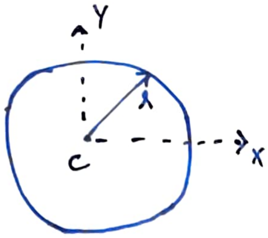
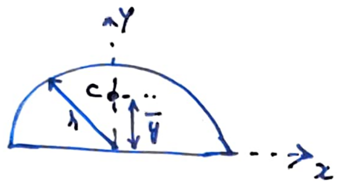
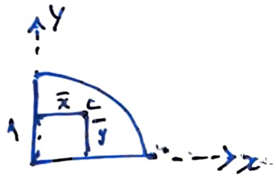
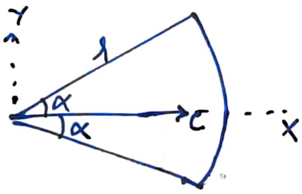
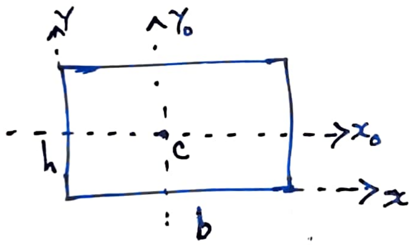
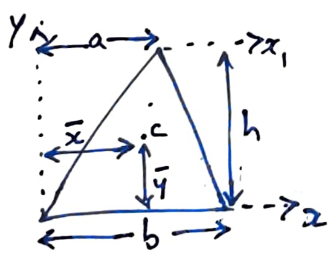
Figure	Centroid	Area MoI
	—	$I_x = I_y = \frac{\pi r^4}{4}$ $I_z = \frac{\pi r^4}{2}$
	$\bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{8}$ $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 = 0.11 r^4$ $I_z = \frac{\pi r^4}{4}$
	$\bar{x} = \bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{8}$
	$\bar{x} = \frac{2}{3} r \frac{\sin\alpha}{\alpha}$	
		$I_x = \frac{bh^3}{3}$ $I_y = \frac{hb^3}{3}$ $I_{x_0} = \bar{I}_x = \frac{bh^3}{12}$ $I_{y_0} = \bar{I}_y = \frac{hb^3}{12}$ $I_z = \frac{bh}{12}(b^2 + h^2)$
	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $I_y = \frac{hb^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $\bar{I}_y = \frac{hb^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$ $\bar{I}_{y_1} = \frac{hb^3}{4}$  $\bar{I}_y = \frac{hb^3}{48}$

Area MoI : The area MoI is the second moment of area around a given axis.

Area MoI around x-axis $I_x = \int y^2 \, dA$

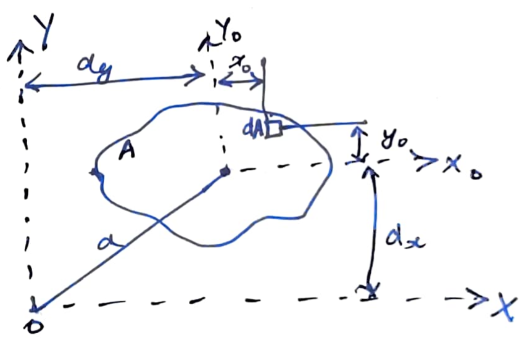
Radius of Gyration : The imaginary distance from centroid at which the area of cross section is imagined to be focused at a point in order to obtain the same MoI. It is denoted by k.
Measure of distribution of area from the ref. axis considered.

Transfer of axis : The MoI of an area about a non centroidal axis can be easily expressed in terms of MoI about a parallel centroidal axis.

$$I_x = \bar{I}_x + Ad_x^2$$

Conditions for // axis theorem :
1. Two axis should be there and must be // to each other
2. Between two axis, one axis has to pass through the centroidal axis.

Derivation of Parallel Axis Theorem



By definition, the moment of inertia of element dA about x-axis is

$$dI_x = (y_0 + d_x)^2 dA$$

$$I_x = \int y_0^2 \, dA + 2d_x \int y_0 \, dA + d_x^2 \int dA$$

first integral is by definition of MoI $\bar{I}_x$ about centroidal x_0 - axis
second integral = 0, since $\int y_0 \, dA = A\bar{y}_0$ & $\bar{y}_0$ is automatically 0 with centroid on x_0 axis

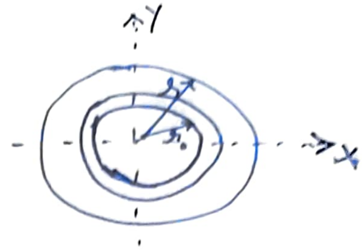
$$\therefore \quad I_x = \bar{I}_x + d_x^2 (A)$$

$$I_x = \bar{I}_x + Ad_x^2 \quad (1)$$

$$//^{ly} \quad I_y = \bar{I}_y + Ad_y^2 \quad (2)$$

$$(1) + (2) \quad I_z = \bar{I}_z + Ad^2$$

q. MOI of area of circle



Sol. $A = \pi r^2$

area of differential element $dA = 2\pi r_0 \, dr_0$

$$I_z = \int r^2 dA$$

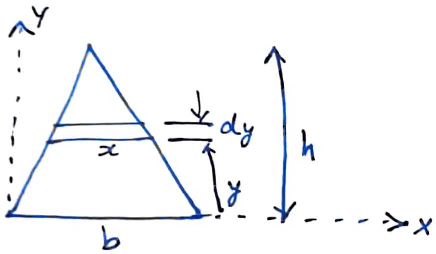
$$= \int_0^r r_0^2 \, 2\pi r_0 \, dr_0 = 2\pi \frac{r^4}{4} = \frac{\pi r^2}{2} = \frac{A r^2}{2}$$

Polar radius of gyration is $k = \sqrt{\frac{I}{A}}$ $\quad k_z = \frac{r}{\sqrt{2}}$

