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UNIT 1: Concepts Leading to Quantum Mechanics

Field → Region of space where presence of something is felt.

Electric field ∝ Electric charge ($E \propto Q$)
Electric field ∝ 1 / Distance from source ($E \propto \frac{1}{r}$)

12/09/2022

Exp 1: Graphs → least square fit for scattered data
Temp v/s Resistance

Ex1:

Trial No.	x (°C) (Temp.)	y (Ω) (Resistance)	xy	x^2	$y' = mx + c$
1	30	4.82	144.60	900	4.81742
2	35	4.84	169.40	1225	4.85887
3	40	4.92	196.80	1600	4.90032
4	45	4.94	222.30	2025	4.94177
5	50	4.99	249.50	2500	4.98322
6	55	5.00	275.00	3025	5.02467
7	60	5.09	305.40	3600	5.08412
8	65	5.11	332.15	4225	5.1076
9	70	5.14	359.80	4900	5.14902
10	75	5.19	389.25	5625	5.19047
n=10	525	50.04	2644.2	29625	

$$m = \frac{\Sigma x \Sigma y - n\Sigma xy}{(\Sigma x)^2 - n\Sigma x^2}$$ $m = 0.00829$

$$c = \frac{\Sigma x \Sigma yx - \Sigma y \Sigma x^2}{(\Sigma x)^2 - n\Sigma x^2}$$ $c = 4.56872$

~~Trial 1, y' = 0~~

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→ Electric field intensity increases with charge and is inversely proportional to distance from source.

$$E \propto Q$$
$$E \propto 1/R^2$$
$$E = \frac{kQ}{r^2}$$

Q- Differentiate $f(x,y) = 2x^2\hat{i} + y\hat{j}$.

$$f_x = 4x\hat{i}$$
$$f_y = 1\hat{j}$$
$$f_r = 4x\hat{i} + 1\hat{j}$$

→ $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

$\downarrow$

Nabla → Differential operator
'del'

→ $\vec{\nabla}\phi = \hat{i}\frac{\partial \phi_x}{\partial x} + \hat{j}\frac{\partial \phi_y}{\partial y} + \hat{k}\frac{\partial \phi_z}{\partial z}$

$\downarrow$ scalar

$\vec{\nabla}\phi$ → Gradient

→ Divergence of a vector → signifies variation of vector field

$$\vec{V} = \hat{i}V_x + \hat{j}V_y + \hat{k}V_z.$$

$$\text{Div}\,V = \vec{\nabla}\cdot\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Divergence of vector field gives scalar.

→ Curl of a vector: $\vec{\nabla}\times\vec{V}$ Gives vector.

$$\vec{\nabla}\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix}$$

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→ $\vec{\nabla}\cdot\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla^2$ (Laplacian operator)

→ $\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot A) - \nabla^2 A$ (vector Identity)

→ Maxwell's equations:

→ 1. $\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$ → Gauss' Law of (Electric field)

→ 2. $\nabla\cdot\vec{B} = 0$ → Gauss' Law (Magnetic field)

→ 3. $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ → faraday's law of induction

→ 4. $\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$ → Ampere-Maxwell's law

$\vec{J}$ → current density (current / Area).

3. $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$$\nabla\times(\nabla\times\vec{E}) = \nabla\times\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$\nabla\cdot(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}(\nabla\times\vec{B})$$ [Interchanged as rate of change <<< speed of light]

$$\nabla\cdot\left(\frac{\rho}{\varepsilon_0}\right) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

In charge-free & current-free region (vacuum),

$\frac{\rho}{\varepsilon_0} = 0$ and $\vec{J} = 0$.

$$\cancel{-}\nabla^2\vec{E} = \cancel{-}\frac{\partial}{\partial t}\left(\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\boxed{\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}$$

↓

wave equation of $\frac{1}{\sqrt{\mu_0\varepsilon_0}} = v$ (velocity) $\simeq 2.99\times10^8$ m/s (speed of light)

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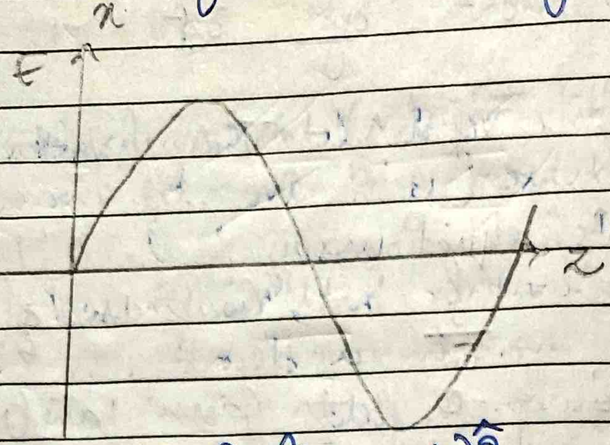
→ Directions of electric & magnetic field in EM wa



$$E_z = E_0 \sin(kz-\omega t)\hat{i}$$

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_z & 0 & 0 \end{vmatrix} = \frac{-\partial B}{\partial t}$$

$$\hat{i}(0) - \hat{j}\left(\frac{-\partial E_z}{\partial z}\right) + \hat{k}\left(\frac{-\partial E_z}{\partial y}\right) = \frac{-\partial B}{\partial t}$$

$$\frac{\partial E_z}{\partial z}\hat{j} - \frac{\partial E_z}{\partial y}\hat{k} = \frac{-\partial B}{\partial t}$$

$$\left[\because \frac{\partial E_z}{\partial y} = 0 \text{ (no component of } y)\right]$$

$$\frac{\partial E_z}{\partial z}\hat{j} = \frac{-\partial B}{\partial t}$$

$$\hat{j}\frac{\partial}{\partial z} E_0 \sin(kz-\omega t)\cancel{\hat{i}} = \frac{-\partial B}{\partial t}$$

$$\hat{j}\, kE_0 \cos(kz-\omega t)\cancel{\hat{i}} = \frac{-\partial B}{\partial t}$$

$$-\int \partial B = \hat{j} E_0 k \int \cos(kz-\omega t)\,\partial t$$

$$-\int \partial B = \hat{j} E_0 k \left(\frac{-\sin(kz-\omega t)}{\omega}\right)$$

$$\cancel{-}B = \hat{j}\frac{E_0 k}{\omega}\cancel{-}\sin(kz-\omega t)$$

$$\boxed{B = \hat{j}\,\frac{E_0}{c}\sin(kz-\omega t)} \qquad \left[\because \frac{k}{\omega} = \frac{1}{c}\right]$$

