

- Mechanism to create population inversion:
- Population inversion → enhances stimulated probability of stimulated emission.
- Metals not good candidate for creating lasers.
- Solid state lasers (Insulators) → optical pumping.
- Semiconductor lasers → forward bias
 - ↓ pumping mechanism
- Gases → placed in enclosures (like glass tube). Gases don't react, they just collide. Energy divided into energy levels as opposed to energy bands in solids.
We require, precisely the right amount of energy to push gas molecules to higher state.

To increase collision → introduce e- by applying potential difference,

Collision energy ↑ as voltage ↑.

Method → Electric discharge.

- Optical Resonator:
- Spontaneous emission → Unstable.
- Characteristic of spontaneous emission → photons emitted by 2 atoms has no correlation to one another.
- We use mirrors to trap these photons.
- We obtain laser oscillating, within few minutes.
- Wave moving back & forth can combine constructively/ destructively.
- So to ensure only constructive interference happens: path length = $n\lambda$

$$2L = n\lambda$$

→ This amplifies light & hence, is called ↑

$$L = \frac{n\lambda}{2}$$

optical resonator

→ To get light out of the system, we make one of the mirrors 95% reflective, 5% transmissive. So 5% light escapes every cycle.

→ Laser systems:

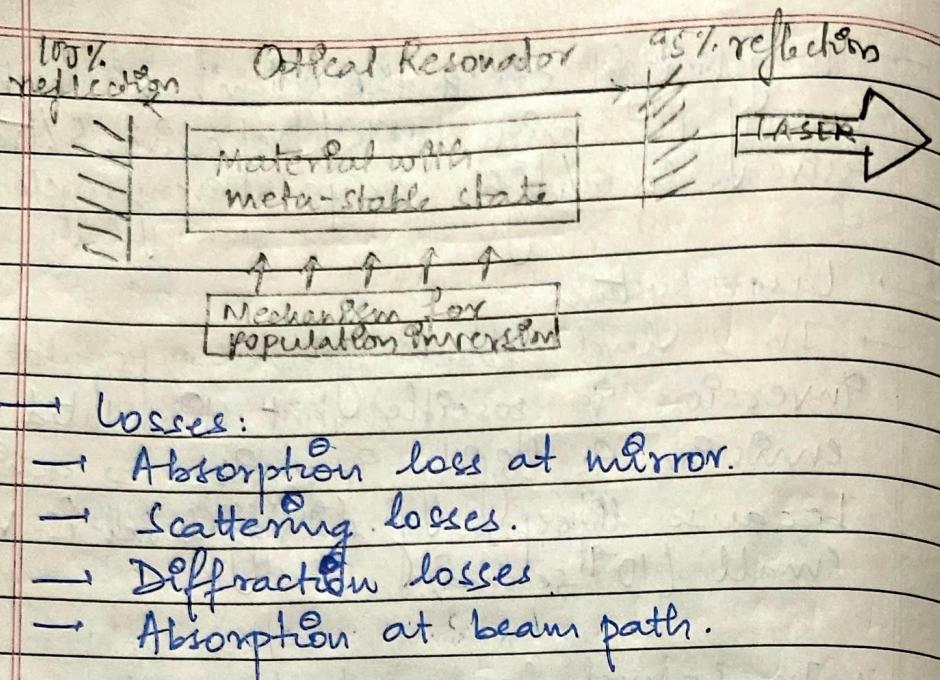
→ In 2-level laser systems → population inversion is possible but stimulated emission is highly impossible to achieve, because lifespan of ^(excited) energy level is very small (10^{-9} seconds).

→ In 3-level laser systems → when atoms are pumped to excited state, it doesn't stay for long (10^{-9} seconds) & drops to intermediate energy level having comparatively higher lifespan (10^{-3} to 10^{-6} seconds). Eventually there is larger population in intermediate (meta-stable) state than in lower state.

Pumping photons & lasing photons → different, which means there is decoupling b/w pumping & lasing photons.

Disadvantage → we need to left more than 50% of atoms from lower state for decoupling to happen. Due to this, light stops, giving us pulsating output.

