18th April, 2023

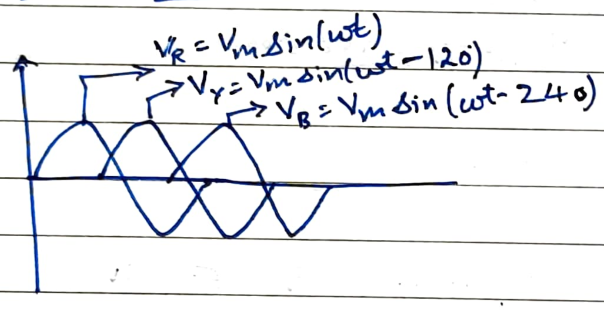
Unit - 3

Three Phase AC

Definition: 3 phases
1 neutral (sharred)
R-Y-B colour coding
Voltage lags w.r.t each other by 120°

20th April, 2023

$$V_R = V\angle 0^\circ$$
$$V_Y = V\angle -120^\circ$$
$$V_B = V\angle -240^\circ$$



Balanced System

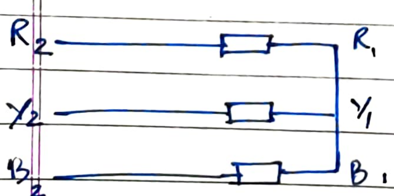
Balanced Source

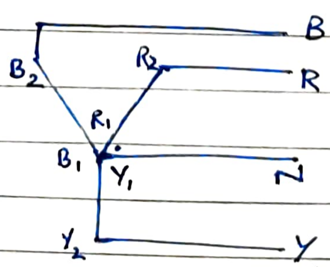
$|V_R| = |V_Y| = |V_B|$

Balanced Load

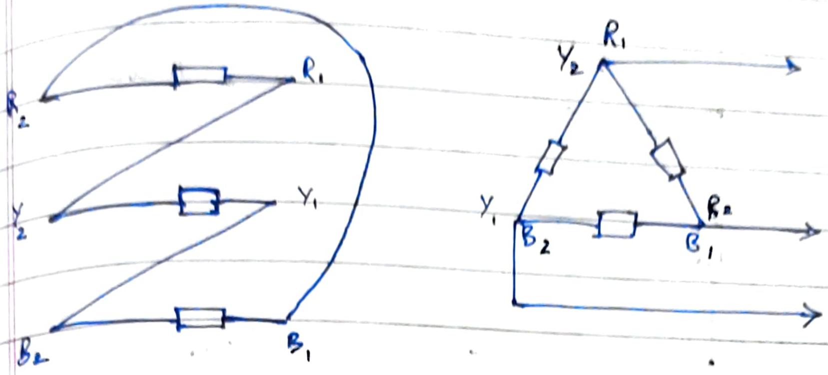
$|Z_R| = |Z_Y| = |Z_B|$

1. Star Connection (Y / Wye) - similar ends are tied together





2. Delta Connection -



24th April, 2024

→ 3-φ AC
- Advantages of 3-φ over 1-φ (4-6M).
- Generation of 3-φ
- Phase sequence < RYB, RBY