∴ Both electric field & magnetic field are in phase (Both are sin waves). So, both electrical & magnetic field vibrate ⊥ to one another but in phase. ($\phi = 0$). Together, they move in a direction ⊥ to both electric & magnetic field.

→ $B_0 = \frac{E_0}{c}$. Amplitude of magnetic field is 1/c times amplitude of electric field.

→ Energy carried by electric & magnetic field in EM Wave

→ Electrical energy stored in a capacitor raised to potential V; $W = \frac{1}{2}CV^2$ — (2)

$C = \frac{A\varepsilon_0}{d}$ — (1)   A → cross-sectional area, d → separation b/w plates

Sub. (1) in (2), $W = \frac{1}{2}\frac{A\varepsilon_0 V^2}{d}$

$$W = \frac{1}{2}\frac{A\varepsilon_0 V^2}{d} \times \frac{d}{d}$$

$$W = \frac{1}{2}\varepsilon_0 E^2 (Ad) \qquad \left[\because V/d = E\right] \text{ (Electric field)}$$

$$W = \frac{1}{2}\varepsilon_0 E^2 V \qquad [V = \text{volume}]$$

$$\boxed{U_e = \frac{1}{2}\varepsilon_0 E^2}$$

(Energy per unit volume)

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→ Magnetic field stored in an inductor

$$W_B = \frac{1}{2} L I^2$$

$$W_B = \frac{1}{2} \frac{\cancel{\mu_0 N^2} A}{\cancel{l}} \times \frac{B^2 \cancel{l^2}}{\mu_0^{\cancel{2}} \cancel{N^2}} \qquad L = \frac{\mu_0 N^2 A}{l}, \quad I = \frac{Bl}{\mu_0 N}$$

$$W_B = \frac{1}{2} \frac{AB^2 l}{\mu}$$

$$\boxed{u_B = \frac{1}{2} \frac{B^2}{\mu_0}}$$

(magnetic field per unit volume)

∴ Energy contained in electro-magnetic wave (radiation):

$$u = u_e + u_B$$

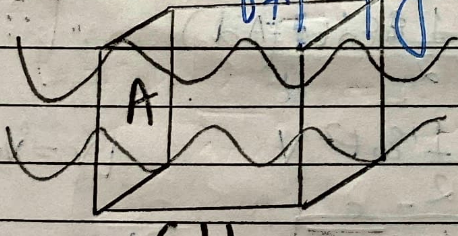
$$u = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0}$$

$$u = \frac{1}{2} \left(\varepsilon_0 E^2 + \frac{E^2}{c^2 \mu_0} \right) \qquad \left[\because B = \frac{E}{c} \right]$$

$$u = \frac{1}{2} \left(\varepsilon_0 E^2 + \varepsilon_0 E^2 \right) \qquad \left[\because \frac{1}{\mu_0 \varepsilon_0} = c^2 \right]$$

$$\boxed{u = \varepsilon_0 E^2}$$

→ Direction of energy transport / propogation of EM Waves:



Cdt

Area = A
time = dt
speed = c
distance = cdt
Volume = ACdt

Energy contained in this volume element

$$u \times (ACdt) = \varepsilon_0 E^2 (ACdt)$$

flux / unit time = Energy/unit area/time

Poynting vector ← $S = \frac{\varepsilon_0 E^2 (A C \cancel{dt})}{\cancel{Adt}}$

$$\boxed{S = C \varepsilon_0 E^2}$$

$S = c\varepsilon_0 E(cB)$ [$\because E = Bc$]

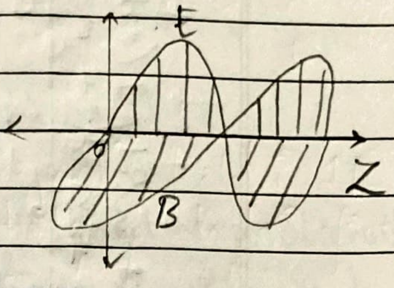
$S = \varepsilon_0 c^2 EB$

$\vec{S} = \varepsilon_0 c^2 \vec{E}\times\vec{B}$

$\vec{S} = \frac{\cancel{\varepsilon_0}}{\mu_0 \cancel{\varepsilon_0}} \vec{E}\times\vec{B}$ [$\because c^2 = 1/\mu_0\varepsilon_0$]

$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$ $|\vec{S}|_{mag} = \frac{1}{\mu_0}EB = c\varepsilon_0 E^2$

→ $E_z = E_0 \sin(kz - \omega t)$



Avg. power dissipated over one cycle:

Power = Flux × Area

(P) Power $= \varepsilon_0 c E^2 \times A$

$\therefore P_{avg} = \frac{1}{T}\int_0^T \varepsilon_0 c E^2 A\,dt$

$P_{avg} = \frac{1}{T} A\varepsilon_0 c E_0^2 \int_0^T \sin^2(kz - \omega t)\,dt$

