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Single Phase AC Systems & Basic Terminology

→ First power distribution system was a DC system → invented by Edison.
DC → not suitable for longer distances.

→ AC → popular because of transformers.
→ An AC waveform → periodic waveforms which alternates.

→ Power system → very high voltage, less power loss
Generation
Transmission } done as three phase AC power
Distribution

→ Basic terminology:-
→ Periodic waveform → repeats after certain time interval.
→ Time period → taken time to complete one cycle (T)
→ Frequency → no. of cycle comp. in 1 sec.
(f) = 1/T

→ A pure AC waveform → +ve area matched by equal -ve area. Avg. value = 0.

→ AC waveform with DC component:
DC component → constant amplitude

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→ Sinusoidal wave can be expressed as function of angle or time.
Accd one cycle completes in $(2\pi)^c$ or T sec.

time (t)		angle (θ)
0	→	0
T/4	→	π/2
T/2	→	π
3T/4	→	3π/2
T	→	2π
1	→	2π/T
t	→	(2π/t)t

Mathematical representation: $E = E_m \sin(\omega t)$

$e(t) = E_m \sin(\omega t \pm \phi)$ → if there exists phase difference

$\omega = \frac{2\pi}{T}$ or $2\pi f$

$\theta = \omega t$, $\omega = \theta/t$ rad/s

1. For sinusoidal ~~wave~~ function of frequency 50Hz, find (i) Half time period (ii) angular frequency

Ans: (i) $T = \frac{1}{f}$

$T = \frac{1}{50}$

$T/2 = 1/100 = 0.01$ sec

(ii) $2\pi f = 2 \times 3.14 \times 50 = 314$ rad/s.

2. Max. value of sinusoidal AC of frequency 50Hz is 25A. Write eqⁿ for Ins. Determine value at 3ms & 14ms.

Ans: $I_{max} = 25A$

$i(t) = I_{max} \sin \omega t$ → instantaneous eqⁿ

$i(t = 3ms) = 25\sin(314 \times 3\times10^{-3})$

$= 25\sin(54^\circ)$

$= 20.225\ A$

$i(t = 14ms) = 25\sin(314 \times 14 \times 10^{-3})$

$= -23.77\ A$

→ Avg. value of sinusoidal function:

Avg. value of AC waveform $f(t)$ is given by

$$F_{avg} = \frac{1}{T}\int_0^T f(t)\,dt$$

$$f_{avg} = \frac{1}{T}\int_0^T A\sin\omega t\,dt$$

$$F_{avg} = \frac{A}{T}\left[\frac{-\cos\omega t}{\omega}\right]_0^T$$

$$F_{avg} = \frac{A}{\omega T}\left[-\cos\omega t\right]_0^T$$

$$f_{avg} = \frac{A}{\omega T}\left[1 - \cos\omega T\right]$$

$$F_{avg} = \frac{A}{\cancel{2\pi \times T}}\left[\cancel{1} - \cancel{\cos\frac{2\pi}{T}T}\right] \qquad \left[\omega = \frac{2\pi}{T}\right] \quad \left[\cos 2\pi = 1\right]$$

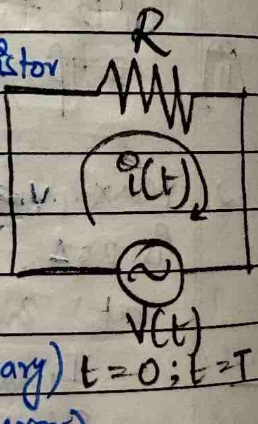
$$F_{avg} = 0$$

NOTE: Net area during one period $= 0$, ∴ $F_{avg} = 0$.

→ Effective/RMS value of AC function:

E_{AC} → Energy consumed by resistor

$$E_{AC} = \int_0^T P(t)\,dt = \int_0^T \frac{[V(t)]^2}{R}\,dt \quad \text{— (1)}$$



For DC source,

$$E_{DC} = \frac{V^2 \cdot T}{R} \quad \text{— (2)}$$

(time doesn't vary) $t = 0;\ t = T$

(power × time = energy)

$E_{AC} = E_{DC}$ [Power developed in resistor by AC & DC source is considered same].

from (1) & (2),

$$\int_0^T \frac{[v(t)]^2}{\cancel{R}} \cdot dt = \frac{V^2 \cdot T}{\cancel{R}}$$

$$\Rightarrow V_{rms} = \sqrt{\frac{1}{T}\int_0^T [v(t)]^2 \cdot dt} \quad -(3)$$

→ Effective/RMS value of sine wave:

Consider sinusoidal voltage $v(t) = V_m \sin(\omega t)$ (4)

Sub. (4) in (3),

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_m \sin \omega t)^2 \cdot dt}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{T}\int_0^T \sin^2(\omega t) \cdot dt}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{T}\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2T}\int_0^T 1-\cos 2\omega t \; dt}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T dt}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2T}\left[(T-0) - \frac{\sin 2\times 2\pi \times \cancel{T}/\cancel{T} - 0}{2\omega}\right]}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2T}[(T-0) - 0 - 0]}$$

[$\omega = 2\pi/T$]

[$\sin(4\pi) = 0$]

$$V_{rms} = \sqrt{\frac{V_m^2 \times \cancel{T}}{2\cancel{T}}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

→ Phase Lag & Phase lead:

$f_1(\theta) = A\sin(\omega t)$ → reference wave

$f_2(\theta) = A\sin(\omega t - \pi/4)$ → Lags reference wave by $\pi/4$

$f_3(\theta) = A\sin(\omega t + \pi/4)$ → Leads reference wave by $\pi/4$.

$f_2(\theta)$ lags $f_3(\theta)$ by $\pi/2$.

Q- Write an eqn to represent the following sine waves of 50 Hz frequency. (i) Sinusoidal current with RMS value 10A & starting at 5ms. (ii) Sinusoidal current with peak value 20A & starting at -2.5ms. Also, comment on the phase relation b/w them.