By symmetry $I_x = I_y$

$(I_z = I_x + I_y) \Rightarrow I_x = \frac{1}{2} I_z = \frac{\pi r^2}{4}$

q. Centroid of Δ



by similarity

$$\frac{x}{b} = \frac{h-y}{h}$$

$A = \frac{1}{2} bh$

area of differential element

$dA = x \, dy$

$dA = \frac{b(h-y)}{h} dy$

$$\bar{y} = \frac{\int y_c \, dA}{A}$$

$$= \frac{1}{A} \int_0^h y \cdot \frac{b(h-y)}{h} dy$$

$$= \frac{b}{hA} \int_0^h hy - y^2 \, dy = \frac{b}{h \frac{1}{2} bh} \left[\frac{hy^2}{2} - \frac{y^3}{3} \right]_0^h$$

$$= \frac{2}{h^2} \left[\frac{h^3}{2} - \frac{h^3}{3} \right]$$

$$= \frac{2}{h^2} \frac{h^3}{6} = \frac{h}{3}$$

q. MOI of Δ

$dA = x \, dy$

$dA = \frac{b(h-y)}{h} dy$

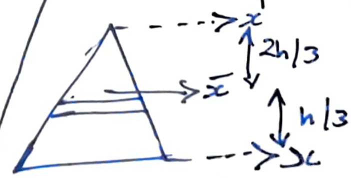
$$I_x = \int y^2 dA$$

$$= \int_0^h y^2 \frac{b(h-y)}{h} dy$$

$$= \frac{b}{h} \int_0^h y^2 h - y^3$$

$$= \frac{b}{h} \left[\frac{y^3}{3}(h) - \frac{y^4}{4} \right]_0^h$$

$$= \frac{b}{h} \frac{h^4}{12} = \frac{bh^3}{12}$$



for centroidal axis which is at a distance $d = \frac{h}{3}$ above x-axis (∥ axis theorem)

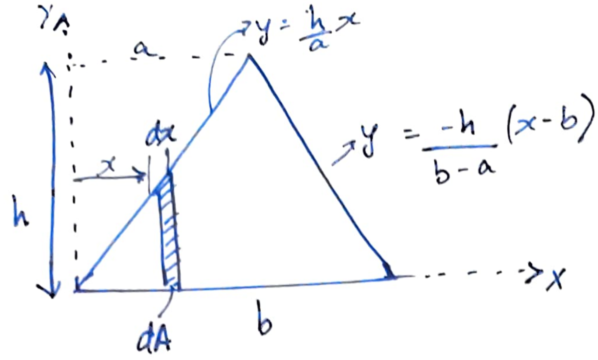
$$\bar{I}_x = I_x - Ad^2$$

$$= \frac{bh^3}{12} - \frac{bh}{2}\left(\frac{h}{3}\right)^2 = \frac{bh^3}{12} - \frac{bh^3}{18}$$

$$= \frac{bh^3}{36}$$

$I_{x'} = \bar{I}_x + Ad^2 \quad d = 2h/3$

$$= \frac{bh^3}{12} + \frac{bh}{2}\left(\frac{2h}{3}\right)^2 = \frac{bh^3}{4}$$



$$A = \frac{1}{2}b(h)$$

$$\bar{x} = \frac{\int x_c \, dA}{A} = \frac{\int_0^a x\left(\frac{h}{a}x\right)dx + \int_a^b x\left(\frac{-h}{b-a}\right)(x-b)\,dx}{\frac{1}{2}bh}$$

$$= \frac{\frac{h}{a}\left[\frac{x^3}{3}\right]_0^a + \frac{-h}{(b-a)}\int_a^b x^2 - bx \, dx}{\frac{1}{2}bh}$$

$$= \frac{\frac{ha^2}{3} - \frac{h}{(b-a)}\left[\frac{x^3}{3} - b\frac{x^2}{2}\right]_a^b}{\frac{1}{2}bh}$$

$$= \frac{\frac{ha^2}{3} - \frac{h}{(b-a)}\left[\frac{(b^3-a^3)}{3} - \frac{b(b^2-a^2)}{2}\right]}{\frac{1}{2}bh}$$

$$= \frac{2a^2}{3b} - \frac{2}{b(b-a)}\left[\frac{(b^3-a^3)}{3} - \frac{b(b^2-a^2)}{2}\right]$$

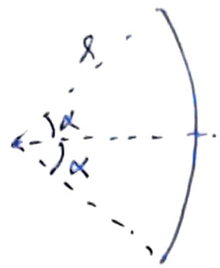
$$= \frac{1}{3}(a+b)$$

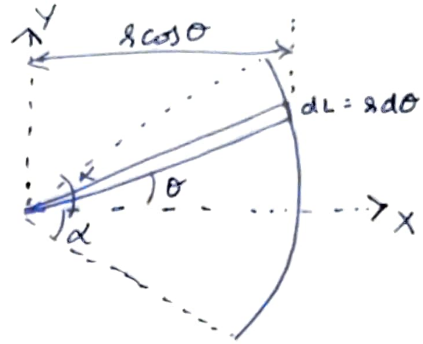
→ Centroid of a circular arc

$dL = r\,d\theta$

$L = 2\alpha r$

$\bar{x} = \dfrac{\int x\,dL}{L}$





$L\bar{x} = \int x\,dL$

$2\alpha r(\bar{x}) = \int_{-\alpha}^{\alpha} (r\cos\theta)\,r\,d\theta$

$2\alpha r(\bar{x}) = r^2\left[\sin\theta\right]_{-\alpha}^{\alpha}$

$= r^2\left[\sin\alpha - \sin(-\alpha)\right]$

$\cancel{2}\alpha \cancel{r}\bar{x} = \cancel{2}r^{\cancel{2}}\left[\sin\alpha\right]$

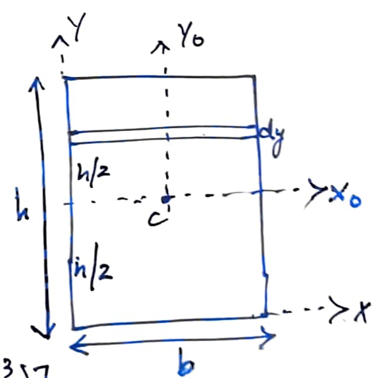
$\bar{x} = \dfrac{r\sin\alpha}{\alpha}$

→ MOI of rectangle

$I_x = \int y^2\,dA$

$dA = b\,dy$

$I_x = \int_{-h/2}^{h/2} y^2\,b\,dy$



$= b\left[\dfrac{y^3}{3}\right]_{-h/2}^{h/2} = \dfrac{b}{3}\left[\dfrac{h^3}{8} - \left(\dfrac{-h^3}{8}\right)\right]$

