UNIT-1

30th March, 2023 ①
18:24

→ Double integral

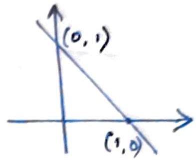
* $\iint_R dx\,dy = \text{Area}$ * $\iint_R f(x,y)\,dx\,dy = \text{Volume}$ (surface)

– Evaluation of double $\int$

inner limit → variable outer limit → constant

dx ⇒

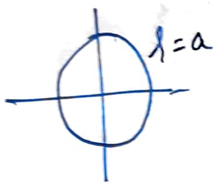

dy ⇒

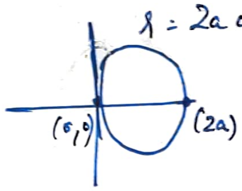

$x+y=1$


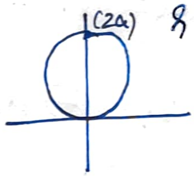
→ Polar coordinates : $x = r\cos\theta$, $y = r\sin\theta$, $dx\,dy = r\,dr\,d\theta$

cylindrical : $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $dx\,dy\,dz = r\,dz\,dr\,d\theta$

spherical : $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, $dz\,dy\,dx = r^2\sin\theta\,dr\,d\theta\,d\phi$

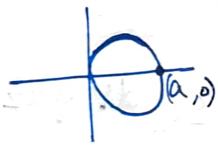
* $r = a$


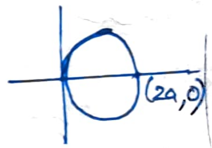
$r = 2a\cos\theta$


$r = 2a\sin\theta$


* Average = $\frac{1}{A}\iint_R f(x,y)\,dx\,dy$

$\iint dx\,dy$

* $x^2+y^2-ax=0$


$x^2+y^2-2ax=0$


* Sphere : $x^2+y^2+z^2=a^2$

→ +ve octant $(x=0, y=0, z=0)$: $r : 0$ to a
$\theta : 0$ to $\pi/2$
$\phi : 0$ to $\pi/2$

* $z = 9-x^2-y^2$
$z = 2-x^2-y^2$


* $z = \sqrt{x^2+y^2}$


* $x^2+y^2 = z$


Application of double integrals

$$M = \iint_D \rho \, dx\,dy = \iint_D f(x, y)\, dx\,dy$$

$x_c =$ C.O.G (Centroid) $\dfrac{\iint x \cdot f(x, y)\,dx\,dy}{M}$ $\qquad y_c = \dfrac{\iint_D y \cdot f(x, y)\, dx\,dy}{M}$

MOI $\quad I_x = \iint_D y^2 f(x, y)\, dx\,dy$

$I_y = \iint_D x^2 f(x, y)\, dx\,dy \qquad I_0 = I_x + I_y$

$= \iint (x^2 + y^2) f(x, y)\, dx\,dy$

Triple integral

$$M = \iiint_V \rho \, dx\,dy\,dz = \iiint f(x, y, z)\, dx\,dy\,dz$$

MOI $\quad I_{zz} = \iiint \rho(x^2 + y^2)\, dx\,dy\,dz = \iiint f(x, y, z)\,(x^2 + y^2)\, dx\,dy\,dz$

$I_{yy} = \iiint \rho(x^2 + z^2)\, dx\,dy\,dz$

$I_{xx} = \iiint \rho(y^2 + z^2)\, dx\,dy\,dz$

C.O.G

$$x_c = \frac{\iiint x \,\rho\, dx\,dy\,dz}{M}$$

$$y_c = \frac{\iiint y \,\rho\, dx\,dy\,dz}{M}$$

$$z_c = \frac{\iiint z \,\rho\, dx\,dy\,dz}{M}$$

Discrete Probability distribution → Bernoulli / Binomial / Poisson

UNIT-2

20th April, 2023
19:37

$p(x) = P(X=x)$

Probability mass function

cdf - cumulative distribution function

→ c.d.f $F(x)$

$F(x) = P(X \le x) = \sum_{i=1}^{n} p(x_i)$

integers

mean $\mu = \sum x_i p(x_i)$

Variance $\sigma^2 = \sum (x_i - \mu)^2 p(x_i) = \sum x_i^2 - \mu^2$

Continuous Probability distribution → Normal

$$P(a \le x \le b) = \int_a^b f(x)\, dx$$

contin. prob. func. / p.d.f

probab density func.