→ In 4-level laser systems → Population inversion happens b/w meta-stable state & low-level intermediate state (just above ground state). Here, if the ground state (reservoir) does not ~~need~~ to be depleted above 50%. Since, there is always more than 50% of atoms in reservoir, the output is a continuous laser.



- Beam broadening:
 - Doppler broadening
 - Pressure broadening
 - Quantum Noise
 - Inhomogeneous broadening
 - Design flaw
 - Frequency comb
- Expression for round-trip gain:

Lambert's Law

$$I = \frac{I_0}{e^{\alpha L}}$$

Intensity at $I_A = I_0$

Intensity at $I_B = I_0 e^{(g-\alpha)L}$

Intensity at $I_C = R_2 I_B$ ~~Gain coefficient~~
 $= R_2 I_0 e^{(g-\alpha)L}$ $\alpha \rightarrow$ loss coefficient

Intensity at $I_D = R_2 I_0 e^{(g-\alpha)2L}$

Intensity at $I_E = R_1 R_2 I_0 e^{(g-\alpha)2L}$

Total Gain = $\frac{\text{Final Intensity}}{\text{Initial Intensity}}$

$$\text{Total Gain} = \frac{R_1 R_2 J_0 e^{(g-\alpha)2L}}{I_0}$$

$$\text{Total Gain}(I) = R_1 R_2 e^{(g-\alpha)2L}$$

If $G = 1$,

$$I = R_1 R_2 e^{(g-\alpha)2L}$$

$$\frac{I}{R_1 R_2} = e^{(g-\alpha)2L}$$

$$R_1 R_2$$

$$\ln\left(\frac{I}{R_1 R_2}\right) = (g - \alpha)2L$$

$$\rightarrow \ln(R_1 R_2) = g2L - \alpha2L$$

$$-\frac{\ln(R_1 R_2) + \alpha2L}{2L} = g$$

$$g = \alpha + \left[\frac{1}{2L} \ln\left(\frac{I}{R_1 R_2}\right) \right]$$

→ He-Ne Laser:

→ Mixture of He-Ne in ratio 10:1 is filled in glass tube at low (~ 1 torr) pressure.

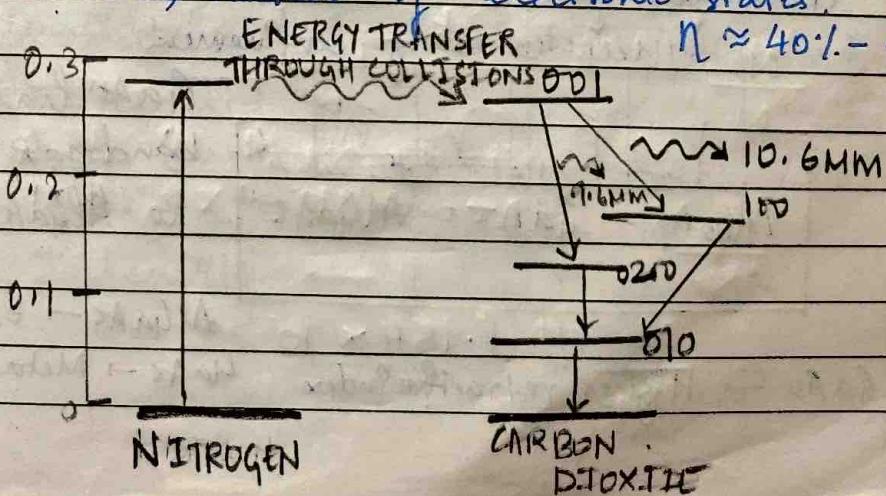
→ Lasing takes place in Ne, whereas He is a facilitator to bring population inversion.

→ Pumping mechanism: electric discharge.

→ He gets hit first by $e^- \rightarrow$ elastic collision.

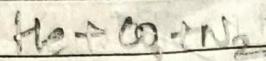
→ CO₂ laser:

→ Uses molecular, rotational or vibrational energy states, instead of electronic states.