Ans: Given: $f = 50Hz$

Angle 1 : $\phi_1 = \omega t_1 = 2\pi f t_1 = 2 \times 3.14 \times 50 \times 5 \times 10^{-3} = \frac{\pi}{2}$

~~Angle~~ $i_1(t) = I_m \sin(\omega t \pm \phi_1)$

$[I_m = \sqrt{2} I_{rms}]$

$i_1(t) = 10 \times 1.414 \times \sin(100\pi t - \pi/2)$

$i_1(t) = 14.14 \sin(100\pi t - \pi/2)$ A

Angle 2: $\phi_2 = \omega t_2 = 2\pi f t_2 = 2\pi \times 50 \times -2.5 \times 10^{-3} = -\pi/4$

$i_2(t) = 20 \sin(100\pi t + \pi/4)$

$i_2(t) = 20 \sin(100\pi t + \pi/4)$

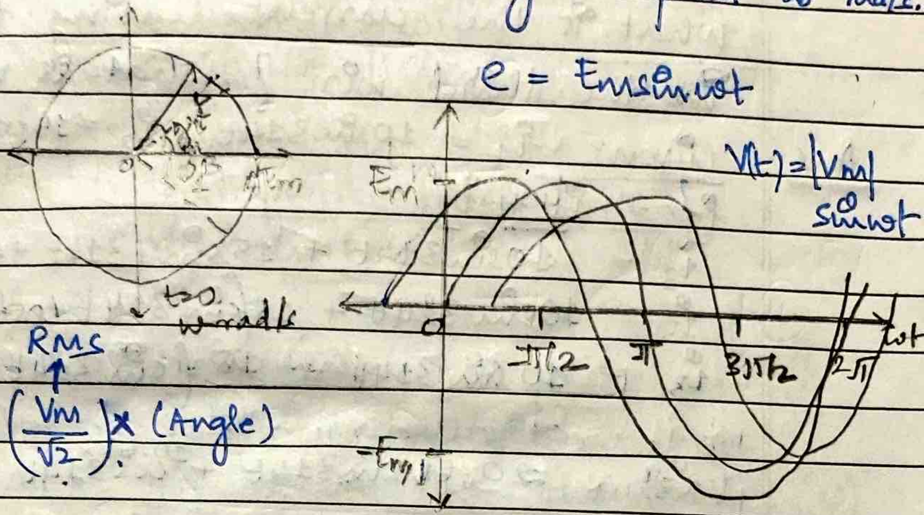
$[I_m = \text{peak} I]$

i_2 leading i_1 by $3\pi/4$

→ Concept of Phasor:

Let us consider a rotating crank length Em lying at 0° position at t=0 & rotating anti-clockwise at an angular speed 'ω' rad/s.

$e = E_m \sin\omega t$



RMS

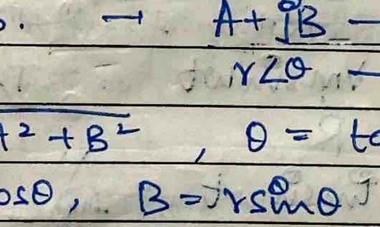
$\bar{V} = \left(\frac{V_m}{\sqrt{2}}\right) \times (\text{Angle})$

Q- Represent in phasor diagram.

(i) $f_1(t) = 100\sin(100\pi t) \longrightarrow \bar{F_1} = 100/\sqrt{2} \times \angle 0°$

$f_2(t) = 200\sin(200\pi t + 60) \rightarrow \bar{F_2} = 200/\sqrt{2} \times \angle 60$

$f_3(t) = 100\cos(100\pi t - 60) \rightarrow \bar{F_3} = 100/\sqrt{2} \times \angle 30°$



→ Complex no. → $A + iB$ → rectangular

$r\angle\theta$ → polar form

$r = \sqrt{A^2 + B^2}$, $\theta = \tan^{-1}(B/A)$

$A = r\cos\theta$, $B = r\sin\theta$

→ Addition and subtraction of phasors → rectangular form

→ Multiplication of phasors → $r_1 \times r_2 \angle \theta_1 + \theta_2$

→ Division of phasors → $r_1/r_2 \angle \theta_1 - \theta_2$

Q- There are three conducting wires connected to form a junction. The currents flowing into the junction in two wires are $i_1 = 10\sin 314t$ A & $i_2 = 15\cos(314t - 45°)$ A. What is the current leaving the junction in the third wire? What is value at t=0?

Ans: Given: $i_1 = 10\sin 314t$, $i_2 = 15\cos(314t - 45°)$

$i_3 = i_1 + i_2$

$i_3 = 10\sin 314t + 15\sin(314t + 45°)$

$i_3 = 10\sin 314t + 15(\sin 314t \cos 45° + \cos 314t \sin 45°)$

$i_3 = 10\sin 314t + \frac{15}{\sqrt{2}}(\sin 314t + \cos 314t)$

$i_3 = 20.61\sin 314t + 10.61\cos 314t$

OR

$i_1 = 10/\sqrt{2}\angle 0°$, $i_2 = 15/\sqrt{2}\angle 45°$

$i_3 = i_1 + i_2$

$i_3 = 16.3879\angle -27.2357$

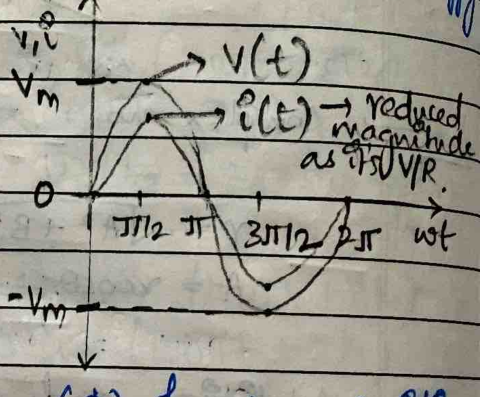
→ Response of resistive load to sinusoidal supply

$v(t) = V_m \sin\omega t$

$i(t) = \frac{v(t)}{R}$

$i(t) = \frac{V_m \sin\omega t}{R}$

$i(t) = I_m \sin\omega t$

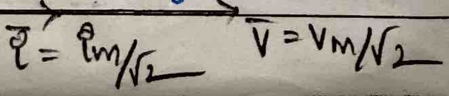


Phase angle difference (ϕ) for pure resistive circuit: $\phi = 0°$

Phasor: $\bar{V} = \frac{V_m}{\sqrt{2}}\angle 0$; $\bar{I} = \frac{I_m}{\sqrt{2}}\angle 0$

Impedance $Z = \frac{\bar{V}}{\bar{I}} = \frac{V_m/\cancel{\sqrt{2}}\angle 0}{I_m/\cancel{\sqrt{2}}\angle 0} = R\,\Omega$

Phasor diagram: Always in phase in pure R circuit.



