Important Questions for Class 12

Physics

Chapter 12 - Atoms

Very Short Answer Questions **1 Mark**

1. Name the series of hydrogen spectrum lying in ultraviolet and visible region.
Ans: Lyman series lies in ultraviolet region while Balmer series lies in visible region.

2. What is Bohr's quantisation condition for the angular momentum of an electron in the second orbit?
Ans: We know that,

$$L = \frac{nh}{2\pi}$$

We are given, $n = 2$

$$\Rightarrow L = \frac{2h}{2\pi}$$

$$\therefore L = \frac{h}{\pi}$$

Therefore, Bohr's quantisation condition for the angular momentum of an electron in the second orbit is found to be, $L = \frac{h}{\pi}$.

Short Answer Questions **2 Marks**

1. Define Bohr's radius.
Ans: The radius of the first orbit of hydrogen atom is termed as Bohr's radius.
Its value if found to be $5.29 \times 10^{-11} m = 0.53 \overset{\circ}{A}$.

2. State the limitations of Bohr's atomic model.
Ans: The limitations of Bohr's atomic model are:
(1) It does not give any indication regarding the arrangement and distribution of electrons in an atom.
(2) It could not account for the wave nature of electrons.

3. Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil.

(Hydrogen is a solid at temperatures below 14K .) What results do you expect?

Ans: In the alpha-particle scattering experiment, when a thin sheet of solid hydrogen is used in place of a gold foil, then the scattering angle would not be large enough. This is because, mass of hydrogen (1.67×10^{-27} kg) is less than that of the mass of incident α -particles (6.64×10^{-27} kg). Thus, the mass of the scattering particle is more than the target nucleus (hydrogen). As a result, the α-particles would not bounce back if solid hydrogen is used in the α-particle scattering experiment.

4. The ground state energy of hydrogen atom is -13.6eV. What are the kinetic and potential energies of the electron in this state?

Ans: We are given,

Ground state energy of hydrogen atom, $E=-13.6\text{eV}$

This is the total energy of a hydrogen atom. Kinetic energy is equal to the negative of the total energy.

Kinetic energy $=-E=-(-13.6)=13.6\text{eV}$

Potential energy is equal to the negative of two times of kinetic energy.

Therefore, Potential energy $=-2\times(13.6)=-27.2\text{eV}$

5. If Bohr's quantisation postulate (angular momentum $=nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Ans: Since, the angular momentum associated with planetary motion is largely relative to the value of Planck's constant (h), we never speak of quantization of orbits of planets around the Sun. The angular momentum of the Earth in its orbit is found to be of the order of $1070h$. This leads to a very high value of quantum levels n of the order of 1070. For large values of n, successive energies and angular momenta are found to be relatively very small. Hence, the quantum levels for planetary motion are always considered continuous.

Short Answer Question **3 Marks**

1. The half-life period of a radioactive substance is 30 days. What is the time for $\frac{3}{4}$th of its original mass to disintegrate?

Ans: We know that, $\frac{N}{N_o}=\left(\frac{1}{2}\right)\frac{t}{T}$

Here, we are given, $N = N_o - \frac{3}{4}N_o$

$$N = \frac{1}{4}N_o$$

$$\Rightarrow \frac{1}{4} = \left(\frac{1}{2}\right)\frac{t}{30}$$

Or $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)\frac{t}{30}$

$$\Rightarrow \frac{t}{30} = 2$$

$$\Rightarrow t = 60\text{days}$$

Therefore, we found the time for $\frac{3}{4}$th of the original mass to disintegrate to be 60days.

2. How many α and β- particles are emitted when $90\overset{232}{\text{Th}}$ changes to $82\overset{208}{\text{Pb}}$?

Ans: The mentioned reaction is as follows:

$$90\overset{232}{\text{Th}} \rightarrow 82\overset{208}{\text{Pb}} + 2\overset{4}{\text{He}} + y\overset{o}{_{-1}e}$$

According to low of conservation of atomic number and mass number

$$90 = 82 + 2x - y$$

$$2x - y = 8 \quad ...... (1)$$

$$232 = 208 + 4x$$

$$\Rightarrow x = 6 \quad ...... (2)$$

From (1) & (2)

$$2(6) - y = 8$$

$$\Rightarrow 12 - 8 = y$$

$$\Rightarrow y = 4$$

The number of α and β- particles emitted when $90\overset{232}{\text{Th}}$ changes to $82\overset{208}{\text{Pb}}$ is found to be 6 and 4 respectively.

