Resolução de Circuito Elétrico com Equação Diferencial de Segunda Ordem

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9 de Novembro de 2024

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Enunciado:

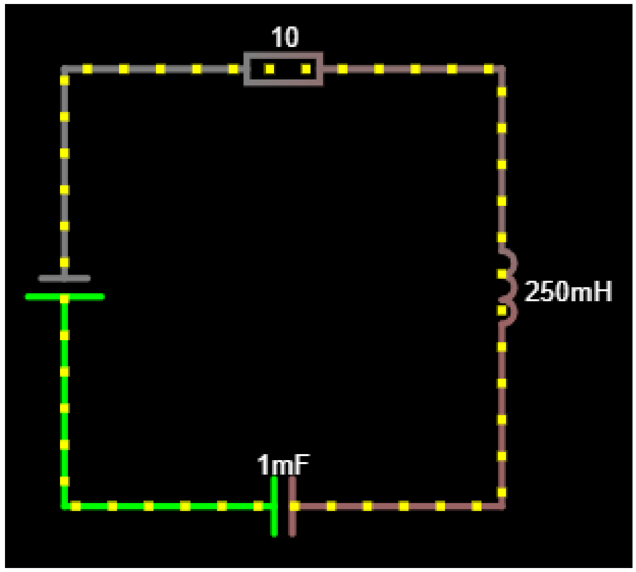

Figura 1 – Figura ilustrativa do circuito

Ache a carga $q(t)$ e a corrente $i(t)$ quando $E(t) = E_0 \sin \alpha t$ e $L = 0,25H$, $R = 10\Omega$, $C = 0,001F$, $q(0) = q_0$, $i(0) = i_0$.

Resolução:

Primeiramente, iremos modelar a EDO e substituir os números decimais por frações, considerando que o circuito pode ser modelado da seguinte maneira: $Lq'' + Rq' + \frac{1}{C}q = E(t)$, então:

$$\frac{1}{4}q'' + 10q' + 1000q = E_0 \sin \alpha t$$

Multiplicando a equação por 4:

$$q'' + 40q' + 4000q = 4E_0 \sin \alpha t$$

Agora iremos resolver a parte homogênea da equação:

$$
\begin{aligned}
m^2+40m+4000&=0\\
\Delta=1600-16000&=-14400\\
\sqrt{\Delta}&=120i\\
m=\frac{-40\pm 120i}{2}&=-20\pm 60i\\
\therefore \alpha=-20,\beta&=60
\end{aligned}
$$

Uma vez que possuimos os valores de α e β podemos montar a solução geral levando em conta que: $q_h=e^{\alpha t}(C_1\cos\beta t+C_2\sin\beta t)$, então:

$$q_h=e^{-20t}(C_1\cos 60t+C_2\sin 60t)$$

Esta é nossa parte homogênea.

Agora iremos calcular o Wronskiano da Equação para que possamos começar a trabalhar na parte particular. Podemos calcular o Wronskiano pela seguinte maneira:

$$W(q_1,...,q_n)=\begin{pmatrix} q_1 & q_2 & \cdots & q_n\\ q_1' & q_2' & \cdots & q_n'\\ \vdots & \vdots & \ddots & \vdots\\ q_1^{(n-1)} & q_2^{(n-1)} & \cdots & q_n^{(n-1)}\end{pmatrix}$$

Sendo $q_1=e^{-20t}\cos 60t$ e $q_2=e^{-20t}\sin 60t$, então:

$$W(q_1,q_2)=\begin{pmatrix} e^{-20t}\cos 60t & e^{-20t}\sin 60t\\ -60e^{-20t}\sin 60t-20e^{-20t}\cos 60t & 60e^{-20t}\cos 60t-20e^{-20t}\sin 60t\end{pmatrix}$$