$I_x = \dfrac{b}{3}\dfrac{h^3}{4} = \dfrac{1}{12}bh^3$

$\parallel^{ly}$ $I_y = \dfrac{1}{12}hb^3$ $\qquad I_z = I_x + I_y$

$I_{x_0} = I_x + Ad^2$ ($\parallel$ axis theorem)

$= \dfrac{1}{12}bh^3 + bh\left(\dfrac{h}{2}\right)^2 = \dfrac{4bh^3}{12} = \dfrac{bh^3}{3}$

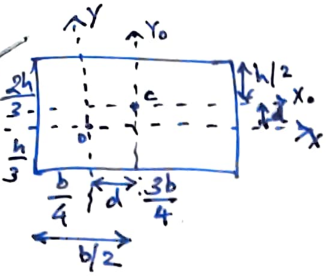
$\parallel^{ly}$ $I_{y_0} = \dfrac{hb^3}{3}$

$I_{z_0} = I_{x_0} + I_{y_0}$

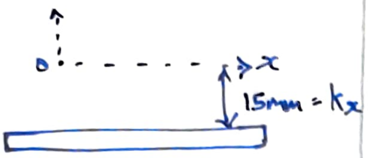
UNIT-4

Area MoI ①

A/1



A/3



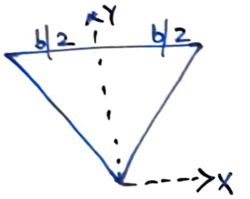
$A = 300$ $\quad k_o = ?$

$I_Y = 35 \times 10^3$ $\quad k_o = \sqrt{\frac{I_o}{A}}$

$I_o = I_X + I_Y$

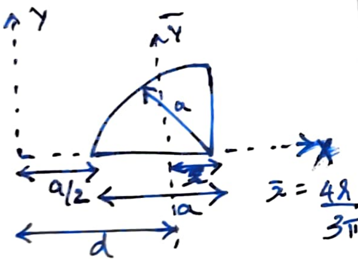
$I_X = k_x^2 A$

A/4



$I_x = \frac{bh^3}{4}$ $\quad I_y = \frac{hb^3}{48}$

A/7

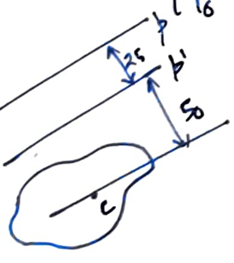


$\bar{x} = \frac{4r}{3\pi}$

$I_Y = \bar{I}_Y + Ad^2$

$= \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4 + \frac{\pi}{4} r^2 \left(\frac{a}{2} + a - \frac{4r}{3\pi}\right)$

A/9.



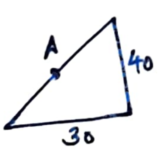
$I_p - I_{p'} = 15 \times 10^6$

$\bar{I}_x + Ad_1^2 - (\bar{I}_x + Ad_2^2) = 15 \times 10^6$

$A(d_1^2 - d_2^2) = 15 \times 10^6$

$d_1 = 75 \quad d_2 = 50$

A/16



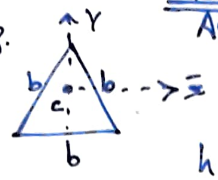
$k = ?$

$I_z = \frac{1}{2}\left(\frac{1}{12}\right) bh (b^2 + h^2)$

$A = \frac{1}{2} bh$

$k = \sqrt{\frac{I_z}{A}}$

A/18.



$h = \frac{\sqrt{3}b}{2}$

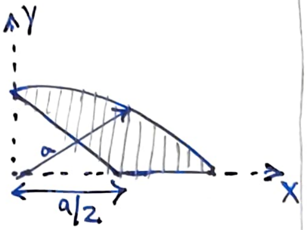
$\bar{I}_x = \frac{bh^3}{36}$

$\bar{I}_Y = \frac{hb^3}{48}$

$I_z = I_x + I_y$

$k = \sqrt{\frac{I_z}{A}}$

A/19.

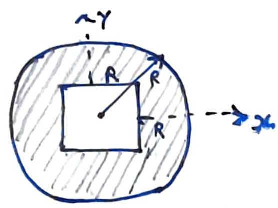


$I_x = I_{x_1} - I_{x_2}$

quater ⊙ $\quad \Delta$

$= \frac{\pi r^4}{16} - \frac{bh^3}{12}$

A/35.



without hole $I_{xx} = I_{YY} = \frac{\pi r^4}{4}$

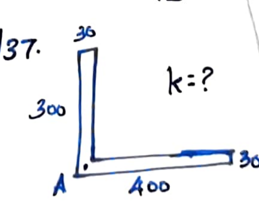
$I_{zz} = I_{xx} + I_{YY}$

with hole

$I_{xx} = \frac{\pi r^4}{4} - \frac{bh^3}{12}$

$I_{YY} = \frac{\pi r^4}{4} - \frac{bh^3}{12}$

A/37.



$k = ?$

$I_{xx} = \frac{bh^3}{3} = \frac{30(300)^3}{3}$

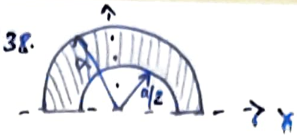
$I_{YY} = \frac{hb^3}{3} = \frac{30(400)^3}{3}$

$I_{zz} = I_{xx} + I_{YY}$

$I_{zz} = k^2 A$

$A = 400(30) + 300(30)$

A/38.



$I_{zz} = \frac{\pi a^4}{4} - \frac{\pi}{4}\left(\frac{a}{2}\right)^4$

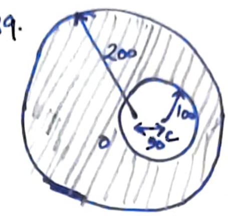
$A = \frac{1}{2}\left(\pi a^2 - \frac{\pi a^2}{4}\right)$

$k_z = \sqrt{\frac{I_{zz}}{A}}$

$k_x^2 + k_y^2 = k_z^2$

$k_x = k_y$

A/39.



