Important Questions for Class 12

Physics

Chapter 13 - Nuclei

Very Short Answer Questions **1 Marks**

1. Complete the following nuclear reactions:

a) $_{4}Be^{9} + {}_{1}H^{1} \rightarrow {}_{3}Li^{6} + ...$

Ans: $_{4}Be^{9} + {}_{1}H^{1} \rightarrow {}_{3}Li^{6} + {}_{2}He^{4}$

b) $_{5}B^{10} + {}_{2}He^{4} \rightarrow {}_{7}N^{13} + ...$

Ans: $_{5}B^{10} + {}_{2}He^{4} \rightarrow {}_{7}N^{13} + {}_{0}n^{1}$

2. What is the Q-value of a nuclear reaction?

Ans: $Q - \text{value} = (\text{Mass of reactants} - \text{Mass of products})$

3. The wavelengths of some of the spectral lines obtained in hydrogen spectrum are $9546\overset{\circ}{A}$, $6463\overset{\circ}{A}$ and $1216\overset{\circ}{A}$. Which one of these wavelengths belongs to the Lyman series?

Ans: $1216\overset{\circ}{A}$ belong to the Lyman series.

4. Write the empirical relation for paschen series lines of hydrogen atom.

Ans: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$ where $n = 4, 5, 6, 7......$

Very Short Answer Questions **2 Marks**

1. What fraction of tritium will remain after 25 years? Given half life of tritium as 12.5 years.

Ans: It is given that,

$t = 25\text{years}$, $T = 12.5\text{years}$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T}} = \left(\frac{1}{2}\right)^{\frac{25}{12.5}}$$

$$\Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$\Rightarrow \frac{N}{N_0} = 0.25$, which is the required fraction.

2. Calculate the kinetic energy and potential energy of an electron in the first orbit of a hydrogen atom. Given $e = 1.6 \times 10^{-19}\,C$ and $r = 0.53 \times 10^{-10}\,m$.

Ans:

I. Kinetic Energy

$$K.E. = \frac{ke^2}{2r}$$

$$\Rightarrow K.E. = \frac{(1.6 \times 10^{-19})^2 \times 9 \times 10^9}{2 \times 0.53 \times 10^{-10}}$$

$$\Rightarrow K.E. = 21.74 \times 10^{-19}\,J$$

$$\Rightarrow K.E. = \frac{21.74 \times 10^{-19}}{1.6 \times 10^{-19}} = 13.59\text{eV}$$

II. Potential Energy

$$\text{P.E.} = -\frac{ke^2}{r} = -2\text{K.E.}$$

$$\Rightarrow \text{P.E.} = -2 \times 13.59 = -27.18\text{eV}$$

Therefore, Kinetic energy is 13.59eV and Potential energy is -27.18eV.

3. Why is nuclear fusion not possible in the laboratory?

Ans: Nuclear fusion is not possible in the laboratory as it is performed in high temperatures. This cannot be attained in the laboratory.

4. Express 16mg mass into equivalent energy in electron volt.

Ans: It is known that,

$$E = mc^2$$

$$\Rightarrow E = 16 \times 10^{-6}\text{kg} \times (3 \times 10^8 \text{m/s})^2$$

$$\Rightarrow E = 16 \times 9 \times 10^{10}\text{Joules}$$

$$\Rightarrow E = \frac{16 \times 9 \times 10^{10}}{1.6 \times 10^{-19}}\text{eV}$$

$$\Rightarrow E = 9 \times 10^{30}\text{eV}$$

5. The three stable isotopes of neon: ${}^{20}_{10}\text{Ne}$, ${}^{21}_{10}\text{Ne}$ and ${}^{22}_{10}\text{Ne}$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99u, 20.99u and 21.99u respectively. Obtain the average atomic mass of neon.

Ans: It is given that,

Atomic mass of ${}^{20}_{10}\text{Ne}$, $m_1 = 19.99\text{u}$

Abundance of ${}^{20}_{10}\text{Ne}$, $\eta_1 = 90.51\%$

Atomic mass of ${}^{21}_{10}\text{Ne}$, $m_2 = 20.99\text{u}$

Abundance of $^{21}_{10}\text{Ne}$, $\eta_2 = 0.27\%$

Atomic mass of $^{22}_{10}\text{Ne}$, $m_3 = 21.99\text{u}$

Abundance of $^{22}_{10}\text{Ne}$, $\eta_3 = 9.22\%$

The average atomic mass of neon is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$\Rightarrow m = \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{90.51 + 0.27 + 9.22}$$

$\Rightarrow m = 20.1771\text{u}$, which is the required average mass.

6. From the relation $R = R_0 A^{\frac{1}{3}}$, where R_0 is a constant and A is the mass number of a nucleus. Show that the nuclear matter density is nearly constant (i.e., independent of A).

Ans: It is known that,

$$R = R_0 A^{\frac{1}{3}}$$

where,

R_0 is a constant

A is the mass number of the nucleus

Nuclear matter density, $\rho = \dfrac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}}$

Let m be the average mass of the nucleus.

Mass of the nucleus $= mA$

$$\rho = \frac{mA}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow \rho = \frac{3mA}{4\pi\left(R_0 A^{\frac{1}{3}}\right)^3} = \frac{3mA}{4\pi R_0^3 A}$$

$$\Rightarrow \rho = \frac{3m}{4\pi R_0^3}$$

Therefore, the nuclear matter density is independent of A . It is nearly constant.

Short Answer Questions **3 Marks**

1. A neutron is absorbed by a $3Li$ nucleus with subsequent emission of alpha particles. Write the corresponding nuclear reaction?

Ans: ${}_0^1n + {}_3^6Li \rightarrow {}_2^4He(\alpha - \text{particle}) + {}_Z^AY$

Conservation of Atomic Number: $0+3=2+Z$

$\Rightarrow Z=1$

Conservation of Mass Number: $1+6=4+A$

$\Rightarrow A=3$

${}_y^AY = {}_1^3Y = {}_1^3H$

$\Rightarrow {}_0^1n + {}_3^6Li \rightarrow {}_2^4He + {}_1^3H$

2. If the activity of a radioactive substance drops to its initial value in 30 years, find its half life period.

Ans: It is known that

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

$$\Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{30}{T}}$$

$$\Rightarrow \left(\frac{1}{2}\right)^{3} = \left(\frac{1}{2}\right)^{\frac{30}{T}}$$

$$\Rightarrow \frac{30}{T} = 3$$

$$\Rightarrow T = 10\text{years}$$

Therefore, required half life period is 10 years.

3. Show that nuclear density is independent of mass number A of a nucleus.

Ans: It is known that,

$$\text{Nuclear density} = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{(\text{Mass of P or N}) \times A}{\text{Volume of nucleus}}$$

$$\Rightarrow P = \frac{\left(1.6 \times 10^{-27}\right) \times A}{\frac{4}{3}\pi R^3} = \frac{\left(1.6 \times 10^{-27}\right) \times A}{\frac{4}{3}\pi\left(R_0 A^{\frac{1}{3}}\right)^3}$$

Here, $R_0 = 1.2 \times 10^{-15}\,\text{m}$

$$\Rightarrow P = \frac{\left(1.6 \times 10^{-27}\right) \times A}{\frac{4}{3}\pi R_0{}^3 A} = \frac{1.6 \times 10^{-27}}{\frac{4}{3} \times 3.14 \times \left(1.2 \times 10^{-15}\right)^3}$$

$$\Rightarrow P = 2.21 \times 10^{17}\,\text{kg/m}^3$$

Therefore, the nuclear density is independent of mass number A.

4. A radioactive nucleus undergoes a series of decay according to the scheme $A \xrightarrow{\alpha} A_1 \xrightarrow{\beta-} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$. If the mass number and atomic

number of A are 180 and 72 respectively, what are these numbers for A_4?

Ans: It is given that,

Mass number of A is 180

Atomic number of A is 72

$${}^{180}_{72}A \xrightarrow{\alpha} {}^{176}_{70}A_1 \xrightarrow{\beta-} {}^{176}_{71}A_2 \xrightarrow{\alpha} {}^{172}_{69}A_3 \xrightarrow{\gamma} {}^{172}_{69}A_4$$

5. Distinguish between isotopes and isobars. Give one example for each of the species.

Ans: The elements which have the same atomic number but different mass number are called Isotopes.

For e.g., $\rightarrow {}^{10}_{6}C\ {}^{11}_{6}C\ {}^{12}_{6}C\ {}^{14}_{6}C$ (Isotopes of carbon)

Thus, nuclides of different elements having the same mass number but different atomic numbers are called isobars.

For e.g., $\rightarrow {}^{3}_{1}H$ and ${}^{3}_{2}He$

${}^{7}_{3}Li$ and ${}^{7}_{4}Be$

6. A radioactive nuclide decays to form a stable nuclide its half life is 3 minutes. What fractions of its 1g will remain radioactive after 9 minutes?