Agora calculamos seu determinante:

$$
\begin{aligned}
=(-20e^{-40t}\cos 60t\sin 60t+60e^{-40t}\cos^2 60t)-(-20e^{-40t}\cos 60t\sin 60t-60e^{-40t}\sin^2 60t)\\
=-20e^{-40t}\cos 60t\sin 60t+60e^{-40t}\cos^2 60t+20e^{-40t}\cos 60t\sin 60t+60e^{-40t}\sin^2 60t\\
=60e^{-40t}\cos^2 60t+60e^{-40t}\sin^2 60t\\
=60e^{-40t}[\cos^2 60t+\sin^2 60t]\\
\cos^2 60t+\sin^2 60t=1\\
\therefore W=60e^{-40t}
\end{aligned}
$$

Com o valor da determinante do Wronskiano nós agora podemos começar a calcular a parte particular que é dada por: $q_p = u_1(t)q_1 + u_2(t)q_2$, sendo $u_1(t) = -\int \frac{q_2 E(t)}{W} dt$ e $u_2(t) = \int \frac{q_1 E(t)}{W} dt$

Portanto:

$$u_1(t) = -\int \frac{e^{-20t} \sin 60t * 4E_0 \sin \alpha t}{60e^{-40t}} dt$$
$$u_1(t) = -\frac{4E_0}{60} \int e^{20t} \sin 60t * \sin \alpha t dt$$

Usando a propriedade de $\sin\theta * \sin\omega = \frac{1}{2}[\cos(\theta - \omega) - \cos(\theta + \omega)]$ temos $\frac{1}{2}[\cos(60t - \alpha t) - \cos(60t + \alpha t)]$ e utilizaremos $60 - \alpha = \theta$ e $60 + \alpha = \omega$ durante o resto da resolução

Sendo assim:

$$u_1(t) = -\frac{4E_0}{60} \int \frac{1}{2}[e^{20t}\cos(\theta t) - e^{20t}\cos(\omega t)]dt$$
$$u_1(t) = -\frac{E_0}{30}\left[\int e^{20t}\cos(\theta t)dt - \int e^{20t}\cos(\omega t)dt\right]$$

Iremos integrar $\int e^{20t}\cos(\theta t)dt$ primeiro utilizando o método de integração por partes:

$$u = \cos(\theta t) \Rightarrow du = -\theta \sin\theta t$$
$$dv = e^{20t} \Rightarrow v = \frac{e^{20t}}{20}$$
$$\int e^{20t}\cos(\theta t)dt = \frac{\cos(\theta t)e^{20t}}{20} - \int \frac{e^{20t}}{20} * (-\theta \sin\theta t)dt$$
$$\int e^{20t}\cos(\theta t)dt = \frac{\cos(\theta t)e^{20t}}{20} + \frac{\theta}{20}\int e^{20t}\sin\theta t dt$$
$$u = \sin(\theta t) \Rightarrow du = \theta\cos\theta t$$
$$dv = e^{20t} \Rightarrow v = \frac{e^{20t}}{20}$$
$$\int e^{20t}\cos(\theta t)dt = \frac{\cos(\theta t)e^{20t}}{20} + \frac{\theta}{20}\left[\sin(\theta t) * \frac{e^{20t}}{20} - \int \frac{e^{20t}}{20} * \theta\cos\theta t\right]$$
$$\int e^{20t}\cos(\theta t)dt = \frac{\cos(\theta t)e^{20t}}{20} + \frac{\theta}{20}\left[\sin(\theta t) * \frac{e^{20t}}{20} - \frac{\theta}{20}\int e^{20t}\cos\theta t\right]$$
$$\int e^{20t}\cos(\theta t)dt = \frac{\cos(\theta t)e^{20t}}{20} + \frac{\theta\sin(\theta t)e^{20t}}{20^2} - \frac{\theta^2}{20^2}\int e^{20t}\cos\theta t$$