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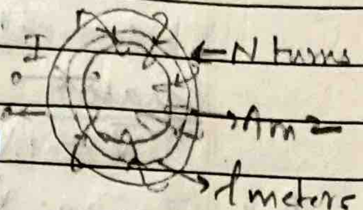
→ Inductor & concept of inductance:

Inductor → obtained by winding the conductor into a coil.

A current carrying coil with magnetic field around it. Expressed in flux (Φ).



$$\Phi = \frac{\text{Magnetomotive Force}}{\text{Reluctance}} = \frac{N * I}{S} \text{ wb}$$

$$S = \frac{l}{\mu A} \qquad \mu = \text{permeability}$$

$\Phi_{mag.} \propto i$ in inductor coil

$N\Phi \propto i$, where $N\Phi \rightarrow$ flux linkages (ψ)

$\psi \propto i$.

$\psi = L i$

$L \rightarrow$ Inductance of the coil

$\rightarrow L = \frac{\psi}{i}$ or $L = \frac{N\Phi}{i}$

L is measured in Henry's (H).

Voltage across terminals of inductor $\propto$ to rate of change of flux linkages.

$$v(t) \propto \frac{d(\psi)}{dt}$$

$$v(t) = \frac{L di}{dt}$$

$$i(t) = \frac{1}{L}\int v(t)\, dt$$

$$i(t) = \frac{1}{L}\int V_m \sin\omega t\, dt$$

$$i(t) = \frac{V_m}{L}\left[\frac{-\cos\omega t}{\omega}\right]$$

$$i(t) = \frac{V_m}{\omega L}\left[-\cos\omega t\right] = \frac{V_m}{\omega L}\left[\sin(\omega t - \pi/2)\right]$$

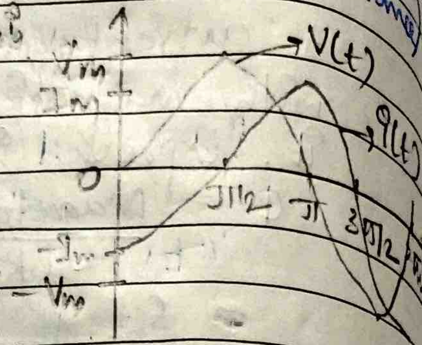
$$i(t) = \frac{V_m}{X_L}\sin(\omega t - \pi/2)$$

$$i(t) = I_m \sin(\omega t - \pi/2) \quad (\because X_L = \omega L \text{ inductive reactance})$$

<u>Phasor</u>

$$\bar{V} = \frac{V_m \angle 0^\circ}{\sqrt{2}}$$

$$\bar{I} = \frac{I_m \angle -90^\circ}{\sqrt{2}}$$

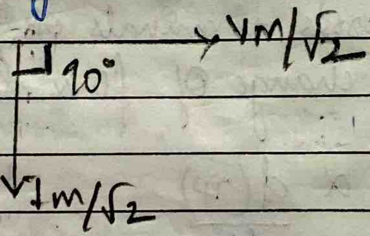


In pure inductive circuit, current lags behind voltage by angle $\pi/2 (90^\circ)$.

Impedance → $Z = \frac{\bar{V}}{\bar{I}} = \frac{V_m \angle 0^\circ / \cancel{\sqrt{2}}}{I_m \angle -90^\circ / \cancel{\sqrt{2}}} = X_L \angle (0-(-90^\circ)) = X_L \angle 90^\circ$

$Z = X_L \angle 90^\circ$ or $Z = jX_L$

Phasor diagram:



→ Capacitor & concept of Capacitance:

Capacitor → obtained by placing dielectric medium b/w conducting plates.

$$C = \frac{A\varepsilon}{d} \text{ Farad}$$

A → Area of plates (m^2)

d → distance b/w plates (m)

ε → permittivity of dielectric medium (F/m)

The charge on the plates of capacitor $\propto$ to voltage across its terminals.

$$q(t) \propto V(t)$$

$$q(t) = C.V(t) \quad \text{— ①}$$

$$i(t) = \frac{d(q(t))}{dt}$$

$$i(t) = \frac{d(CV(t))}{dt}$$

$$i(t) = C\frac{d(V(t))}{dt}$$

$V(t) = V_m \sin\omega t$ — ②

$$i(t) = \frac{C\,d(V_m \sin\omega t)}{dt}$$

$$i(t) = CV_m\omega\cos\omega t$$

$$i(t) = \frac{V_m}{(1/\omega C)}\sin(\pi/2 + \omega t)$$

$$i(t) = \frac{V_m}{X_C}\sin(\omega t + \pi/2)$$

$\left[\because X_C = \frac{1}{\omega C}\right.$ capacitive reactance $\left.\right]$

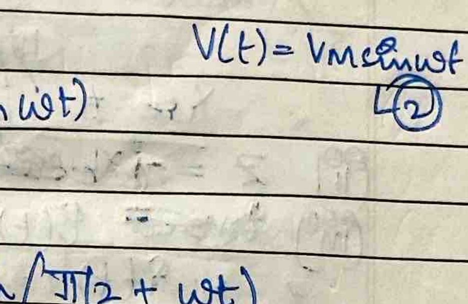
$$i(t) = I_m \sin(\omega t + \pi/2)$$

$[\because I_m = V_m/X_C]$

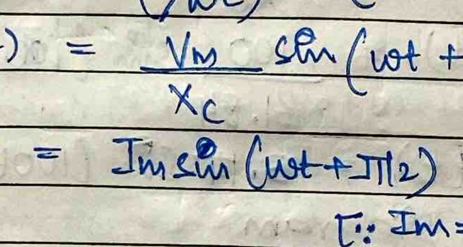
Phasor

$$\bar{V} = \frac{V_m}{\sqrt{2}}\angle 0^\circ$$

$$\bar{I} = \frac{I_m}{\sqrt{2}}\angle 90^\circ$$



Phasor diagram.



$$Z = \frac{\bar{V}}{\bar{I}} = \frac{V_m/\sqrt{2}\angle 0^\circ}{I_m/\sqrt{2}\angle 90^\circ} = X_C(0^\circ - 90^\circ)$$

$$Z = X_C\angle -90^\circ \quad \text{or} \quad Z = -jX_C$$

In pure capacitive circuit, current leads voltage by $\pi/2$

Q- A capacitor of capacitance 100 μF is connected across an AC voltage source $100 \sin(100\pi t)$ V. Determine:

(i) Capacitive reactance

(ii) Impedance

(iii) Instantaneous expression of current.