$P_{avg} = \frac{1}{\cancel{T}} A\varepsilon_0 c E_0^2 \times \frac{\cancel{T}}{2}$

$P_{avg} = \frac{A\varepsilon_0 c E_0^2}{2}$

→ Intensity of EM radiation $= \frac{P_{avg}}{Area}$

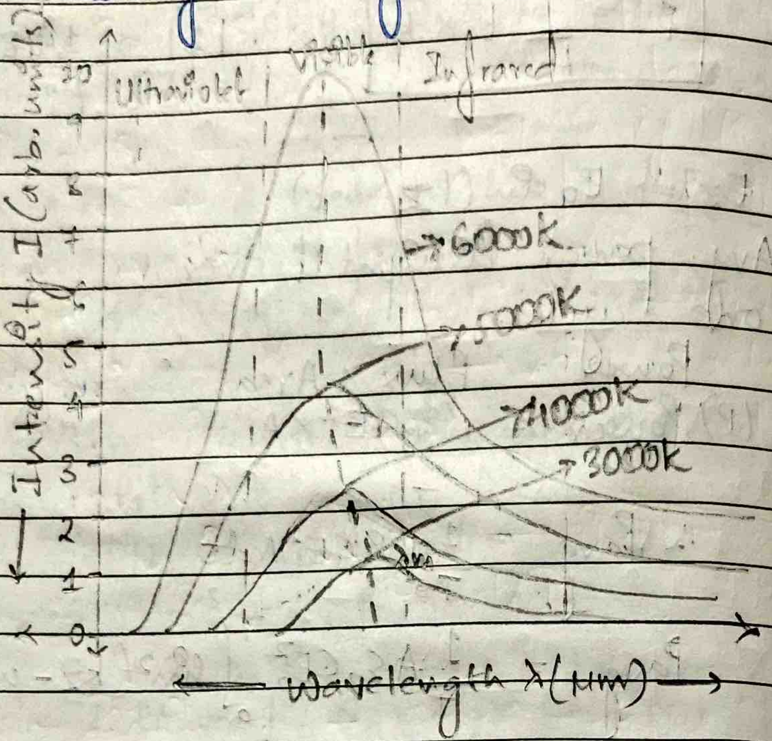
Intensity $= \frac{\varepsilon_0 c E_0^2}{2}$

→ Intensity ∝ $|\text{Amplitude}|^2$ (here, intensity ∝ $|E_0|^2$)

↓

This suggests that energy is contained in the amplitude.

→ Blackbody → Hypothetical body which absorbs all the incident radiations & emits radiation depending upon the temperature to which it is raised. Emissivity is unity. Ex: Incandescant bulb, sun



→ Observations:

→ There are ~~different~~ distinct curves for different temperature.

→ The most probable wavelength λ_m shifts to shorter wavelength side

$\lambda_m \propto \frac{1}{T}$, T → absolute temperature

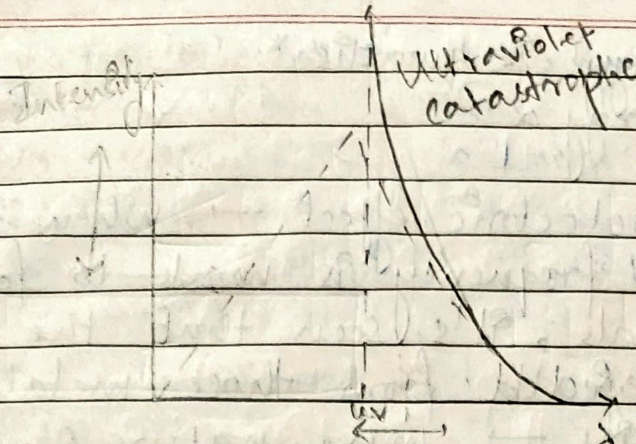
$T \propto \frac{1}{\lambda_{max}} \Rightarrow \lambda_{max} T = \text{constant}$

→ Rayleigh-Jeans' law:

$$\underline{u(\lambda)d\lambda} = \frac{8\pi kT}{\lambda^4} d\lambda$$

k_B = Boltzman's constant $= 1.38 \times 10^{-23}$ J/K

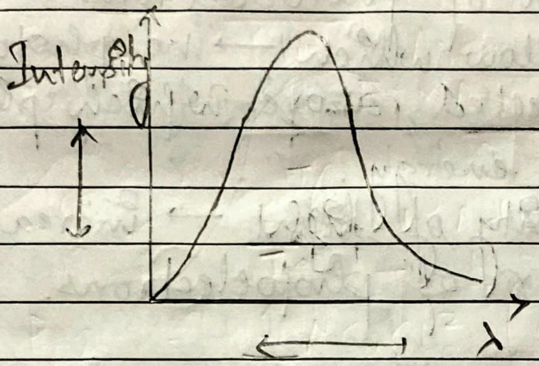
Intensity kT → Thermal energy



Matched exp. observation from longer wavelength side.

→ **Wein's law:**

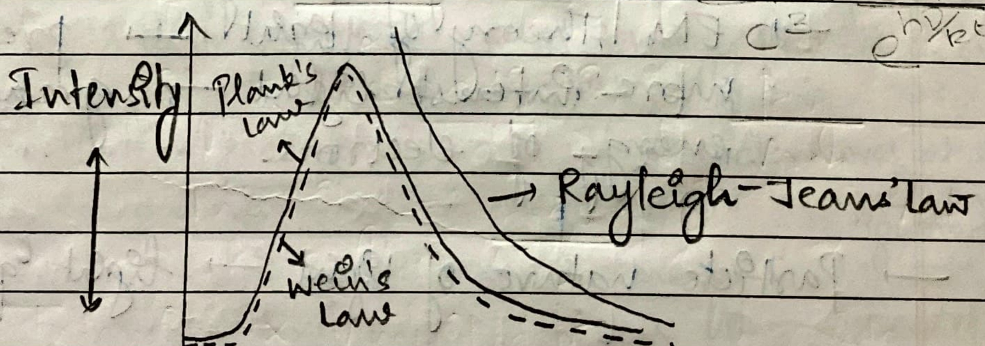
$$u(\lambda)d\lambda = C_1\lambda^{-5}e^{-C_2/\lambda}\,d\lambda$$



Matched exp. observation from shorter wavelength side.

