} \text{ or } \omega^2=\frac{1}{LC}$$

$$\therefore \omega=\frac{1}{\sqrt{LC}}$$

Where ω is the natural frequency of the circuit.

Therefore, a series LCR circuit connected to an a.c. source exhibits resonance at its natural frequency equal to $\frac{1}{\sqrt{LC}}$.

2. In a step up transformer, transformation ratio is 100. The primary voltage is $200V$ and input is $1000watt$. The number of turns in primary is 100. Calculate

a) Number of turns in the secondary

Ans: Given that, $k=100$

$$E_P=200V$$

$$E_PI_P=1000W$$

$$N_P=100$$

$$K=100=\frac{N_S}{N_P}$$

$$\Rightarrow N_S=100\times N_P$$

$$\Rightarrow N_S=100\times 100$$

$$\therefore N_S=10000$$

b) Current in the primary

Ans: From, $E_P I_P = 1000W$

$$\Rightarrow I_P = \frac{1000}{E_P}$$

$$\Rightarrow I_P = \frac{1000}{200} = 5A$$

$$\therefore I_P = 5A$$

c) The voltage across the secondary

Ans: We know, $\frac{E_S}{E_P} = \frac{N_S}{N_P}$

$$\Rightarrow E_S = E_P \times \frac{N_S}{N_P}$$

$$\Rightarrow E_S = 200 \times 100$$

$$\therefore E_S = 20000\text{Volt}$$

d) Current in the secondary

Ans: We know, $\frac{E_S}{E_P} = \frac{I_P}{I_S}$

$$\therefore I_S = \frac{I_P E_P}{E_S}$$

$$\Rightarrow I_S = \frac{1000}{20000} = \frac{1}{20}$$

$$\therefore I_S = 0.05A$$

e) Write the formula for transformation ratio.

Ans: For step up transformer; $K > 1$, that is, $K = \frac{N_S}{N_P}$. Therefore, the transformation ratio is $\frac{N_S}{N_P}$.

3. Derive an expression for the average power consumed in a.c. series LCR circuit. Hence define power factor.

Ans: Consider an a.c. series circuit,

$E = E_0 \sin \omega t$

And $I = I_0 \sin(\omega t + \phi)$

Where ϕ is the phase angle by which current leads the emf.

Now using, $dw = EIdt$

$\Rightarrow dw = (E_0 \sin \omega t)(I_0 \sin \omega t + \phi)dt$

$\Rightarrow dw = E_0 I_0 \sin \omega t(\sin \omega t \cos \phi + \cos \omega t \sin \phi)dt$

$\Rightarrow dw = E_0 I_0 (\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi)dt \quad [\because \sin 2\omega t = 2\sin \omega t \cos \omega t]$

$$\Rightarrow dw = E_0 I_0 \left(\frac{1 - \cos 2\omega t}{2} \cos \phi + \frac{\sin 2\omega t}{2} \sin \phi \right)$$

$$\left[\therefore \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} \right]$$

$$\Rightarrow dw = \frac{E_0 I_0}{2} (\cos \phi - \cos \phi \cos 2\omega t + \sin \phi \sin 2\omega t)dt$$

Integrating within limits $t = 0$ to $t = T$

$$W = \frac{E_0 I_0}{2} \left[\cos \phi \int_0^T dt - \cos \phi \int_0^T \cos 2\omega t dt + \sin \phi \int_0^T \sin 2\omega t dt \right]$$

$$\Rightarrow W = \frac{E_0 I_0}{2} \cos \phi \int_0^T dt \qquad \left[\because \int_0^T \sin 2\omega t dt = \int_0^T \cos 2\omega t dt = 0 \right]$$

$$\Rightarrow W = \frac{E_0 I_0}{2} T \cos \phi$$

Therefore, average power consumed in a.c circuit is given by

$$P_{av} = \frac{W}{T} = \frac{E_0 I_0}{2} \cos \phi$$

$\Rightarrow P_{av} = E_V I_V \cos \phi \qquad$ ....(1)

In the above expression, $\cos \phi$ is termed as power factor.