Ans: (i) $X_c = \frac{1}{\omega C}$

$$X_C = \frac{10^4/2}{100\pi} = 100/\pi$$

$$X_c = 31.84\ \Omega$$

(ii) $Z = -jX_c = -j.31.84\ \Omega$

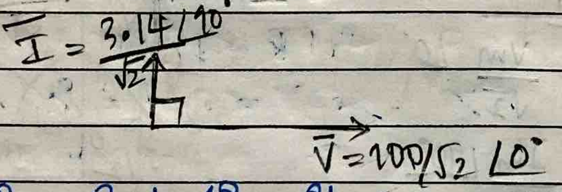
(iii) $i(t) = I_m \sin(\omega t + \pi/2)$

$$i(t) = \frac{V_m}{X_C} \sin(\omega t + \pi/2)$$

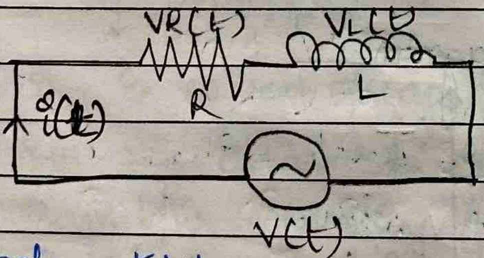
$$i(t) = \frac{100}{31.84} \sin(100t + \pi/2)$$

$$i(t) = 3.14 \sin(100t + \pi/2)$$

Phasor Diagram -



→ Series R-L Circuit,



Apply KVL,

$$v(t) = V_R(t) + V_L(t)$$

$$\bar{V} = \bar{V}_R + \bar{V}_L$$

$$\bar{V}_R = \bar{I} R$$

$$\bar{V}_L = \bar{I}(jX_L)$$

$$\bar{V} = \bar{I}(R + jX_L)$$

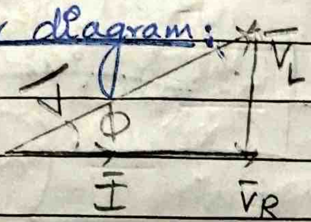
$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = (R + jX_L)$$

$$|\bar{Z}| = \sqrt{R^2 + X_L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

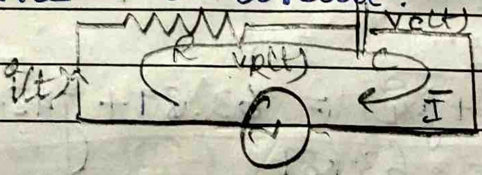
In series circuit, Impedance angle = Phase angle.

Phasor diagram:



$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

→ Series R-C circuit:



Apply KVL,

$$V(t) = V_R(t) + V_C(t)$$

$$\bar{V} = \bar{V}_R + \bar{V}_C$$

$$\bar{V}_R = \bar{I}R$$

$$\bar{V}_C = \bar{I}(-jX_C)$$

$$\bar{V} = \bar{I}(R - jX_C)$$

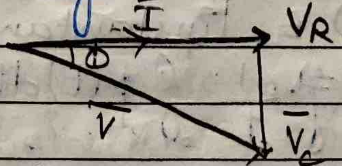
$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = (R - jX_C)$$

$$|\bar{Z}| = \sqrt{R^2 + X_C^2}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

Impedance angle = Phase angle.

Phasor diagram:



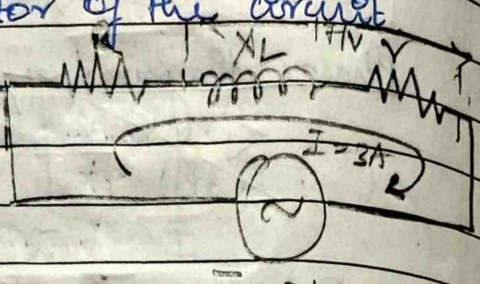
$$\phi = -\tan\left(\frac{V_C}{V_R}\right)$$

↓

-ve because voltage lagging behind current

Q- When a resistor & an inductor in series are connected to 240V, a current of 3A flows lagging 37° behind the supply voltage, while voltage across inductor is 171V. find resistance of resistor, the resistance & reactance of inductor. Find the power factor of the circuit

Ans: Given:



$Z = \frac{\bar{V}}{\bar{I}}$

$Z = \frac{240\angle 0^\circ}{3\angle -37^\circ}$

$Z = 80\angle 37^\circ$

$Z = (R + r + jX_L) = 63.89 + j48.14$

$R + r = 63.89$

$Z_L = jX_L = \frac{V_{inductor}}{I_{current}} = \frac{171}{3} = 57\Omega$

$\sqrt{r^2 + X_L^2} = 57$

$r = \sqrt{57^2 + (48.14)^2}$

$r = 30.52\Omega$

$R + r = 63.89\Omega$

$R = 33.37\Omega$

Power factor (series circuits) : $\cos\phi = R/Z$

$\cos\phi = \frac{63.89}{80}$

$\cos\phi = 0.798$ (lagging)

→ Power in AC Circuits:

Case 1: $v(t).i(t)$ = instantaneous

Resistive load

$P_{avg} = \frac{1}{T}\int p(t)dt$

$v(t) = V_m \sin\omega t$

$i(t) = I_m \sin\omega t$

$P_{avg} = \frac{1}{T}\int_0^T v(t).i(t)dt$

$P_{avg} = \frac{V_m I_m}{T}\int_0^T \sin^2\omega t\, dt$

$P_{avg} = \frac{V_m I_m}{2} = V_{rms} I_{rms}$

In resistive load, instantaneous power p(t) is always +ve because, resistor consumes the power given to it by source. Resistor dissipates power but doesn't generate.

<u>Case 2</u>: Pure Inductive load

$v(t) = V_m \sin \omega t$, $i(t) = i_m \sin(\omega t - 90°)$.

$p(t) = v(t) \cdot i(t)$.

$P_{avg} = 0$ here, as inductor can store energy

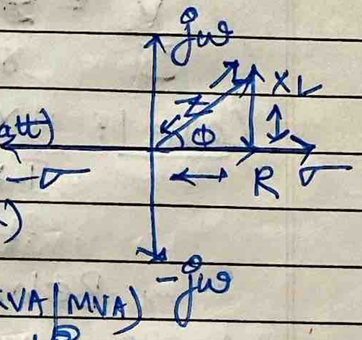
<u>Case 3</u>: Pure capacitive load

$v(t) = V_m \sin \omega t$, $i(t) = I_m \sin(\omega t + 90°)$.