Ans: Let the number of atoms/gram $= N_0$

$t = 9$ minutes

$T_{1/2} = 3$ minutes

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} = \left(\frac{1}{2}\right)^{9/3} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$\Rightarrow N = \frac{N_0}{8}$$

Fraction decayed $= \frac{N_0 - N}{N_0}$

$$\Rightarrow \frac{N_0 - \frac{N_0}{8}}{N_0} = 1 - \frac{1}{8} = \frac{7}{8} = 0.875$$

Therefore, Fraction remain undecayed $= 1 - 0.875 = 0.125$

7. Obtain the binding energy (in MeV) of a nitrogen nucleus $^{14}_{7}N$, given $m\left(^{14}_{7}N\right) = 14.00307u$

Ans: Atomic mass of nitrogen $\left(^{14}_{7}N\right)$, $m = 14.00307u$

A nucleus of nitrogen $^{14}_{7}N$ contains 7 protons and 7 neutrons.

Therefore, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

$\Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

$\Rightarrow \Delta m = 7.054775 + 7.06055 - 14.00307$

$\Rightarrow \Delta m = 0.11236u$

It is known that, $1u = 931.5 \mathrm{MeV}/c^2$

$\Delta m = 0.11236 \times 931.5 \mathrm{MeV}/c^2$

Thus, the binding energy of the nucleus is $E_b = \Delta mc^2$

Where,

c is the speed of light

$$E_b = 0.11236 \times 931.5\left(\frac{MeV}{c^2}\right) \times c^2$$

$$\Rightarrow E_b = 104.66334 MeV$$

Therefore, the binding energy of a nitrogen nucleus is 104.66334 MeV.

8. Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.

Ans: Given,

Nuclear radius of the gold isotope $^{197}_{79}Au = R_{Au}$

Nuclear radius of the silver isotope $^{107}_{47}Ag = R_{Ag}$

Mass number of gold, $A_{Au} = 197$

Mass number of silver, $A_{Ag} = 107$

The ratio of the radii of the two nuclei is: $\frac{R_{Au}}{R_{Ag}} = \left(\frac{R_{Au}}{R_{Ag}}\right)^{\frac{1}{3}}$

$$\Rightarrow \frac{R_{Au}}{R_{Ag}} = \left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.2256$$

Therefore, the ratio of the nuclear radii of the gold and silver isotopes is about 1.23.

9. Suppose, we think of fission of a $^{56}_{26}Fe$ nucleus into two equal fragments, $^{28}_{13}Al$. Is fission energetically possible? Argue by working out Q of the process. Given $m\left(^{56}_{26}Fe\right) = 55.93494u$ and $m\left(^{28}_{13}Al\right) = 27.98191u$

Ans: The fission of $^{56}_{26}Fe$ can be given as:

It is given that:

Atomic mass of $m\left({}_{26}^{56}Fe\right) = 55.93494u$

Atomic mass of $m\left({}_{13}^{28}Al\right) = 27.98191u$

The Q-value of this nuclear reaction is: $Q = \left[m\left({}_{26}^{56}Fe\right) - 2m\left({}_{13}^{28}Al\right)\right]c^2$

$\Rightarrow Q = [55.93494 - 2 \times 27.98191]c^2$

$\Rightarrow Q = \left(-0.02888c^2\right)u$

It is known that, $1u = 931.5 MeV/c^2$

$Q = -0.02888 \times 931.5 = -26.902 MeV$

The Q-value of the fission is negative.

Thus, the fission is not possible energetically. For an energetically-possible fission reaction, the Q-value must be positive.

10. The fission properties of $^{239}_{94}Pu$ are very similar to those of $^{235}_{92}U$. The average energy released per fission is $180\ MeV$. How much energy, in MeV, is released if all the atoms in $1kg$ of pure $^{239}_{94}Pu$ undergo fission?

Ans: It is given that,

Average energy released per fission of $^{239}_{94}Pu$, $E_{av} = 180\ MeV$

Amount of pure $^{239}_{94}Pu$, $m = 1kg = 1000g$

$N_A =$ Avogadro number $= 6.023 \times 10^{23}$

Mass number of $^{239}_{94}Pu = 239g$

1mole of $^{239}_{94}Pu$ contains N_A atoms.

Therefore, $1kg$ of $^{239}_{94}Pu$ contains $\left(\frac{N_A}{\text{Mass number}} \times m\right)$ atoms

$$\Rightarrow \frac{6.023\times10^{23}}{239}\times1000 = 2.52\times10^{24}\text{atoms}$$

Thus, Total energy released during the fission of 1kg of ${}^{239}_{94}\text{Pu}$:
$E = E_{av}\times2.52\times10^{24}$

$$\Rightarrow E = 180\times2.52\times10^{24} = 4.536\times10^{26}\,\text{MeV}$$

Therefore, $4.536\times10^{26}\,\text{MeV}$ is released if all the atoms in 1kg of pure ${}^{239}_{94}\text{Pu}$ undergo fission.

11. Calculate the height of the potential barrier for a head-on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0fm.)

Ans: If two deuterons collide head-on then, the distance between their centres is:
$d = \text{Radius of first deuteron} + \text{Radius of second deuteron}$

Radius of a deuteron nucleus $= 2f_m = 2\times10^{-15}\,\text{m}$

$$\Rightarrow d = 2\times10^{-15} + 2\times10^{-15} = 4\times10^{-15}\,\text{m}$$

Charge on a deuteron nucleus $=$ Charge on an electron $= e = 1.6\times10^{-19}\,\text{C}$

Potential energy of the two-deuteron system: $V = \dfrac{e^2}{4\pi\varepsilon_0 d}$

Where,

ε_0 is the Permittivity of free space

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{Mm}^2\text{c}^{-2}$$

$$\Rightarrow V = \frac{9\times10\times\left(1.6\times10^{-19}\right)^2}{4\times10^{-15}}\,\text{J}$$

$$\Rightarrow V=\frac{9\times 10\times\left(1.6\times 10^{-19}\right)^2}{4\times 10^{-15}\times\left(1.6\times 10^{-19}\right)}eV$$

$$\Rightarrow V=360keV$$

Therefore, the height of the potential barrier of the two-deuteron system is $360keV$.

Long Answer Questions **5 Marks**

1. The wavelength of the first member of Balmer series in the hydrogen spectrum is $6563\overset{\circ}{A}$. Calculate the wavelength of the first member of Lyman series in the same spectrum.

Ans: It is known that,

$$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{{n_i}^2}\right),\ n=3,4,5,.....$$

For first member $n_i=3$ (Balmer series)

$$\Rightarrow\frac{1}{\lambda_1}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$\Rightarrow\frac{1}{\lambda_1}=R\left(\frac{1}{4}-\frac{1}{9}\right)$$

$$\Rightarrow\lambda_1=\frac{36}{5R}\quad ....(1)$$

For first member of Lyman series

$$\Rightarrow\frac{1}{\lambda_1'}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

$$\Rightarrow\frac{1}{\lambda_1'}=R\left(1-\frac{1}{4}\right)$$

$$\Rightarrow \lambda_1' = \frac{4}{3R} \quad ....(2)$$

From (1) and (2)

$$\Rightarrow \frac{\lambda_1'}{\lambda_1} = \frac{4}{3R} \times \frac{5R}{36}$$

$$\Rightarrow \lambda_1' = \frac{5}{27}\lambda_1$$

$$\Rightarrow \lambda_1' = \frac{5}{27} \times 6563$$

$$\Rightarrow \lambda_1' = 1215.4\overset{\circ}{A}$$

Therefore, the wavelength of first member of Lyman series is $1215.4\overset{\circ}{A}$.

2. A neutron is absorbed by a $^{6}_{3}Li$ nucleus with subsequent emission of $\alpha-$ particle. Write the corresponding nuclear reaction. Calculate the energy released in this reaction. Given mass of $^{6}_{3}Li = 6.015126 a.m.u.$, Mass of $^{4}_{2}He = 4.00026044$ a.m.u., Mass of neutron $^{1}_{0}n = 1.0086654$ a.m.u. Mass of tritium $^{1}_{1}H = 3.016049$ a.m.u.

Ans: Nuclear reaction is: $^{1}_{0}n + ^{6}_{3}Li \rightarrow ^{4}_{2}He + ^{3}_{1}H$

Mass of reactants $= m\left(^{1}_{0}n\right) + m\left(^{6}_{3}Li\right) =$

$$\Rightarrow \text{Mass} = 1.0086654 + 6.015126 = 7.0237914 \text{ a.m.u}$$

Mass Defect, $\Delta m = \text{mass of reactant} - \text{mass of product}$

$$\Rightarrow \Delta m = 7.0237194 - 7.0186534$$

$$\Rightarrow \Delta m = 0.005138 \text{ a.m.u.}$$

It is known that, $1 a.m.u. = 931 MeV$

Energy released, $E = \Delta m \times 931 MeV$

$\Rightarrow E = 0.005138 \times 931$

$\Rightarrow E = 4.783 MeV$

3. Define decay constant of a radioactive sample. Which of the following radiation $\alpha -$ rays, $\beta -$ rays and $\gamma -$ rays.

a) Are they similar to X – rays?

Ans: Radioactive decay constant (λ) is the reciprocal of time during which the number of atoms in the radioactive substance reduced to 36.8% of the original number of atoms in it.