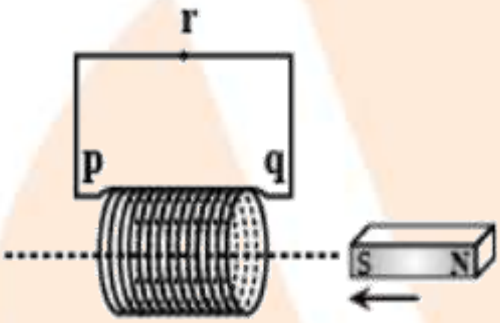
When the power factor, $\cos \phi = 1 \phi = 0^{\circ}$, It means circuit is purely resistive and $P_{av} = E_V I_V$.

When $\cos\phi = 0$, $\phi = 90^\circ$,

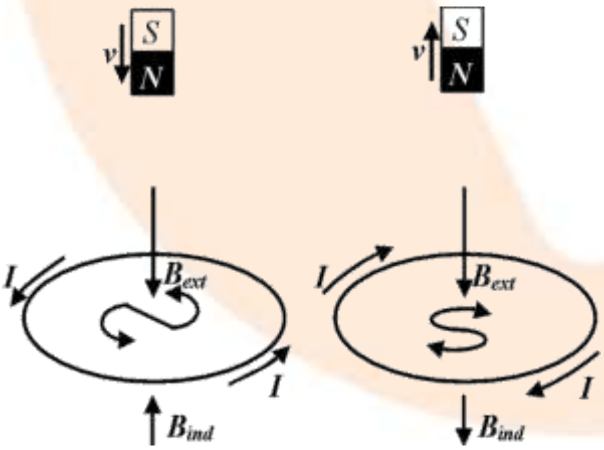
It means the circuit is purely capacitive or inductive. That is $P_{av} = 0$.

4. Predict the direction of induced current in the situations described by the following figures (a) to (f).

a)

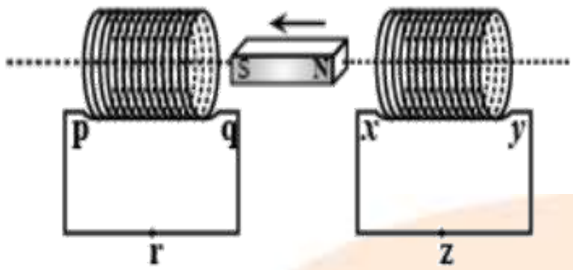


Ans: The direction of the induced current in a closed loop could be given by Lenz’s law. The following pairs of figures show the direction of the induced current when the North pole of a bar magnet is moved towards and away from a closed loop respectively.



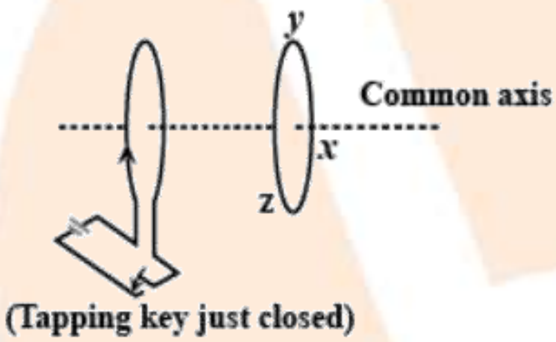
Now, by using Lenz’s rule, the direction of the induced current in the given situation is found to be along qrpq.

b)



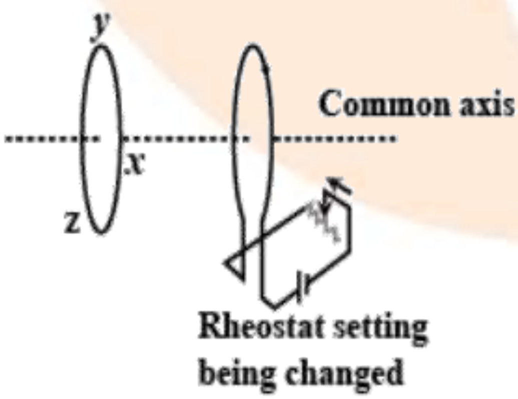
Ans: On using Lenz's law, we find the direction of the induced current here to be long prqp.

