Problem Set - 2

Introduction to quantum computing using QSim

Instruction

- This is a practice problem set. Answers need not be submitted.
- Discussions with other course attendees is encouraged (use the *open-discussions* channel on the course discord).
- Questions addressed to the instructors can be should be posted on the *questions-and-queries* channel.
- The problems in the upcoming quiz will be based on these problems.

Problem 1: Multi-qubit transformations

The following questions are based on the properties of the Kronecker (Tensor) product and transformations on multi-qubit systems.

a) If A and B are two single qubit gates, prove the following relations: [I is the 2×2 identity matrix and $\circ$ denotes matrix multiplication. All other symbols carry their usual meanings]

   i. $(A \otimes I) \circ (I \otimes B) = (A \otimes B)$

   ii. $(A \circ B)^{-1} = B^{\dagger} \circ A^{\dagger}$

b) Let U be a single qubit gate with actions on the standard basis given as follows:

$$U|0\rangle = |a\rangle$$
$$U|1\rangle = |b\rangle$$

Given V be a single qubit gate related to U through the following relation: $V^{\dagger} X V = U$

Find the action of the following circuit (i.e. find $|\psi\rangle$), on the two-qubit computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

c) Show that there <u>exists no</u> pair of single qubit gates, A, B that satisfy the following relation:

$$A \otimes B = \text{CNOT}_1^0$$

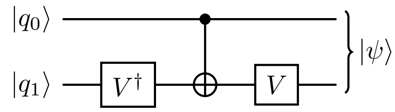

Figure 1: Circuit diagram for Problem 1 b).

d) If a two-qubit gate has the following action on the two-qubit standard basis:

$$\begin{aligned} U\,|00\rangle &= |v_0\rangle \\ U\,|01\rangle &= |v_1\rangle \\ U\,|10\rangle &= |v_2\rangle \\ U\,|11\rangle &= |v_3\rangle \end{aligned}$$

i. Prove the following relation:

$$|v_0\rangle\langle 00| + |v_1\rangle\langle 01| + |v_2\rangle\langle 10| + |v_3\rangle\langle 11| = U$$

ii. Using the above relation, find the matrix form of CNOT_0^1.

Problem 2: Boolean functions and quantum gates

The following questions are based on quantum implementations of Boolean functions.

a) Find the quantum circuits (bit oracles) for the following Boolean functions: [$\oplus$ - XOR, $\vee$ - OR, $^{-}$ - NOT]

1. $F(x_0, x_1) = x_0 \oplus x_1$
ii. $Fx_0, x_1 = x_0 \vee x_!$
iii. $F(x) = \overline{x}$

b) Prove the following statements for a n-bit Boolean function bit oracle: [+ denoted the XOR operation]

$$U_F \underbrace{|\mathbf{x}\rangle}_{n\text{-qubit}} \underbrace{|y\rangle}_{1\text{-qubit}} \rightarrow |\mathbf{x}\rangle\,|F(\mathbf{x}) + y\rangle$$

i. The bit oracle causes entanglement between the input qubits ($|\mathbf{x}\rangle$) and the target qubit ($|y\rangle$).
ii. $U_F^\dagger = U_F$