3. Binding energies of $^{16}_{8}\text{O}$ and $^{35}_{17}\text{Cl}$ are 127.35MeV and 289.3MeV respectively. Which of the two nuclei are more stable?

Ans: We know that the stability of a nucleus is proportional to binding energy per nucleon

B.E / nucleon of $^{16}_{8}O = \dfrac{127.35}{8} = 15.82 MeV / nucleon$

B.E / nucleon of $^{35}_{17}Cl = \dfrac{289.3}{17} = 17.02 MeV / nucleon$

$\therefore$ $^{35}_{17}Cl$ is found to be more stable than $^{16}_{8}O$.

4. What is the shortest wavelength present in the Paschen series of spectral lines?

Ans: We know that Rydberg's formula is given as:

$$\frac{hc}{\lambda} = 21.76\times10^{-19}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Where,

$h =$ Planck's constant $= 6.6\times10^{-34} Js$

$c =$ Speed of light $= 3\times10^{8} m/s$

(n_1 and n_2 are integers)

Now, the shortest wavelength present in the Paschen series of the spectral lines is given for values $n_1 = 3$ and $n_2 = \infty$.

$$\frac{hc}{\lambda} = 21.76\times10^{-19}\left[\frac{1}{(3)^2} - \frac{1}{(\infty)^2}\right]$$

$$\Rightarrow \lambda = \frac{6.6\times10^{-34}\times3\times10^{8}\times9}{21.76\times10^{-19}}$$

$$\Rightarrow \lambda = 8.189\times10^{7} m$$

$$\therefore \lambda = 818.9 nm$$

Therefore, the shortest wavelength present in the Paschen series of spectral lines is found to be $\lambda = 818.9 nm$.

5. A difference of $2.3 eV$ separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?

Ans: We are given the separation of two energy levels in an atom,

$E = 2.3 eV = 2.3\times1.6\times10^{-19} = 3.68\times10^{-19} J$

Now, let v be the frequency of radiation emitted when the atom transits from the upper level to the lower level.

We have the relation for energy as:

$E = hv$

Where,

$h =$ Planck's constant $= 6.62\times10^{-4} Js$

$\Rightarrow v = \frac{E}{h}$

Substituting the given values,

$$v = \frac{3.68\times10^{-19}}{6.62\times10^{-32}} = 6.62\times10^{-32} = 5.55\times10^{14}\,Hz$$

Hence, the frequency of the radiation is found to be $5.55\times10^{14}\,Hz$.

6. The radius of the innermost electron orbit of a hydrogen atom is $5.3\times10^{-11}\,m$. What are the radii of the $n=2$ and $n=3$ orbits?

Ans: The radius of the innermost orbit of a hydrogen atom, $r_1 = 5.3\times10^{-11}\,m$

Let r_2 be the radius of the orbit at $n=2$. It is related to the radius of the innermost orbit as:

$$r_2 = (n)^2 r_1 = 4\times5.3\times10^{-11} = 2.12\times10^{-10}\,m$$

For $n=3$, we can write the corresponding electron radius as:

$$r_3 = (n)^2 r_1 = 9\times5.3\times10^{-11} = 4.77\times10^{-10}\,m$$

Hence, the radii of an electron for $n=2$ and $n=3$ orbits are found to be equal to $2.12\times10^{-10}\,m$ and $4.77\times10^{-10}\,m$ respectively.

7. In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius $1.5\times10^{11}\,m$ with orbital speed $3\times10^{4}\,m/s$ (Mass of earth $= 6.0\times10^{24}\,kg$.)