$p(t) = v(t) \cdot i(t)$

$P_{avg} = 0$.

$Z = R + jX_L$



Active/Real power (P) = $VI\cos\phi$ (watt)

Reactive Power (Q) = $VI\sin\phi$ (VAR)

Apparent Power (S) = $\sqrt{P^2+Q^2}$

= VI (VA/KVA/MVA)

<u>NOTE</u>: Conventionally, Inductive reactive power → +ve
Capacitive reactive power → −ve.

Power Factor = P/S = $\cos\phi$

For series AC circuits, power factor = $\frac{R_T}{|Z_T|}$

Q- Series RL circuit is connected to sinusoidal voltage source $v(t) = 100\sin(\omega t)$ V. It draws current of $10\sin(\omega t - 60°)$ A.
Determine: (i) Active, Reactive & Apparent power
(ii) Power factor of circuit

Ans: $v(t) = 100 \sin \omega t$ V

$i(t) = 10 \sin(\omega t - 60^\circ)$ A

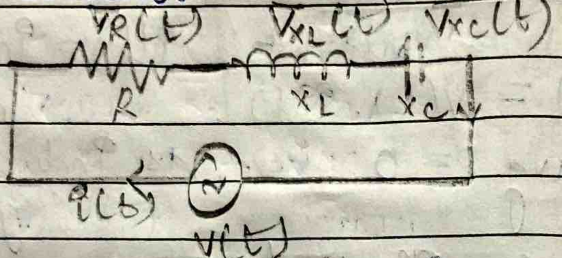
$P = VI\cos\phi = 250$ W

$Q = VI\sin\phi = 433.01$ VAR

$S = VI = 500$ VA

$\cos\phi = 1/2 = 0.5$ (lag)

→ RLC circuit:



$$\bar{V} = \bar{v}(t) = V_R(t) + \bar{V}_L(t) + \bar{V}_C(t)$$

$$\cancel{i(t) = i(t)R + i(t)_{X_L} + i(t)_{X_C}}$$

$$\bar{I} = \bar{i}(t) = \bar{i}(t)R + \hat{j}X_L\bar{i}(t) + -\hat{j}X_C\bar{i}(t)$$

$$Z = R + \hat{j}X_L - \hat{j}X_C$$

$$\frac{\bar{V}}{\bar{I}} = Z = R + \hat{j}(X_L - X_C)$$

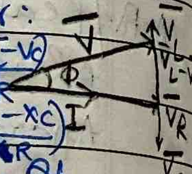
or $Z = \sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Case 1: $X_L > X_C$ → Phasor:

$\bar{I}X_L > \bar{I}X_C$ $\quad \phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$

$V_L > V_C$ $\quad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Behaves like series RL - Circuit

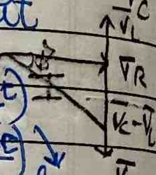


Case 2: $X_L < X_C$ $\quad$ Phasor:

$\bar{I}X_L < \bar{I}X_C$ $\quad \phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$

$V_L < V_C$ $\quad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$ → for negative phase difference

Behaves like series RC - Circuit



Case 3: $X_L = X_C$ → Series Resonance condition

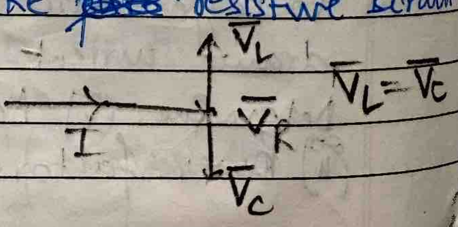
$\bar{I}X_L = \bar{I}X_C$

$V_L = V_C$

Behaves like ~~pure~~ series/pure resistive circuit

Phasor:

$\phi = 0$



classmate

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Q- Series RLC circuit draws current of 20A when connected to 200V, 50Hz supply. If total active power drawn from source is 500W & circuit behaves effectively like inductive circuit (series RL) determine: (i) Power factor, (ii) L if $C = 100 \times 10^{-6}$ F

Ans: $I = 20A$

$V = 200V$

$f = 50Hz$

$S = VI = 4000VA$

$P = 500W$ (given)

(i) Power factor $= P/S = \frac{\cancel{500}^{1}}{\cancel{4000}_{8}} = 0.125$ (lag)

(ii) $|Z| = \frac{V}{I} = \frac{\cancel{200}^{10\Omega}}{\cancel{20}}$

$P = I^2R$

$\cancel{500} = \cancel{20} \times 20 \times R$

$R = 1.25\Omega$

$\omega = 2\pi f$

$\omega = 100\pi$

$X_C = \frac{1}{\omega C}$

$X_C = \frac{10^6}{100\pi \times 100} = 31.8\Omega$

$Z^2 = R^2 + (X_L - X_C)^2$

$\sqrt{Z^2 - R^2} = X_L - X_C$

$\sqrt{100 - (1.25)^2} + X_C = X_L$

$9.92 + 31.8 = X_L$

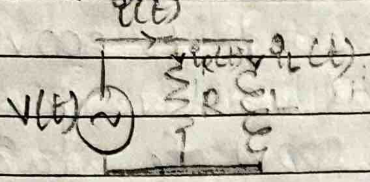
$X_L = 41.72\Omega$

$X_L = \omega L$

$L = \frac{41.72}{100\pi}$

$L = 132.86\ mH$

→ RL & RC ||el circuit :



Parallel R-L circuit

Apply KCL,

$$i(t) = i_R(t) + i_L(t)$$

Phasor: $\bar{I} = \bar{I}_R + \bar{I}_L$

$$\bar{I} = \frac{\bar{V}}{\bar{R}} + \frac{\bar{V}}{jX_L}$$

$$\frac{\bar{I}}{\bar{V}} = \frac{1}{\bar{R}} + \frac{1}{jX_L}$$

$$Y = 1/Z = \bar{I}/\bar{V}$$

↓

Admittance (inverse of impedance)

$G = 1/R$

↓

conductance (inverse of resistance)

$$B_L = \frac{1}{X_L}$$

↓

Inductive susceptance (inverse of inductive reactance)

$Y = G - jB_L$

↳ unit : ohm^{-1} or mho ($\mho$), siemens (S)