→ **Plank's law:**

$$u(\lambda)d\lambda = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{(h\nu/kT)}-1}\right]d\lambda \qquad \frac{8\pi h\nu^3}{c^3}\,\frac{d\nu}{e^{h\nu/kT}-1}$$



→ **Plank's method:**

→ Avg. energy of oscillators using K_b distribution function:

$$E = \frac{\Sigma E.P(E)}{\Sigma P(E)} = \frac{h\nu}{\exp^{h\nu/kT}-1}$$

→ No. of standing waves $= \dfrac{1}{e^{h\nu/kt}-1}$

- Energy is quantised.
- Energy $\propto \nu$

- Photoelectric effect → when light of certain frequency is made to fall on some materials, it's seen that the electrons are ejected from those metals.
  It defies → wave nature of light.

- Observations:
  - ejection of photoelectrons → instantaneous
  - ν_{min} below which → no photoelectrons are ejected, above which photoelectrons range energy.
  - Intensity of light → increases the number of photoelectrons.

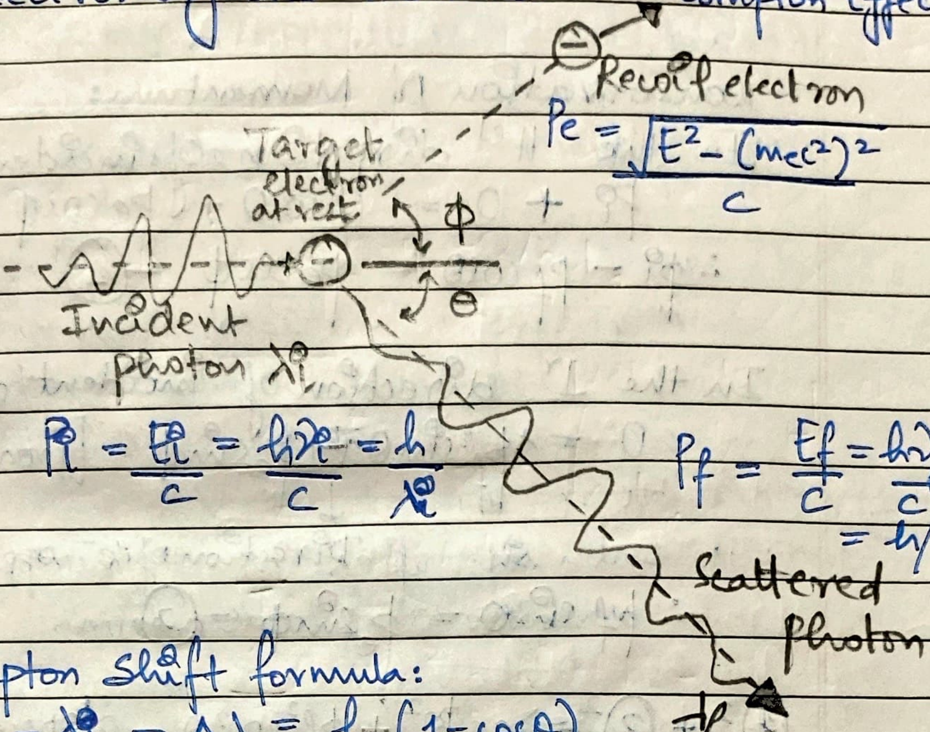
- Predictions:
  - Electrons should be emitted with time delay
  - Electrons should be emitted irrespective of ν / λ of incident light.
  - EM Theory of light → predicts more intense light → greater energy of electrons.

- Particle nature of light → light 'quantum' → photon.

- Energy of photon $E = h\nu = \frac{hc}{\lambda}$

- Kinetic energy of photon $KE = h\nu - \Phi$ or $KE = h(\nu - \nu_0)$.

→ Compton Effect → Process of scattering of radiation from a loosely bound electron, which is released from the atom. The remainder of energy is reradiated as electromagnetic radiation → 'Compton Effect'.



$$P_e = \frac{\sqrt{E^2 - (m_e c^2)^2}}{c}$$

$$P_i = \frac{E_i}{c} = \frac{h\nu_i}{c} = \frac{h}{\lambda_i}$$

$$P_f = \frac{E_f}{c} = \frac{h\nu_f}{c} = h/\lambda_f$$

Compton shift formula:

$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

→ These electrons move at high speed (close to speed of light) → Newton's laws don't work. So, Einstein gave Total Energy (~~[illegible]~~ Relativistic formula)

$$E = \sqrt{\underbrace{p^2c^2}_{KE} + \underbrace{(m_0c^2)^2}_{\text{Rest mass energy}}}$$

If particle moves at speed at light → m_0c^2 can be ignored.