Ans: We are given:

Radius of the orbit of the Earth around the Sun, $r = 1.5\times10^{11}\,m$

Orbital speed of the Earth, $v = 3\times10^{4}\,m/s$

Mass of the Earth, $m = 6.0\times10^{24}\,kg$

According to Bohr's model, angular momentum is quantized and could be given as:

$$mvr = \frac{nh}{2\pi}$$

Where,

$h =$ Planck's constant $= 6.62\times10^{-34}\,Js$

$n =$ Quantum number

$$\Rightarrow n = \frac{mvr2\pi}{h}$$

$$\Rightarrow n = \frac{2\pi\times6\times10^{24}\times3\times10^{4}\times1.5\times10^{11}}{6.62\times10^{-34}}$$

$$\therefore n = 25.61\times10^{73} = 2.6\times10^{74}$$

Hence, the quanta number that characterizes the Earth' revolution is found to be 2.6×10^{74}.

8. The total energy of an electron in the first excited state of the hydrogen atom is about -3.4eV.

a) What is the kinetic energy of the electron in this state?

Ans: (a) We are given,

Total energy of the electron, $E=-3.4\text{eV}$

Kinetic energy of the electron is equal to the negative of the total energy.

$\Rightarrow K.E=-E$

$\therefore K.E=-\left(-3.4\right)=+3.4\text{eV}$

Hence, the kinetic energy of the electron in the given state is found to be $+3.4\text{eV}$.

b) What is the potential energy of the electron in this state?

Ans: We know that, the potential energy (U) of the electron is found to be equal to the negative of twice of its kinetic energy.

$\Rightarrow U=-2K.E$

$\therefore U=-2\times3.4=-6.8\text{eV}$

Hence, the potential energy of the electron in the given state is found to be -6.8eV.

c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Ans: We know that, the potential energy of a system would depend on the reference point taken. Here, the potential energy of the reference point is taken to be zero. On changing the reference point, then the value of the potential energy of the system would also change. Since, we know that total energy is the sum of kinetic and potential energies, total energy of the system will also change.

Long Answer Questions **5 Marks**

1. The total energy of an electron in the first excited state of hydrogen atom is -3.4eV. Calculate:

a) K.E. of the electron in this state.

Ans: We know that,

$K.E=-E$

$\therefore K.E=3.4\text{eV}$

b) P.E. of the electron in this state.

Ans: $P.E = 2 \times K.E$
$\therefore P.E = 2 \times 3.4 = 6.8eV$

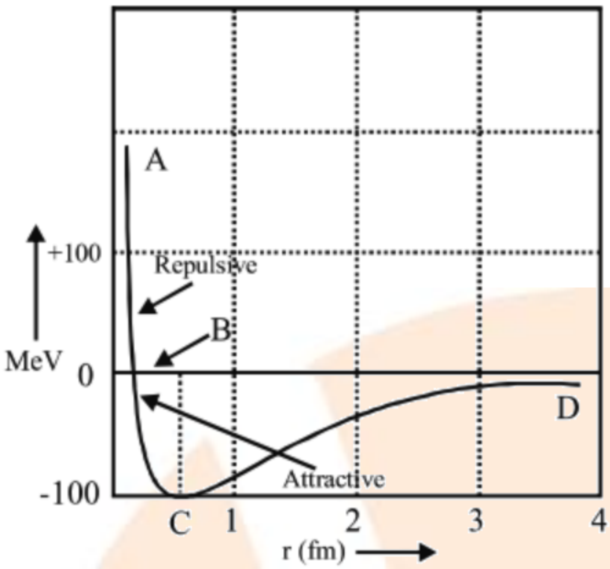
c) Which of the answer would change if zero of PE is changed? Justify your answer?
Ans: If the zero of the P.E is changed, K.E would remain unchanged but the P.E will change, so will the total energy.