eg: In a 3-φ balanced AC system 415V, 50Hz is loaded with $5+j10\ \Omega$

$Z_R = 5+j10 = Z_Y = Z_B$

$V_R = 415\angle 0^\circ$

$V_Y = 415\angle -120^\circ$

$V_B = 415\angle -240^\circ$

$Z_R = 5+j10 = 11.78\angle 63.4$

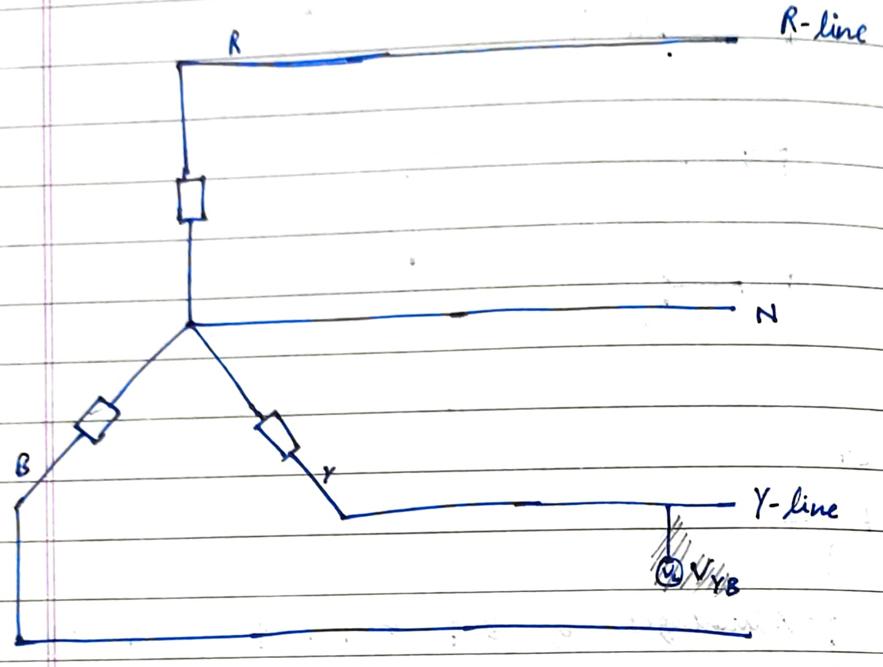
$I_R = 2\angle -63.4^\circ$

$I_Y = 2\angle -63.4-120^\circ$

$I_B = 2\angle -63.4-240^\circ$

- → Derivation $V_L = \sqrt{3}\, V_{ph}$ star (4M)
- → Derivation $I_L = \sqrt{3}\, I_{ph}$ Delta
- → Derivation $P = \sqrt{3}\, V_L I_L \cos\phi$ (8M)
- → Derivation $\phi = \tan^{-1}\left[\frac{\sqrt{3}(\omega_2-\omega_1)}{\omega_2+\omega_1}\right]$ (2M)

Star/Y/WYE connection



Line Voltage & Phase Voltage

V_L I_L V_{ph} I_{ph}

V_L → Line voltage : voltage b/w two lines.
I_L → Line current : current flowing through a line

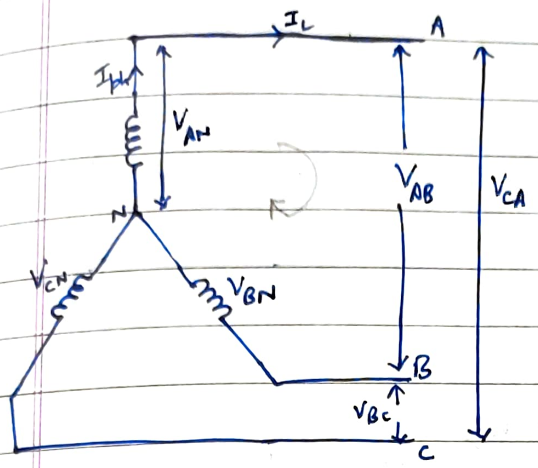
V_{ph} → Phase voltage : Voltage across a phase
(or) Voltage b/w a line & neutral
I_{ph} → Phase current : current flowing through a phase

$$Y \rightarrow \begin{cases} I_L = I_{ph} \\ V_L \neq V_{ph} \; ; \; V_L = \sqrt{3}\, V_{ph} \end{cases}$$

$$\Delta \rightarrow \begin{cases} V_L = V_{ph} \\ I_L \neq I_{ph} \; ; \; I_L = \sqrt{3}\, I_{ph} \end{cases}$$

25th April, 2023

→ To prove that $V_L = \sqrt{3}\, V_{ph}$ in star connection



From circuit diagram $I_L = I_{ph}$

Let $\overrightarrow{V_{AN}} = V_{ph} \angle 90^\circ$ (assumption)

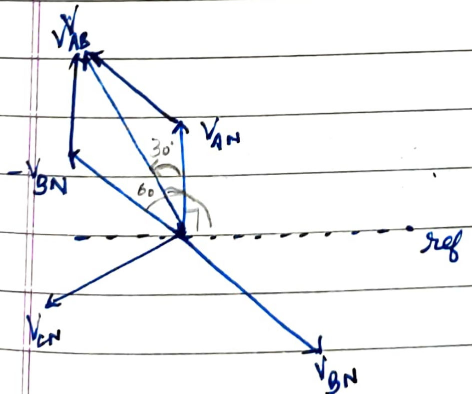
$\overrightarrow{V_{BN}} = V_{ph} \angle 90 - 120^\circ = V_{ph} \angle -30^\circ$

