SDS → Unit 3

→ Factors that affect CI →
- → Pop. size → ~~[illegible]~~ factor when size ↓.
- → Sample size → ↓ sample less confident that it reflects true pop. paramy
- → Percentage → extreme have better accuracy

→ 0% CI → no faith that if survey repeated you'll get same results.

→ 100% CI → no doubt that if survey repeated you'll get same results

→ CI → how much uncertainty there is with a particular stat. Often used with margin of error.

→ Confidence level → %, Confidence interval → no.

→ Confidence coefficient → confidence level in terms of proportion. ↑ coeff. more accurate

Confidence coeffi.	→	Confidence level
$(1-\alpha)$		$[(1-\alpha)\times 100]\%$
0.90	→	90%

and so on.

where α → significance level.

→ Margin of error → provides upper bound to the diff. b/w particular estimate & the parameter of estimates.

$$MOE = \pm Z_{\alpha/2} \frac{s}{\sqrt{n}}$$

→ For quantitative pop., MOE $(n \geq 30) = \pm 1.96\, s/\sqrt{n}$
→ for binomial pop., MOE $= \pm 1.96 \sqrt{\frac{\hat{p}\hat{q}}{n}}$

tail area $\alpha/2$	→ $z_{\alpha/2}$	
.05	→ 1.645	(90% confidence interval)
.025	→ 1.96	(95% ")
.005	→ 2.58	(99% ")

Point estimator $\pm z_{\alpha/2}$ SE

→ Confidence level associated with a confidence interval is the success rate of the confidence interval

Confidence interval (quant. pop.) $= \bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$

Confidence interval (binomial pop.) $= \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

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→ Narrower CI → gives more info. about pop. param.

→ width of CI = Upper limit − lower limit
$= 2 z_{\alpha/2} \frac{s}{\sqrt{n}}$

mean ↑,↓ → width same.
$s \propto$ width
↓ confidence level → $z_{\alpha/2}$ ↓ → width ↓
n ↑, width ↓

→ Estimating the diff b/w 2 means →

CI for $\mu_x - \mu_y$:

$$\bar{X} - \bar{Y} \pm Z_{\alpha/2}\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}}$$

$X+Y \sim N(\mu_x+\mu_y, \sigma_x^2+\sigma_y^2)$.

$X-Y \sim N(\mu_x-\mu_y, \sigma_x^2-\sigma_y^2)$.

→ Two sided CI :

CI : $\left(\bar{X} - Z_{\alpha/2}\frac{s}{\sqrt{n}}, \bar{X} + Z_{\alpha/2}\frac{s}{\sqrt{n}}\right)$

only for this α used Z_α not $\alpha/2$

lower interval : $\left(-\infty, \bar{X} + Z_\alpha \frac{s}{\sqrt{n}}\right)$ → upper confidence bound

upper interval : $\left(\bar{X} - Z_\alpha \frac{s}{\sqrt{n}}, \infty\right)$ → lower confidence bound

Sample 1: $W_1 = 2Z_{\alpha/2}\frac{s}{\sqrt{400}}$

Sample 2: $W_2 = 2Z_{\alpha/2}\frac{s}{\sqrt{100}}$

$\cancel{\frac{W_2}{W_1} = \frac{\sqrt{400}}{\sqrt{100}}}$ $\quad \frac{W_1}{W_2} = \cancel{\frac{\sqrt{100}}{\sqrt{400}}}\ \frac{1}{2}$

→ Agresti Coull Confidence Interval →
→ Appropriate for any sample size n.

$x \sim Bin(n, p)$ such that
$\tilde{n} = n+4$, $\tilde{p} = \frac{x+2}{\tilde{n}}$

confidence interval for p:
$$\tilde{p} \pm z_{\alpha/2} \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}}$$

If lower limit < 0 → replace with 0
If upper limit > 1 → replace with 1.

~~slide 65~~ ~~22~~ when $\tilde{p}$ not present assume $\tilde{p} = 0.5$ when calculating width.

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$$\pm 1.96 \sqrt{\frac{0.5 \times 0.5}{n+4}} = \pm 0.08$$

$n \approx 146.0625$

↑ use when n small

→ Student's t-distribution →

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ → has $(n-1)$ degrees of freedom

→ Prop.
→ mound shaped & symmetric about 0.
— more variable than z with heavier tails.
→ shape dep. on n, or degree of freedom
→ $n\uparrow$ → shape of t & z → almost id. $n \uparrow t$

→ Degrees of freedom → no. of ind. pieces of info. that went into calculating the estimate. $df = n-1$.