→ Relation b/w relativistic & rest mass:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

→ For photon $E = \sqrt{p^2c^2 + 0}$ (no m_0c^2).

$$E = pc$$

→ Compton-Shift → $\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_0 c}(1-\cos\theta)$

final & initial wavelengths

scattering angle

$\frac{h}{m_0 c}$ → Compton wavelength

→ this is inelastic collision → KE not conserved

→ Conservation of Momentum:

In the $\parallel^{el}$ direction of incident photon

$$p_i + 0 = p_f\cos\theta + p_e\cos\phi \quad \text{[from figure]}$$

$$\therefore p_i - p_f\cos\theta = p_e\cos\phi \quad \text{— (1)}$$

In the $\perp^r$ direction of incident photon

$$0 = p_f\sin\theta \ominus p_e\sin\phi \quad \text{[from figure]}$$

Direction of e^- opp. to photon

$$\therefore p_f\sin\theta = p_e\sin\phi \quad \text{— (2)}$$

$$(1)^2 + (2)^2 \Rightarrow p_i^2 + p_f^2\cos^2\theta - 2p_ip_f\cos\theta + p_f^2\sin^2\theta = p_e^2\cos^2\phi + p_e^2\sin^2\phi$$

$$\Rightarrow p_i^2 + p_f^2 - 2p_ip_f\cos\theta = p_e^2 \quad \text{— (3)}$$

Loss in photon energy = Gain in electron's KE

$$E_i - E_f = K_e \quad \text{(from figure)}$$

$$p_ic - p_fc = K_e$$

$$c(p_i - p_f) = K_e$$

Relativistic (Total Energy) formula,

$$E^2 = p^2c^2 + (m_0c^2)^2$$

Electron's energy has part KE & part rest-mass energy

$$(K_e + m_0c^2)^2 = E^2 = p_e^2c^2 + (m_0c^2)^2$$

$$K_e^2 + \cancel{(m_0c^2)^2} + 2K_em_0c^2 = p_e^2c^2 + \cancel{(m_0c^2)^2}$$

$$K_e^2 + 2m_0c^2K_e = p_e^2c^2$$

$$\div \text{ by } c^2 \Rightarrow \frac{K_e^2}{c^2} + 2m_0K_e = p_e^2 \quad \text{— (4)}$$

$$(3)\ \&\ (4) \Rightarrow p_i^2 + p_f^2 - 2p_ip_f\cos\theta = \frac{K_e^2}{c^2} + 2m_0K_e$$

compton scattering, angle b/w direction of incident photon is, recoil e^-,

$$\cot\phi = \left(1 + \frac{h}{m_e c} \times \frac{1}{\lambda}\right)\tan\theta/2$$

$$p_i^2 + p_f^2 - 2p_i p_f \cos\theta = \cancel{\frac{ke^2}{c^2}} \frac{\cancel{c^2}(p_i - p_f)^2 + 2m_0(p_i - p_f)c}{\cancel{c^2}}$$

$[\because ke = c(p_i - p_f)]$

$$\cancel{p_i^2} + \cancel{p_f^2} - 2p_i p_f \cos\theta = \cancel{p_i^2} + \cancel{p_f^2} - 2p_i p_f + 2m_0 c(p_i - p_f)$$

$$2p_i p_f - 2p_i p_f \cos\theta = 2m_0 c(p_i - p_f)$$

$$\cancel{2}p_i p_f(1 - \cos\theta) = \cancel{2}m_0 c(p_i - p_f)$$

$$\frac{1-\cos\theta}{m_0 c} = \frac{p_i - p_f}{p_i p_f}$$

$$\frac{1-\cos\theta}{m_0 c} = \frac{1}{p_f} - \frac{1}{p_i}$$

$$\cancel{\frac{\Delta\lambda}{h} = \frac{1}{p_f} - \frac{1}{p_i}} \qquad \left[\because \Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)\right]$$

$$\cancel{\therefore \Delta\lambda = h\left(\frac{1}{p_f} - \frac{1}{p_i}\right)}$$

$$\times h \Rightarrow \frac{h(1-\cos\theta)}{m_0 c} = \frac{h}{p_f} - \frac{h}{p_i}$$

$$\therefore \frac{h(1-\cos\theta)}{m_0 c} = \lambda_f - \lambda_i$$

Accd to this formula, shift in wavelength is only dependant on the scattering angle.

When scattering angle → 0°, then there is no shift and the incident photon misses target e^-.

When scattering angle → 180°, then ~~electron~~ the wavelength shift is $\frac{2h}{m_0 c}$ and it represents a head-on collision.

When scattering angle → 90°, $\lambda_f - \lambda_i = h/m_0 c$.

$\frac{h}{m_0 c}$ for electron $= 2.43 \times 10^{-12}$ m.

$= 0.0243 Å$.

→ from Relativistic (Total energy) formula

$$E^2 = p^2c^2 + (m_0c^2)^2$$

Since photon is massless,

$$E = pc$$

Also, $E = h\nu = \frac{hc}{\lambda}$

$$\therefore E = pc = \frac{hc}{\lambda}$$

$$p\cancel{c} = \frac{h\cancel{c}}{\lambda}$$

$$p = \frac{h}{\lambda}$$

$$\therefore \boxed{\lambda = \frac{h}{p} = \frac{h}{mc}}$$

This is the wave-particle duality of light. From experimental observations, light is known to have momentum although photon is considered mass-less.

→ wave-particle duality of any particle (matter).

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

Given by Louis De Broglie.

These waves associated with matter 'matter waves'

→ Matter waves exist only with moving particles as if $v = 0$, $\lambda = \infty$.

Q- A golf-ball weighing 250g & moving with speed of 200m/s is associated with matter wave. Calculate λ.

Ans: $\lambda = \frac{h}{mv}$

$$\lambda = \frac{6.63 \times 10^{-34}}{25 \times 10^{-2} \times 200}$$

$$\lambda = \frac{\overset{1.32}{\cancel{6.63}} \times \cancel{10^{-34}}\ 10^{-35}}{\cancel{5.25} \times \cancel{10^{-2}} \times \cancel{2} \times \cancel{10^{2}}}$$

$$\lambda = 1.32 \times 10^{-35}\ m$$

Q- Calculate λ associated with e^- moving with 10^5 m/s.