$I_{o_1} = \frac{\pi (200)^4}{2}$

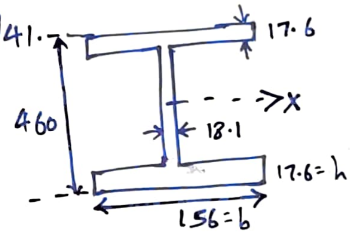
$I_{o_2} = \frac{\pi}{2}(100)^4 + \pi(100)^2(50)^2$

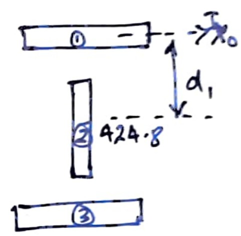
$I_o = I_{o_1} - I_{o_2}$

$A = \pi(200^2 - 100^2)$

$k = \sqrt{\frac{I_o}{A}}$

A/41.



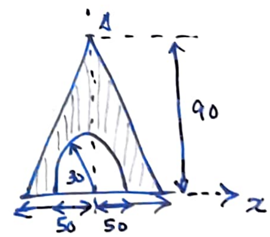


$I_{xx_1} = I_{xx_3} = \bar{I}_{xx_1} + Ad_1^2$ $\quad d_1 = \frac{17.6}{2} + \frac{424.8}{2}$

$= \frac{bh^3}{12} + bh(d_1^2)$

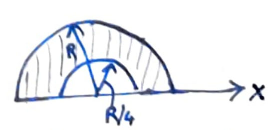
$I_{xx_2} = \frac{bh^3}{12} = \frac{(18.1)(424.8)^3}{12}$

A/42.



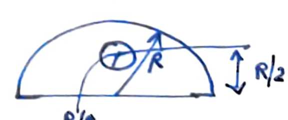
$I_{xx} = \frac{bh^3}{12} - \frac{\pi r^4}{8}$

A/44. i) $h = 0$



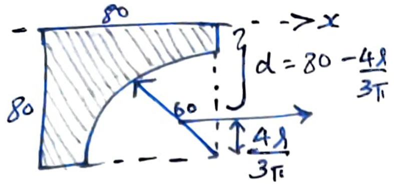
$I_{xx} = \frac{\pi R^4}{8} - \frac{\pi}{8}(R/4)^4$

ii) $h = R/2$



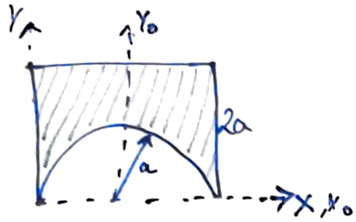
$I_{xx} = \frac{\pi R^4}{8} - \left[\frac{\pi}{8}(R/4)^2 + \pi(R/4)^2(R/2)^2\right]$

A/45.



$$I_x = \frac{bh^3}{3} - \left[\frac{\pi r^4}{16} + \frac{\pi}{4} r^2 (d)^2\right]$$

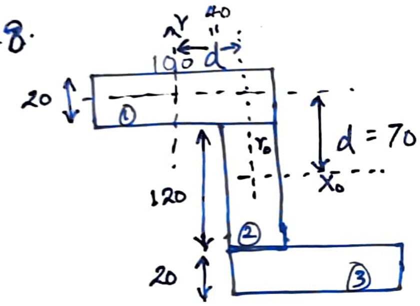
A/46.



$$I_{xx} = \frac{bh^3}{3} - \frac{\pi r^4}{8}$$

$$I_{yy} = \frac{hb^3}{3} - \left[\frac{\pi r^4}{8} + \frac{\pi r^2}{2}(a^2)\right]$$

A/48.



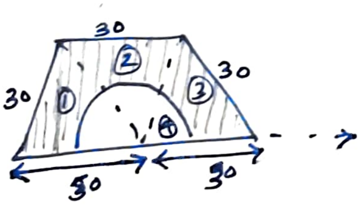
$$I_{xx_1} = I_{xx_3} = \frac{bh^3}{12} + bh(70^2)$$

$$I_{xx_2} = \frac{bh^3}{12}$$

$$I_{yy_1} = I_{yy_3} = \frac{b^3h}{12} + bh(40^2)$$

$$I_{yy_2} = \frac{hb^3}{12}$$

A/51

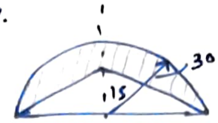


$$I_{xx_1} = I_{xx_3} = \frac{bh^3}{12} \qquad h = \frac{b\sqrt{3}}{2}$$

$$I_{xx_2} = \frac{bh^3}{4}$$

$$I_{xx_4} = \frac{\pi r^4}{8}$$

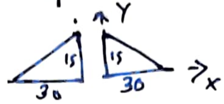
A/57.



$$I_o = I_{o_1} - I_{o_2}$$

$$I_{o_1} = \frac{\pi r^4}{4}$$
(I_z)

I_{o_2}



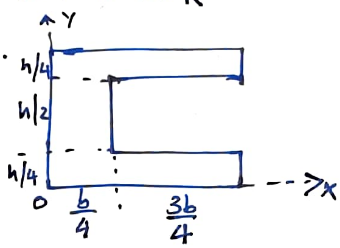
$$I_{o_2 x} = \frac{bh^3}{12}$$

$$I_{o_2 y} = \frac{hb^3}{12}$$

$$I_{o_2 Y} = 2(I_{o_2 x} + I_{o_2 y})$$

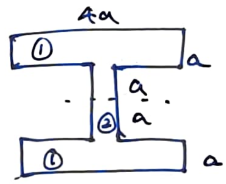
REMEMBER

A/40.



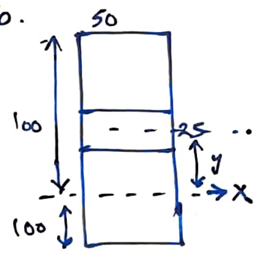
$\boxed{\begin{array}{l} \%\ \text{Area} = 37.5\% \\ \%\ I_y = 49.2\% \end{array}}$

A/49.