2. Prove that the speed of election in the ground state of hydrogen atom is equal to the speed of electron in the first excited state of hydrogen like Li^{++} atom.
Ans: We have the following expression,
$$\upsilon_n = \frac{2\pi Ke^2}{nh}$$
For ground state of hydrogen atom $x = 1$; $\upsilon_1 = \frac{2\pi Ke^2}{h}$

From hydrogen like atom $\left(\upsilon_n\right)_\mu = \frac{Z \times 2\pi Ke^2}{nh}$ ......(1)

Now, for Li^{++} atom $z = 3$ $n = 2$
$$\Rightarrow \left(\upsilon_n\right)_{Li^{++}} = \frac{2 \times 2\pi Ke^2}{2h}$$
$$\Rightarrow \left(\upsilon_n\right)_{Li^{++}} = \frac{2\pi Ke^2}{h} \quad ......(2)$$
Now, from (1) and (2), we have,
$$\left(\upsilon_n\right)_H = \left(\upsilon_n\right)_{Li^{++}}$$
Hence, we proved that the speed of election in the ground sate of hydrogen atom is equal to the speed of electron in the first excited state of hydrogen like Li^{++} atom.

3. Draw a graph showing variation of potential energy of a pair of nucleon as a function of their separation indicate the region in which the nuclear force is attractive and repulsive. Also write two characteristics features which distinguish it from the coulomb's force.
Ans: The required graph is:



1. Nuclear forces are known to be charge independent.
2. They are non – central forces.

4. Choose the correct alternative from the clues given at the end of each statement:

a) The size of the atom in Thomson's model is .... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

Ans: The sizes of the atoms taken in Thomson's model and Rutherford's model have the same order of magnitude.

b) In the ground state of .... electrons are in stable equilibrium, while in .... electrons would always experience a net force.(Thomson's model/ Rutherford's model.)

Ans: In the ground state of Thomson's model, the electrons are in stable equilibrium. However, in Rutherford's model, the electrons would always experience a net force.

c) A classical atom based on .... is doomed to collapse.(Thomson's model/ Rutherford's model.)

Ans: A classical atom based on Rutherford's model is doomed to collapse.

d) An atom has a nearly continuous mass distribution in a .... but is also known to have a highly non- uniform mass distribution in ....(Thomson's model/ Rutherford's model.)

Ans: An atom has a nearly continuous mass distribution in Thomson's model, but is also known to have a highly non-uniform mass distribution in Rutherford's model.

e) The positively charged part of the atom possesses most of the mass in ....(Rutherford's model/Thomson's model/both the models.)

Ans: The positively charged part of the atom possesses most of the mass in both the models.

5. A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n=4$ level. Determine the wavelength and frequency of the photon.

Ans: We have, For ground level, $n_1=1$

Let E_1 be the energy of this level and it is known that E_1 is related with n_1 as:

$$E_1=\frac{-13.6}{n_1}$$

$$\Rightarrow E_1=\frac{-13.6}{1^2}=-13.6\text{eV}$$

We are said that the atom is excited to a higher level, $n_2=4$.

Let E_2 be the energy of this level.

$$E_2=\frac{-13.6}{n_2^2}\text{eV}$$

$$\Rightarrow E_2=\frac{-13.6}{4^2}=-13.6\text{eV}$$

Now, the amount of energy absorbed by the photon could be given as:

$$E=E_2-E_1$$

$$\Rightarrow E=\frac{-13.6}{16}-\left(-\frac{13.6}{1}\right)$$

$$\Rightarrow E=\frac{13.6\times 15}{16}\text{eV}$$

$$\therefore E=\frac{13.6\times 15}{16}\times 1.6\times 10^{-19}=2.04\times 10^{-18}\text{J}$$

For a photon of wavelength λ, the expression of energy could be written as:

$$E=\frac{hc}{\lambda}$$

Where,

$h=$ Planck's constant $=6.6\times 10^{-34}\text{Js}$

$c=$ Speed of light $=3\times 10^{8}\text{m/s}$

$$\Rightarrow \lambda=\frac{hc}{E}$$

$$\Rightarrow \lambda = \frac{6.6\times10^{-34}\times3\times10^{8}}{2.04\times10^{-18}}$$

$\therefore \lambda = 9.7\times10^{-8}\,\text{m} = 97\,\text{nm}$

And, frequency of a photon is given by the relation,

$$v = \frac{c}{\lambda}$$

$$\therefore v = \frac{3\times10^{8}}{9.7\times10^{-8}} \approx 3.1\times10^{15}\,\text{Hz}$$

Hence, the wavelength of the photon is found to be $97\,\text{nm}$ while the frequency is found to be $3.1\times10^{15}\,\text{Hz}$.