$\gamma -$ rays are similar to X-rays.

b) Are they easily absorbed by matter?

Ans: Penetration power of $\alpha -$ rays is less than that of $\beta -$ rays and $\gamma -$ rays. So $\gamma -$ rays are easily absorbed by matter.

4. State radioactive decay law and hence derive the relation $N = N_0 e^{-\lambda t}$ where symbols have their usual meanings.

Ans: From the radioactive decay law, the rate of disintegration of a radioactive substance at an instant is directly proportional to the number of nuclei in the radioactive substance at that time i.e.

$N = N_0 e^{-\lambda t}$ where symbols have their usual meanings

Consider a radioactive substance having N_0 atoms initially at time $(t = 0)$. After time (t) number of atoms left undecayed be N.

If dN is the number of atoms decayed in time dt, then

From the law of radioactive decay: $\frac{-dN}{dt} \prec N$ or $\frac{-dN}{dt} = \lambda N$ ....(1)

Where,

λ is the decay constant and negative sign indicates that a radioactive sample goes on decreasing with time.

$$(1) \Rightarrow \frac{dN}{N} = -\lambda dt$$

Integrating both the sides

$$\log_e N = -\lambda t + K \quad \text{........}(2)$$

Where K is constant of integration

For $t = 0$, $N = N_0$

$$\Rightarrow K = \log_e N_0$$

Substituting K in equation (2)

$$\Rightarrow \log_e N = -\lambda t + \log_e N_0$$

$$\Rightarrow \log_e N - \log_e N_0 = -\lambda t \quad \left[\log_e m - \log_e n = \log_e\left(\frac{m}{n}\right)\right]$$

$$\Rightarrow \log_e\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t}$$

$$\Rightarrow N = N_0 e^{-\lambda t}$$

Hence derived.

5. Define half life and decay constant of a radioactive element. Write their S.I. unit. Define expression for half life.

Ans: The time during which half of the atoms of the radioactive substance disintegrate is called half life of a radioactive substance.

It is known that, $N = N_0 e^{-\lambda t}$

If $t = T_{1/2}$ (Half life) , $N = \frac{N_0}{2}$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\Rightarrow e^{\lambda T_{1/2}} = 2$$

$$\Rightarrow \lambda T_{1/2} = \log_e 2$$

$$\Rightarrow \lambda T_{1/2} = 2.303 \times \log_{10} 2$$

$$\Rightarrow \lambda T_{1/2} = 2.303 \times 0.3010$$

$$\Rightarrow T_{1/2} = \frac{0.6931}{\lambda}$$

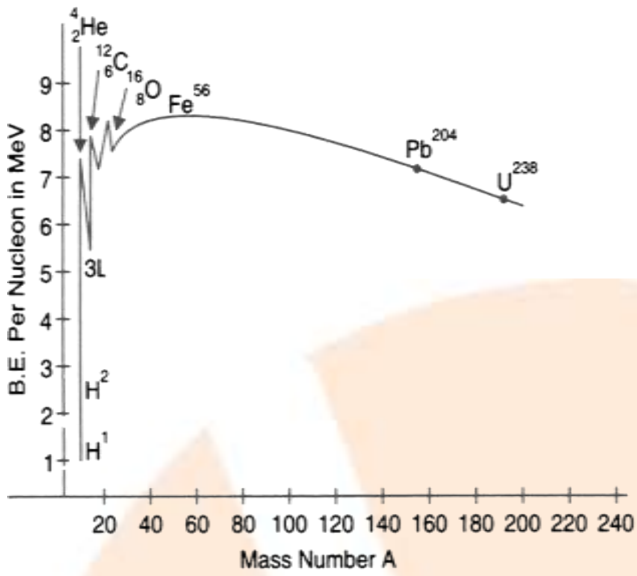
S.I. unit – second (s)

Radioactive decay constant (λ) is the reciprocal of the time during which the number of atoms in the radioactive substance reduces to 36.8% of the original number of atoms in it.

S.I. unit – s^{-1} or min^{-1}

6. Draw a curve between mass number and binding energy per nucleon. Give two salient features of the curve. Hence define binding energy.

Ans: The total energy required to disintegrate the nucleus into its constituent particles is called binding energy of the nucleus.



Salient features of the curve

(I) The intermediate nuclei have a large value of binding energy per nucleon, so they are most stable. (For $30 < A > 63$)

(II) The binding energy per nucleon has low value for both the light and heavy nuclei. So, they are unstable nuclei.

7.

a) Two stable isotopes of lithium $_{3}^{6}Li$ and $_{3}^{7}Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses $6.01512u$ and $7.01600u$ respectively. Find the atomic mass of lithium.

Ans: Given that,

Mass of lithium isotope $_{3}^{6}Li$, $m_1 = 6.01512 \text{ u}$

Mass of lithium isotope $_{3}^{7}Li$, $m_2 = 7.01600 \text{ u}$

Abundance of $_{3}^{6}Li$, $\eta_1 = 7.5\%$

Abundance of $_{3}^{7}Li$, $\eta_2 = 92.5\%$

The atomic mass of lithium atom, $m = \dfrac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$

$$\Rightarrow m = \frac{6.0512 \times 7.5 + 7.01600 \times 92.5}{7.5 + 92.5}$$

$$\Rightarrow m = 6.940934u$$

Therefore, the atomic mass of lithium is $6.940934u$.

b) Boron has two stable isotopes, ${}^{10}_{5}B$ and ${}^{11}_{5}B$. Their respective masses are $10.01294u$ and $11.00931u$, and the atomic mass of boron is $10.811u$. Find the abundances of ${}^{10}_{5}B$ and ${}^{11}_{5}B$.

Ans: It is given that,

Mass of boron isotope ${}^{10}_{5}B$, $m_1 = 10.01294u$

Mass of boron isotope ${}^{11}_{5}B$, $m_2 = 11.00931u$

Abundance of ${}^{10}_{5}B$, $\eta_1 = x\%$

Abundance of ${}^{11}_{5}B$, $\eta_2 = (100 - x)\%$

Atomic mass of boron, $m = 10.811u$

The atomic mass of boron atom, $m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$

$$\Rightarrow 10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + 100 - x}$$

$$\Rightarrow 108.11 = 10.01294x + 1100.931 - 11.00931x$$

$$\Rightarrow x = \frac{19.821}{0.99637} = 19.89\%$$

$$\Rightarrow 100 - x = 80.11\%$$

Therefore, the abundance of ${}^{10}_{5}B$ is 19.89% and abundance of ${}^{11}_{5}B$ is 80.11%.

8. Obtain the binding energy of the nuclei ${}^{56}_{26}Fe$ and ${}^{209}_{83}Bi$ in units of MeV from the following data: $m\left({}^{56}_{26}Fe\right) = 55.934939u$, $m\left({}^{209}_{83}Bi\right) = 208.980388u$

Ans: Given that,

Atomic mass of $^{56}_{26}\mathrm{Fe}$, $m_1 = 55.934939\mathrm{u}$

$^{56}_{26}\mathrm{Fe}$ nucleus has 26 protons and $(56-26) = 30$ neutrons

Therefore, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where,

Mass of proton, $m_H = 1.007825\mathrm{u}$

Mass of a neutron, $m_n = 1.008665\mathrm{u}$

$\Rightarrow \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$

$\Rightarrow \Delta m = 26.20345 + 30.25995 - 55.934939$

$\Rightarrow \Delta m = 0.528461\mathrm{u}$

It is known that, $1\mathrm{u} = 931.5\dfrac{\mathrm{MeV}}{c^2}$

The binding energy of this nucleus is $E_{b_1} = \Delta mc^2$

Where,

c is the speed of light

$\Rightarrow E_{b_1} = 0.528461 \times 931.5\left(\dfrac{\mathrm{MeV}}{c^2}\right) \times c^2$

$\Rightarrow E_{b_1} = 492.26\mathrm{MeV}$

Average binding energy per nucleon $= \dfrac{492.26}{56} = 8.79\mathrm{MeV}$

Atomic mass of $^{209}_{83}\mathrm{Bi}$, $m_2 = 208.980388\mathrm{u}$

$^{209}_{83}\mathrm{Bi}$ nucleus has 83 protons and $(209-83) = 126$ neutrons.