c)



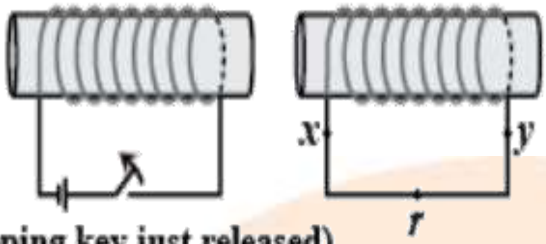
Ans: Using Lenz's law, we find the direction of the induced current to be along yzxy.

d)



Ans: Using Lenz's law, we find the direction of the induced current to be along zyxz.

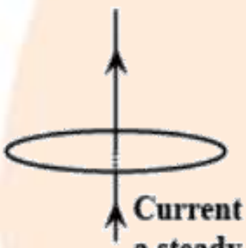
e)



(Tapping key just released)

Ans: Using Lenz's law, we found the direction of the induced current to be along xryx.

f)



Ans: Here we find that no current is induced since the field lines are lying in the same plane as that of the closed loop.

5. A long solenoid with 15 turns per cm has a small loop of area $2.0cm^2$ placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from $2.0A$ to $4.0A$ in $0.1s$, what is the induced emf in the loop while the current is changing?

Ans: We are given the following information:

Number of turns on the solenoid $= 15 turns/cm = 1500 turns/m$

Number of turns per unit length, $n = 1500$ turns

The solenoid has a small loop of area, $A = 2.0cm^2 = 2 \times 10^{-4} m^2$

Current carried by the solenoid changes from $2A$ to $4A$.

Now, the change in current in the solenoid, $di = 4 - 2 = 2A$

Change in time, $dt = 0.1\ s$

Induced emf in the solenoid could be given by Faraday's law as:

$$\varepsilon = \frac{d\phi}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where, induced flux through the small loop, $\phi = BA \ldots\ldots\ldots\ldots\ldots (2)$

Equation (1) would now reduce to:

$$\varepsilon = \frac{d}{dt}(BA) = A\mu_0 n \times \left(\frac{di}{dt}\right)$$

Substituting the given values into this equation, we get,

$$\varepsilon = 2\times10^{-4} \times 4\pi \times 10^{-7} \times 1500 \times \frac{2}{0.1}$$

$$\therefore \varepsilon = 7.54\times10^{-6}\,V$$

Therefore, the induced voltage in the loop is found to be, $\varepsilon = 7.54\times10^{-6}\,V$.

6. A circular coil of radius $8.0cm$ and 20 turns is rotated about its vertical diameter with an angular speed of $50rads^{-1}$ in a uniform horizontal magnetic field of magnitude $3.0\times10^{-2}T$. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Ans: We are given:

Max induced emf $= 0.603V$

Average induced emf $= 0V$

Max current in the coil $= 0.0603A$

Average power loss $= 0.018W$ (Power comes from the external rotor)

Radius of the circular coil, $r = 8cm = 0.08m$

Area of the coil, $A = \pi r^2 = \pi \times (0.08)^2\,m^2$

Number of turns on the coil, $N = 20$

Angular speed, $\omega = 50rad/s$

Magnetic field strength, $B = 3\times10^{-2}T$

Resistance of the loop, $R = 10\ \Omega$

Maximum induced emf could be given as:

$$\varepsilon = N\omega AB = 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2}$$

$$\therefore \varepsilon = 0.603V$$

The maximum emf induced in the coil is found to be $0.603V$.

Over a full cycle, the average emf induced in the coil is found to be zero.

Maximum current is given as:

$$I = \frac{\varepsilon}{R}$$

$$\Rightarrow I = \frac{0.603}{10}$$

$$\Rightarrow I = 0.0603A$$

Average power loss due to joule heating:

$$\therefore P = \frac{eI}{2} = \frac{0.603 \times 0.0603}{2} = 0.018W$$

We know that the current induced in the coil would produce a torque opposing the rotation of the coil. Since, the rotor is an external agent, it must supply a torque to counter this torque in order to keep the coil rotating uniformly. Hence, dissipated power comes from the external rotor.