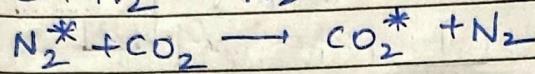


100%

95%

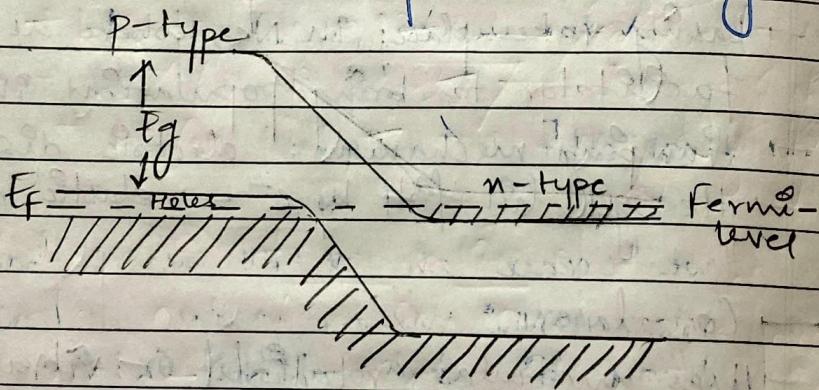


F.V.



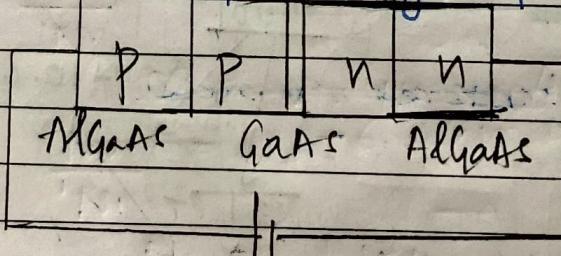
→ Semiconductor lasers:

- Direct band-gap semiconductor → High of valence band is right below low of conduction band. EX: GaAs, ZnO
- Indirect band-gap semiconductor → High of valence band is misaligned with low of conduction band.
- When recombination takes place,
in direct → photon generated
in indirect → photon + heat generated



→ Heterojunction diode:

Helps in charge confinement



GaAs has lower band gap compared to AlGaAs.

GaAs → Higher refractive index

AlGaAs → Excited state
GaAs → Metastable state

→ Ruby laser:

- Al_2O_3 doped with Cr^{3+} ions.
 - Active medium → Ruby crystal / rod.
 - Band gap of Al_2O_3 → large (Insulator)
 - Around 1.74 eV → Meta-stable state (lifetime 10^{-3} sec)
 - Transition from upper absorption energy levels to intermediate state is through non-radiative transitions.
 - Leads to increase in population of intermediate state while population of ground state is being depleted rapidly.
-

→ Energy pump:

- Two types → Concentric & Elliptical
- Concentric arrangement of flash lamp & ruby rods
- Elliptical arrangement of flash lamp & ruby rods are at the two foci of an elliptical mirror arrangement.

→ Applications of Laser:

- Holography: To retain phase information we create an interference pattern on a photographic plate (hologram) of light scattered from object & a reference beam.

Consider an object wave (scattered from the obj.) represented by $E_o(x, y)$ & reference beam (unscattered) represented by ER which remains constant for all (x, y) .

Intensity at holographic plate:

$$I = |E_o(x, y) + ER|^2$$

$$I = (E_o^*(x, y) + ER^*)(E_o(x, y) + ER)$$

$$I = |E_0(x, y)|^2 + |E_R|^2 + E_0^*(x, y)E_R + \\ E_0(x, y)E_R^*$$

Imagine that holographic plate's transparency is proportional to light intensity.