$$
\begin{aligned}
\int e^{20t}\cos(\theta t)dt + \frac{\theta^2}{20^2}\int e^{20t}\cos\theta t &= \frac{\cos(\theta t)e^{20t}}{20} + \frac{\theta\sin(\theta t)e^{20t}}{20^2} \\
\left(1+\frac{\theta^2}{20^2}\right)\int e^{20t}\cos(\theta t)dt &= \frac{\cos(\theta t)e^{20t}}{20} + \frac{\theta\sin(\theta t)e^{20t}}{20^2} \\
\frac{20^2+\theta^2}{20^2}\int e^{20t}\cos(\theta t)dt &= \frac{\cos(\theta t)e^{20t}}{20} + \frac{\theta\sin(\theta t)e^{20t}}{20^2} \\
\int e^{20t}\cos(\theta t)dt &= \frac{20^2}{20^2+\theta^2}\left[\frac{\cos(\theta t)e^{20t}}{20} + \frac{\theta\sin(\theta t)e^{20t}}{20^2}\right] \\
\int e^{20t}\cos(\theta t)dt &= \frac{e^{20t}20^2}{20^2+\theta^2}\left[\frac{\cos(\theta t)}{20} + \frac{\theta\sin(\theta t)}{20^2}\right] \\
\int e^{20t}\cos(\theta t)dt &= \frac{e^{20t}}{20^2+\theta^2}\left[20\cos(\theta t)+\theta\sin(\theta t)\right]
\end{aligned}
$$

Como $\int e^{20t}\cos(\theta t)dt$ e $\int e^{20t}\cos(\omega t)dt$ são muito parecidos tendo como única diferença o valor dentro do cosseno, os únicos valores que se diferenciam dentre cada resultado são θ e ω, Portanto:

$$
\begin{aligned}
\therefore \int e^{20t}\cos(\theta t)dt &= \frac{e^{20t}}{20^2+\theta^2}\left[20\cos(\theta t)+\theta\sin(\theta t)\right] \\
\therefore \int e^{20t}\cos(\omega t)dt &= \frac{e^{20t}}{20^2+\omega^2}\left[20\cos(\omega t)+\omega\sin(\omega t)\right]
\end{aligned}
$$

Agora podemos completar $u_1(t)$:

$$
\begin{aligned}
u_1(t) = -\frac{E_0}{30}\left[\frac{e^{20t}}{20^2+\theta^2}(20\cos(\theta t)+\theta\sin(\theta t)) - \frac{e^{20t}}{20^2+\omega^2}(20\cos(\omega t)+\omega\sin(\omega t))\right] \\
\therefore u_1(t) = -\frac{E_0e^{20t}}{30}\left[\frac{20\cos(\theta t)+\theta\sin(\theta t)}{20^2+\theta^2} - \frac{20\cos(\omega t)+\omega\sin(\omega t)}{20^2+\omega^2}\right]
\end{aligned}
$$

Seguindo para a resolução de $u_2(t)$:

$$
\begin{aligned}
u_2(t) &= \int \frac{e^{-20t}\cos 60t * 4E_0\sin\alpha t}{60e^{-40t}}dt \\
u_2(t) &= \frac{4E_0}{60}\int e^{20t}\cos 60t * \sin\alpha t dt
\end{aligned}
$$

Como feito em $u_1(x)$, também utilizaremos uma relação trigonométrica na resolução, sendo $\cos\theta * \sin\omega = \frac{1}{2}[\sin(\theta+\omega) - \sin(\theta-\omega)]$, então temos $\frac{1}{2}[\sin(60t+\alpha t) - \sin(60t-\alpha t)]$, chamaremos $60+\alpha = \theta$ e $60-\alpha = \omega$

$$u_2(t) = \frac{4E_0}{60}\int \frac{1}{2}\left[e^{20t}\sin(\theta t) - e^{20t}\sin(\omega t)\right]dt$$
$$u_2(t) = \frac{E_0}{30}\left[\int e^{20t}\sin(\theta t)dt - \int e^{20t}\sin(\omega t)dt\right]$$