Ans: $$\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{5}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-26}}$$

$$\lambda = 7.28 \times 10^{-9}\ m$$

→ Microscopic particles → have detectable wave nature but macroscopic particles do not.

→ Although macroscopic ~~par~~ particles have λ, it's highly impossible to set up a diffraction apparatus/experiment for them. Hence, they do not have detectable wave nature.

→ Superposition of waves:

$$y_1 = A \sin(kx - \omega t)$$

$$y_2 = A \sin[(k+\Delta k)x - (\omega + \Delta\omega)t]$$

$$y = y_1 + y_2 \quad \text{(Resultant wave)}$$

$$y = A\left[2 \sin\left(\frac{(2k+\Delta k)x - (2\omega + \Delta\omega)t}{2}\right)\right] \cdot \cos\left(\frac{-\Delta k x + \Delta\omega t}{2}\right)$$

$$y = 2A\left[\sin\left(\frac{(2k+\Delta k)x - (2\omega+\Delta\omega)t}{2}\right) \cdot \cos\left(\frac{\Delta k x - \Delta\omega t}{2}\right)\right]$$

$2k + \Delta k \approx 2k$
$2\omega + \Delta\omega \approx 2\omega$ [Δk & $\Delta\omega$ negligible]

$$y = 2A\left[\sin\left(\frac{2kx - 2\omega t}{2}\right) \cdot \cos\left(\frac{\Delta k x - \Delta\omega t}{2}\right)\right]$$

$$y = 2A\left[\sin(kx - \omega t) \cdot \cos\left(\frac{\Delta k x - \Delta\omega t}{2}\right)\right]$$

$$\therefore y = 2A\cos\left(\frac{\Delta k x - \Delta\omega t}{2}\right) \cdot \sin(kx - \omega t)$$

$$Y = \left[2A\cos\left(\frac{\Delta kx - \Delta\omega t}{2}\right)\right] \cdot \sin(kx - \omega t)$$

$\underbrace{\qquad}_{\text{Modulation}}$

$2A\cos\left(\frac{\Delta kx - \Delta\omega t}{2}\right) \rightarrow$ Resultant amplitude which is time dependant.

→ Phase & Group Velocity:

$kx - \omega t = \text{constant}$

Differentiating w.r.t t,

$$k\frac{dx}{dt} - \omega = 0$$

$$k\frac{dx}{dt} = \omega$$

$\omega \rightarrow$ angular frequency

$k \rightarrow$ wave number (no. of waves in wave packet)

$$V_p = \frac{dx}{dt} = \frac{\omega}{k}$$

$\left[\text{velocity} = \frac{dx}{dt}\right]$

↳ Phase Velocity $V_p = \omega/k$

Phase velocity → It's the velocity with which an individual wave enclosed in a wave packet moves.

$$\frac{\Delta kx - \Delta\omega t}{2} = \text{constant}$$

$$\Delta kx - \Delta\omega t = 2\,\text{constant}$$

Differentiating w.r.t t,

$$\Delta k\frac{dx}{dt} - \Delta\omega = 0$$

$$\Delta k\frac{dx}{dt} = \Delta\omega$$

$$V_g = \frac{dx}{dt} = \lim_{\Delta k \to 0}\frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$

↳ Group velocity $V_g = d\omega/dk$

Group velocity → It's the velocity with which a wave packet formed by the superimposition of wave, moves.

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→ Relation b/w Group & Phase velocity:

$$V_p = \frac{\omega}{k} \quad -(1)$$

$$V_g = \frac{d\omega}{dk}$$

(1) ⟹ $k \cdot V_p = \omega$

Differentiating w.r.t k,

$$\frac{d\omega}{dk} = V_p + k\frac{dV_p}{dk}$$

$$\cancel{\frac{d\omega}{dk} = k\frac{dV_p}{dk}\cdot\frac{dk}{d\lambda}}$$

$$\frac{d\omega}{dk} = k\frac{dV_p}{d\lambda}\cdot\frac{d\lambda}{dk} + V_p$$

$$k = \frac{2\pi}{\lambda}$$

$$\frac{dk}{d\lambda} = \frac{-2\pi}{\lambda^2}$$

$$\frac{d\omega}{dk} = k\frac{dV_p}{d\lambda}\cdot\frac{-\lambda^2}{2\pi} + V_p$$

$$\frac{d\omega}{dk} = k\cdot\frac{-\lambda^2}{2\pi}\frac{dV_p}{d\lambda} + V_p$$

$$\frac{d\omega}{dk} = \frac{\cancel{2\pi}}{\cancel{\lambda}} \times \frac{-\lambda^{\cancel{2}}}{\cancel{2\pi}}\frac{dV_p}{d\lambda} + V_p$$

$$\frac{d\omega}{dk} = -\lambda\frac{dV_p}{d\lambda} + V_p$$

$$V_g = -\lambda\frac{dV_p}{d\lambda} + V_p \qquad \left[\because V_g = \frac{d\omega}{dk}\right]$$

$$\therefore \boxed{V_g = V_p - \lambda\frac{dV_p}{d\lambda}}$$

Phase velocity (V_p) is always greater than the Group velocity (V_g).

$\frac{dV_p}{d\lambda}$ → signifies nature of the medium the wave travels in.