→ c.d.f $F(x)$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\, dx$$

cumulative distribution func

$\| \quad P(-\infty < X < x)$

probability density

$$P(X \ge \lambda) = \int_{\lambda}^{\infty} f(x)\, dx$$

$$P(X < \lambda) = 1 - P(X \ge \lambda)$$

$$\mu = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\, dx$$

Bernoulli

$X \sim Bernoulli(p)$

$$P(X=x) = \begin{cases} p & x=1 \\ 1-p & x=0 \end{cases}$$

$\mu_x = p$

$\sigma^2 = p(1-p)$

Binomial

$X \sim Bin(p, n)$

$$p(x) = P(X=x) = \begin{cases} {}^nC_x\, p^x (1-p)^{n-x} & x: 0,1,2,\cdots \\ 0 & \text{otherwise} \end{cases}$$

$\mu_x = np$

$\sigma^2 = npq$

proportion $\hat{p} = \frac{X}{n} = \frac{\text{no. of success}}{\text{no. of trials}}$ uncertainty $= \sqrt{\frac{p(1-p)}{n}}$

Poisson

$X \sim Poisson(\lambda)$

avg.

$$p(x) = P(X=x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & \text{if } x \text{ is non-ve } \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$$

$\mu_x = \lambda = np$

$\sigma^2 = \lambda = np$

Normal

$X \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad \sigma > 0$$

probab. density func.

$$P(a \le x \le b) = \int_a^b f(x)\, dx$$

$$z = \frac{x-\mu}{\sigma}$$

$$P(a \le x \le b) = P(z_1 \le Z \le z_2) = \int_{z_1}^{z_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2/2} \cdot \sigma\, dz$$

$\frac{1}{\sqrt{2\pi}} e^{-z^2/2} \rightarrow$ normal density func.

if $\mu = 0$ $\sigma = 1$ $\Rightarrow$ standard

Central Limit Theorem

1. $\mu_{\bar{x}} = \mu$

$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

2. $S_n = n\bar{X}$ $\qquad \mu_{S_n} = n\mu$

$\sigma_{S_n}^2 = n\sigma^2$

$\cancel{X}$ $S_n \sim N(n\mu, n\sigma^2)$

UNIT - 3

3rd May, 2023
18:12

LAPLACE TRANSFORMS

→

1. $L[k] = \frac{k}{s}$ $L^{-1}\left[\frac{k}{s}\right] = k$

2. $L[e^{at}] = \frac{1}{s-a}$ $L^{-1}\left[\frac{1}{s-a}\right] = e^{at}$

3. $L[e^{-at}] = \frac{1}{s+a}$ $L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$

4. $L[\sin at] = \frac{a}{s^2+a^2}$ $L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$

5. $L[\cos at] = \frac{s}{s^2+a^2}$ $L^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{\sinh at}{a}$

6. $L[\sinh at] = \frac{a}{s^2-a^2}$ $L^{-1}\left[\frac{s}{s^2-a^2}\right] = \cosh at$

7. $L[\cosh at] = \frac{s}{s^2-a^2}$ $L^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{\sin at}{a}$

8. $L[t^n] = \frac{n!}{s^{n+1}}$ $L^{-1}\left[\frac{1}{s^{n+1}}\right] = \frac{t^n}{n!}$

(or) (or)

$L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$ $L^{-1}\left[\frac{1}{s^{n+1}}\right] = \frac{t^n}{\Gamma(n+1)}$

→ PROPERTIES

1. Shifting

$L[e^{at}f(t)] = F(s-a)$; $L^{-1}[F(s-a)] = e^{at}L^{-1}[F(s)]$

$L[e^{-at}f(t)] = F(s+a)$; $L^{-1}[F(s+a)] = e^{-at}L^{-1}[F(s)]$

– using P.F

a) non-repeated linear factors $F(s) = \frac{A}{(\ )} + \frac{B}{(\ )} + \cdots$

b) repeated linear factors $F(s) = \frac{A}{(\ )} + \frac{B}{(\ )^2} + \frac{C}{(\ )^3} + \cdots$

c) non-repeated quadratic factor $F(s) = \frac{As+B}{(\ )} + \frac{Cs+D}{(\ )} + \cdots$

d) repeated quadratic factor $F(s) = \frac{As+B}{(\ )} + \frac{Cs+D}{(\ )^2}$

2. Differentiation

$$L[t f(t)] = -\frac{d}{ds}\left(L[f(t)]\right) \quad ; \quad L^{-1}[F^{(n)}(s)] = (-1)^n t^n f(t)$$

$$L[t^2 f(t)] = (-1)^2 \frac{d^2}{ds^2}[L[f(t)])$$

3. Integration

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty L[f(t)]\, ds \quad ; \quad L^{-1}\left[\int_s^\infty F(s)\,ds\right] = \frac{L^{-1}[F(s)]}{t}$$

$$L\left[\frac{f(t)}{t^2}\right] = \int_s^\infty \int_s^\infty L[f(t)]\, ds\, ds$$

4. Derivative

$$L[f'(t)] = sF(s) - f(0) \quad ; \quad L^{-1}[sF(s)] = \frac{d}{dt} f(t)$$

$$L[f''(t)] = s^2 F(s) - sf(0) - f'(0)$$

$$L[f'''(t)] = s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$$

5. Integral

$$L\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s} \quad ; \quad L^{-1}\left[\frac{F(s)}{s}\right] = \int_0^t f(t)\,dt$$

$$L\left[\int_0^t\int_0^t f(t)\,dt\,dt\right] = \frac{F(s)}{s^2}$$

→ Unit Step func. (Heavyside func.)