Começaremos Integrando $\int e^{20t}\sin(\theta t)dt$:

$$u = \sin(\theta t) \Rightarrow du = \theta\cos\theta t$$
$$dv = e^{20t} \Rightarrow v = \frac{e^{20t}}{20}$$
$$\int e^{20t}\sin(\theta t)dt = \frac{\sin(\theta t)e^{20t}}{20} - \int \frac{e^{20t}}{20} * \theta\cos\theta t dt$$
$$\int e^{20t}\sin(\theta t)dt = \frac{\sin(\theta t)e^{20t}}{20} - \frac{\theta}{20}\int e^{20t}\cos\theta t dt$$
$$u = \cos(\theta t) \Rightarrow du = -\theta\sin\theta t$$
$$dv = e^{20t} \Rightarrow v = \frac{e^{20t}}{20}$$
$$\int e^{20t}\sin(\theta t)dt = \frac{\sin(\theta t)e^{20t}}{20} - \frac{\theta}{20}\left[\frac{\cos(\theta t)e^{20t}}{20} - \int \frac{e^{20t}}{20} * (-\theta\sin\theta t)dt\right]$$
$$\int e^{20t}\sin(\theta t)dt = \frac{\sin(\theta t)e^{20t}}{20} - \frac{\theta}{20}\left[\frac{\cos(\theta t)e^{20t}}{20} + \frac{\theta}{20}\int e^{20t}\sin\theta t dt\right]$$
$$\int e^{20t}\sin(\theta t)dt = \frac{\sin(\theta t)e^{20t}}{20} - \frac{\theta\cos(\theta t)e^{20t}}{20^2} - \frac{\theta^2}{20^2}\int e^{20t}\sin\theta t dt$$
$$\int e^{20t}\sin(\theta t)dt + \frac{\theta^2}{20^2}\int e^{20t}\sin\theta t dt = \frac{\sin(\theta t)e^{20t}}{20} - \frac{\theta\cos(\theta t)e^{20t}}{20^2}$$
$$(1 + \frac{\theta^2}{20^2})\int e^{20t}\sin\theta t dt = \frac{\sin(\theta t)e^{20t}}{20} - \frac{\theta\cos(\theta t)e^{20t}}{20^2}$$
$$\frac{20^2+\theta^2}{20^2}\int e^{20t}\sin\theta t dt = \frac{\sin(\theta t)e^{20t}}{20} - \frac{\theta\cos(\theta t)e^{20t}}{20^2}$$
$$\int e^{20t}\sin\theta t dt = \frac{20^2}{20^2+\theta^2}\left[\frac{\sin(\theta t)e^{20t}}{20} - \frac{\theta\cos(\theta t)e^{20t}}{20^2}\right]$$
$$\int e^{20t}\sin\theta t dt = \frac{e^{20t}20^2}{20^2+\theta^2}\left[\frac{\sin(\theta t)}{20} - \frac{\theta\cos(\theta t)}{20^2}\right]$$
$$\int e^{20t}\sin\theta t dt = \frac{e^{20t}}{20^2+\theta^2}\left[20\sin(\theta t) - \theta\cos(\theta t)\right]$$

Como anteriormente, os únicos valores que se alteram dentro de $\int e^{20t}\sin(\theta t)dt$ e $\int e^{20t}\sin(\omega t)dt$ são θ e ω, Portanto:

$$\therefore \int e^{20t}\sin\theta t dt = \frac{e^{20t}}{20^2+\theta^2}[20\sin(\theta t)-\theta\cos(\theta t)]$$
$$\therefore \int e^{20t}\sin\omega t dt = \frac{e^{20t}}{20^2+\omega^2}[20\sin(\omega t)-\omega\cos(\omega t)]$$