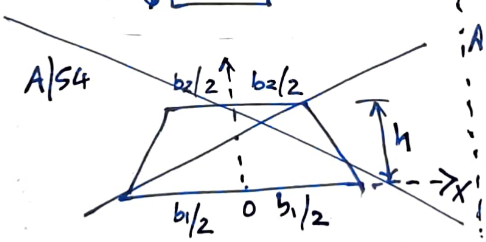
$\boxed{I_x = \frac{58}{3} a^4}$

A/50.



$\boxed{\%\ \text{change} = 9.57\%}$

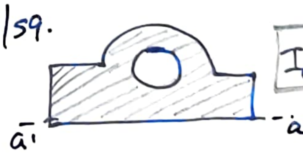
A/54



A/58

$\boxed{I_x = 16.27 \times 10^6}$

A/59.

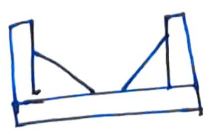


$\boxed{I_{aa'} = 346 \times 10^6}$

A/60

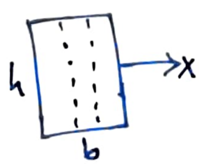
$\boxed{k_c = 261}$

A/55.

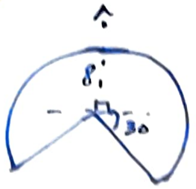


$\boxed{I_x = 38 \times 10^6}$

A/56.



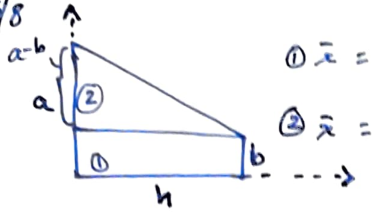
$\boxed{\%\ \text{Inc} = 176\%}$



$\alpha = 120$, $r = 8$

$$\bar{x} = \frac{2}{3} r \frac{\sin \alpha}{\alpha}$$

5/8

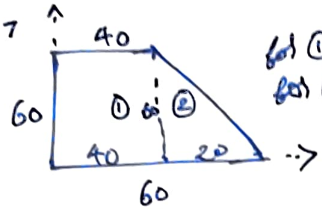


① $\bar{x} = \frac{h}{2}$ $\bar{y} = \frac{b}{2}$ $A = bh$

② $\bar{x} = \frac{h}{3}$ $\bar{y} = b + \frac{a-b}{3}$

$A = \frac{1}{2}(a-b)(h)$

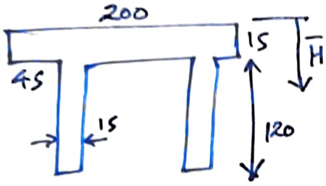
5/47



for ① $\bar{x} = 20$ $\bar{y} = 30$

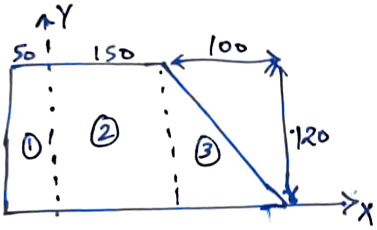
for ② $\bar{x} = 40 + \frac{20}{3}$ $\bar{y} = \frac{60}{3}$

5/48.



$$\bar{H} = \frac{200(15)\left(\frac{15}{2}\right) + 2(15)(120)\left(15 + \frac{120}{2}\right)}{200(15) + 2(15)(120)}$$

5/49.

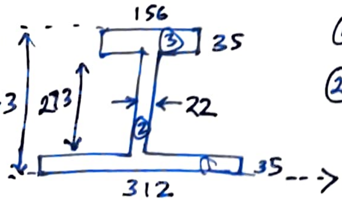


① $A = 15(120)$ $\bar{x} = -50/2$ $\bar{y} = \frac{120}{2}$

② $A = 150(120)$ $\bar{x} = 150/2$ $\bar{y} = 120/2$

③ $A = \frac{1}{2}(100)(120)$ $\bar{x} = 150 + \frac{100}{3}$ $\bar{y} = \frac{120}{3}$

5/50

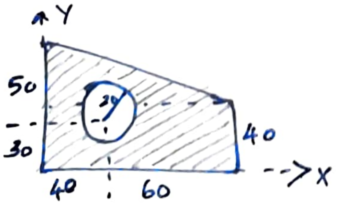


① $\bar{y}_1 = 35/2$ $A = 312(35)$

② $\bar{y}_2 = 35 + \frac{273}{2}$ $A = 273(22)$

③ $\bar{y}_3 = 35 + 273 + \frac{35}{2}$ $A = 156(35)$

5/51



$\bar{x} = \frac{100}{3}$ $\bar{y} = 40 + \frac{40}{3}$

$A = \frac{1}{2}(100)(40)$

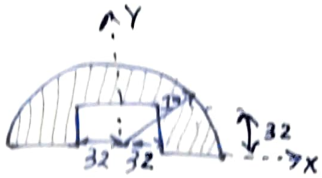
$\bar{x} = \frac{100}{2}$ $\bar{y} = \frac{40}{2}$

$A = 100(40)$

$\bar{x} = 40$ $\bar{y} = 30$

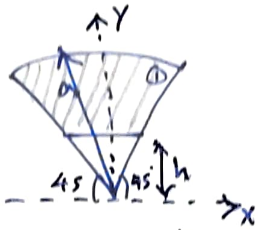
$A = \pi(20)^2$

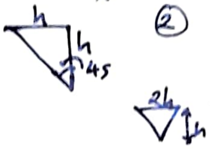
5/53



$$\bar{y} = \frac{\frac{\pi}{2}(74)^2\left(\frac{4(74)}{3\pi}\right) - 64(32)(32/2)}{\frac{\pi}{2}(74)^2 - 64(32)}$$

5/55.