6.

a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the $1, 2,$ and 3 levels.

Ans: Let v_1 be the orbital speed of the electron in a hydrogen atom in the ground state level, $n_1 = 1$. For charge (e) of an electron, v_1 could be given by the relation,

$$v_1 = \frac{e^2}{n_1 4\pi \in_0 \left(\frac{h}{2\pi}\right)} = \frac{e^2}{2\in_0 h}$$

Where,

$e = 1.6\times10^{-19}\,\text{C}$

$\in_0 =$ Permittivity of free space $= 8.85\times10^{-12}\,\text{N}^{-1}\text{C}^2\text{m}^{-2}$

$h =$ Planck's constant $= 6.62\times10^{-34}\,\text{Js}$

Substituting the given values, we get,

$$\Rightarrow v_1 = \frac{\left(1.6\times10^{-19}\right)^2}{2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$\Rightarrow v_1 = 0.0218\times10^{8} = 2.18\times10^{6}\,\text{m/s}$$

For level $n_2 = 2$, we can write the relation for the corresponding orbital speed as:

$$v_2 = \frac{e^2}{n_2 2\in_0 h}$$

$$\Rightarrow v_2 = \frac{\left(1.6\times10^{-19}\right)^2}{2\times2\times8.85\times10^{-22}\therefore 6.62\times10-34}\,\text{s}$$

$$\Rightarrow v_2 = 1.09\times10^{6}\,\text{m/s}$$

And, for $n_3 = 3$, we can write the relation for the corresponding orbital speed as:

$$v_3 = \frac{e^2}{n_2 2 \epsilon_0 h}$$

$$\Rightarrow v_3 = \frac{\left(1.6\times10^{-19}\right)^2}{3\times2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$\therefore v_3 = 7.27\times10^6 m/s$$

Hence, the speed of the electron in a hydrogen atom in $n=1, n=2$, and $n=3$ is $2.18\times10^6 m/s, 1.09\times10^6 m/s, 7.27\times10^5 m/s$, respectively.

b) Calculate the orbital period in each of these levels.

Ans: Let T_1 be the orbital period of the electron when it is in level $n_1 = 1$.

Orbital period is related to orbital speed as:

$$T_1 = \frac{2\pi r_1}{v_1}$$

Where,

$r_1 = \frac{n_1^2 h^2 \epsilon_0}{\pi m e^2}$ Radius of the orbit

$h =$ Planck's constant $= 6.62\times10^{-34} Js$

$e =$ Charge on an electron $= 1.6\times10^{-19} C$

$\epsilon_0 =$ Permittivity of free space $= 8.85\times10^{-12} N^{-1}C^2m^{-2}$

$m =$ Mass of an electron $= 9.1\times10^{-31} Kg$

$$\Rightarrow T_1 = \frac{2\pi r_1}{v_1}$$

$$\Rightarrow T_1 = \frac{2\pi\times(1)^2\times\left(6.62\times10^{-34}\right)^2\times8.85\times10^{-12}}{2.18\times10^6\times\pi\times9.1\times10^{31}\times\left(1.6\times10^{-19}\right)^2}$$

$$\therefore T_1 = 15.27\times10^{-17} = 1.527\times10^{-16}$$

Now, for level $n_2 = 2$, we can write the period as:

$$T_2 = \frac{2\pi r_2}{v_2}$$

Where,

$r_2 = \frac{(n_2)^2 h^2 \epsilon_0}{\pi m e^2}$ Radius of the electron in $n_2 = 2$

$$\Rightarrow T_2 = \frac{2\pi \times (2)^2}{v_2}$$

$$\Rightarrow T_2 = \frac{2\pi \times (2)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{1.09 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2}$$

$$\therefore T_2 = 1.22 \times 10^{-15}$$

And, for level $n_3 = 3$, we could write the period as:

$$T_3 = \frac{2\pi r_2}{v_3}$$

Where,

$$r_3 = \text{ Radius of the electron in } n_3 = 3\frac{(n_3)^2 h^2 \in_0}{\pi m e^2}$$