Therefore, the mass defect of this nucleus, $\Delta m' = 83 \times m_H + 126 \times m_n - m_2$

Where,

Mass of proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

$\Rightarrow \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

$\Rightarrow \Delta m' = 83.649475 + 127.091790 - 208.980388$

$\Rightarrow \Delta m' = 1.760877u$

It is known that, $1u = 931.5\frac{MeV}{c^2}$

The binding energy of this nucleus is $E_{b_2} = \Delta m' c^2$

Where,

c is the speed of light

$\Rightarrow E_{b_2} = 1.760877 \times 931.5\left(\frac{MeV}{c^2}\right) \times c^2$

$\Rightarrow E_{b_2} = 1640.26MeV$

Clearly, average binding energy per nucleon $= \frac{1640}{209} = 7.848MeV$

9. A given coin has a mass of $3.0g$. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ atoms (of mass 62.92960 u).

Ans: It is given that,

Mass of a copper coin, $m' = 3g$

Atomic mass of $^{63}_{29}Cu$ atom, $m = 62.92960\text{ u}$

The total number of $^{63}_{29}Cu$ atoms in the coin, $N = \frac{N_A \times m'}{\text{Mass number}}$

Where,

Avogadro's number, $N_A = 6.023\times10^{23}$ atoms/g

Mass number $= 63$g

$$\Rightarrow N = \frac{6.023\times10^{23}\times3}{63} = 2.868\times10^{22}\text{atoms}$$

$^{63}_{29}Cu$ nucleus has 29 protons and $(63-29) = 34$ neutrons

Thus, the mass defect of this nucleus, $\Delta m = 29\times m_H + 34\times m_n - m_1$

Where,

Mass of proton, $m_H = 1.007825u$

Mass of a neutron, $m_n = 1.008665u$

$$\Rightarrow \Delta m' = 29\times1.007825 + 34\times1.008665 - 62.9296$$

$$\Rightarrow \Delta m' = 0.591935u$$

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935\times2.868\times10^{22}$

$$\Rightarrow \Delta m = 1.69766958\times10^{22}u$$

It is known that, $1u = 931.5\frac{MeV}{c^2}$

$$\Rightarrow \Delta m = 1.69766958\times10^{22}\times931.5\frac{MeV}{c^2}$$

The binding energy of this nucleus is $E_b = \Delta mc^2$

Where,

c is the speed of light

$$\Rightarrow E_b = 1.69766958\times10^{22}\times931.5\frac{MeV}{c^2}\times c^2$$

$$\Rightarrow E_b = 1.581\times10^{25}MeV$$

It is known that, $1MeV = 1.6\times10^{-13}J$

$\Rightarrow E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13} J$

$\Rightarrow E_b = 2.5296 \times 10^{12} J$

Clearly, energy required to separate all the neutrons and protons from the given coin is $2.5296 \times 10^{12} J$.

10. Write nuclear reaction equations for

a) α-decay of $^{226}_{88}Ra$

Ans: α is a nucleus of Helium $\left(^{4}_{2}He\right)$. In every α-decay, there is a loss of 2 protons and 4 neutrons.

$^{226}_{88}Ra \to ^{222}_{86}Ra + ^{4}_{2}He$

b) α-decay of $^{242}_{94}Pu$

Ans: α is a nucleus of Helium $\left(^{4}_{2}He\right)$. In every α-decay, there is a loss of 2 protons and 4 neutrons.

$^{242}_{94}Pu \to ^{238}_{92}U + ^{4}_{2}He$

c) β^--decay of $^{32}_{15}P$

Ans: β is an electron (e^- for β^- and e^+ for β^+). In every β^--decay, there is a gain of a proton while an antineutrino is emitted from the nucleus.

$^{32}_{15}P \to ^{32}_{16}S + e^- + \bar{\nu}$

d) β^--decay of $^{210}_{83}Bi$

Ans: β is an electron (e^- for β^- and e^+ for β^+). In every β^--decay, there is a gain of a proton while an antineutrino is emitted from the nucleus.

$^{32}_{15}P \to ^{32}_{16}S + e^- + \bar{\nu}$

e) β^+-decay of ${}^{11}_{6}C$

Ans: β is an electron (e^- for β^- and e^+ for β^+). In every β^+-decay, there is a loss of a proton while a neutrino is emitted from the nucleus.

$${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^+ + \nu$$

f) β^+-decay of ${}^{97}_{43}Tc$

Ans: β is an electron (e^- for β^- and e^+ for β^+). In every β^+-decay, there is a loss of a proton while a neutrino is emitted from the nucleus.

$${}^{97}_{43}Tc \rightarrow {}^{97}_{42}Mo + e^+ + \nu$$

g) **Electron capture of ${}^{120}_{54}Xe$**

Ans: ${}^{120}_{54}Xe + e^+ \rightarrow {}^{120}_{53}I + \nu$

11. A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to

a) 3.125%

Ans: Let the half-life of the radioactive isotope be T years.

Original amount of the radioactive isotope be N_0

After decay, the amount of the radioactive isotope be N

It is given that only 3.125% of N_0 remains after decay.

$$\Rightarrow \frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

It is known that, $\frac{N}{N_0} = e^{-\lambda t}$

Where,

λ is the Decay constant

t is the Time

$$\Rightarrow e^{-\lambda t} = \frac{1}{32}$$

$$\Rightarrow -\lambda t = \ln 1 - \ln 32$$

$$\Rightarrow -\lambda t = 0 - 3.4567$$

$$\Rightarrow t = \frac{3.4567}{\lambda}$$

It is known that, $\lambda = \frac{0.693}{T}$

$$\Rightarrow t = \frac{3.4567}{\frac{0.693}{T}} = 5T$$

Therefore, the isotope will take about $5T$ years to reduce to 3.125% of its original value.

b) 1% of its original value

Ans: Suppose that after decay, the amount of the radioactive isotope be N.

It is given that only 1% of N_0 remains after decay.

$$\Rightarrow \frac{N}{N_0} = 1\% = \frac{1}{100}$$

It is known that, $\frac{N}{N_0} = e^{-\lambda t}$

Where,

λ is the Decay constant

t is the Time

$$\Rightarrow e^{-\lambda t} = \frac{1}{100}$$

$$\Rightarrow -\lambda t = \ln 1 - \ln 100$$

$\Rightarrow -\lambda t = 0 - 4.6052$

$\Rightarrow t = \dfrac{4.6052}{\lambda}$

It is known that, $\lambda = \dfrac{0.693}{T}$

$\Rightarrow t = \dfrac{4.6052}{\frac{0.693}{T}} = 6.645T$

Therefore, the isotope will take about $6.645T$ years to reduce to 1% of its original value.

12. The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ present with the stable carbon isotope ${}^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of ${}^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Ans: It is given that,

Decay rate of living carbon-containing matter, $R = 15 \text{decay}/\text{min}$

Let N be the number of radioactive atoms present in a normal carbon- containing matter.

Half life of ${}^{14}_{6}C$, $T_{1/2} = 5730 \text{years}$

The decay rate of the specimen obtained from the Mohenjodaro site: $R' = 9 \text{decays}/\text{min}$

Let N' be the number of radioactive atoms present in the specimen during the Mohenjodaro period.

Thus, decay constant (λ) and time (t) is related as: $\frac{N}{N'} = \frac{R}{R'} = e^{-\lambda t}$

$$\Rightarrow e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$$

$$\Rightarrow -\lambda t = \log_e \frac{3}{5} = -0.5108$$

$$\Rightarrow t = \frac{0.5108}{\lambda}$$

It is known that, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730}$

$$\Rightarrow t = \frac{0.5108}{\frac{0.693}{5730}} = 4223.5$$

Therefore, the approximate age of the Indus-Valley civilisation is 4223.5 years.

13. Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.

Ans: The strength of the radioactive source is $\frac{dN}{dt} = 8.0 \text{mCi}$

$$\Rightarrow \frac{dN}{dt} = 8 \times 10^{-3} \times 3.7 \times 10^{10}$$

$$\Rightarrow \frac{dN}{dt} = 29.6 \times 10^{7} \text{ decay/s}$$

Where,

N is the required number of atoms

Half-life of ${}_{27}^{60}Co$, $T_{1/2} = 5.3\text{years}$

$\Rightarrow T_{1/2} = 5.3 \times 365 \times 24 \times 60 \times 60$

$\Rightarrow T_{1/2} = 29.6 \times 10^8 s$

For decay constant λ,

The rate of decay is $\frac{dN}{dt} = \lambda N$

Where, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{29.6 \times 10^8} s^{-1}$

$\Rightarrow N = \frac{1}{\lambda}\frac{dN}{dt}$

$\Rightarrow N = \frac{29.6 \times 10^7}{\frac{0.693}{29.6 \times 10^8}} = 7.133 \times 10^{16} \text{atoms}$

For ${}_{27}^{60}Co$: Mass of 6.023×10^{23} (Avogadro's number) atoms $= 60g$

$\therefore$Mass of 7.133×10^{16} atoms $= \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6} g$

Therefore, the amount of ${}_{27}^{60}Co$ necessary for the purpose is $7.106 \times 10^{-6} g$.

14. The half-life of ${}_{38}^{90}Sr$ is 28 years. What is the disintegration rate of 15mg of this isotope?

Ans: It is given that,

Half -life of ${}_{38}^{90}Sr$, $T_{1/2} = 28\text{years}$

$\Rightarrow T_{1/2} = 28 \times 365 \times 24 \times 60 \times 60$

$\Rightarrow T_{1/2} = 8.83 \times 10^8 s$

Mass of the isotope, $m = 15\text{mg}$

90g of $^{90}_{38}\text{Sr}$ atom contains 6.023×10^{23} (Avogadro's number) atoms.