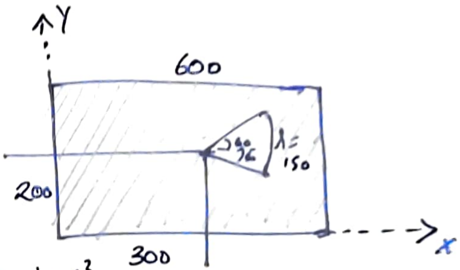
① $\bar{y} = \frac{2}{3} a \frac{\sin\frac{\pi}{4}}{\pi/4}$

$A = \frac{1}{4}\pi a^2$

② $\bar{y} = \frac{2}{3}h$

$A = \frac{1}{2}(2h)(h)$

5/57



area of sector

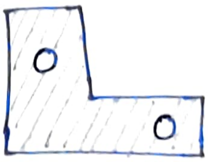
$r = 120$ $\frac{\theta}{360}\pi r^2 = \frac{1}{3}\pi r^2$

$$\bar{x} = \frac{600(400)(300) - \frac{1}{3}\pi(150)^2\left[300 + \frac{2(150)}{3}\frac{\sin(60)}{\frac{\pi}{3}}\right]}{600(400) - \frac{1}{3}\pi(150)^2}$$

$$\bar{y} = \frac{400}{2} = 200$$

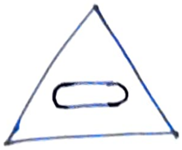
REMEMBER

5/52.



$\boxed{\bar{X} = \bar{Y} = 103.6}$

5/56



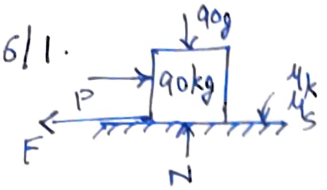
$\boxed{\bar{Y} = 63.9}$

* always assume eqm.
* prove it right / wrong

UNIT - 5 FRICTION

6th February, 2023 ①

6/1.

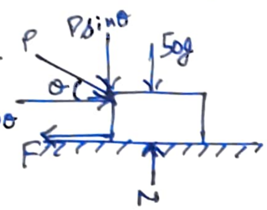


$\Sigma F_x = 0$
$\Sigma F_y = 0$

$f_{s\,max} = \mu_s N$

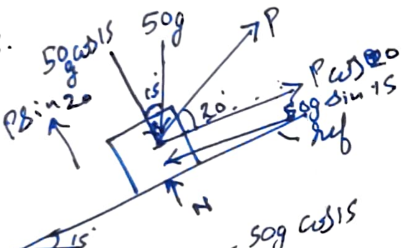
if $F < f_{s\,max} \Rightarrow$ eqm
if $F > f_{s\,max} \Rightarrow$ motion $\Downarrow$ $F = \mu_k N$

6/2.

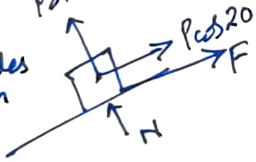


$\Sigma F_x = 0$
$\Sigma F_y = 0$ $\quad \mu_s = \frac{F}{N}$

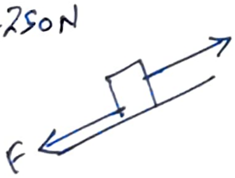
6/3.



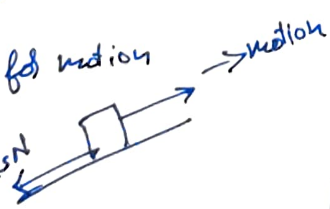
a) $P = 0$
$\Rightarrow$ box slides down



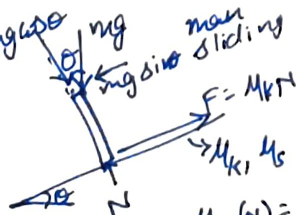
b) $P = 250N$ — assume eqm; if $F > f_{s\,max}$ use f_k



c) $P = ?$ for motion



6/4.



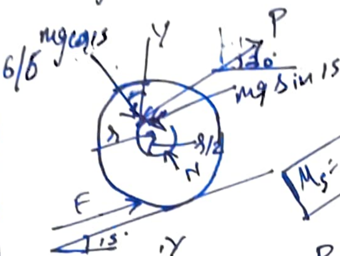
$\mu_k(N) = mg\sin\theta$
$N = mg\cos\theta$
$\mu_k = $ ___

6/5

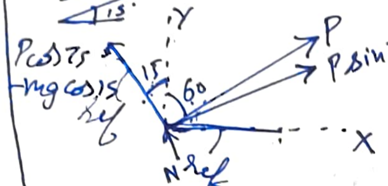
$\mu_s = ?$ $\quad \boxed{\mu_s = 0.08}$

$\Sigma F_x = 0$ $\quad F_A = \mu_s N_A$
$\Sigma F_y = 0$ $\quad F_B = \mu_s N_B$

6/6

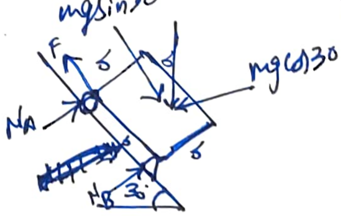


$\boxed{\mu_s = 0.0959}$



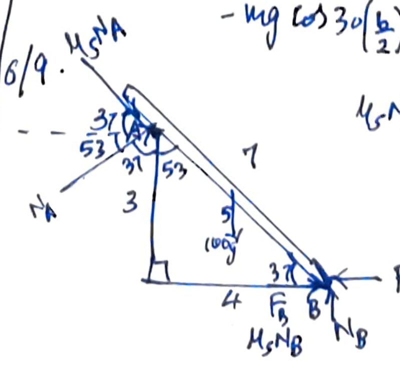
$P\cos 75 - mg\cos 15 + N = 0$
$P\sin 75 + F - mg\sin 15 = 0$
$\Sigma M_G = 0$
$-P(r/2) + F(r) = 0$

6/8



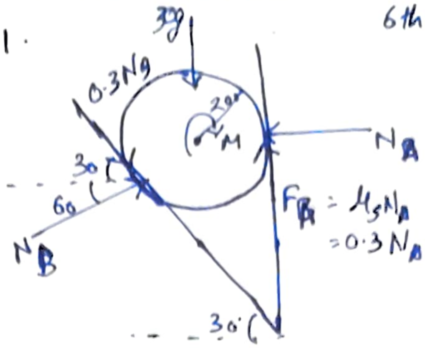
$F = \mu_k N_B$ $\quad \mu_k = ?$
$\mu_k N_B = mg\sin 30$
$N_A + N_B = mg\cos 30$
$\Sigma M_A = 0$
$N_B(b) - mg\sin 30\left(\frac{b}{2}\right) - mg\cos 30\left(\frac{b}{2}\right) = 0$

6/9.