$B_C = 1/X_C = \omega C$, $jB_C = j/X_C = j\omega C$

↓

Capacitive susceptance

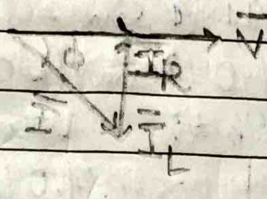
$Y = G + jB_C$

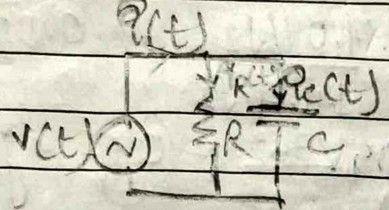
$Y_{magnitude} = \sqrt{G^2 + B_L^2} \angle \tan^{-1}(-B_L/G)$

In ||el circuit, total admittance is equal to the sum of individual branch admittances

Phasor diagram:

$$\phi = -\tan^{-1}\left(\frac{\bar{I}_L}{\bar{I}_R}\right)$$





Parallel R-C circuit

Apply KCL,

$$i(t) = i_R(t) + i_C(t)$$

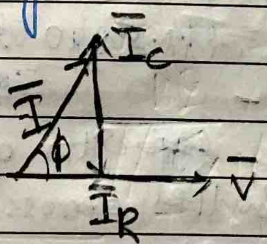
Phasor: $\bar{I} = \bar{I}_R + \bar{I}_C$

$$\bar{I} = \frac{\bar{V}}{R} + \frac{\bar{V}}{-jX_C}$$

$$Y = G + jB_C$$

$$Y_{magnitude} = \sqrt{G^2 + B_C^2} / \tan^{-1}(B_C/G)$$

Phasor diagram:



$$\phi = \tan^{-1}(\bar{I}_C/\bar{I}_R)$$

NOTE: Phase angle of network is equal to impedance angle or negative of admittance angle.

Q- Terminal voltage & current for ||el circuit are $141.4 \sin 2000t$ V & $7.07 \sin(2000t + 36°)$. Obtain simplest two element ||el circuit which would have the above relationship.

Ans:
$$v(t) = 141.4 \sin 2000t \text{ V}$$
$$i(t) = 7.07 \sin(2000t + 36°) \text{ A}$$

$$|Y| = \frac{|I|}{|V|} = \frac{7.07/\sqrt{2}}{141.4/\sqrt{2}} = 0.05 \text{ S}$$

$$Y_{magnitude} = 0.05\angle 36° = (0.04 + j0.029) \text{ S}$$

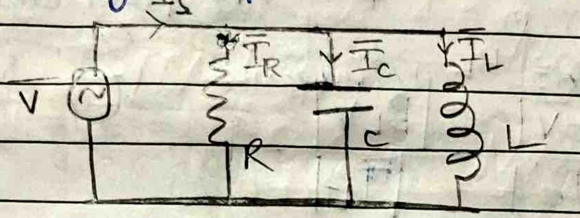
$$G = 0.04 \text{ S}$$
$$R = 25\ \Omega \quad (1/G)$$
$$B_C = 0.029 \text{ S}$$
$$X_C = \frac{1}{0.029} \quad (1/B_C)$$
$$\frac{1}{\omega C} = \frac{1}{0.029}$$
$$\cancel{2 \times 3.14 \times C = 0.029}$$
$$\cancel{C =}$$
$$2000 \times C = 0.029$$
$$C = 14.5 \times 10^{-6} \text{ F}$$

Q- Resistor of 30Ω, capacitor of unknown value are connected in ||el across 110V, 50Hz supply. Combination draws current of 5A from the supply. Find unknown capacitance.

Ans:
$$|Y| = \frac{|I|}{|V|}$$
$$|Y| = \frac{5}{110} = \frac{1}{22}$$
$$Y = 0.045 \text{ S}$$
$$\cancel{\frac{1}{22} = \frac{1}{30} + B_C}. \quad G = \frac{1}{30} = 0.033 \text{ S}$$

$$B_C = \sqrt{Y^2 - G^2}$$
$$B_C = 0.0306\, S$$
$$B_C = \omega C$$
$$\frac{0.0306}{100\pi} = C$$
$$C = 97.38 \times 10^{-6} F$$

→ **Analysis of $\parallel^{el}$ R-L-C circuit:**

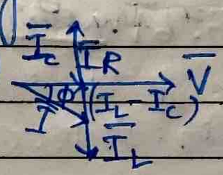


Apply KCL,
$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C \quad — ①$$
$$i(t) = i_R(t) + i_L(t) + i_C(t)$$
$$\bar{I}_R = \bar{V}/R = \bar{V}G$$
$$\bar{I}_L = \bar{V}/jX_L = \bar{V}(-jB_L) \quad ②$$
$$\bar{I}_C = \bar{V}/-jX_C = \bar{V}(jB_C)$$
② in ① $\Rightarrow$ $\bar{I} = \bar{V}G + \bar{V}(-jB_L) + \bar{V}(jB_C)$
$$\bar{I} = \bar{V}(G - jB_L + jB_C)$$
$$Y = \frac{\bar{I}}{\bar{V}} = G + j(B_C - B_L)$$

Admittance of parallel RLC circuit is:
$$|Y| = \sqrt{G^2 + (B_C - B_L)^2}$$
$$\phi = \tan^{-1}\left(\frac{B_C - B_L}{G}\right)$$

Case 1: $B_C < B_L$
$$|B_C \bar{V}| < |B_L \bar{V}|$$
$$|\bar{I}_C| < |\bar{I}_L|$$
circuit effectively behaves as $\parallel^{el}$ RL circuit
Phasor diagram:

$|I_C| < |I_L|$



$$\phi = \tan^{-1}\left(\frac{\bar{I}_C - \bar{I}_L}{\bar{I}_R}\right)$$
as ϕ should be negative from diagram

Case 2: $B_C > B_L$

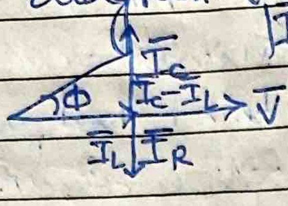
$$|B_C\bar{V}| > |B_L\bar{V}|$$

$$|\bar{I}_C| > |\bar{I}_L|$$

Circuit effectively behaves as ||el RC circuit

Phasor diagram:

$|I_C| > |I_L|$



$$\phi = \tan^{-1}\left(\frac{\bar{I}_C - \bar{I}_L}{\bar{I}_R}\right)$$

$\phi \rightarrow$ +ve from diagram

Case 3: $B_C = B_L$

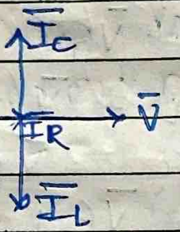
$$|B_C\bar{V}| = |B_L V|$$

$$|\bar{I}_C| = |\bar{I}_L|$$

Circuit effectively behaves as pure resistive circuit. (Parallel resonance condition)