7. A square loop of side $12cm$ with its sides parallel to X and Y axes is moved with a velocity of $18cm$ in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of $10^{-3}Tcm^{-1}$ along the negative x-direction (that is it increases by $10^{-3}Tcm^{-1}$ as one moves in the negative x-direction), and it is decreasing in time at the rate of $10^{-3}Ts^{-1}$. Determine the direction and magnitude of the induced current in the loop if its resistance is $4.50m\Omega$.

Ans: We are given,

Side of the square loop, $s = 12cm = 0.12m$

Area of the square loop, $A = 0.12 \times 0.12 = 0.0144m^2$

Velocity of the loop, $v = 8\text{cm}/\text{s} = 0.08\text{m}/\text{s}$

Gradient of the magnetic field along negative x-direction,

$$\frac{dB}{dx} = 10^{-3}\text{Tcm}^{-1} = 10^{-1}\text{Tm}^{-1}$$

And, rate of decrease of the magnetic field,

$$\frac{dB}{dt} = 10^{-3}\text{Ts}^{-1}$$

Resistance of the loop,

$$R = 4.5\text{m}\Omega = 4.5\times10^{-3}\Omega$$

Rate of change of the magnetic flux due to the motion of the loop in a non-uniform magnetic field is given as:

$$\frac{d\phi}{dt} = A\times\frac{dB}{dx}\times v$$

$$\Rightarrow \frac{d\phi}{dt} = 144\times10^{-4}\text{m}^2\times10^{-1}\times0.08$$

$$\Rightarrow \frac{d\phi}{dt} = 11.52\times10^{-5}\text{Tm}^2\text{s}^{-1}$$

Rate of change of the flux due to explicit time variation in field B is given as:

$$\frac{d\phi'}{dt} = A\times\frac{dB}{dx}$$

$$\Rightarrow \frac{d\phi'}{dt} = 144\times10^{-4}\times10^{-3} = 1.44\times10^{-5}\text{Tm}^2\text{s}^{-1}$$

Since the rate of change of the flux is the induced emf, the total induced emf in the loop can be calculated as:

$$e = 1.44\times10^{-5} + 11.52\times10^{-5}$$

$$= 12.96\times10^{-5}\text{V}$$

$\therefore$ Induced current, $i = \dfrac{e}{R}$

$$\Rightarrow i = \frac{12.96\times10^{-5}}{4.5\times10^{-3}}$$

$$\therefore i = 2.88\times10^{-2}\text{A}$$

Therefore, the direction of the induced current is such that there is an increase in the flux through the loop along the positive z-direction.

8. It is desired to measure the magnitude of field between the poles of a powerful loudspeaker magnet. A small flat search coil of area 2cm^2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5mC. The combined resistance of the coil and the galvanometer is 0.50Ω. Estimate the field strength of the magnet.

Ans: We are given the following:

Area of the small flat search coil, . $A = 2\text{cm}^2 = 2\times10^{-4}\text{m}^2$

Number of turns on the coil, $N = 25$

Total charge flowing in the coil, $Q = 7.5\text{mC} = 7.5\times10^{-3}\text{C}$

Total resistance of the coil and galvanometer, $R = 0.50\Omega$

Induced current in the coil,

$$I = \frac{\text{Induced emf}\,(\varepsilon)}{R} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Induced emf is given as:

$$\varepsilon = -N\frac{d\phi}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where, $d\phi = \text{Induced flux}$

Combining equations (1) and (2), we get

$$I = -\frac{N\dfrac{d\phi}{dt}}{R}$$

$$Idt = -\frac{N}{R}d\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Initial flux through the coil, $\phi_i = BA$