Então podemos continuar com $u_2(t)$:

$$u_2(t) = \frac{E_0}{30}\left[\frac{e^{20t}}{20^2+\theta^2}[20\sin(\theta t)-\theta\cos(\theta t)] - \frac{e^{20t}}{20^2+\omega^2}[20\sin(\omega t)-\omega\cos(\omega t)]\right]$$
$$\therefore u_2(t) = \frac{E_0e^{20t}}{30}\left[\frac{20\sin(\theta t)-\theta\cos(\theta t)}{20^2+\theta^2} - \frac{20\sin(\omega t)-\omega\cos(\omega t)}{20^2+\omega^2}\right]$$

Agora que temos os valores de $u_1(t)$ e $u_2(t)$ iremos montar nossa parte particular $q_p = u_1(t)*q_1 + u_2(t)*q_2$:

$$\begin{aligned} q_p = &-\frac{E_0e^{20t}}{30}\left[\frac{20\cos(\theta t)+\theta\sin(\theta t)}{20^2+\theta^2} - \frac{20\cos(\omega t)+\omega\sin(\omega t)}{20^2+\omega^2}\right]e^{-20t}\cos 60t \\ &+\frac{E_0e^{20t}}{30}\left[\frac{20\sin(\theta t)-\theta\cos(\theta t)}{20^2+\theta^2} - \frac{20\sin(\omega t)-\omega\cos(\omega t)}{20^2+\omega^2}\right]e^{-20t}\sin 60t \end{aligned}$$

$$\begin{aligned} \therefore q_p = [&-\frac{E_0}{30}\left[\frac{20\cos(\theta t)+\theta\sin(\theta t)}{20^2+\theta^2} - \frac{20\cos(\omega t)+\omega\sin(\omega t)}{20^2+\omega^2}\right]\cos 60t \\ &+\frac{E_0}{30}\left[\frac{20\sin(\theta t)-\theta\cos(\theta t)}{20^2+\theta^2} - \frac{20\sin(\omega t)-\omega\cos(\omega t)}{20^2+\omega^2}\right]\sin 60t] \end{aligned}$$

Finalmente podemos montar a solução geral da nossa equação, a solução geral de uma equação de ordem superior não homogênea é dada por $q = q_h + q_p$, portanto:

$$\begin{aligned} \therefore q = e^{-20t}(C_1\cos 60t + C_2\sin 60t) + [&-\frac{E_0}{30}\left[\frac{20\cos(\theta t)+\theta\sin(\theta t)}{20^2+\theta^2} - \frac{20\cos(\omega t)+\omega\sin(\omega t)}{20^2+\omega^2}\right]\cos 60t \\ &+\frac{E_0}{30}\left[\frac{20\sin(\theta t)-\theta\cos(\theta t)}{20^2+\theta^2} - \frac{20\sin(\omega t)-\omega\cos(\omega t)}{20^2+\omega^2}\right]\sin 60t] \end{aligned}$$

Agora que temos a carga, precisamos encontrar os coeficientes, temos que $q(0) = q_0$ e $i(0) = i_0$, a corrente elétrica é a derivada da carga ou seja $\frac{dq}{dt} = i(t)$, Portanto:

$$q'(t) = {q_h}' + {q_p}'$$

$$\begin{aligned}{q_h}' &= [e^{-20t}(C_1\cos 60t + C_2\sin 60t)]' \\ {q_h}' &= -20e^{-20t}(C_1\cos 60t + C_2\sin 60t) + e^{-20t}(-60C_1\sin 60t + 60C_2\cos 60t)\end{aligned}$$