$\overrightarrow{V_{CN}} = V_{ph} \angle 90 - 240 = V_{ph} \angle -150^\circ$

KVL : $-\overrightarrow{V_{AB}} - \overrightarrow{V_{BN}} + \overrightarrow{V_{AN}} = 0$

$$\overrightarrow{V_{AB}} = \overrightarrow{V_{AN}} - \overrightarrow{V_{BN}}$$

let $|V_{AB}| = |V_{BC}| = |V_{CA}| = V_L$



$$|V_{AB}|^2 = |V_{AN}|^2 + |V_{BN}|^2 + 2|V_{AN}|\,|V_{BN}| \cos(\text{angle b/w } V_{AN} \text{ \& } -V_{BN})$$

$$V_L^2 = V_{ph}^2 + V_{ph}^2 + 2\, V_{ph} \cdot V_{ph} \cos(60)$$

$$V_L^2 = 2V_{ph}^2 + 2V_{ph}^2 \cdot \left(\frac{1}{2}\right)$$

$$V_L^2 = 3\, V_{ph}^2$$

$V_{AB} = V_L \angle 120^\circ$

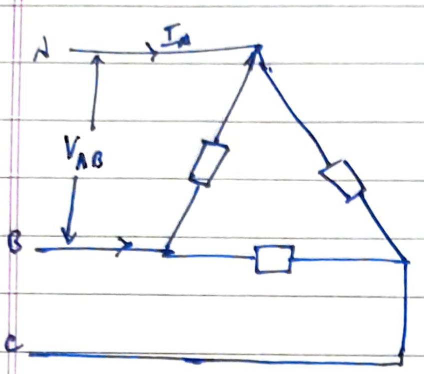
$V_{BC} = V_L \angle 0^\circ$

$V_{CA} = V_L \angle -120^\circ$

$$V_L = \sqrt{3}\, V_{ph}$$

* Line voltage leads corresponding phase ~~voltage~~ voltage by 30°

→ To prove that $I_L = \sqrt{3}\, I_{ph}$ in Δ connection



From circuit diagram,

$$|V_L| = |V_{ph}|$$

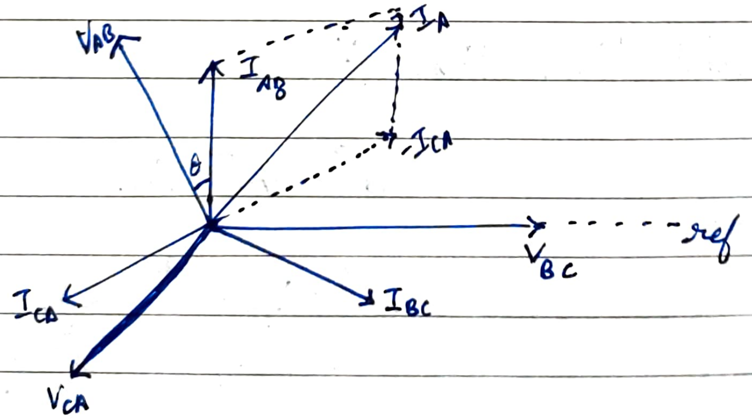
Let $\vec{V_{AB}} = V_L \angle 120^\circ$ $\therefore$ $\vec{V_{BC}} = V_L \angle 0^\circ$

$\vec{V_{CA}} = V_L \angle -120^\circ$

$$\vec{I_{AB}} = \frac{\vec{V_{AB}}}{Z\angle\theta} = \frac{V_L \angle 120^\circ}{Z\angle\theta} = \frac{V_{ph}}{Z} \angle 120-\theta$$

$$\vec{I_{BC}} = I_{ph} \angle -\theta$$

$$I_{CA} = I_{ph} \angle \theta - 120^\circ$$



KCL at node m

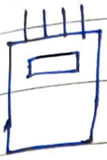
$$\vec{I_A} - \vec{I_{AB}} + \vec{I_{CA}} = 0$$