Phasor diagram:

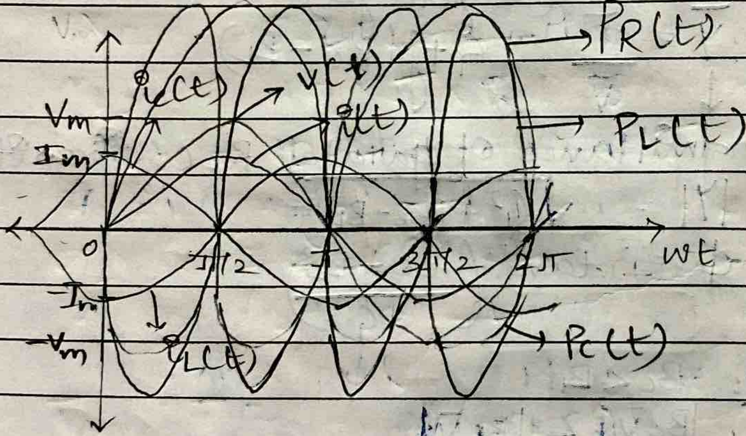
$|I_C| = |I_L|$



$$\bar{I} = \bar{I}_R$$

$$\phi = 0$$

→ Power:



$$v(t) = V_m \sin\omega t$$

$$i_R(t) = I_m \sin\omega t$$

$$i_L(t) = I_m \sin(\omega t - \pi/2)$$

$$i_C(t) = I_m \sin(\omega t + \pi/2)$$

Q- Three circuit elements $R = 2.5\,\Omega$, $X_L = 4\,\Omega$, $X_C = 10\,\Omega$ are connected ||el, reactances = 50Hz.
(a) Determine admittance of each element & obtain input admittance.
(b) If circuit is connected across 10V, 50Hz AC source, determine the current in each branch & total ~~output~~ input current.

Ans: Given: $R = 2.5\,\Omega$, $X_L = 4\,\Omega$, $X_C = 10\,\Omega$.

(a) $G = 1/R = 1/2.5 = 0.4\ S$

$$-jB_L = \frac{1}{jX_L} = -j\left(\frac{1}{4}\right) = -j(0.25)\,S$$

$$jB_C = \frac{1}{-jX_C} = j\left(\frac{1}{10}\right) = j(0.1)\,S$$

Total admittance $Y = G + j(B_C - B_L)$

$$Y = [0.4 + j(0.1 - 0.25)]\,S$$

$$Y = [0.4 - j(0.15)]\,S$$

(b) $\bar{I}_R = \bar{V}G$

$\bar{I}_L = \bar{V}(-jB_L)$

$\bar{I}_C = \bar{V}(jB_C)$

$$\bar{I}_R = \frac{\cancel{10} \times 4}{\cancel{10}} = 4\,A$$

$\bar{I}_L = 10 \times 0.25 \angle -90^\circ = 2.5 \angle -90^\circ\ A$

$\bar{I}_C = 10 \times 0.1 \angle 90^\circ = 1 \angle 90^\circ\ A$

$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$

$\bar{I} = \bar{V}(Y)$

$\bar{I} = 10\,[0.4 - j(0.15)]\ A$

$\bar{I} = 4 - 1.5j\ A$

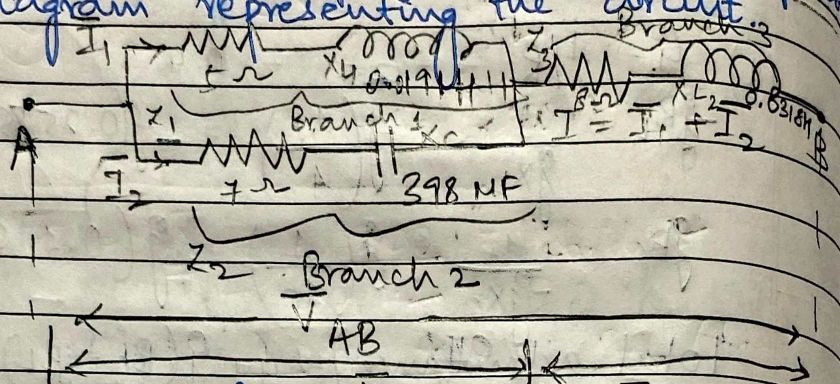
$\bar{I} = 4.27 \angle -20.55^\circ\ A$

→ Analysis of Series - Parallel circuit:

Series - Parallel AC circuit → few elements connected in series & few in parallel.

Q - In the circuit shown, what voltage of 50 Hz frequency is to be applied across A & B that will cause current 10A to flow in the capacitor. Also, draw phasor diagram representing the circuit.

Ans:



$\overline{V}_{AC}$ $\overline{V}_{CB}$

$X_{L1} = \omega L_1$

$X_{L1} = 100 \times \pi \times 0.0191$

$X_{L1} = 6\,\Omega$

$X_C = \dfrac{1}{\omega C}$

$X_C = 8\,\Omega$

$X_{L2} = 100 \times \pi \times 0.0318$

$X_{L2} = 10\,\Omega$

$Z_1 = (5 + j6)\,\Omega$

$Z_2 = (7 - 8j)\,\Omega$

$Z_3 = (8 + 10j)\,\Omega$

$\overline{V}_{AC} = \overline{I}_2 Z_2 = 10(7 - 8j)$

$\overline{V}_{AC} = 106.301 \angle -48.81°\,V$

$\overline{I}_1 = \dfrac{\overline{V}_{AC}}{Z_1} = \dfrac{106.301 \angle -48.81}{(5+6j)}$

$\overline{I}_1 = 13.61 \angle -99°\,A$

Total current $\overline{I} = \overline{I}_1 + \overline{I}_2$

$\overline{I} = 15.574 \angle -59.64\,A$

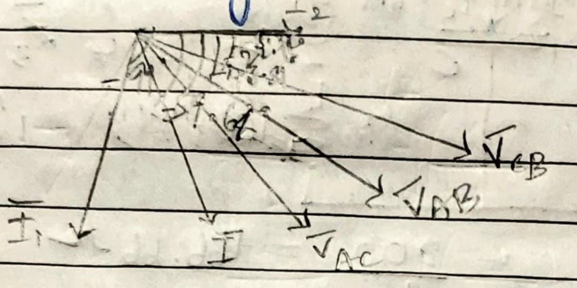
$\bar{V}_{CB} = \bar{I} Z_3 = 199.47 \angle -8.31^\circ V$