$$\text{Transparency } T(x, y) = a + bI(x, y)$$

$a, b \rightarrow \text{constants}$

Light passing through the hologram when illuminated only

$$E = aE_R + bE_R(|E_0(x, y)|^2 + |E_R|^2 + E_0^*(x, y)E_R + E_0(x, y)E_R^*)$$

$$E(x, y) = aE_R + bE_R|E_R|^2 + bE_R|E_0(x, y)|^2 + bE_0^*(x, y)|E_R|^2 + E_0(x, y)E_R^2$$

$$E = aE_R + bE_R|E_R|^2 \rightarrow \text{constant term} \\ E_R \rightarrow \text{constant} \\ + bE_R|E_0(x, y)|^2 \rightarrow \text{Scattered} \rightarrow \text{negligible} \\ + bE_0^*(x, y)|E_R|^2 \rightarrow \text{Image of object} \\ + bE_0(x, y)E_R^2 \rightarrow \text{Image of object}$$

Q- Find ratio of population of two energy levels out of which one corresponds to meta-stable state, of $\lambda = 632.8 \text{ nm}$ at room temp.

$$\frac{N_2}{N_1} = e^{-h\nu/kT}$$

~~$$\nu = 4.74 \times 10^{14} \text{ Hz} = hc/\lambda = 4.74 \times 10^{14} \text{ Hz}$$~~

$$\frac{N_2}{N_1} = e^{-6.63 \times 10^{-34} \times 4.74 \times 10^{14} / 1.38 \times 10^{-23}}$$

$$\frac{N_2}{N_1} = 1.154 \times 10^{-83}$$

Q- Pulsed laser with power per pulse is 1 mW lasts for 10 ns. If no. of photons/pulse is 3.94×10^7 , calculate λ of laser.

Solu:

$$P = \frac{nhc}{\lambda T}$$

$$10^{-3} = \frac{3.94 \times 10^7 \times 6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 10 \times 10^{-9}}$$

$$\lambda = \frac{3.94 \times 10^7 \times 6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-3} \times 10^{-8}}$$

$$\lambda = 783 \text{ nm}$$

Q- for ν of $9 \times 10^{14} \text{ Hz}$, find temp. at which the rates of spontaneous to stimulated emission are equal.

Solu:

$$\frac{B}{A} = e^{h\nu/kT} - 1$$

$$1 = e^{h\nu/kT} - 1$$

$$2 = e^{h\nu/kT}$$

$$0.693 = \frac{h\nu}{kT}$$

$$0.693 = \frac{6.63 \times 10^{-34} \times 9 \times 10^{14}}{1.38 \times 10^{-23} \times T}$$

$$T = 62394.12 \text{ K}$$

Q- Calculate the threshold gain factor, for a laser, which has loss coefficient as $0.05/\text{m}$ if configuration of laser system has a tube of 50 cm length, 1 mirror & 99% reflecting & other 90% reflecting.

Solu:

$$g_{th} = \alpha + \frac{1}{2L} \left[\ln \left(\frac{1}{R_1 R_2} \right) \right]$$

$$g_{th} = 0.05 + \frac{1}{2 \times 0.5} \left[\ln \left(\frac{1}{0.99 \times 0.9} \right) \right]$$

$$g_{th} = 0.1654/\text{m}$$

Unit 5: Application of Quantum Mechanics to Materials (Magnetic & Dielectric)

- Magnetic materials:
- Magnetic monopoles don't exist
- From Maxwell's eqns, $\nabla \cdot \mathbf{B} = 0$

\mathbf{B} → magnetic field, \mathbf{H} → magnetic field strength

$$\mathbf{B} = \mu_0 \mathbf{H} \rightarrow \text{vacuum}$$

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H} \rightarrow \text{free space}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \rightarrow \text{permeability of free space}$$

$\mu_r \rightarrow \text{relative permeability}$

- Lorentz's force:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- magnetic dipole moment = IA

$$\mathbf{M} = -\frac{e}{4\pi} \mathbf{T} \mathbf{r}^2$$

$$\mathbf{M} = -ev \times \frac{\mathbf{r} \mathbf{r}^2}{2\pi r^3}$$

[∴ angular velocity $\omega = 2\pi/T$]

$$\mathbf{M} = -evr \times \frac{\mathbf{m}}{m}$$

$$\mathbf{M} = -(\mathbf{mv}\omega) \mathbf{e}$$

angular (orbital)

$$\text{magnetic moment } \mathbf{M} = -\frac{Le}{2m} \quad [L \rightarrow \text{angular momentum}]$$