Para facilitar a derivação da parte particular iremos chamar
$-\frac{E_0}{30}\left[\frac{20\cos(\theta t)+\theta\sin(\theta t)}{20^2+\theta^2} - \frac{20\cos(\omega t)+\omega\sin(\omega t)}{20^2+\omega^2}\right]\cos 60t$ de $-\frac{E_0}{30}P(t)\cos 60t$
e $\frac{E_0}{30}\left[\frac{20\sin(\theta t)-\theta\cos(\theta t)}{20^2+\theta^2} - \frac{20\sin(\omega t)-\omega\cos(\omega t)}{20^2+\omega^2}\right]\sin 60t$ de $\frac{E_0}{30}Q(t)\sin 60t$, Portanto:

$$\begin{aligned}{q_p}' &= \left[-\frac{E_0}{30}P(t)\cos 60t + \frac{E_0}{30}Q(t)\sin 60t\right]' \\ {q_p}' &= \frac{E_0}{30}[-P(t)\cos 60t + Q(t)\sin 60t]' \\ \therefore {q_p}' &= \frac{E_0}{30}[-P'(t)\cos 60t + 60P(t)\sin 60t + Q'(t)\sin 60t + 60Q(t)\cos 60t]\end{aligned}$$

$$\begin{aligned}\therefore i(t) = -20e^{-20t}(C_1\cos 60t + C_2\sin 60t) + e^{-20t}(-60C_1\sin 60t + 60C_2\cos 60t) + \\ \frac{E_0}{30}[-P'(t)\cos 60t + 60P(t)\sin 60t + Q'(t)\sin 60t + 60Q(t)\cos 60t]\end{aligned}$$

Encontrando $q(0)$ e $i(0)$:

$$\begin{aligned}q(0) &= e^{-20*0}(C_1\cos(60*0) + C_2\sin(60*0)) + \frac{E_0}{30}[-P(0)\cos(60*0) + Q(0)\sin(60*0)] \\ q(0) &= e^{0}(C_1\cos(0) + C_2\sin(0)) + \frac{E_0}{30}[-P(0)\cos(0) + Q(0)\sin(0)] \\ \therefore q(0) &= C_1 - \frac{E_0P(0)}{30}\end{aligned}$$

$$i(0) = q_h' + q_p'$$
$$q_h'(0) = -20e^{-20*0}(C_1 \cos 60*0 + C_2 \sin 60*0) + e^{-20*0}(-60C_1 \sin 60*0 + 60C_2 \cos 60*0)$$
$$q_h'(0) = -20e^{0}(C_1 \cos 0 + C_2 \sin 0) + e^{0}(-60C_1 \sin 0 + 60C_2 \cos 0)$$
$$\therefore q_h'(0) = -20C_1 + 60C_2$$

$$q_p'(0) = \frac{E_0}{30}[-P'(0)\cos 60*0 + 60P(0)\sin 60*0 + Q'(0)\sin 60*0 + 60Q(0)\cos 60*0]$$
$$q_p'(0) = \frac{E_0}{30}[-P'(0)\cos 0 + 60P(0)\sin 0 + Q'(0)\sin 0 + 60Q(0)\cos 0]$$
$$\therefore q_p'(0) = \frac{E_0}{30}[-P'(0) \ + 60Q(0)]$$

$$\therefore i(0) = -20C_1 + 60C_2 + \frac{E_0}{30}[-P'(0) \ + 60Q(0)]$$

Encontrando $P(0)$ e $Q(0)$:

$$P(0) = \frac{20\cos(\theta*0) + \theta\sin(\theta*0)}{20^2+\theta^2} - \frac{20\cos(\omega*0) + \omega\sin(\omega*0)}{20^2+\omega^2}$$
$$P(0) = \frac{20}{20^2+\theta^2} - \frac{20}{20^2+\omega^2}$$
$$\therefore P(0) = 20\left(\frac{1}{20^2+\theta^2} - \frac{1}{20^2+\omega^2}\right)$$

$$Q(0) = \frac{20\sin(\theta*0) - \theta\cos(\theta*0)}{20^2+\theta^2} - \frac{20\sin(\omega*0) - \omega\cos(\omega*0)}{20^2+\omega^2}$$
$$\therefore Q(0) = -\frac{\theta}{20^2+\theta^2} + \frac{\omega}{20^2+\omega^2}$$