$$\Rightarrow T_3 = \frac{2\pi r_3}{v_3} = \frac{(n_3)^2 h^2 \in_0}{\pi m e^2}$$

$$\Rightarrow T_3 = \frac{2\pi \times (3)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{7.27 \times 10^5 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2} = 4.12 \times 10^{-15}$$

$$\therefore T_3 = \frac{2\pi r_3}{v_3} = 4.12 \times 10^{-15}\,\text{s}$$

Hence, the orbital period in each of these levels are found to be $1.52 \times 10^{-16}\text{s}, 1.22 \times 10^{15}\text{s},$ and $4.12 \times 10^{-15}\text{s}$ respectively.

7. A 12.5eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Ans: We are given that the energy of the electron beam that is used to bombard gaseous hydrogen at room temperature is found to be 12.5eV. Also, the energy of the gaseous hydrogen in its ground state at room temperature is known to be -13.6eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen would become $-13.6 + 12.5\text{eV}$ i.e., -1.1eV.

Orbital energy is related to orbit level (n) as:

$$E = \frac{-13.6}{(n)^2}\text{eV}$$

For $n=3, E=\frac{-13.6}{9}=-1.5eV$

This energy is found to be approximately equal to the energy of gaseous hydrogen. It can be concluded that the electron has jumped from $n=1$ to $n=3$ level.

During its de-excitation, the electrons can jump from $n=3$ to $n=1$ directly, which would form a line of the Lyman series of the hydrogen spectrum.

We have the relation for wave number for Lyman series which could be given as:

$$\frac{1}{\lambda}=R_y\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$$

Where,

$R_y=$ Rydberg constant $=1.097\times107m^{-1}$

$\lambda=$ Wavelength of radiation emitted by the transition of the electron

Now, for $n=3$, we can obtain λ as:

$$\frac{1}{\lambda}=1.097\times10^7\left(\frac{1}{1^2}-\frac{1}{3^2}\right)$$

$$\Rightarrow\frac{1}{\lambda}=1.097\times10^7\left(1-\frac{1}{9}\right)=1.097\times10^7\times\frac{8}{9}$$

$$\Rightarrow\lambda=\frac{9}{8\times1.097\times10^7}=102.55nm$$

If the electron jumps from $n=2$ to $n=1$, then the wavelength of the radiation could be given as:

$$\frac{1}{\lambda}=1.097\times10^7\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

$$\Rightarrow\frac{1}{\lambda}=1.097\times10^7\left(1-\frac{1}{4}\right)=1.097\times10^7\times\frac{3}{4}$$

$$\therefore\lambda=\frac{4}{1.097\times10^7\times3}=121.54nm$$

If the transition takes place from $n=3$ to $n=2$, then the wavelength of the radiation could be given as:

$$\frac{1}{\lambda}=1.097\times10^7\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$\Rightarrow\frac{1}{\lambda}=1.097\times10^7\left(\frac{1}{4}-\frac{1}{9}\right)=1.097\times10^7\times\frac{5}{36}$$

$$\therefore\lambda=\frac{4}{5\times1.097\times10^7}=656.33nm$$

This radiation corresponds to the Balmer series of the hydrogen spectrum.
Hence, in Lyman series, two wavelengths i.e., 102.5nm and 121.5nm would be emitted. And in the Balmer series, one wavelength i.e., 656.33nm would be emitted.

8. Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

Ans: About the same;
The average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model is found to be about the same size as predicted by that of Rutherford's model. And this is because the average angle was taken in b both these models.

b) Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

Ans: Much less;
The probability of scattering of α-particles at angles that are greater than 90° predicted by Thomson's model is much less than that predicted by Rutherford's model.

c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

Ans: We know that scattering is mainly due to single collisions. The chances of a single collision increases linearly with the number of target atoms. Since the number of target atoms increase with an increase in thickness, the collision probability would depend linearly on the thickness of the target.

d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Ans: Thomson's model;
It is wrong to ignore multiple scattering in Thomson's model for the calculation of average angle of scattering of α-particles by a thin foil. This is because a single collision could cause very little deflection in this model. Hence, the observed average scattering angle could be explained only by considering multiple scattering.