$\frac{dV_p}{d\lambda} = 0$ → medium is non-dispersive or homogeneous. ($V_g = V_p$, no superposition)

$\frac{dV_p}{d\lambda} \neq 0$ → medium is dispersive ($V_g < V_p$)

→ Relation b/w particle & Group Velocity (V_g):

$$V_g = \frac{d\omega}{dk}$$

$$\omega = 2\pi\nu \quad -(2)$$

$$E = h\nu \quad -(1)$$

Sub. (2) in (1) ⟹ $\omega = \frac{2\pi E}{h}$

$$k = \frac{2\pi}{\lambda} \quad -(3)$$

$$\lambda = \frac{h}{p} \quad -(4)$$

Sub. (4) in (3) ⟹ $k = \frac{2\pi p}{h}$

Now, $V_g = \frac{d(2\pi/h)E}{d(2\pi/h)p}$

$$V_g = \frac{dE}{dp}$$

$$E = KE + PE$$

$$E = \frac{p^2}{2m} + V(x)$$ [PE → function of position]

$$\frac{dE}{dp} = \frac{\cancel{2}p}{\cancel{2}m} + 0$$

$$\frac{dE}{dp} = V_{particle}$$ [∵ $p = mv$, $p/m = v$]

→ $V_g \cdot V_p = c^2$

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Q- If a photon & an ~~neutron~~ electron are both associated with a wavelength of 2A°, calculate the ratio of their energies. Also, calculate the phase & group velocities.

Soln: $E_{photon} = \frac{hc}{\lambda_{photon}}$

$E_{photon} = \frac{6.63 \times 10^{-34} \times \cancel{3}^{1.5} \times 10^{8}}{\cancel{2} \times 10^{-10}}$

$E_{photon} = 9.945 \times 10^{-16} J$

$$\begin{array}{r} 663 \\ \times 15 \\ \hline 3315 \\ 663\times \\ \hline 9945 \end{array}$$

$E_{electron} = \frac{p^2}{2m}$

$p = \frac{h}{\lambda}$

$E_{electron} = \frac{h^2}{\lambda^2 \times 2m}$

$E_{electron} = \frac{6.63 \times 10^{-34} \times 6.63 \times 10^{-34}}{2 \times 10^{-10} \times 2 \times 10^{-10} \times 2 \times 9.1 \times 10^{-31}}$

$E_{electron} = 6.03 \times 10^{-16} J$

Ratio of their energies is $\simeq 3:2$.

$v_p = \omega / k$.

$v_p = \frac{\cancel{2\pi} E}{\cancel{h}} \times \frac{\cancel{h}}{\cancel{2\pi} p}$

$v_p = \frac{E}{h}$

proton rest energy $= 1.24 \times 10^{9}$ eV

electron rest energy $= 511 \times 10^{3}$ eV

→ Double-slit experiment with waves:

$I_1 \propto |E_1|^2$ ~~Iα~~ $I \rightarrow$ intensity

$E \rightarrow$ amplitude

$I_2 \propto |E_2|^2$

$I_{12} = |E_1 + E_2|^2$

$I_{12} = |E_1|^2 + |E_2|^2 + \underbrace{2E_1E_2\cos\delta}_{\text{phase difference}}$

phase difference
↓
differentiates particle & wave interference

→ For particles:

$I \propto N(h\nu)$ $I \rightarrow$ intensity.

$I \propto \frac{1}{\mu_0 \varepsilon_0}|E_0|^2$ $N \rightarrow$ no.

$E \rightarrow$ amplitude.

$\therefore N \propto |E_0|^2$.

$P \propto N \propto |E_0|^2$ $P \rightarrow$ probability of finding particle

$\therefore P \propto |E_0|^2$

Amplitude → helps to find probability of finding a particle.

→ Double-slit experiment with electrons:

$P_{12} = |\psi_1 + \psi_2|^2$

$P_{12} \rightarrow$ probability of finding electrons

Initially, electrons on the screen seem to appear in hap-hazard pattern, but with time, they seem to form an interference pattern.

$\psi_1 = A_1 \sin(k_1 x - \omega_1 t) = A_1 \cos(k_1 x - \omega_1 t)$

$\psi_1 = A_1 \sin(k_1 x - \omega_1 t) + A_1 \cos(k_1 x - \omega_1 t)$

$\psi_1 = A_1 e^{i(k_1 x - \omega_1 t)}$

lly $\psi_2 = A_2 e^{i(k_2 x - \omega_2 t)}$ $[e^{i\theta} = \cos\theta + i\sin\theta]$

$\left[\frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta,\right.$

$\left.\frac{e^{i\theta} - e^{-i\theta}}{2i} = \sin\theta\right]$

$$P_{12} = |\psi_1 + \psi_2|^2$$

$$P_{12} = |\psi_1|^2 + |\psi_2|^2 + \psi_1^* \psi_2 + \psi_1 \psi_2^*$$

physically 'unperceivable'

→ ψ (psi) $= Ae^{i(kx - \omega t)}$ → no meaning by itself → contains necessary information, obtained by squaring

↓

wave function → state of quantum entity

$Ae^{i(kx-\omega t)} \times Ae^{-i(kx-\omega t)} = |A|^2$

$|\psi^* \psi| = |\psi|^2 = |A|^2$ → probability

→ Quantum waves → waves of probability

→ Superposition principle:

$$\psi_p = a\psi_{p_x} + b\psi_{p_y} + c\psi_{p_z}$$

→ $P = |\psi|^2 = \psi^* \psi$

$\psi^* = Ae^{-i(kx-\omega t)}$, $\psi = Ae^{i(kx-\omega t)}$

$\psi^* \psi = Ae^{-i(kx-\omega t)} \cdot Ae^{i(kx-\omega t)}$

$\psi^* \psi = A^2$

$\therefore P = |\psi|^2 = A^2$

→ By increasing no. of waves, span of the wave can be decreased.