4th May, 2023
19:53

1. $L[u(t-a)] = \frac{e^{-as}}{s}$

2. $L[f(t-a)u(t-a)] = e^{-as} L[f(t)]$; $L^{-1}[e^{-as} F(s)] = f(t-a)u(t-a)$

find $L^{-1}(f(t))$
$t \to t-a$
followed by $u(t-a)$

3. $L[f(t)u(t-a)] = e^{-as} L[f(t+a)]$

1. $f(t) = \begin{cases} f_1(t) & t < a \\ f_2(t) & t \geq a \end{cases}$

$f \in [0,\infty)$

$$f(t) = f_1(t) + \{f_2(t) - f_1(t)\} u(t-a)$$

2. $f(t) = \begin{cases} f_1(t) & t \leq a \\ f_2(t) & a < t \leq b \\ f_3(t) & t > b \end{cases}$

$$f(t) = f_1(t) + \{f_2(t) - f_1(t)\} u(t-a) + \{f_3(t) - f_2(t)\} u(t-b)$$

(2)

→ Unit impulse func.

$$L[\delta(t-a)] = e^{-as}$$

$$L[f(t)\delta(t-a)] = e^{-as} f(a)$$

$$\int_0^{\infty} f(t)\,\delta(t-a)\,dt = f(a)$$

→ ILT of log & trig.

$$L^{-1}[F(s)] = \frac{-1}{t} L^{-1}[F'(s)]$$

→ Convolution Theorem

$$\int e^{ax} \sin bx\,dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$$

$$\int e^{ax} \cos bx\,dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$$

*choose G(s) as easier func.

$$L^{-1}[F(s)G(s)] = f(t) * g(t) = \int_0^t f(u)\,g(t-u)\,du$$

→ Periodic Function

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$$

→ Change of State

$$L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$$

$$L^{-1}[F(ks)] = \frac{1}{k} f\left(\frac{t}{k}\right)$$

$$\cosh t = \frac{e^t + e^{-t}}{2}$$

$$\sinh t = \frac{e^t - e^{-t}}{2}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

MATHS UNIT-5

20th June, 2023

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta$$

$$\frac{e^{i\theta} - e^{-i\theta}}{2i} = \sin\theta$$

$$\int e^{ax}\cos bx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$$

$$\int e^{ax}\sin bx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$F[f(t-t_0)] = e^{-i\omega t_0} F(\omega)$$

$$F^{-1}[e^{-i\omega t_0} f(t)] = f(t-t_0)$$

$$F[e^{i\omega_0 t} f(t)] = F(\omega-\omega_0)$$

$$F[at] = \frac{1}{a} F\left(\frac{\omega}{a}\right)$$

$$F[f(t)\cos\omega_0 t] = \frac{1}{2}\left[F(\omega+\omega_0) + F[\omega-\omega_0]\right]$$

$$F[f(t)\sin\omega_0 t] = \frac{i}{2}\left[F(\omega+\omega_0) - F(\omega-\omega_0)\right]$$

$$F[f^{(n)}(t)] = (i\omega)^n F(\omega)$$

$$F[e^{-a|t|}] = \frac{2a}{a^2+\omega^2} \quad ; \quad F^{-1}\left[\frac{2a}{a^2+\omega^2}\right] = e^{-a|t|}$$

$$F[e^{-at} H(t)] = \frac{1}{a+i\omega} \quad ; \quad F^{-1}\left[\frac{1}{a+i\omega}\right] = e^{-at} H(t)$$

$$F[e^{at} H(-t)] = \frac{1}{a-i\omega} \quad ; \quad F^{-1}\left[\frac{1}{a-i\omega}\right] = e^{at} H(-t)$$

$$F[\delta(t-a)] = e^{-i\omega a}$$

$$F[\delta(t)] = 1$$

$$F[k] = 2\pi k\,\delta(\omega)$$

$$F[e^{iat}] = 2\pi\,\delta(\omega-a)$$

$$F[e^{-a^2t^2}] = \frac{\sqrt{\pi}}{a} e^{-\omega^2/4a^2}$$

FT: $F[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt$

$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{i\omega t}\, d\omega$

FST: $F_s[f(t)] = \int_0^{\infty} f(t)\sin\omega t\, dt$

$f(t) = \frac{2}{\pi}\int_0^{\infty} F_s(\omega)\sin\omega t\, d\omega$

FFCT: $F_c(n) = \int_0^{\pi} f(t)\cos nt\, dt$

$f(t) = \frac{1}{\pi}\left[F_c(0) + 2\sum_{n=1}^{\infty} F_c(n)\cos nt\right]$

FFST: $F_s(n) = \int_0^{\pi} f(t)\sin nt\, dt$

$f(t) = \frac{2}{\pi}\sum_{n=1}^{\infty} F_s(n)\sin nt$

FCT: $F_c[f(t)] = \int_0^{\infty} f(t)\cos\omega t\, dt$

$f(t) = \frac{2}{\pi}\int_0^{\infty} F_c(\omega)\cos\omega t\, d\omega$