Where,

$B =$ Magnetic field strength

Final flux through the coil, $\phi_f = 0$

Integrating equation (3) on both sides, we have

$$\int I dt = -\frac{N}{R}\int_{\phi_i}^{\phi_f} d\phi$$

But total charge could be given as, $Q = \int I dt$

$$\Rightarrow Q = \frac{-N}{R}(\phi_f - \phi_i) = \frac{-N}{R}(-\phi_i) = \frac{+N\phi_i}{R}$$

$$Q = \frac{NBA}{R}$$

$$\Rightarrow B = \frac{QR}{NA}$$

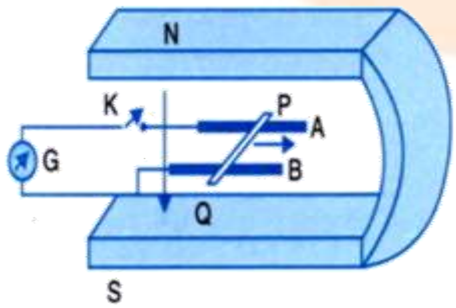
Substituting the given values, we get,

$$\Rightarrow B = \frac{7.5 \times 10^{-3} \times 0.5}{25 \times 2 \times 10^{-4}}$$

$$\therefore B = 0.75T$$

Therefore, the field strength of the magnet is found to be $0.75T$.

9. Figure shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod $= 15\text{cm}$, $B = 0.50T$, resistance of the closed loop containing the rod $= 9.0\text{m}\Omega$. Assume the field to be uniform.



a) Suppose K is open and the rod is moved with a speed of $12\text{cm}\cdot\text{s}^{-1}$ in the direction shown. Give the polarity and magnitude of the induced emf .

Ans: We are given:

Length of the rod, $l = 15\text{cm} = 0.15\text{m}$

Magnetic field strength, $B = 0.50\text{T}$

Resistance of the closed loop, $R = 9\text{m}\Omega = 9\times10^{-3}\Omega$

Induced emf $= 9\text{mV}$

Here, polarity of the induced emf is such that end P shows positive while end Q shows negative ends.

Speed of the rod, $v = 12\text{cm/s} = 0.12\text{m/s}$

We know that the induced emf could be given as: $\varepsilon = Bvl$

Substituting the given values, we get,

$\varepsilon = 0.5\times0.12\times0.15$

$\Rightarrow \varepsilon = 9\times10^{-3}\text{v}$

$\therefore \varepsilon = 9\text{mV}$

Therefore, the magnitude of the induced emf is found to be $\varepsilon = 9\text{mV}$ and the polarity of the induced emf is such that end P shows positive while end Q shows negative.

b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

Ans: Yes; when key K is closed, excess charge could be maintained by the continuous flow of current. When key K is open, there is excess charge built up at both rod ends but when key K is closed, excess charge is maintained by the continuous flow of current.

c) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.

Ans: Magnetic force is cancelled by the electric force that is set-up due to the excess charge of opposite nature at both rod ends. There is no net force on the electrons in rod PQ when key K is open and the rod would move uniformly. This is because magnetic force is cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rods.

d) What is the retarding force on the rod when K is closed?

Ans: We know that the retarding force exerted on the rod could be given by,

$F = IBl$

Where,

$I =$ Current flowing through the rod

Substituting the given values, we get,

$$I = \frac{e}{R} = \frac{9\times10^{-3}}{9\times10^{-3}} = 1A$$

$\Rightarrow F = 1\times0.5\times0.15$

$\therefore F = 75\times10^{-3}N$

Therefore, the retarding force on the rod when the key K is closed is:

$F = 75\times10^{-3}N$

e) How much power is required (by an external agent) to keep the rod moving at the same speed ($=12cm\cdot s^{-1}$) when K is closed? How much power is required when K is open?

Ans: We are given:

Speed of the rod, $v = 12cm/s = 0.12m/s$

Now, power could be given as:

$P = Fv$

Substituting the given values, we get,

$\Rightarrow P = 75\times10^{-3}\times0.12$

$\Rightarrow P = 9\times10^{-3}W$

$\therefore P = 9mW$

Therefore, we found the power that is required (by an external agent) to keep the rod moving at the same speed

$(=12cm\cdot s^{-1})$ when K is closed to be

$P = 9mW$ and when key K is open, no power is expended.

f) How much power is dissipated as heat in the closed circuit? What is the source of this power?

