Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**ANS**

MU = 45 MIN

STD DEV = 8 min

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|  | Pr(X > 50) = 1 - Pr(X ≤ 50). |
|  | Z = (X - )/ = (X - 45)/8.0 |
|  | Thus the question can be answered by using the normal table to find |
|  | Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4% |
|  | Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676 |

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

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| MU = 38  SD = 6  Let X be the number of employees. |
|  | a)Probabilty of employees greater than age of 44= Pr(X>44) |
|  | Pr(X > 44) = 1 - Pr(X ≤ 44). |
|  | Z = (X - )/ = (X - 38)/6 |
|  | Thus the question can be answered by using the normal table to find |
|  | Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6) = Pr(Z ≤ 1)=84.1345% |
|  | Probabilty that the employee will be greater than age of 44 = 100-84.1345=15.86% |
|  | So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345% |
|  | Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE. |
|  | b) Probabilty of employees less than age of 30 = Pr(X<30). |
|  | Z = (X - )/ = (30 - 38)/6 |
|  | Thus the question can be answered by using the normal table to find |
|  | Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12% |
|  | So the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48( or 36 employees). |
|  | Therefore the statement B of the question is also TRUE. |

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS**

if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables

Then

X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 )

similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 )

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| Therefore in the question |
|  | 2X1~ N(2 u,4 σ^2) and |
|  | X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 ) |
|  | 2X1-(X1+X2) = N( 4µ,6 σ^2) |

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z=(X- μ) / σ

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|  | For Probability 0.005 the Z Value is -2.57 (from Z Table). |
|  | Z \* σ + μ = X |
|  | Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4 |
|  | Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6 |
|  | So, option D is correct. |

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?