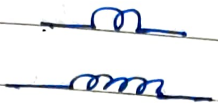
$$\vec{I_A} = \vec{I_{AB}} - \vec{I_{CA}}$$

26th April, 2023

Power Measurement

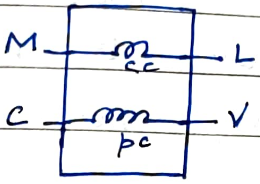
Wattmeter





current coil

pressure coil / potential coil



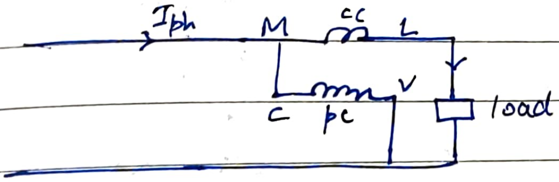
$M \rightarrow$ mains

$L \rightarrow$ Load

$C \rightarrow$ common

$V \rightarrow$ voltage

1 - ϕ Power



$$P = \quad \cos\phi = V_{ph} I_{ph} \cos\phi$$

3 - ϕ Power

$$P_{3\phi} = P_R + P_Y + P_B$$

$$= V_{ph} I_{ph} \cos\phi + V_{ph} I_{ph} \cos\phi + V_{ph} I_{ph} \cos\phi$$

$$\boxed{P_{3\phi} = 3 V_{ph} I_{ph} \cos\phi}$$

Y connected $V_{ph} = \frac{V_L}{\sqrt{3}}$, $I_{ph} = I_L$

$$P_{3-\phi} = \frac{3 V_L I_L \cos\phi}{\sqrt{3}}$$

$$P_{3-\phi} = \sqrt{3} V_L I_L \cos\phi$$

Δ connected $V_{ph} = V_L$

$I_{ph} = \frac{I_L}{\sqrt{3}}$

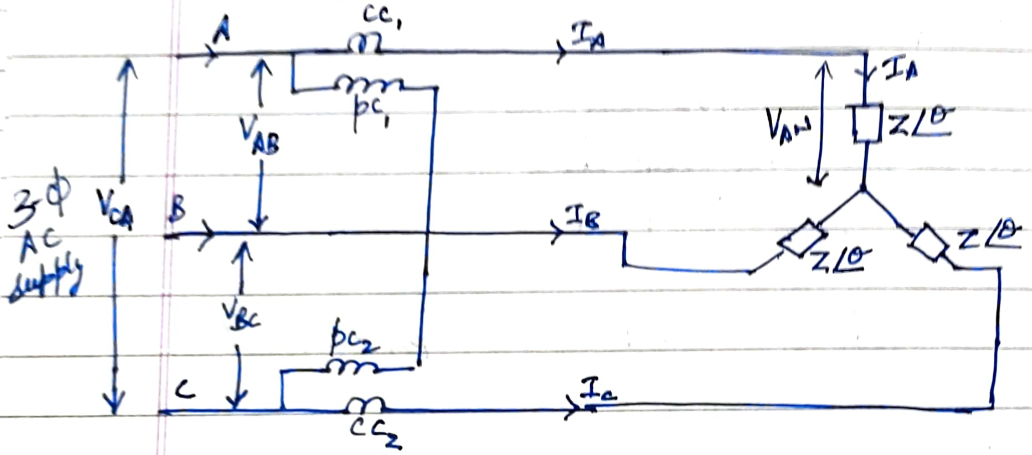
$$P_{3-\phi} = \frac{3 V_L I_L \cos\phi}{\sqrt{3}}$$

$$P_{3-\phi} = \sqrt{3} V_L I_L \cos\phi$$

Power measurement

- One Wattmeter method
- Two Wattmeter method ✓ (in syllabus)
- Three Wattmeter method

→ Prove that 2 wattmeters are enough to measure power in a 3-ϕ system, when a load is star connected. [8M]