Encontrando $P'(0)$:

$$P'(t) = \left[\frac{20\cos(\theta t) + \theta\sin(\theta t)}{20^2+\theta^2} - \frac{20\cos(\omega t) + \omega\sin(\omega t)}{20^2+\omega^2}\right]'$$
$$P'(t) = \frac{-20\theta\sin(\theta t) + \theta^2\cos(\theta t)}{20^2+\theta^2} + \frac{20\omega\sin(\omega t) - \omega^2\cos(\omega t)}{20^2+\omega^2}$$

$$P'(0) = \frac{-20\theta\sin(\theta*0)+\theta^2\cos(\theta*0)}{20^2+\theta^2} + \frac{20\omega\sin(\omega*0)-\omega^2\cos(\omega*0)}{20^2+\omega^2}$$
$$\therefore P'(0) = \frac{\theta^2}{20^2+\theta^2} - \frac{\omega^2}{20^2+\omega^2}$$

Portanto:

$$i(0) = -20C_1 + 60C_2 + \frac{E_0}{30}\left[-\frac{\theta^2}{20^2+\theta^2} - \frac{\omega^2}{20^2+\omega^2} \ +60\left[-\frac{\theta}{20^2+\theta^2} + \frac{\omega}{20^2+\omega^2}\right]\right]$$
$$i(0) = -20C_1 + 60C_2 + \frac{E_0}{30}\left[-\frac{\theta^2}{20^2+\theta^2} - \frac{\omega^2}{20^2+\omega^2} \ -\frac{60\theta}{20^2+\theta^2} + \frac{60\omega}{20^2+\omega^2}\right]$$
$$\therefore i(0) = -20C_1 + 60C_2 + \frac{E_0}{30}\left[-\frac{\theta^2-60\theta}{20^2+\theta^2} + \frac{\omega^2-60\omega}{20^2+\theta^2}\right]$$
$$\therefore q(0) = C_1 - \frac{2E_0}{3}\left(\frac{1}{20^2+\theta^2} - \frac{1}{20^2+\omega^2}\right)$$

Agora apenas precisamos resolver o seguinte sistema linear:

$$\begin{cases} C_1 - \frac{2E_0}{3}\left(\frac{1}{20^2+\theta^2} - \frac{1}{20^2+\omega^2}\right) = q_0 \\ -20C_1 + 60C_2 + \frac{E_0}{30}\left[-\frac{\theta^2-60\theta}{20^2+\theta^2} + \frac{\omega^2-60\omega}{20^2+\theta^2}\right] = i_0 \end{cases}$$

Então temos:

$$\therefore C_1 = q_0 + \frac{2E_0}{3}\left(\frac{1}{20^2+\theta^2} - \frac{1}{20^2+\omega^2}\right)$$

Substituindo C_1 na segunda equação:

$$-20\left[q_0 + \frac{2E_0}{3}\left(\frac{1}{20^2+\theta^2} - \frac{1}{20^2+\omega^2}\right)\right] + 60C_2 + \frac{E_0}{30}\left[-\frac{\theta^2-60\theta}{20^2+\theta^2} + \frac{\omega^2-60\omega}{20^2+\theta^2}\right] = i_0$$
$$60C_2 = i_0 - \frac{E_0}{30}\left[-\frac{\theta^2-60\theta}{20^2+\theta^2} + \frac{\omega^2-60\omega}{20^2+\theta^2}\right] + 20q_0 + \frac{40E_0}{3}\left(\frac{1}{20^2+\theta^2} - \frac{1}{20^2+\omega^2}\right)$$
$$\therefore C_2 = \frac{i_0}{60} - \frac{E_0}{1800}\left[-\frac{\theta^2-60\theta}{20^2+\theta^2} + \frac{\omega^2-60\omega}{20^2+\theta^2}\right] + \frac{q_0}{3} + \frac{2E_0}{9}\left(\frac{1}{20^2+\theta^2} - \frac{1}{20^2+\omega^2}\right)$$