Therefore, 15mg of $^{90}_{38}\text{Sr}$ contains: $\dfrac{6.023\times10^{23}\times15\times10^{-3}}{90}$ i.e. 1.0038×10^{20}

number of atoms

Rate of disintegration, $\dfrac{dN}{dt} = \lambda N$

Where,

λ is the Decay constant, $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{8.83\times10^{8}}\text{s}^{-1}$

$$\Rightarrow \frac{dN}{dt} = \frac{0.693\times1.0038\times10^{20}}{8.83\times10^{8}} = 7.878\times10^{10}\ \text{atoms/s}$$

Therefore, the disintegration rate of 15mg of the given isotope is

7.878×10^{10} atoms/s.

15. Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay if $m\left(^{226}_{88}\text{Ra}\right) = 226.02540\ \text{u}$, $m\left(^{222}_{86}\text{Rn}\right) = 222.01750\ \text{u}$, $m\left(^{220}_{86}\text{Rn}\right) = 220.01137\ \text{u}$, $m\left(^{216}_{84}\text{Po}\right) = 216.00189\ \text{u}$ of

a) $^{226}_{88}\text{Ra}$

Ans: Alpha particle decay of $^{226}_{88}\text{Ra}$ emits a helium nucleus. As a result, its mass number reduces to $(226-4) = 222$ and its atomic number reduces to $(88-2) = 86$.

Nuclear reaction: $^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Ra} + {}^{4}_{2}\text{He}$

$Q -$ value of emitted $\alpha -$ particle

$= (\text{Sum of initial mass} - \text{Sum of final mass}) \times c^2$

Where,

c is the speed of light

It is given that:

$m\left({}^{226}_{88}\text{Ra}\right)=226.02540 \text{ u}$

$m\left({}^{222}_{86}\text{Rn}\right)=222.01750 \text{ u}$

$m\left({}^{4}_{2}\text{He}\right)=4.002603 \text{ u}$

$Q-\text{value}=\left[226.02540-\left(222.01750+4.002603\right)\right]uc^2$

$\Rightarrow Q=0.005297uc^2$

It is known that, $1u=931.5\ \dfrac{\text{MeV}}{c^2}$

$\Rightarrow Q=0.005297\times 931.5\approx 4.94\text{MeV}$

Kinetic energy of the α-particle $=\left(\dfrac{\text{Mass number after decay}}{\text{Mass number before decay}}\right)\times Q$

$\Rightarrow \text{K.E.}=\dfrac{222}{226}\times 4.94=4.85\text{MeV}$

Therefore, Q-value is 4.94MeV and Kinetic Energy is 4.85MeV.

b) ${}^{220}_{86}\text{Rn}$

Ans: Alpha particle decay of ${}^{220}_{86}\text{Rn}$ is ${}^{220}_{86}\text{Rn}\rightarrow {}^{216}_{84}\text{Rn}+{}^{4}_{2}\text{He}$

It is given that:

Mass of $m\left({}^{220}_{86}\text{Rn}\right)=220.01137 \text{ u}$

Mass of $m\left({}^{216}_{84}\text{Po}\right)=216.00189 \text{ u}$

$Q-\text{value}=\left[220.01137-\left(216.00189+4.00260\right)\right]\times 931.5$

$\Rightarrow Q \approx 641 MeV$

Kinetic energy of the α-particle $= \left(\frac{220-4}{220}\right) \times 6.41$

$\Rightarrow K.E = 6.29 MeV$

Therefore, Q-value is $641 MeV$ and Kinetic Energy is $6.29 MeV$.

16. The radionuclide 11C decays according to $^{11}_{6}C \to ^{11}_{5}B + e^{+} + \nu$: $T_{1/2} = 20.3 min$. The maximum energy of the emitted positron is $0.960 MeV$. Given the mass values: $m\left(^{11}_{6}C\right) = 11.011434u$ and $m\left(^{11}_{5}B\right) = 11.009305u$ Calculate Q and compare it with the maximum energy of the positron emitted.

Ans: The given nuclear reaction is: $^{11}_{6}C \to ^{11}_{5}B + e^{+} + \nu$

Half life of $^{11}_{6}C$ nuclei, $T_{1/2} = 20.3 min$

Atomic mass of $m\left(^{11}_{6}C\right) = 11.011434u$

Atomic mass of $m\left(^{11}_{5}B\right) = 11.009305u$

Maximum energy possessed by the emitted positron $= 0.960$ MeV

The change in the Q-value (ΔQ) of the nuclear masses of the $^{11}_{6}C$ nucleus is

$$\Delta Q = \left[m'\left(^{11}_{6}C\right) - \left[m'\left(^{11}_{5}B\right) + m_{\varepsilon}\right]\right]c^{2} \quad ....(1)$$

Where,

Mass of an electron or positron $m_{\varepsilon} = 0.000548$ u

c is the speed of light

m' is respective nuclear mass

If atomic masses are used instead of nuclear masses, then $6m_\varepsilon$ is added in case of ${}^{11}_{6}C$ and $5m_\varepsilon$ in the case of ${}^{11}_{5}B$.

Therefore, equation (1) reduces to: $\Delta Q=\left[m\left({}^{11}_{6}C\right)-m\left({}^{11}_{5}B\right)-2m_\varepsilon\right]c^2$

Where, $m\left({}^{11}_{6}C\right)$ and $m\left({}^{11}_{5}B\right)$ are the atomic masses.

$\Delta Q=[11.011434-11.009305-2\times 0.000548]c^2$

$\Delta Q=\left(0.001033c^2\right)u$

It is known that, $1u=931.5\ \dfrac{MeV}{c^2}$

$\Delta Q=0.001033\times 931.5\approx 0.962MeV$

It is known that, $Q=E_d+E_e+E_\nu$

The daughter nucleus is too heavy compared to e^+ and ν. Therefore, it carries negligible energy $\left(E_d\approx 0\right)$.

If the kinetic energy $\left(E_\nu\right)$ carried by the neutrino is minimum $\left(E_\nu\approx 0\right)$ then the positron carries maximum energy, which gives $E_\varepsilon\approx Q$.

Therefore, the value of Q is almost comparable to the maximum energy of the emitted positron.

17. The nucleus ${}^{23}_{10}Ne$ decays by β^- emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that: $m\left({}^{23}_{10}Ne\right)=22.994466\ u$, $m\left({}^{23}_{11}Na\right)=22.989770\ u$.

Ans: During β^- emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

β^- emission of the nucleus ${}^{23}_{10}Ne$ is: ${}^{23}_{10}Ne\rightarrow {}^{23}_{11}Na+e^-+\bar{\nu}+Q$

It is given that:

Atomic mass of $m\left({}_{10}^{23}\text{Ne}\right)=22.994466$ u

Atomic mass of $m\left({}_{11}^{23}\text{Na}\right)=22.989770$ u

Mass of an electron, $m_{\varepsilon}=0.000548$ u

Q-value of the given reaction is: $Q=\left[m\left({}_{10}^{23}\text{Ne}\right)-\left[m\left({}_{11}^{23}\text{Na}\right)+m_{\varepsilon}\right]\right]c^2$

There are 10 electrons in ${}_{10}^{23}\text{Ne}$ and 11 electrons in ${}_{11}^{23}\text{Na}$.

Therefore, the mass of the electron is cancelled in the Q-value equation.

$\Rightarrow Q=\left[22.994466-22.989770\right]c^2$

$\Rightarrow Q=\left(0.004696c^2\right)u$

It is known that, $1u=931.5\ \dfrac{\text{MeV}}{c^2}$

$\Rightarrow Q=\left(0.004696c^2\right)u\times 931.5\ \dfrac{\text{MeV}}{c^2}=4.374\text{ MeV}$

The daughter nucleus is too heavy as compared to e^- and $\bar{\nu}$.

Therefore, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero.

Thus, the maximum kinetic energy of the emitted electrons is almost equal to the Q-value, i.e. 4.374 MeV.