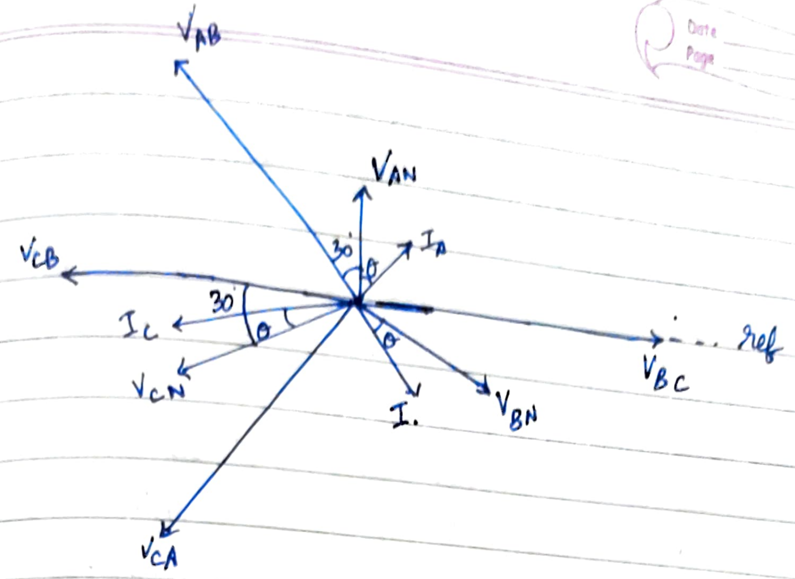


$-30°$

Let $\vec{V_{AB}} = V_L \angle 120°$ | $\vec{V_{AN}} = V_{ph} \angle 90$
$\vec{V_{BC}} = V_L \angle 0°$ | $\vec{V_{BN}} = V_{ph} \angle -30$ $-120°$
$\vec{V_{CA}} = V_L \angle -120°$ | $\vec{V_{CN}} = V_{ph} \angle -150°$

I_A lags V_{AN} by θ
I_B lags V_{BN} by θ
I_C lags V_{CN} by θ

P-3φ delta → b/w



Wattmeter reading W_1

$W_1 = \left| \text{current through } CC_1 \right| \times \left| \text{voltage across } PC_1 \right| * \cos\left(\text{angle b/w } I_{CC_1} \text{ \& } V_{PC_1}\right)$

$$W_1 = |I_A| |V_{AB}| \cos(30+\theta)$$
$$= I_L V_L \cos(30+\theta)$$
$$\therefore W_1 = I_L V_L \cos(30+\theta) \quad \text{--- (1)}$$

Wattmeter reading W_2

$$W_2 = |I_C| |V_{CB}| \cos(\text{angle b/w } I_C \text{ \& } V_{CB})$$
$$= V_L I_L \cos(30-\theta) \quad \text{--- (2)}$$

$$W_1 + W_2 = V_L I_L \cos(30+\theta) + V_L I_L \cos(30-\theta)$$
$$= \sqrt{3} V_L I_L \cos\theta = P_{3-\phi}$$

Hence Proved

$$W_2 - W_1 = V_L I_L \sin\theta$$

$$\therefore \frac{W_2 - W_1}{W_2 + W_1} = \frac{V_L I_L \sin\theta}{\sqrt{3} V_L I_L \cos\theta}$$

$$\therefore \tan\theta = \frac{\sqrt{3}(W_2 - W_1)}{W_2 + W_1}$$

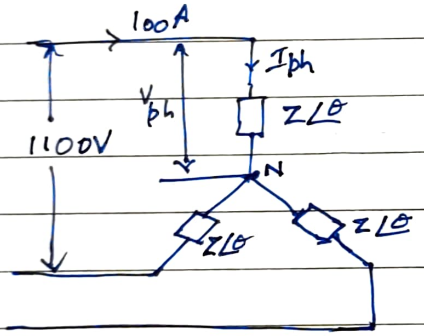
$$\theta = \tan^{-1}\left[\frac{\sqrt{3}(W_2 - W_1)}{W_2 + W_1}\right]$$

$$\therefore P.f = \cos\theta = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_2 - W_1)}{W_2 + W_1}\right]\right\}$$

28th April, 2023

q.1. Balanced RC / star connected load of 150 kW takes a leading current of 100 A with a line voltage of 1100V, 50Hz. Find circuit constants of the load per phase. 'R, L, C

Sol. * In 3-ϕ by default parameters mentioned is line (line current).



