Properties of DFT

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DFT pair

- $x[n] \overset{DFT}{\longleftrightarrow} X(k)$

Periodicity

- $x[n] \overset{DFT}{\longleftrightarrow} X(k)$
- $x[n] = x[n + mN]$ where, m=… –1, 0, 1, 2, …
- $X[k] = X[k + mN]$ where, m=… –1, 0, 1, 2, …

Linearity

If

$x_1[n] \overset{DFT}{\longleftrightarrow} X_1(k)$ and $x_2[n] \overset{DFT}{\longleftrightarrow} X_2(k)$

Then,

$$ax_1[n] + bx_2[n] \overset{DFT}{\longleftrightarrow} aX_1(k) + bX_2(k)$$

Circular Time Shift

- $x[n] \overset{DFT}{\longleftrightarrow} X(k)$

- $x[(n \mp l)_N] \overset{DFT}{\longleftrightarrow} e^{\mp j\frac{2\pi kl}{N}} X(k)$

Circular Frequency Shift

- $x[n] \overset{DFT}{\longleftrightarrow} X(k)$

- $x[n]e^{\pm j\frac{2\pi nl}{N}} \overset{DFT}{\longleftrightarrow} X((k \mp l)_N)$

Time Reversal

- $x[n] \xleftrightarrow{DFT} X(k)$

- $x[(-n)_N] \xleftrightarrow{DFT} X((-k)_N)$

Complex Conjugate

- $x[n] \overset{DFT}{\longleftrightarrow} X(k)$

- $x^*[n] \overset{DFT}{\longleftrightarrow} X^*\big((-k)_N\big)$

- $x^*[(-n)_N] \overset{DFT}{\longleftrightarrow} X^*(k)$

Complex Conjugate

- Two real time sequences are given as
  $x1[n]$ and $x2[n]$

- Let's write as $x[n]=x1[n] + j\, x2[n]$ – – – (1)
  and find DFT of $x[n]$ i.e. $X(k)$

- Now take complex conjugate of $x[n]$ i.e.
  $x^*[n]=x1[n] - j\, x2[n]$ – – – – (2)

  Add (1) and (2)
  $x[n]+x^*[n]=2\, x1[n]$
  $x1[n]=(1/2)\ (x[n]+x^*[n])$ → $X1(k)=$?

Complex Conjugate

- x1[n]=(1/2) (x[n]+x*[n]) → X1(k)= ?

- $x[n] \overset{DFT}{\longleftrightarrow} X(k)$
- $x^*[n] \overset{DFT}{\longleftrightarrow} X^*((-k)_N)$

Complex Conjugate

- x1[n]=(1/2) (x[n]+x*[n]) → X1(k)= ?

- $x[n] \overset{DFT}{\longleftrightarrow} X(k)$
- $x^*[n] \overset{DFT}{\longleftrightarrow} X^*((-k)_N)$

$$X_1(k) = \frac{X(k) \ + X^*((-k)_N)}{2}$$

Complex Conjugate

$x[n]=x1[n] + j\, x2[n]$ ---- (1)

$x^*[n]=x1[n] - j\, x2[n]$ ---- (2)

Subtract (2) from (1)

$x[n]-x^*[n]=2\, j\, x2[n]$

- $x2[n]=(1/2j)\ (x[n]-x^*[n])$ → $X2(k)=$?

Complex Conjugate

x[n]=x1[n] + j x2[n]   - - - - (1)

x*[n]=x1[n] - j x2[n]   - - - - (2)

Subtract (2) from (1)

x[n]-x*[n]=2 j x2[n]

- x2[n]=(1/2j) (x[n]-x*[n]) → X2(k)= ?

$$X_2(k) = \frac{X(k) \ - X^*((-k)_N)}{2j}$$

Circular Convolution

If $x[n] \overset{DFT}{\longleftrightarrow} X(k)$ and $h[n] \overset{DFT}{\longleftrightarrow} H(k)$

Then,

$x[n] \circledast h[n] \overset{DFT}{\longleftrightarrow} X(k).H(k)$

Circular Correlation

If $x[n] \overset{DFT}{\longleftrightarrow} X(k)$ and $y[n] \overset{DFT}{\longleftrightarrow} Y(k)$

Then,

$\mathcal{R}_{xy}(l) \overset{DFT}{\longleftrightarrow} X(k).\, Y^*(k)$

Parseval's Theorem

- If $x[n] \overset{DFT}{\longleftrightarrow} X(k)$ then
- $\sum_{n=0}^{N-1} x[n]\, x^*[n] \overset{DFT}{\longleftrightarrow} \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, X^*(k)$
- $\sum_{n=0}^{N-1} |x[n]|^2 \overset{DFT}{\longleftrightarrow} \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$

Exercise

Problem

Two finite sequences h and x have the following DFT's:

$$X = \text{DFT}\{x\} = [1,\ -2,\ 1,\ -2]$$
$$H = \text{DFT}\{h\} = [1,\ j,\ 1,\ -j]$$

Let $y = h \otimes x$ be the four point circular convolution of the two sequences. Using the properties of the DFT (do not compute $x[n]$ and $h[n]$),

a) determine $\text{DFT}\{x[(n-1)_4]\}$ and $\text{DFT}\{h[(n+2)_4]\}$;

b) determine $y[0]$ and $y[1]$.

Exercise

Problem

Let x be a finite sequence with DFT

$$X = \text{DFT}\{x\} = [0,\ 1 + j,\ 1,\ 1 - j]$$

Using the properties of the DFT determine the DFT's of the following:

a) $y[n] = e^{j(\pi/2)\,n}\, x[n]$

b) $y[n] = \cos(\frac{\pi}{2}\, n)\, x[n]$

c) $y[n] = x[(n-1)_4]$

d) $y[n] = [0,\ 0,\ 1,\ 0] \oplus x[n]$ with $\oplus$ denoting circular convolution

Exercise

- Two finite sequences $x[n]=\{x[0], x[1], x[2], x[3]\}$ and $h[n]=\{h[0], h[1], h[2], h[3]\}$ have DFT as

  $X(k)=DFT\{x[n]\}=\{1, j, -1, -j\}$
  $H(k)=DFT\{h[n]\}=\{0, 1+j, 1, 1-j\}$

Find
(a) DFT $\{x[3], x[0], x[1], x[2]\}$
(b) DFT $\{h[0], -h[1], h[2], -h[3]\}$
(c) DFT $\{h$ (*) $x\}$, where (*) denotes circular convolution
(d) DFT $\{x[0], h[0], x[1], h[1], x[2], h[2], x[3], h[3]\}$