18. The Q value of a nuclear reaction $A+b\rightarrow c+d$ is defined by $Q=\left[m_A+m_b-m_c-m_d\right]c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

Atomic masses are given to be $m\left({}_1^2\text{H}\right)=2.014102\text{u}$, $m\left({}_1^3\text{H}\right)=3.016049\text{u}$, $m\left({}_6^{12}\text{C}\right)=12.000000\text{u}$, $m\left({}_{10}^{20}\text{Ne}\right)=19.992439\text{u}$

a) $\mathbf{{}^{1}_{1}H + {}^{3}_{1}H \rightarrow {}^{2}_{1}H + {}^{2}_{1}H}$

Ans: The given nuclear reaction is: ${}^{1}_{1}H + {}^{3}_{1}H \rightarrow {}^{2}_{1}H + {}^{2}_{1}H$

It is given that,

Atomic mass, $m\left({}^{1}_{1}H\right) = 1.007825u$

Atomic mass, $m\left({}^{3}_{1}H\right) = 3.016049u$

Atomic mass, $m\left({}^{2}_{1}H\right) = 2.014102u$

Q-value of the reaction: $\Delta Q = \left[m\left({}^{1}_{1}H\right) + m\left({}^{3}_{1}H\right) - 2m\left({}^{2}_{1}H\right)\right]c^2$

$\Rightarrow \Delta Q = [1.007825 + 3.016049 - 2 \times 2.014102]c^2$

$\Rightarrow \Delta Q = (-0.00433)c^2u$

It is known that, $1u = 931.5 \dfrac{MeV}{c^2}$

$\Rightarrow \Delta Q = (-0.00433) \times c^2 \times 931.5 \dfrac{MeV}{c^2} = -4.0334MeV$

The negative Q-value of the reaction shows that the reaction is endothermic.

b) $\mathbf{{}^{12}_{6}C + {}^{12}_{6}C \rightarrow {}^{20}_{10}Ne + {}^{4}_{2}He}$

Ans: The given nuclear reaction is: ${}^{12}_{6}C + {}^{12}_{6}C \rightarrow {}^{20}_{10}Ne + {}^{4}_{2}He$

It is given that,

Atomic mass $m\left({}^{12}_{6}C\right) = 12.000000u$

Atomic mass $m\left({}^{20}_{10}Ne\right) = 19.992439u$

Atomic mass $m\left({}^{4}_{2}He\right) = 4.002603u$

Q-value of the reaction: $\Delta Q = \left[2m\left({}^{12}_{6}C\right) - m\left({}^{20}_{10}Ne\right) - m\left({}^{4}_{2}He\right)\right]c^2$

$\Rightarrow \Delta Q = [2 \times 12.0 - 19.992439 - 4.002603]c^2$

$\Rightarrow \Delta Q = (0.004958)c^2 u$

It is known that, $1u = 931.5\ \frac{MeV}{c^2}$

$\Rightarrow \Delta Q = (0.004958)c^2 \times 931.5\ \frac{MeV}{c^2} = 4.618377 MeV$

The positive Q-value of the reaction shows that the reaction is exothermic.

19. A $1000MW$ fission reactor consumes half of its fuel in $5.00y$. How much $^{235}_{92}U$ did it contain it initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

Ans: It is given that,

Half life of the fuel of the fission reactor, $t_{1/2} = 5 years$

It is known that in the fission of $1g$ of $^{235}_{92}U$ nucleus, the energy released is equal to $200MeV$

$1 mole$ i.e., $235g$ of $^{235}_{92}U$ contains 6.023×10^{23} atoms.

$1g$ of $^{235}_{92}U$ contains $\frac{6.023 \times 10^{23}}{235}$ atoms.

The total energy generated per gram of is: $E = \frac{6.023 \times 10^{23}}{235} \times 200 MeV/g$

$\Rightarrow E = \frac{200 \times 6.023 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^6}{235} = 8.20 \times 10^{10} J/g$

The reactor operates only 80% of the time.

Hence, the amount of ${}^{235}_{92}\mathrm{U}$ consumed in 5 years by the 1000MW fission reactor is: $\frac{5\times80\times60\times60\times365\times24\times1000\times10^{6}}{100\times8.20\times10^{10}}\mathrm{g}\approx1538\mathrm{kg}$

Therefore, initial amount of ${}^{235}_{92}\mathrm{U}=2\times1538=3076\mathrm{kg}$

20. How long can an electric lamp of 100W be kept glowing by fusion of 2.0kg of deuterium? Take the fusion reaction as

$\mathbf{{}^{2}_{1}H+{}^{2}_{1}H\rightarrow{}^{3}_{2}He+n+3.27MeV}$.

Ans: The given fusion reaction is: ${}^{2}_{1}\mathrm{H}+{}^{2}_{1}\mathrm{H}\rightarrow{}^{3}_{2}\mathrm{He}+\mathrm{n}+3.27\mathrm{MeV}$

Amount of deuterium, $m=2\mathrm{kg}$

1 mole, i.e., 2g of deuterium contains 6.023×10^{23} atoms.

2kg of deuterium contains $=\frac{6.023\times10^{23}}{2}\times2000=6.023\times10^{26}\mathrm{atoms}$

It can be concluded from the given reaction that when two atoms of deuterium fuse, 3.27MeV energy is released.

Thus, total energy per nucleus released in the fusion reaction:

$$E=\frac{3.27}{2}\times6.023\times10^{23}\mathrm{MeV}$$

$$\Rightarrow E=\frac{3.27}{2}\times6.023\times10^{23}\times1.6\times10^{-19}\times10^{6}$$

$$\Rightarrow E=1.576\times10^{14}\mathrm{J}$$

Power of the electric lamp, $P=100\mathrm{W}=100\ \mathrm{J/s}$

Hence, the energy consumed by the lamp per second $=100\mathrm{J}$

The total time for which the electric lamp will glow is:

$$\frac{1.576\times10^{14}}{100\times60\times60\times24\times365}\approx4.9\times10^{4}\,\mathrm{years}$$

Therefore, the total time for which the lamp glows is 4.9×10^{4} years.

21. For the β^{+} (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K - shell, is captured by the nucleus and a neutrino is emitted). $e^{+}+{}_{Z}^{A}X\rightarrow{}_{Z-1}^{A}Y+\nu$. Show that if β^{+} emission is energetically allowed, electron capture is necessarily allowed but not vice -versa.

Ans: Let the amount of energy released during the electron capture process be Q_1 . The nuclear reaction can be written as: $e^{+}+{}_{Z}^{A}X\rightarrow{}_{Z-1}^{A}Y+\nu+Q_1$ ......(1)

Let the amount of energy released during the positron capture process be Q_2. The nuclear reaction can be written as: ${}_{Z}^{A}X\rightarrow{}_{Z-1}^{A}Y+e^{+}+\nu+Q_1$ ......(2)

$m_N\left({}_{Z}^{A}X\right)$ is the nuclear mass of ${}_{Z}^{A}X$

$m_N\left({}_{Z-1}^{A}Y\right)$ is the nuclear mass of ${}_{Z-1}^{A}Y$

$m\left({}_{Z}^{A}X\right)$ is the atomic mass of ${}_{Z}^{A}X$

$m\left({}_{Z-1}^{A}Y\right)$ is the atomic mass of ${}_{Z-1}^{A}Y$

m_ε is the mass of an electron

c is the speed of light

Q-value of the electron capture reaction is: $Q_1=\left[m_N\left({}_{Z}^{A}X\right)+m_\varepsilon-m_N\left({}_{Z-1}^{A}Y\right)\right]c^2$

$$\Rightarrow Q_1=\left[m\left({}_{Z}^{A}X\right)+Zm_\varepsilon+m_\varepsilon-m\left({}_{Z-1}^{A}Y\right)-(Z-1)m_\varepsilon\right]c^2$$

$$\Rightarrow Q_1=\left[m\left({}_{Z}^{A}X\right)-m\left({}_{Z-1}^{A}Y\right)\right]c^2 \quad ....(3)$$

Q-value of the positron capture reaction is: $Q_2=\left[m\left({}_{Z}^{A}X\right)-m\left({}_{Z-1}^{A}Y\right)-2m_\varepsilon\right]c^2$

$$\Rightarrow Q_2=\left[m_N\left({}_{Z}^{A}X\right)-m_N\left({}_{Z-1}^{A}Y\right)-m_\varepsilon\right]c^2$$

$$\Rightarrow Q_2 = \left[m\left({}^{A}_{Z}X\right) - Zm_{\varepsilon} - m\left({}^{A}_{Z-1}Y\right) + (Z-1)m_{\varepsilon} - m_{\varepsilon} \right]c^2$$

$$\Rightarrow Q_2 = \left[m\left({}^{A}_{Z}X\right) - m\left({}^{A}_{Z-1}Y\right) - 2m_{\varepsilon} \right]c^2 \quad ....(4)$$

It is understood that if $Q_2 > 0$, then $Q_1 > 0$; Also, if $Q_1 > 0$, it does not necessarily mean that $Q_2 > 0$.

Also, this means that if β^+ emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is because the Q-value must be positive for an energetically-allowed nuclear reaction.

22. In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}^{24}_{12}Mg(23.98504u)$, ${}^{25}_{12}Mg(24.98584u)$ and ${}^{26}_{12}Mg(25.98259u)$. The natural abundance of ${}^{24}_{12}Mg$ is 78.99% by mass. Calculate the abundances of the other two isotopes.

Ans: It is given that,

Average atomic mass of magnesium, $m = 24.312\text{ u}$

Mass of magnesium isotope ${}^{24}_{12}Mg$, $m_1 = 23.98504\text{ u}$

Mass of magnesium isotope ${}^{25}_{12}Mg$, $m_2 = 24.98584\text{ u}$

Mass of magnesium isotope ${}^{26}_{12}Mg$, $m_3 = 25.98259\text{ u}$ Abundance of ${}^{24}_{12}Mg$, $\eta_1 = 78.99\%$

Abundance of ${}^{25}_{12}Mg$, $\eta_2 = x\%$

Abundance of ${}^{26}_{12}Mg$, $\eta_3 = (100 - x - 78.99)\% = (21.01 - x)\%$

Relation for the average atomic mass is: $m = \dfrac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$

$$\Rightarrow 24.312 = \frac{23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 - x)}{100}$$

$$\Rightarrow 2431.2 = 23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 - x)$$

$\Rightarrow 0.99675x = 9.2725255$

$\Rightarrow x \approx 9.3\%$

$(21.01 - x)\% \Rightarrow 11.71\%$

Therefore, the abundance of ${}_{12}^{25}\text{Mg}$ is 9.3% and that of ${}_{12}^{26}\text{Mg}$ is 11.71%.