$$P_{3-\phi} = 150 \times 10^3 \text{ W}$$

$$150 \times 10^3 = \sqrt{3}(1100)(100)\cos\theta$$

$$\theta = 38^\circ$$

$$I_{ph} = 100 \angle 38^\circ$$

$$Z = \frac{V_{ph}}{I_{ph}} = \cancel{100}$$

$V_L = 1100 \text{ V} \angle 0$ (we can either assume V_L at $\angle 0^\circ$ / V_{ph} at $\angle 0$)

$$V_{ph} = \frac{1100}{\sqrt{3}} = 635 \cancel{\angle 30^\circ}$$

here we assumed $V_{ph} \angle 0^\circ$

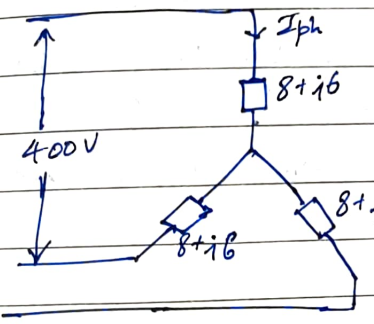
$$Z = \frac{V_{ph}}{I_{ph}} = \frac{635 \angle 0^\circ}{100 \angle 35} = 6.35 \angle -35^\circ \ \Omega$$

$$Z = \underset{R}{5} - j\,\underset{X_c}{3.9}$$

$$X_c = \frac{1}{2\pi f C} \Rightarrow C = 819\ \mu F$$

Q2. A balanced star connected load $(8 + j6)$ ohm/phase is connected to a balanced 3-phase 400V supply. Find line current, power factor, power and total volt-amperes.

Sol.



$$8 + j6 = 10 \angle 36.86^\circ$$

Let $V_{ph} = \frac{400}{\sqrt{3}} \angle 0^\circ = 230.9 \angle 0^\circ$

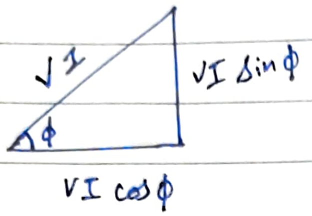
$$\therefore I_{ph} = \frac{V_{ph}}{Z}$$

$$I_{ph} = \frac{230.9 \angle 0^\circ}{10 \angle 36.86^\circ} = 23.09 \angle -36.86^\circ$$

Y connection $I_L = 23.09$ A

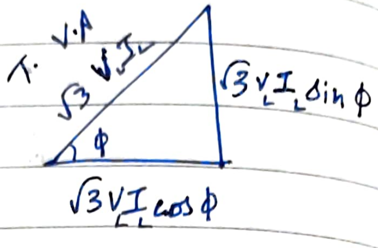
$p.f = \cos(36.86) = 0.8$ lag

$1-\phi$



$3-\phi$

Power Δ

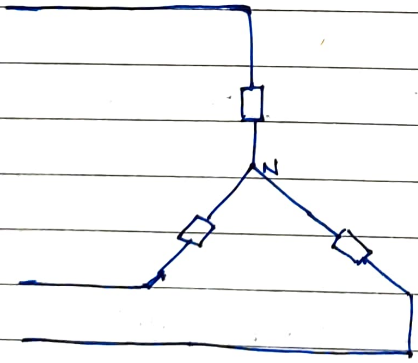


$$P_{3-\phi} = \sqrt{3}\, V_L I_L \cos\phi$$
$$= \sqrt{3}\,(400)(23.09)(0.8) = 12.79\,kW$$

$$T.V.A = \sqrt{3}\, V_L I_L = \sqrt{3}(400)(23.09) = 16.21\,kVA$$

Q.3. Three equal star connected inductors take 8 kW at a power factor 0.8 when connected across a 460 V, $3-\phi$, $3-\phi$, 3 wire supply. Find circuit constants of load per phase

Sol



Sol.
$$P = \sqrt{3}\, V_L I_L \cos\phi$$
$$8000 = \sqrt{3} \times 460 \times I_L \times 0.8$$

$$I_L = 12.55A$$
$$I_{ph} = 12.55A$$

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$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{460}{\sqrt{3}} = 265\,V$

$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{265}{12.55} = 21.1$ Ohm

$R_{ph} = Z_{ph} \cos\phi = 21.1 \times 0.8 = 16.9\,\Omega$

$X_{ph} = Z_{ph} \sin\phi = 21.1 \times 0.6 = 12.66\,\Omega$

$L = 40.39$ mH

Q4 A balanced delta connected load of $(8+j6)$ Ohm/phase is connected to a balanced 3-ϕ 230V supply.
Find line current, power factor, power and reactive volt-amperes, total volt-amperes.