→ Relationship b/w t-curve & normal curve →

→ df ↑ → t dist. approaches standard normal dist. with $\mu = 0$, $\sigma = 1$.

→ standard normal curve → t dist. when $df = \infty$

→ $df = 30$, both almost id.

→ t-dist. symmetric & bell shaped but has heavier tails than normal so it is more prone to prod. values far from the mean

→ t-stat. shouldn't be used when outliers present. Only used when dist. approx. normal → such samples no outliers.

Confidence Interval:

$$\bar{x} \pm t_{\alpha/2} \, s/\sqrt{n}, \quad df = n-1.$$

assumption: population normal

Upper confidence bound

$$\bar{x} + t_{n-1,\alpha} \frac{s}{\sqrt{n}}$$

lower confidence bound

$$\bar{x} - t_{n-1,\alpha} \frac{s}{\sqrt{n}}$$

Again use α, not $\alpha/2$

If σ known use z & not t.

→ CI for paired data → compare two pop/ means where obs/ in one sample is paired with obs in other sample.

$$t = \frac{\text{mean diff}}{SE(d)} \qquad SE(d) = \frac{sd}{\sqrt{n}}$$

Mean diff/ μ_D, CI:

$$\bar{D} \pm t_{n-1, \alpha/2} \frac{SD}{\sqrt{n}} \quad \text{(small sample size)}$$

$$\bar{D} \pm z_{\alpha/2} \frac{SD}{\sqrt{n}} \quad \text{(large sample size)}$$

→ Hypothesis testing → ~~measures~~ prob/ no/ b/w 0 & 1. Measures degree of certainity we have in truth of hypothesis of a quantity.

→ When CI computed → hypothesis test can also be performed.

→ Hypothesis → assumption about pop/ param/

- null hypothesis: H_0 → assumption we wish to test
- alternate hypothesis: H_1 → all possible alternates other than null hypothesis

Steps → define H_0, H_1 → assume H_0 true → compute test stat/ → find P value of test stat/ → state conclusion based on evidence against H_0

→ Types of hypothesis test:

→ Left tailed  → $H_0: \geq$, $H_1: <$

→ Right tailed  → $H_0: \leq$, $H_1: >$

→ Two tailed  → $H_0: =$, $H_1: \neq$

Others → z-test, t-test, chi-square test

when $n < 30$ ~~...~~ → SD known → z test, else t test

when $n > 30$ → z test, when SD unknown z or t test

→ Z score when $n \to$ large,

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

Alt. hypothesis → P-value

$H_1: \mu > \mu_0$ → Area to the right of z.

$H_1: \mu < \mu_0$ → Area to the left of z

$H_1: \mu = \mu_0$ → Sum of areas in tails cut off by z & -z

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coin tossed 400 times → head 216 times, find if unbiased.

$H_0: p = 1/2$

$H_1: p \neq 1/2$

$$z = \frac{x - np}{\sqrt{npq}} \Rightarrow \cancel{\frac{216 - 400/2}{\sqrt{400 \times 1/2 \times 1/2}}} \; 1.6$$

$P(z = 1.6) = 0.11$ → sufficiently large

accept null hypothesis, coin unbiased.

→ Conclusions from hypothesis tests →

→ Ho false or Ho plausible

→ Ho can never be concluded as true

If $P \leq 0.05$ → reject else plausible.

$\uparrow P$ → more plausible Ho becomes

If $P \leq 0.05$ → result stat. significant at 5% level

$H_0: \mu = 0$. P value $= 0.03$

Result statistically significant at 10% level, 5% level but not 1% level

Null hypothesis rejected at 10% level, 5% level but not at 1% level

→ P value doesn't measure practical significance. It measures degree of confidence we have that true value is really diff. from value in null hypothesis.

P value $\downarrow$ ~~the~~ ~~real~~ true value really diff.

→ Critical point → value of test stat. that prod. P-value equal to α.

Region on the side of the critical point towards rej. → rej. reg.

→ For pop. prop. :

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

→ Sample tests for diffⁱ b/w two means →

$H_0: \mu_x - \mu_y = \Delta_0$,

$H_0: \mu_x - \mu_y > \Delta_0$,

$H_0: \mu_x - \mu_y < \Delta_0$

diffⁱ far from 0 → mean diffⁱ

else, might be same

$$z = \frac{(\bar{x} - \bar{y}) - \Delta_0}{\sqrt{\sigma_x^2/n_x + \sigma_y^2/n_y}}$$