23. The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}_{20}^{41}\text{Ca}$ and ${}_{13}^{27}\text{Al}$ from the following data: $m\left({}_{20}^{40}\text{Ca}\right) = 39.962591\ \text{u}$, $m\left({}_{20}^{41}\text{Ca}\right) = 40.962278\ \text{u}$, $m\left({}_{13}^{26}\text{Al}\right) = 25.986895\ \text{u}$, $m\left({}_{13}^{27}\text{Al}\right) = 26.981541\ \text{u}$

Ans: For ${}_{20}^{41}\text{Ca}$: Separation energy $= 8.363007\text{MeV}$

For ${}_{13}^{27}\text{Al}$: Separation energy $= 13.059\text{MeV}$

A neutron ${}_{0}^{1}\text{n}$ is removed from a ${}_{20}^{41}\text{Ca}$ nucleus. The corresponding nuclear reaction can be written as: ${}_{20}^{41}\text{Ca} \to {}_{20}^{40}\text{Ca} + {}_{0}^{1}\text{n}$

It is given that,

Mass $m\left({}_{20}^{40}\text{Ca}\right) = 39.962591\ \text{u}$

Mass $m\left({}_{20}^{41}\text{Ca}\right) = 40.962278\ \text{u}$

Mass $m\left({}_{0}^{1}\text{n}\right) = 1.008665\text{u}$

The mass defect of this reaction is: $\Delta m = m\left({}_{20}^{40}\text{Ca}\right) + m\left({}_{0}^{1}\text{n}\right) - m\left({}_{20}^{41}\text{Ca}\right)$

$\Rightarrow \Delta m = 39.962591 + 1.008665 - 40.962278 = 0.008978\text{u}$

It is known that, $1\text{u} = 931.5\ \dfrac{\text{MeV}}{c^2}$

$\Rightarrow \Delta m = 0.008978 \times 931.5\ \dfrac{\text{MeV}}{c^2}$

Therefore, the energy required for neutron removal is: $E = \Delta mc^2$

$$\Rightarrow E = 0.008978 \times 931.5\ \frac{MeV}{c^2}c^2$$

$$\Rightarrow E = 0.008978 \times 931.5 MeV$$

For ${}^{27}_{13}Al$, the neutron removal reaction can be written as: ${}^{27}_{13}Al \to {}^{26}_{13}Ca + {}^{1}_{0}n$

It is given that:

Mass $m\left({}^{26}_{13}Al\right) = 25.986895\ u$

Mass $m\left({}^{27}_{13}Al\right) = 26.981541\ u$

The mass defect of this reaction is: $\Delta m = m\left({}^{26}_{13}Al\right) + m\left({}^{1}_{0}n\right) - m\left({}^{27}_{13}Al\right)$

$$\Delta m = 25.986895 + 1.00866 - 26.98154140$$
$$\Delta m = 0.014019u$$

It is known that, $1u = 931.5\ \frac{MeV}{c^2}$

$$\Delta m = 0.014019 \times 931.5\ \frac{MeV}{c^2}$$

$$E = \Delta mc^2 = 0.014019 \times 931.5\ \frac{MeV}{c^2}c^2 = 13.059MeV$$

Therefore, the energy required for neutron removal is $13.059MeV$.

24. A source contains two phosphorous radio nuclides ${}^{32}_{15}P\ \left(T_{1/2} = 14.3d\right)$ and ${}^{32}_{15}P\ \left(T_{1/2} = 25.3d\right)$. Initially, 10% of the decays come from ${}^{33}_{15}P$. How long one must wait until 90% do so?

Ans: Half life of ${}^{32}_{15}P$, $T_{1/2} = 14.3days$

Half life of $_{15}^{33}P$, $T_{1/2}' = 25.3$days

$_{15}^{33}P$ nucleus decay is 10% of the total amount of decay.

The source has initially 10% of $_{15}^{33}P$ nucleus and 90% of $_{15}^{32}P$ nucleus.

Suppose after t days, the source has 10% of $_{15}^{32}P$ nucleus and 90% of $_{15}^{33}P$ nucleus.

Initially:

Number of $_{15}^{33}P$ nucleus $= N$

Number of $_{15}^{32}P$ nucleus $= 9N$

Finally:

Number of $_{15}^{33}P$ nucleus $= 9N'$

Number of $_{15}^{32}P$ nucleus $= N'$

For $_{15}^{32}P$ nucleus, the number ratio is: $\frac{N'}{9N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$$\Rightarrow N' = 9N\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$\Rightarrow N' = 9N(2)^{\frac{-t}{14.3}} \quad ....(1)$$

For $_{15}^{33}P$ nucleus, the number ratio is: $\frac{9N'}{N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$$\Rightarrow 9N' = N\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$\Rightarrow 9N' = N(2)^{\frac{-t}{25.3}} \quad ....(2)$$

On dividing equation (1) by equation (2): $\frac{1}{9} = 9 \times 2^{\left(\frac{t}{25.3} - \frac{t}{14.3}\right)}$

$$\Rightarrow \frac{1}{81} = 2^{\left(\frac{t}{25.3} - \frac{t}{14.3}\right)}$$

$$\Rightarrow 81 = 2^{\left(\frac{t}{14.3} - \frac{t}{25.3}\right)}$$

$$\Rightarrow \log 81 = \left(\frac{t}{14.3} - \frac{t}{25.3}\right)\log 2$$

$$\Rightarrow t\left(\frac{25.3 - 14.3}{14.3 \times 25.3}\right) = \frac{1.9085}{0.3010}$$

$$\Rightarrow t = \frac{1.9085}{0.3010} \times \frac{14.3 \times 25.3}{25.3 - 14.3} = 208.54\text{days}$$

Therefore, it will take about 208.5days for 90% decay of ${}^{33}_{15}P$.

25. Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes: ${}^{223}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C$, ${}^{223}_{88}Ra \rightarrow {}^{219}_{86}Rn + {}^{4}_{2}He$. Calculate the Q-values for these decays and determine that both are energetically allowed.

Ans: Considering a ${}^{14}_{6}C$ emission nuclear reaction: ${}^{223}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C$

It is known that,

Mass of ${}^{223}_{88}Ra$, $m_1 = 223.01850u$

Mass of ${}^{209}_{82}Pb$, $m_2 = 208.98107\ u$

Mass of ${}^{14}_{6}C$, $m_3 = 14.00324\ u$

Thus, the Q-value of the reaction is: $Q = (m_1 - m_2 - m_3)c^2$

$$\Rightarrow Q = (223.01850 - 208.98107 - 14.00324)c^2$$

$$\Rightarrow Q = 0.03419c^2u$$

It is known that, $1u = 931.5\ \frac{MeV}{c^2}$

$$\Rightarrow Q = 0.03419c^2 \times 931.5\ \frac{\text{MeV}}{c^2}$$

$$\Rightarrow Q = 31.848\ \text{MeV}$$

Therefore, the Q-value of the nuclear reaction is $31.848\ \text{MeV}$. Since the value is positive, the reaction is energetically allowed.

Considering a ${}^{4}_{2}\text{He}$ emission nuclear reaction: ${}^{223}_{88}\text{Ra} \to {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He}$

It is known that:

Mass of ${}^{223}_{88}\text{Ra}$, $m_1 = 223.01850\text{u}$

Mass of ${}^{219}_{86}\text{Rn}$, $m_2 = 219.00948\ \text{u}$

Mass of ${}^{4}_{2}\text{He}$, $m_3 = 4.00260\ \text{u}$

Q-value of this nuclear reaction is $Q = (m_1 - m_2 - m_3)c^2$

$$\Rightarrow Q = (223.01850 - 219.00948 - 4.00260)c^2$$

$$\Rightarrow Q = (0.00642c^2)\text{u}$$

It is known that, $1\text{u} = 931.5\ \frac{\text{MeV}}{c^2}$

$$\Rightarrow Q = (0.00642c^2) \times 931.5\ \frac{\text{MeV}}{c^2}$$

$$\Rightarrow Q = 5.98\ \text{MeV}$$

Therefore, the Q-value of the second nuclear reaction is $5.98\ \text{MeV}$. Since the value is positive, the reaction is energetically allowed.