Sol Δ connection $V_L = V_{ph}$
$I_L = \sqrt{3} I_{ph}$

ans: $I_L = 39.83A$ ✓
$p.f = 0.8$ lag ✓
$P = 12.69$ kW ✓
$R.V.A = 9.521$ kVAR ✓
$T.V.A = 15.86$ kV.A ✓

$Z = 8+6j = 10\angle 36.86^\circ$

$V_L = V_{ph} = 230\,V$

$I_{ph} = \frac{V_{ph}}{Z} = 23\angle -36.86$

$I_L = \sqrt{3} I_{ph} = \sqrt{3}(23) = 39.83A$

$P_f = \cos(36.86) = 0.80$ (lag)

$P = \sqrt{3} V_L I_L \cos\phi$
$= \sqrt{3}(230)(39.83)(0.80) = 12.69$ kW

$T.VA = \sqrt{3} V_L I_L = 15.86$ kV.A

$R.VA = \sqrt{3} V_L I_L \sin\phi = 9.518$ kVAR

2nd May, 2023

Q5. Two wattmeters are connected to measure Power in 3-ϕ balanced circuit. What would be its rating of each wattmeter, If

i) $\theta = 60°$ ii) $\theta = 30°$ iii) $\theta = 0°$ iv) $\theta = 90°$

Sol. i) $\theta = 60°$

$W_1 = V_L I_L \cos(30+\theta)$

$= V_L I \cos(90)$

$W_1 = 0$

$W_2 = V_L I_L \cos(30-\theta)$

$= V_L I_L \cos(-30)$

$= \frac{\sqrt{3}}{2} V_L I_L$

W_1 reads 0 & W_2 reads +ve Values

ii) $\theta = 30°$

$W_1 = V_L I_L \cos(30+\theta)$

$= \frac{V_L I_L}{2}$

$W_2 = V_L I_L \cos(30-\theta)$

$= V_L I_L$

. W_1 reads 1/2 W_2

iii) $\theta = 0°$

$W_1 = V_L I_L \cos(30+0)$

$= \frac{\sqrt{3}}{2} V_L I_L$

$W_2 = V_L I_L \cos(30-0)$

$= \frac{\sqrt{3}}{2} V_L I_L$

. Both read equal

iv) $\theta = 90°$

$W_1 = V_L I_L \cos(30+90)$

$= -\frac{1}{2} V_L I_L$

either current/pressure coil is connected in reverse. Pointer goes below 0, kickback action

$W_2 = V_L I_L \cos(30-90)$

$= \frac{1}{2} V_L I_L$

W_1 & W_2 read equal

W_1 is -ve

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96. Each of two wattmeters are connected to measure the input to a 3-ϕ circuit reads 10kW on a balanced load, when the p.f is unity what does each instrument read when the p.f falls to 0.866 (lag), the total 3-ϕ power remaining unchanged.

Sol. $W_1 = 10kW$ $\cos\phi = 1$
$W_2 = 10kW$
$W_1 + W_2 = 20$

$W_1 + W_2 = 20$ — $\cos\phi = 0.866$
$\theta = 30°$

$$\tan 30 = \frac{\sqrt{3}(W_2 - W_1)}{20 \times 10^3}$$

$W_2 - W_1 = 6666.67$

$2W_1 = 26666.67$
$W_1 = 13333.33$ $W_2 = 6666.67$

97. In a 3-ϕ delta connection wattmeter 100kW taking a lagging current of 200A at a line voltage of 400V, 50Hz. Find circuit parameters of each phase. What could be the power consumed if the load was star connected.