Ans: We know that,

Power dissipated as heat , $P = I^2R$

$\Rightarrow P = (1)^2 \times 9 \times 10^{-3}$

$\therefore P = 9mW$

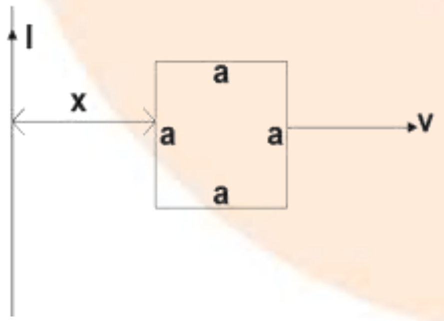
The power dissipated as heat in the closed circuit is found to be $P = 9mW$ and the source of this power is found to be an external agent.

g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

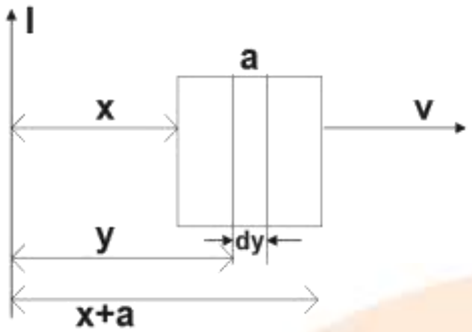
Ans: In this case, no emf would be induced in the coil because the motion of the rod does not cut across the field lines.

10.

a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in below figure.



Ans: Consider a small element dy in the loop at a distance y from the long straight wire (as shown in the given figure).



Magnetic flux associated with element $dy, d\phi = BdA$

Where, dA = Area of element $dy = a\,dy$

B = magnetic field at distance $y = \dfrac{\mu_0 I}{2\pi y}$

I = Current in the wire

μ_0 = Permeability of free space $= 4\pi \times 10^{-7}\,H/m$

Carrying out the substitutions accordingly, we get,

$$\Rightarrow d\phi = \frac{\mu_0 Ia}{2\pi}\frac{dy}{y}$$

$$\Rightarrow \phi = \frac{\mu_0 Ia}{2\pi}\int \frac{dy}{y}$$

Now, the limit of y will be from x to $a + x$, on applying the limits we get,

$$\Rightarrow \phi = \frac{\mu_0 Ia}{2x}\int_x^{a+x}\frac{dy}{y}$$

$$\Rightarrow \phi = \frac{\mu_0 Ia}{2\pi}\left[\log_e y\right]_x^{a+x}$$

$$\Rightarrow \phi = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a+x}{x}\right)$$

For mutual inductance M, the flux could be given as:

$\phi = MI$

$$\Rightarrow MI = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a}{x}+1\right)$$

$$\therefore M = \frac{\mu_0 a}{2\pi}\log_e\left(\frac{a}{x}+1\right)$$

Therefore, the expression for the mutual inductance between the given long straight wire and the square loop of side a is found to be,

$$M = \frac{\mu_0 a}{2\pi}\log_e\left(\frac{a}{x}+1\right)$$

b) Now assume that the straight wire carries a current of $50A$ and the loop is moved to the right with a constant velocity, $v = 10m/s$. Calculate the induced emf in the loop at the instant when $x = 0.2m$. Take $a = 0.1m$ and assume that the loop has a large resistance.

Ans: We know that, the Emf induced in the loop, $\varepsilon = B'av = \left(\frac{\mu_0 I}{2\pi x}\right)av$

We are given the following,

$I = 50A$

$x = 0.2m$

$a = 0.1m$

$v = 10m/s$

On substituting the given values into the equation, we get,

$$\varepsilon = \frac{4\pi\times10^{-7}\times50\times0.1\times10}{2\pi\times0.2}$$

$\therefore \varepsilon = 5\times10^{-5}V$

Therefore, induced emf in the loop at the given instant is found to be,

$\varepsilon = 5\times10^{-5}V$.