$\mu_s N_A\cos 37 + N_A\cos 53 - P + \mu_s N_B = 0$
$N_A + N_A\cos 37 - 100g - \mu_s N_A\sin 37 = 0$
$\Sigma M_B = 0$
$\boxed{P = 775N}$

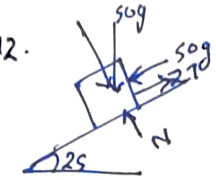
6/11.



$F_B = \mu_s N_B = 0.3 N_B$

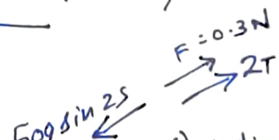
$\Sigma M_G = 0$
$0.3N_A(0.2) + 0.3N_B(0.2) - M = 0$
$\Sigma F_x = 0 \quad 0.3N_B\cos 30 + N_B\cos 60 - N_A = 0$
$\Sigma F_y = 0 \quad N_B\sin 60 - 0.3N_B\sin 30 - 30g + 0.3N_A = 0$

6/12.

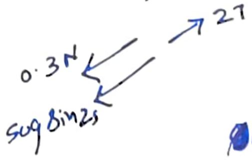


$N = 50g\cos 25$

i) motion down

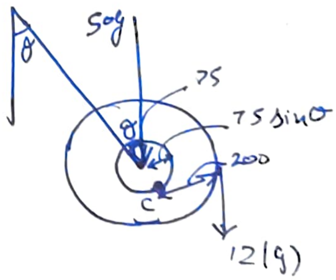


ii) motion up



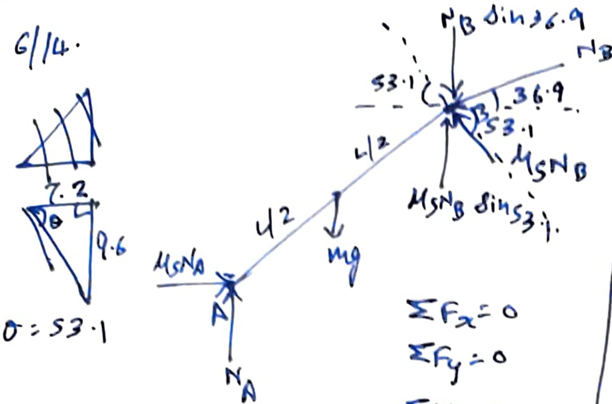
$m_B g = 3T$

6/13



$\Sigma M_c = 0$
$50g(75\sin\theta) - 12g(200 - 75\sin\theta) = 0$
$\mu_{min} = \tan\theta$

6/14.



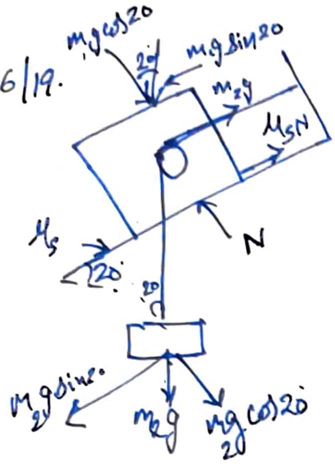
$\theta = 53.1$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_B = 0$

6/19.



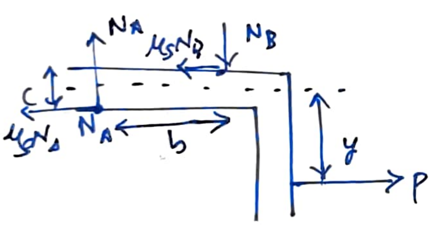
motion

$$m_2 g + \mu_s N - m_1 g \sin 20 - m_2 g \sin 20 = 0$$

$$N = m_1 g \sin 20 + m_2 g \sin \theta$$

motion ↗ friction sign reverses

6/20.

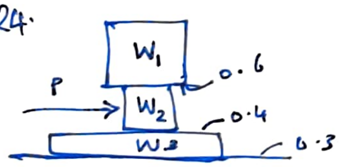


$\Sigma F_x = 0$ $\quad P = \mu_s (N_A + N_B)$

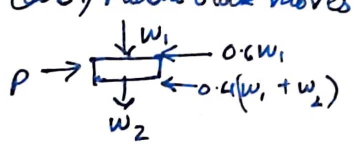
$\Sigma F_y = 0$ $\quad N_A = N_B$

$\Sigma M_A = 0$ $\quad \mu_s N_B (c) - N_B (b) + P\left(y - \frac{c}{2}\right) = 0$

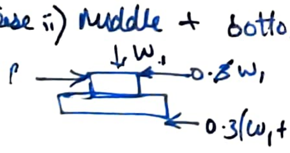
6/24.



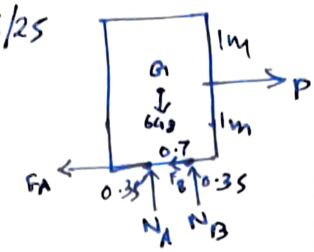
Case i) Middle block moves, bottom doesn't



Case ii) Middle + bottom moves



6/25



Case i) A frozen

$\Rightarrow F_B = 0$

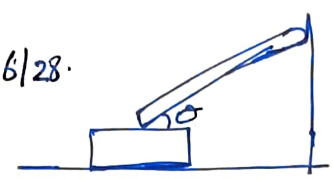
$\Sigma M_B = 0$ & $P = \mu_s N_A$

Case ii) B frozen

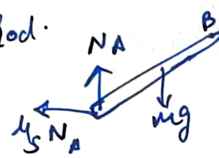
$\Rightarrow F_A = 0$

$\Sigma M_B = 0$ & $P = \mu_s N_B$

6/28.