Sol. $I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{200}{\sqrt{3}} = 115.47$ $V_L = V_{ph} = 400V$

$100 \times 10^3 = \sqrt{3} V_L I_L \cos\phi$
$10^5 = \sqrt{3}(400)(200)\cos\phi$
$\phi = 43.80$

Let $V_{ph} = 400 \angle 0°$
$I_{ph} = 115.4 \angle -43.8°$

$$Z = \frac{V_{ph}}{I_{ph}} = \frac{400 \angle 0°}{115.4 \angle -43.8} = 3.46 \angle 43.8$$

$$Z = \underbrace{2.5}_{R} + j\underbrace{2.4}_{X_L}$$

$L = 7.63 mH$

ii) V_L & Z remain same
(source)

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94$$

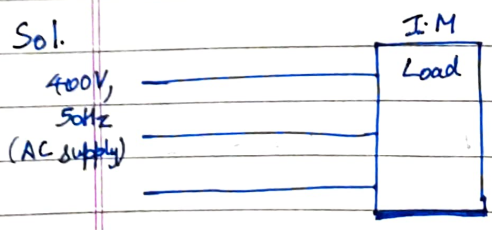
$$I_L = I_{ph} = \frac{V_{ph}}{Z} = \frac{230.94}{3.46 \angle 43.8} = 66.75 \ A$$

$$P = \sqrt{3} V_L I_L \cos\phi$$
$$= \sqrt{3}(400)(66.75) \cos(43.8)$$
$$= 33.37 \ kW$$

3rd May, 2023

q8. A 5HP, 400V, 3-ϕ, 50Hz induction motor (I.M) is working at full load with an efficiency of 90% at a power factor of 0.866. Determine the power input and line current.

Sol.



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o/p of I.M = 5 H.P

$$\eta = \frac{o/p \text{ Power}}{I/P \text{ Power}}$$

$$0.9 = \frac{5 \times 746}{I/P \text{ power}}$$

I/P Power = 4.14 kW

$$P_{3-\phi} = \sqrt{3} V_L I_L \cos\phi$$

$$4144 = \sqrt{3} (400) I_L (0.866)$$

$$I_L = 6.9 A$$

Q9. Two wattmeters are connected to measure input of a 15 HP, 50Hz, 3-ϕ I.M at full load. The Full load η and p.f are 0.9 and 0.8 (lag) respectively. Find the readings of two wattmeters

$$0.9 = \frac{15 \times 746}{I/P \text{ power}}$$

I/P power = 12.433 kW = $W_1 + W_2$

$\phi = 36.86°$

$$\tan 36.86 = \frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2}$$

$$W_2 - W_1 = 5.381$$

$$2W_2 = 17.814$$

$\boxed{W_2 = 8.9 \text{ kW}}$ $\boxed{W_1 = 3.526 \text{ kW}}$

q10. Two wattmeters readings were 3400W & -1200W calculate Total Power and Power factor of circuit

Sol Total Power $= W_1 + W_2$

$= 2200\ W$

$$pf = \cos\left(\tan^{-1}\left(\frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2}\right)\right)$$

$pf = 0.266$

q11. Two wattmeters are connected to measure power in a 3-ϕ circuit read 5kW, 1kW the second reading is obtained after reversing the current coil, calculate power factor, power consumed in circuit

Sol. $W_1 = 5kW$ $\qquad W_2 = -1kW$

Power consumed $= W_1 + W_2$

$= 4kW$

$p.f = 0.359$

q12. Three coils each of impendence $20\angle 60°$ are star connected to a 400V, 50Hz 3-ϕ supply. Find the reading of each of two wattmeters connected to measure the input.

Sol $Z = 20\angle 60°$ $\qquad V_L = 400V$ $\qquad V_{ph} = \frac{400}{\sqrt{3}} = 230.94V$

$$I_{ph} = \frac{V_{ph}}{Z} = \frac{230.94}{20} = 11.547\ A = I_L$$

$$W_1 = V_L I_L \cos(30 + \theta)$$
$$= 400(11.547)\cos(30 + 60)$$
$$= 0 \text{ W}$$

$$W_2 = V_L I_L \cos(30 - \theta)$$
$$= 400(11.547)\cos(30 - 60)$$
$$= 3999.99 \text{ W}$$

Advantages of AC