Total voltage $\bar{V}_{AB} = \bar{V}_{AC} + \bar{V}_{CB}$

$\bar{V}_{AB} = (106.3 \angle -48.81^\circ) V + (199.3 \angle -8.31^\circ) V$

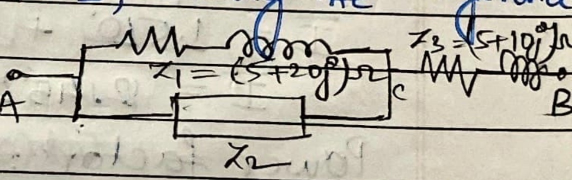
$\bar{V}_{AB} = 288.68 \angle -22.15^\circ V$

Phasor diagram:



Q- When 220V AC supply is applied across terminals A & B of circuit shown, total power input is 3.25 kW & total current 20A, lag. Find the complex expressions for current through Z_1 & Z_2, taking V_{AC} → reference

Ans: Given:



$\bar{V}_{AB} = 220 \angle 0^\circ V$

$P = 3.25 kW$

$\bar{I} = 20A$

$$\cos\phi = \frac{P}{VI} = \frac{3.25 \times 10^3}{220 \times 20} = 0.738 (lag)$$

$\phi = -42.38^\circ$

$\bar{V}_{AB} = \bar{V}_{AC} + \bar{V}_{CB}$

$\bar{V}_{CB} = \bar{I} \times Z_3$

$\bar{V}_{CB} = 223.60 \angle 21^\circ V$

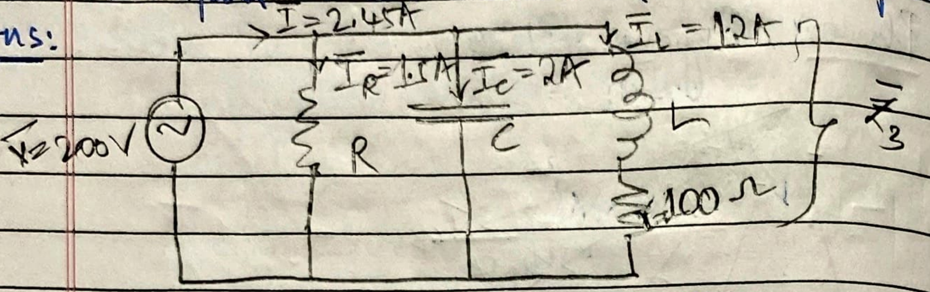
$\bar{V}_{AC} = \bar{V}_{AB} - \bar{V}_{CB} = 80.91 \angle -82^\circ V$

$$\bar{I}_1 = \frac{\bar{V}_{AC}}{Z_1} = 3.92 \angle -157^\circ A$$

$\bar{I}_2 = \bar{I} - \bar{I}_1 = 21.78 \angle -33.1^\circ A$

Q- A voltage of 200 V is applied to pure resistor, pure capacitor (C) & lossy inductor coil with resistance of 100 Ω, all of them connected in ||el. Total current 2.45A, while component currents are 1.5A, 2A, ~~2.5A~~ 1.2 A respectively. Find total power factor & power factor of coil. Also find total active & reactive power.

Ans:



$$\bar{Z}_3 = \frac{\bar{V}}{\bar{I}_L} = \frac{200}{1.2} = 166.66\,\Omega$$

$$\cos\phi_3 = \frac{r}{Z_3} = \frac{100}{166.6} = 0.6 \text{ (lag)}$$

$$\phi_3 = \cos^{-1}(0.6) \text{ (lag)}$$

$$\phi_3 = -53.128^\circ$$

$$\bar{I} = \bar{I}_R + \bar{I}_C + \bar{I}_L$$

$$\bar{I} = 1.5\angle 0^\circ + 2\angle 90^\circ + 1.2\angle -53.128^\circ$$

$$\bar{I} = 2.45\angle 25.10^\circ \text{ A}$$

Power factor : $\cos\phi = \cos(25.10^\circ) = 0.905$ (lead)

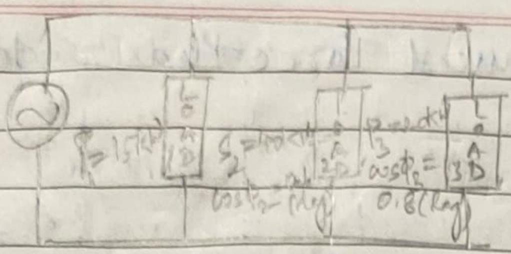
~~$\cos\phi = 0.905$~~

Active power : $\bar{V}\bar{I}\cos\phi = 443.45\,W$

Reactive power : $\bar{V}\bar{I}\sin\phi = 207.85$ VAR

Q- Load connected across an AC supply consists of a heating load of 15 kW, motor load of 40 kVA at 0.6 lag & load of 20 kW at 0.8 lag. Calculate the total power drawn from the supply and its power factor. What would be the KVAR rating of a capacitor to bring power factor to unity & how must the capacitor be connected?

Ans:



Load 1: $P_1 = 15kW$

$\phi_1 = 0$

$S_1 = 15kVA$

Load 2: $S_2 = 40KVA$

$\cos\phi_2 = 0.6$

$P_2/S_2 = \cos\phi_2$

$P_2 = 24KW$

$S_2 = \sqrt{P_2^2 + Q_2^2}$

$Q_2 = \sqrt{S_2^2 - P_2^2}$

$Q_2 = 32 KVAR$

Load 3: $P_3 = 20kW$

$\cos\phi_3 = 0.8$

$S_3 = P_3/\cos\phi_3$

$S_3 = 25kVA$

$Q_3 = \sqrt{S_3^2 - P_3^2}$

$Q_3 = 15 KVAR$

$P_T = P_1 + P_2 + P_3$

$P_T = (15 + 24 + 20)\ kW$

$P_T = 59 kW$

$Q_T = (0 + 32 + 15) KVAR$

$Q_T = 47 KVAR$

~~$S_T = (15 + 40 + 25) kVA$~~

~~$S_T = 80 kVA$~~

$S_T = \sqrt{P_T^2 + Q_T^2}$

$S_T = 75.432 kVA$

At $\cos\phi = 1$, $\phi = 0$, Power factor $= 1$.

Add capacitor in ||el to circuit