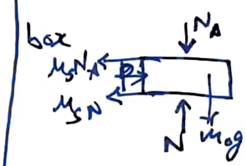


Rod.

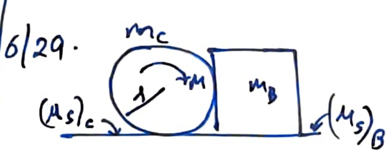


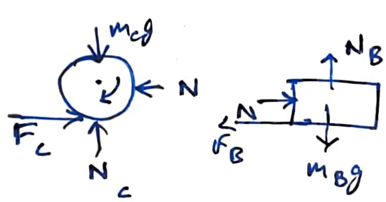
$\Sigma M_B = 0$

box

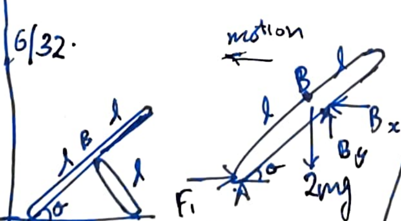


6/29.





6/32.

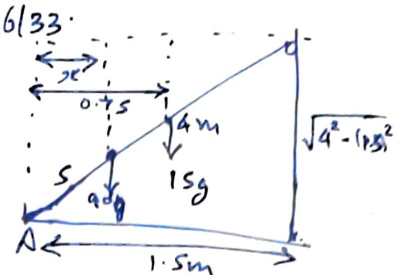


$\Sigma F_x = 0$

$\Sigma F_y = 0$

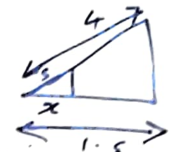
$\Sigma M_A = 0$

6/33.



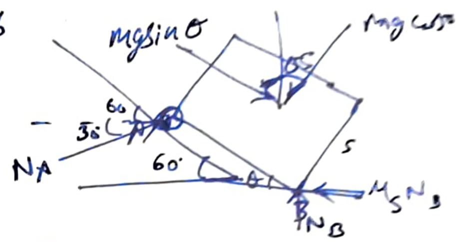
$\Sigma F_y = 0$

$\Sigma M_B = 0$



$\frac{s}{x} = \frac{4}{1.5}$

6/36



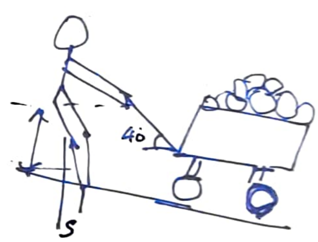
$\Sigma F_x = 0 \quad N_A \cos 30 - \mu_s N_B = 0$

$\Sigma F_y = 0 \quad -mg + N_B + N_A \sin 30 = 0$

$\Sigma M_A = 0 \quad N_B (s\cos\theta) - mg\sin\theta\left(\frac{s}{2}\right) - \mu_s N_B (s\sin\theta) - mg\cos\theta\left(\frac{s}{2}\right) = 0$

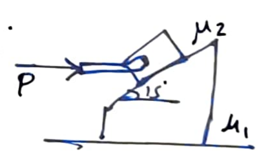
REMEMBER

6/23.



$\boxed{\mu_s = 0.408}$ $\boxed{s = 126.2\text{ mm}}$

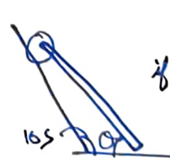
6/30.



a) slips b/w A & B

b) slips b/w A & Ground

6/18.



if $\mu_s = 0.25$ $\theta = 61.8$

$\mu_s = 0.5$ $\theta = 40.9$

...es of Friction

...y Friction : # Dry friction occurs when the unlubricated surfaces of two solids are in contact under a condition of sliding / tendency to slide.

* A frictional force tangent to surface of contact occurs both during the interval ~~and~~ leading upto impending slippage and when slippage takes place.
* The direction of this friction always opposes the ~~direction of~~ motion.
* Also called Coulomb Friction

2. Fluid Friction : * Fluid friction occurs when adjacent layers in a fluid (liq / gas) are moving at diff. velocities.
* This motion causes frictional forces b/w fluid elements, and these forces depend on relative velocity b/w layers.
* No relative velocity ⇒ No fluid friction
* Fluid friction depends on ─┬ velocity gradients within the fluid
  └ viscosity of liquid.

3. Internal Friction: * Internal friction occurs in all solid materials which are subjected to cyclical loading.
* For highly elastic materials the recovery from deformation occurs with very little loss of energy due to internal friction

Static Friction

* Static friction force is determined by the equations of eqm.. This frictional force may have any value from 0 upto including the max. value. For a given value of mating surfaces the experiment shows that the max. value of static friction is proportional to the normal force N.

$$F_{max} = \mu_s N$$

μ_s is the proportionality constant, called coefficient of static friction

Kinetic Friction

* After slippage occurs, the frictional force which accompanies the ensuing motion is called kinetic friction. Kinetic Friction is usually < max static friction force.

F_k is also $\propto N$

$$F_k = \mu_k N$$

μ_k : coefficient of kinetic friction

Angle of Friction

* Angle made by the resultant of normal reaction and limiting friction with the normal reaction is called angle of friction.
  Coefficient of static friction is equal to the tangent of angle of friction.

Angle of Repose

* It is the minimum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide down.

  lets say an inclined plane with θ angle with horizontal & a body of mass m

$$mg\sin\theta = \mu N$$

$$N = mg\cos\theta$$

$$\tan\theta = \mu$$

$$\underbrace{\theta}_{\text{angle of repose}} = \tan^{-1}(\mu)$$

Cone of Friction

* If the resultant reaction is rotated about the normal reaction force, it will form a cone known as cone of friction. It is a right circular cone with vertex at the point of contact of the two bodies (or surfaces), axis in direction of normal reaction and semi vertical angle = angle of friction.

Factors affecting friction

* generation of high local temp and adhesion at contact points
* relative hardness of mating surfaces
* presence of thin surface films of oxide, oil, dirt or other substances
* molecular attraction
* Roughness/smoothness of the sliding object
* roughness/smoothness of the ~~sliding~~ surface
* shape/design of objects
* Normal Force
* Type of friction involved (rolling/sliding)