9. The gravitational attraction between electron and proton in a hydrogen

atom is weaker than the coulomb attraction by a factor of about 10^{-14}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Ans: Radius of the first Bohr orbit could be given by the relation,

$$r_1 = \frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2} \quad ......(1)$$

Where,

$\in_0 =$ Permittivity of free space $= 6.63 \times 10^{-34}$ Js

$h =$ Planck's constant $= 9.1 \times 10^{-31}$ Kg

$me =$ Mass of an electron $= 1.9 \times 10^{-19}$ C

$e =$ Charge of an electron $= 1.67 \times 10^{-27}$ Kg

$mp =$ Mass of a proton

$r =$ Distance between the electron and the proton

Coulomb attraction between an electron and a proton could be given as:

$$F_G = \frac{e^2}{4\pi \in_0 r^2} \quad ......(2)$$

Gravitational force of attraction between an electron and a proton could be given as:

$$F_G = \frac{Gm_p m_e}{r_2} \quad ......(3)$$

Where,

$G =$ Gravitational constant $= 6.67 \times 10^{-11}\ Nm^2/kg^2$

If the electrostatic (Coulomb) force and the gravitational force between an electron and a proton are equal, then we can write:

$$F_G = F_C$$

$$\Rightarrow \frac{Gm_p m_c}{r_2} = \frac{e^2}{4\pi \in_0 r^2}$$

$$\Rightarrow \frac{e^2}{4\pi \in_0} = Gm_p m_c \quad ......(4)$$

Putting the value of equation (4) in equation (1), we get:

$$r_1 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_p m_c{}^2}$$

$$\therefore r_1 = \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{6.67\times10^{-11}\times1.67\times10^{-27}\times\left(9.1\times10^{-31}\right)^2} \approx 1.21\times10^{29}\,m$$

It is known that the universe is about 156 billion light years wide or 1.5×10^{27} wide.

Therefore, we can conclude that the radius of the first Bohr orbit is much greater than the estimated size of the whole universe.

10. Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level $(n-1)$. For large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Ans: It is given that a hydrogen atom de-excites from an upper level (n) to a lower level $(n-1)$. We have the relation for energy (E_1) of radiation at level n which is given as:

$$E_1 = hv_1 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \times \left(\frac{1}{n^2}\right) \quad ......(i)$$

Where,

$v_1 =$ Frequency of radiation at level n

$h =$ planck's constant

$m =$ mass of hydrogen atom

$e =$ charge of an electron

$\in_0 =$ Permittivity of free space

Now, the relation for energy (E_2) of radiation at level $(n-1)$ can be given as:

$$E_2 = hv_2 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \times \frac{1}{(n-1)^2} \quad ......(ii)$$

Where,

$v_2 =$ Frequency of radiation at level $(n-1)$

Energy (E) released as a result of de-excitation:

$E = E_2 - E_1$

$hv = E_2 - E_1 \quad ......(iii)$

Where,

$v =$ Frequency of radiation emitted

Putting values from equations (i) and (ii) in equation (iii), we get:

$$v = \frac{me^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$\Rightarrow v = \frac{me^4(2n-1)}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^2 (n-1)^2}$$

For large n, we can write $(2n-1) \approx 2n$ and $(n-1) \approx n$

$$\Rightarrow v = \frac{me^4}{32\pi \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad ......(iv)$$

Classical relation of frequency of revolution of an electron is given as:

$$v_c = \frac{v}{2\pi r} \quad ......(v)$$

Where,
Velocity of the electron in the nth orbit is given as:

$$v = \frac{e^2}{4\pi \epsilon_0 \left(\frac{h}{2\pi}\right) n} \quad ......(vi)$$

And, radius of the nth orbit is given as:

$$r = \frac{4\pi \epsilon_0 \left(\frac{h}{2\pi}\right)^2}{me^2} n^2 \quad ......(vii)$$

Putting the values of equations (vi) and (vii) in equation (v), we get:

$$v_c = \frac{me^4}{32\pi^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad ......(viii)$$

Therefore, the frequency of radiation emitted by the hydrogen atom is equal to its classical orbital frequency.

11. Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to

the known size of an atom $\left(\sim 10^{-10}\mathrm{m}\right)$.

a) **Construct a quantity with the dimensions of length from the fundamental constants e, me, and c. Determine its numerical value.**

Ans: We are given:

Charge on an electron, $e = 1.6 \times 10^{-19}\mathrm{C}$

Mass of an electron, $me = 9.1 \times 10^{-31}\mathrm{kg}$

Speed of light, $c = 3 \times 10^{8}\mathrm{m/s}$

Let us take a quantity involving the given quantities as $\left(\frac{e^2}{4\pi \in_0 m_e c^2}\right)$

Where,

$\in_0 =$ Permittivity of free space

And, $\frac{1}{4\pi \in_0} = 9 \times 10^{9}\mathrm{Nm^2C^{-2}}$

The numerical value of the taken quantity will be:

$$\frac{1}{4\pi \in_0} \times \frac{e^2}{m_e c^2} = 9 \times 10^9 \times \frac{\left(1.6 \times 10^{-19}\right)^2}{9.1 \times 10^{-31} \times \left(3 \times 10^8\right)^2}$$

$$\therefore \left(\frac{e^2}{4\pi \in_0 m_e c^2}\right) = 2.81 \times 10^{-15}\mathrm{m}$$

Hence, we found that the numerical value of the taken quantity is much smaller than the typical size of an atom.

b) **You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, me, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, me, and e and confirm that its numerical value has indeed the correct order of magnitude.**

Ans: We are given:

Charge on an electron $e = 1.6 \times 10^{-19}\mathrm{C}$

Mass of an electron, $me = 9.1 \times 10^{-31}\mathrm{kg}$

Planck’s constant, $h = 6.63 \times 10^{-34}\mathrm{Js}$

Let us take a quantity involving the given quantities as $\left(\dfrac{e^2}{4\pi \in_0 m_e c^2}\right)$.

Where,

$\in_0 =$ Permittivity of free space

And, $\dfrac{1}{4\pi \in_0} = 9\times10^9\,Nm^2C^{-2}$

The numerical value of the taken quantity will be:

$$\left(\frac{e^2}{4\pi \in_0 m_e c^2}\right) = \frac{1}{4\pi \in_0}\times\frac{\left(1.6\times10^{-19}\right)^2}{9.1\times10^{-31}\times\left(3\times10^8\right)^2}$$

$$\Rightarrow \left(\frac{e^2}{4\pi \in_0 m_e c^2}\right) = 2.81\times10^{-15}\,m$$

Hence, we found the value of the quantity taken is of the order of the atomic size.

12. Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon $\left(\mu^-\right)$ of mass about $207me$ orbits around a proton].

Ans: It is known that,

Mass of a negatively charged muon, $m_{\mu^-} = 207m_e$

According to Bohr's model we have,

Bohr radius, $r_e \infty \left(\dfrac{1}{m_e}\right)$

And, energy of a ground state electronic hydrogen atom, $E_e \propto m$.

We have the value of the first Bohr orbit known to be,

$r_e = 0.53A = 0.53\times10^{-10}\,m$

Let r_μ be the radius of muonic hydrogen atom.

At equilibrium, we could write the relation as:

$m_\mu r_\mu = m_e r_e$

$\Rightarrow 208m_e \times r_\mu = m_e r_e$

$$\Rightarrow r_\mu = \frac{0.53\times10^{-10}}{207} = 2.56\times10^{-13}\,m$$

Therefore, the value of the first Bohr radius of a muonic hydrogen atom is $2.56\times10^{-13}\,m$.

Now, we have,

$E_e = -13.6\text{eV}$

Take the ratio of these energies as:

$$\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu} = \frac{m_e}{207m_e}$$

$$\Rightarrow E_\mu = 207E_e$$

$$\Rightarrow E_\mu = 207 \times (-13.6) = -2.81\text{keV}$$

Hence, the ground state energy of a muonic hydrogen atom is found to be -2.81keV.