26. Consider the fission of ${}^{238}_{92}\text{U}$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are ${}^{140}_{58}\text{Ce}$ and ${}^{99}_{44}\text{Ru}$. Calculate Q for this fission process. The relevant atomic and particle masses are $m({}^{238}_{92}\text{U}) = 238.05079\ \text{u}$, $m({}^{140}_{58}\text{Ce}) = 139.90543\ \text{u}$, $m({}^{99}_{44}\text{Ru}) = 98.90594\ \text{u}$

Ans: In the fission of ${}_{92}^{238}U$, 10 β^- particles decay from the parent nucleus. The nuclear reaction can be written as: ${}_{92}^{238}U + {}_{0}^{1}n \to {}_{58}^{140}Ce + {}_{44}^{99}Ru + 10\,{}_{-1}^{0}e$

It is given that:

Mass of a nucleus ${}_{92}^{238}U$, $m_1 = 238.05079$ u

Mass of a nucleus ${}_{58}^{140}Ce$, $m_2 = 139.90543$ u

Mass of a nucleus ${}_{44}^{99}Ru$, $m_3 = 98.90594$ u

Mass of a neutron ${}_{0}^{1}n$, $m_4 = 1.008665$ u

Q-value of the above equation is:

$$Q = \left[m'\left({}_{92}^{238}U\right) + m\left({}_{0}^{1}n\right) - m'\left({}_{58}^{140}Ce\right) - m'\left({}_{44}^{99}Ru\right) - 10m_\varepsilon\right]c^2$$

Where,

$m' =$ Represents the corresponding atomic masses of the nuclei

$$m'\left({}_{92}^{238}U\right) = m_1 - 92m_\varepsilon$$

$$m'\left({}_{58}^{140}Ce\right) = m_2 - 58m_\varepsilon$$

$$m'\left({}_{44}^{99}Ru\right) = m_3 - 44m_\varepsilon$$

$$m\left({}_{0}^{1}n\right) = m_4$$

$$\Rightarrow Q = [m_1 - 92m_\varepsilon + m_4 - m_2 + 58m_\varepsilon - m_3 + 44m_\varepsilon - 10m_\varepsilon]c^2$$

$$\Rightarrow Q = [m_1 + m_4 - m_2 - m_3]c^2$$

$$\Rightarrow Q = [238.0507 + 1.008665 - 139.90543 - 98.90594]c^2$$

$$\Rightarrow Q = \left[0.247995c^2\right]u$$

It is known that, $1u = 931.5\ \dfrac{MeV}{c^2}$

$$\Rightarrow Q=\left[0.247995c^2\right]\times 931.5\ \frac{MeV}{c^2}=231.007MeV$$

Therefore, the Q-value of the fission process is 231.007 MeV.

27. Consider the D-T reaction (deuterium - tritium fusion)

a) Calculate the energy released in MeV in this reaction from the data:

$$\mathbf{m\left({}_1^2H\right)=2.014102\ u,\ m\left({}_1^3H\right)=3.016049\ u}$$

Ans: Consider the D-T nuclear reaction: ${}_1^2H+{}_1^3H\rightarrow{}_2^4He+{}_0^1n$

It is given that:

Mass of ${}_1^2H$, $m_1=2.014102\ u$

Mass of ${}_1^3H$, $m_2=3.016049\ u$

Mass of ${}_2^4He$, $m_3=4.002603\ u$

Mass of ${}_0^1n$, $m_4=1.008665\ u$

Q-value of the given D-T reaction is: $Q=\left[m_1+m_2-m_3-m_4\right]c^2$

$$\Rightarrow Q=\left[2.014102+3.016049-4.002603-1.008665\right]c^2$$

$$\Rightarrow Q=\left[0.018883\right]c^2u$$

It is known that, $1u=931.5\ \frac{MeV}{c^2}$

$$\Rightarrow Q=\left[0.018883\right]c^2\times 931.5\ \frac{MeV}{c^2}$$

$$\Rightarrow Q=17.59MeV$$

Therefore, the energy released is $17.59MeV$.

b) Consider the radius of both deuterium and tritium to be approximately 2.0fm . What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event= average thermal kinetic energy available with the interacting particles $= 2(3kT/2)$; $k =$ Boltzmann's constant, $T =$ absolute temperature.)

Ans: It is given that,

Radius of deuterium and tritium, $r \approx 2.0\text{fm} = 2\times10^{-15}\text{m}$

Distance between the two nuclei at the moment when they touch each other, $d = r + r = 4\times10^{-15}\text{m}$

Charge on the deuterium nucleus $= e$

Charge on the tritium nucleus $= e$

Therefore, the repulsive potential energy between the two nuclei is: $V = \dfrac{e^2}{4\pi\varepsilon_0(d)}$

Where,

$\varepsilon_0 =$ Permittivity of free space

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{Nm}^2\text{C}^{-2}$$

$$\Rightarrow V = \frac{9\times10^9\times(1.6\times10^{-19})^2}{4\times10^{-15}} = 5.76\times10^{-14}\,\text{J}$$

$$\Rightarrow V = \frac{5.76\times10^{-14}}{1.6\times10^{-19}} = 3.6\times10^5\,\text{eV} = 360\text{keV}$$

Therefore, $5.76\times10^{-14}\,\text{J}$ or 360keV kinetic energy (KE) is needed to overcome the Coulomb repulsion between the two nuclei.

It is given that,

$$\text{K.E.} = 2\times\frac{3}{2}kT$$

Where, $k = \text{Boltzmann constant} = 1.38 \times 10^{-23} m^2 kg s^{-2} K^{-1}$

$T =$ Temperature required for triggering the reaction

$$\Rightarrow T = \frac{K.E.}{3k}$$

$$\Rightarrow T = \frac{5.76 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 1.39 \times 10^9 K$$

Therefore, the gas must be heated to a temperature of $1.39 \times 10^9 K$ to initiate the reaction.

28. Calculate and compare the energy released by

a) fusion of $1.0 kg$ of hydrogen deep within Sun

Ans: It is given that,

Amount of hydrogen, $m = 1kg = 1000g$

1 mole, i.e., $1g$ of hydrogen $\left({}_1^1H\right)$ contains 6.023×10^{23} atoms.

$1000g$ of ${}_1^1H$ contains $6.023 \times 10^{23} \times 1000$ atoms.

Within the sun, four ${}_1^1H$ nuclei combine and form one ${}_2^4He$ nucleus. In this process $26 MeV$ of energy is released.

Therefore, the energy released from the fusion of $1kg$ is:

$$E_1 = \frac{6.023 \times 10^{23} \times 26 \times 10^3}{4} = 39.1495 \times 10^{26} MeV$$

Therefore, the energy released is $39.1495 \times 10^{26} MeV$.

b) the fission of $1.0 kg$ of ${}_{92}^{235}U$ in a fission reactor.

Ans: It is given that,

Amount of ${}_{92}^{235}U$, $m = 1kg = 1000g$

1 mole, i.e., 235g of $^{235}_{92}U$ contains 6.023×10^{23} atoms.

1000g of $^{235}_{92}U$ contains $\frac{6.023\times10^{23}\times1000}{235}$ atoms.

It is known that the amount of energy released in the fission of one atom of is 200 MeV.

Therefore, the energy released from the fission of 1kg of $^{235}_{92}U$ is:

$$E_2=\frac{6\times10\times200\times1000}{235}=5.106\times10^{26}\text{MeV}$$

Therefore, the energy released is $5.106\times10^{26}\text{MeV}$.

$$\frac{E_1}{E_2}=\frac{39.1495\times10^{26}}{5.106\times10^{26}}=7.67\approx8$$

Therefore, the energy released in the fusion of 1kg of hydrogen is nearly 8 times the energy released in the fission of 1kg of uranium.

29. Suppose India had a target of producing by 2020 AD , 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e., conversion to electric energy) of thermal energy produced in a reactor was 25% . How much amount of fissionable uranium would our country need per year by 2020 ? Take the heat energy per fission of ^{235}U to be about 200MeV .

Ans: It is given that,

Amount of electric power to be generated, $P=2\times10^5\text{MW}$

10% of this amount has to be obtained from nuclear power plants.

Therefore, Amount of nuclear power, $P_1=\frac{10}{100}\times2\times10^5$

$$\Rightarrow P_1=2\times10^4\text{MW}$$

$$\Rightarrow P_1=2\times10^4\times10^6\text{J/s}$$

$\Rightarrow P_1 = 2\times10^4 \times10^6 \times60\times60\times24\times365 J/y$

Heat energy released per fission of a ^{235}U nucleus, $E = 200$ MeV

Efficiency of a reactor $= 25\%$

Therefore, the amount of energy converted into the electrical energy per fission is: $\frac{25}{100}\times200 = 50 MeV$

$\Rightarrow E = 50\times1.6\times10^{-19}\times10^6 = 8\times10^{-12} J$

Number of atoms required for fission per year:

$$\frac{2\times10^4\times10^6\times60\times60\times24\times365}{8\times10^{-12}} = 78840\times10^{24} \text{atoms}$$

1 mole, i.e., 235g of ^{235}U contains 6.023×10^{23} atoms.

Mass of 6.023×10^{23} atoms of $^{235}U = 235g = 235\times10^{-3} kg$

Mass of 78840×10^{24} atoms of $^{235}U = \frac{235\times10^{-3}}{6.023\times10^{23}}\times78840\times10^{24}$

$\Rightarrow \text{Mass} = 3.076\times10^4 kg$

Therefore, the mass of uranium needed per year is $3.076\times10^4 kg$.