$$k \propto \frac{1}{x}$$

$$k_n - k_0 = \frac{1}{x_n - x_0}$$

$$\Delta k \propto \frac{1}{\Delta x}$$

$$\Delta\left(\frac{2\pi}{\lambda}\right) \propto \frac{1}{\Delta x} \quad \left[\because k = 2\pi/\lambda,\ \lambda = h/p\right]$$

$$\Delta\left(\frac{2\pi p}{h}\right) \propto \frac{1}{\Delta x}$$

$$\frac{2\pi}{h} \Delta p \propto \frac{1}{\Delta x}$$

$$\therefore \boxed{\Delta p \Delta x \propto \frac{h}{2\pi}} \rightarrow \text{Heisenberg's Uncertainity Principle}$$

Δp & Δx → cannot be known simultaneously

$\frac{h}{2\pi} \rightarrow \hbar = 1.05 \times 10^{-34}$ Js (Reduced Planck's constant)

$$\boxed{\Delta x . \Delta p_x \geq \frac{h}{4\pi}} \text{ or } \boxed{\Delta x . \Delta p_x \geq \frac{\hbar}{2}}$$

Δx & Δp → conjugate variables

Heisenberg's Uncertainity principle → also called position-momentum uncertainity principle.

NOTE: Some other uncertainity principles:

→ Energy-time → $\Delta E . \Delta t \geq \hbar/2$

→ Angular momentum - angle → $\Delta L . \Delta \theta \geq \hbar/2$

→ To show that electrons don't exist in the nucleus.

$\Delta x \simeq 10^{-15}$ m

$$\Delta p_x \geq \frac{\hbar}{2\Delta x} = \frac{1.05 \times 10^{-34}}{2 \times 10^{-15}} = 0.525 \times 10^{-19}$$

$$\Delta p_x \geq \frac{\hbar}{2\Delta x} = 5.25 \times 10^{-20} \text{ kgm/s}$$

$$E = \frac{p^2}{2m} \simeq \frac{\Delta p^2}{2m} = \frac{27.56 \times 10^{-40}}{2 \times 9.1 \times 10^{-31}}$$

$$9.43 \times 10^{9} \text{ eV} = \frac{27.56 \times 10^{-9}}{18.2}$$

$$9.43 \text{ GeV} = 1.514 \times 10^{-9} \text{ J}$$

e^- cannot be inside the nucleus

Wave function → State of the particle
→ $|\psi|^2$ → tells about the probability of finding the particle at some 'x' & 't'.

↓

physically 'unperceivable' particle

$$\psi(x,t) = Ae^{i(kx-\omega t)}$$

$$\psi(x,t) = Ae^{i/\hbar(px-Et)}$$

$$k = \frac{2\pi}{\lambda}$$

$$\frac{d\psi(x,t)}{dx} = A\left(\frac{ip}{\hbar}\right)e^{i/\hbar(px-Et)}$$

$$k = \frac{2\pi}{h} \times p$$

$$= \left(\frac{i}{\hbar}p\right) Ae^{i/\hbar(px-Et)}$$

$k = p/\hbar$ [$\because \hbar = h/2\pi$]

$\omega = E/\hbar$

$$\frac{d\psi(x,t)}{dx} = \frac{i}{\hbar}p\,\psi(x,t)$$

1D → $\hat{p} = \frac{\hbar}{i}\frac{d}{dx}$ or $\hat{p} = -\hbar i\frac{d}{dx}$ [$i \times i = -1$] [Multiply & divide by i]

3D → $\hat{p} = -i\hbar\nabla$

|||ly $\frac{d}{dt}\psi(x,t) = \frac{-iE}{\hbar}\psi(x,t)$

1D → $\hat{E} = \frac{-\hbar}{i}\frac{d}{dt}$ or $\hat{E} = \hbar i\frac{d}{dt}$

$$KE = \frac{p^2}{2m}$$

$$\hat{T} = KE = \frac{-h^2}{2m}\frac{d^2}{dx^2}$$

Total Energy = KE + PE

(H) Total Energy = $\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$
(Hamiltonian)

$$\boxed{\left(\frac{d}{dx}\right)\psi(x,t) = \frac{i}{\hbar}p\,\psi(x,t)}$$ → Eigenvalue equation

Ex: ψ ← function doesn't change

Original

constants → eigen values

↓ (experiment or instrument) ↓ operator → acts on ψ to give set of constants & ψ itself.

↓ physical system → remains same

Eigenvalue eqn $\rightarrow \dfrac{d(e^{ax})}{dx} = a e^{ax}$

function doesn't change

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→ Probability of finding wave :

Normalisation formula : $\displaystyle\int_{-\infty}^{+\infty} \psi^* \psi \, \underset{\substack{\downarrow \\ \text{volume}}}{dv} = 1$

$\psi(x) = A\sin\left(\frac{2\pi}{L}x\right)$

$\psi^* = A\sin\left(\frac{2\pi}{L}x\right)$

$\psi^* \psi = A^2 \sin^2\left(\frac{2\pi x}{L}\right)$.

$\displaystyle\int_0^L \psi^* \psi \, dx = \int_0^L A^2 \sin^2\left(\frac{2\pi}{L}x\right) dx$

$\displaystyle 1 = A^2 \int_0^L \sin^2\left(\frac{2\pi x}{L}\right) dx$

$\displaystyle 1 = A^2 \int_0^L \frac{1 - \cos 2\left(\frac{2\pi x}{L}\right)}{2} dx$

$\displaystyle 1 = \frac{A^2}{2}\left[\int_0^L dx - \int_0^L \cos 2\left(\frac{2\pi x}{L}\right) dx\right]$

$\displaystyle 1 = \frac{A^2}{2}\left[L - 0 - \cancel{\frac{L}{4\pi} \sin\left(\frac{2\pi x}{L}\right)}\right]_0^L$

$1 = \dfrac{A^2 L}{2}$

$A = \sqrt{\dfrac{2}{L}}$