$$L = l(l+1) \hbar$$

$l \rightarrow$ azimuthal quantum number

$L \rightarrow$ angular momentum

$$\hbar \rightarrow \frac{h}{2\pi}$$

$$\gamma_e = \frac{e}{2me} \rightarrow \text{Gyroscopic ratio (electron)}$$

$$\vec{M}_e = \Theta \gamma_e \vec{L}$$

comes from charge

$$\gamma_p = \frac{e}{2mp} \rightarrow \text{Gyroscopic ratio (proton)}$$

$$\vec{M}_p = \gamma_e \vec{L}$$

neutrons → no magnetic moment

Magnetisation: $M = \frac{\text{magnetic dipole moment}}{V}$

$$M = \frac{Si m^2}{V}$$

\boxed{M} → depends on how many dipoles are aligned in particular direction, p. e. in direction of external \vec{B} .

$M \propto H$ → mag. field strength.

$\frac{M}{H} = \chi_m$ → mag. susceptibility

$$B = \mu_0(M + H)$$

How easy to magnetise material

$$B = \mu_0(1 + \frac{M}{H})H$$

$$B = \mu_0 \mu_r H$$

$$\mu_r = 1 + \chi_m \rightarrow \text{material dependent property}$$

As time ~~increases~~ progresses, how many dipoles
orient w.r.t \vec{B} determines M .

For materials like Fe, $M \rightarrow$ good.

$\chi_m \rightarrow$ +ve high (10^3 to 10^5)

↓
Ferromagnetic substances

$\chi_m \rightarrow$ -ve small (-10^{-6} to -10^{-3})

↓
Diamagnetic

Ex: Ad, superconductors

$\chi_m \rightarrow$ +ve small (10^{-3} to 10^{-6})

↓
Paramagnetic material

Ferromagnetic

Paramagnetic

Diamagnetic

→ Quantum Theory of ~~Electron~~ Magnetism

→ Angular momentum is quantised.

$$\vec{L} = \underline{n\hbar} = n\hbar$$

$$2\pi$$

$$M = \frac{-e\vec{L}}{2mc} = -\left(\frac{e\hbar}{2m}\right)n = -n\mu_B$$

$$(M_B) \rightarrow \text{Bohr magneton}$$

$$= 9.27 \times 10^{-24} \text{ A m}^2$$

Fundamental magnetic moment

$$L = \sqrt{l(l+1)}\hbar$$

$l \rightarrow$ azimuthal quantum number

$$\mu_l = \frac{-e\sqrt{l(l+1)}\hbar}{2me} = \frac{-e\hbar\sqrt{l(l+1)}}{2me}$$

$$\mathbf{M} = \mu_B \sqrt{l(l+1)}$$

for $n=1$, $l=0$ so M will have no orbital moment

→ Spin magnetic moment

$$\mu_s = \frac{e\hbar}{2me} |s|$$

$$S = \sqrt{s(s+1)} \hbar$$

$$s = \pm \hbar/2$$

angular
Total momentum $\vec{T} = \vec{L} + \vec{S}$

$$\mu_e + \mu_s = \mu_p$$

$$(M_B)_{\text{proton}} = 5.05 \times 10^{-27} \text{ Am}^2$$

→ $\mu_e = \mu_B \sqrt{l(l+1)}$

$$\mu_{\text{proton}} = (M_B)_{\text{proton}} \sqrt{l(l+1)}$$

$$\mu_f = g_e \sqrt{j(j+1)} = g_e \frac{e\hbar}{2me}$$

$g_e \rightarrow$ Lande's g -factor

$$g_e' = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

only spin for s -shell, $l=0$, $s=\pm \hbar/2$

$$g_e = 1 + \frac{1}{2} \left(\frac{3}{2} \right) + \frac{1}{2} \left(\frac{3}{2} \right)$$

$$2 \times \frac{1}{2} \left(\frac{3}{2} \right)$$

$$g_e = 1 + \cancel{\frac{3}{2} + \frac{3}{2}} = 2$$

$$g_e \approx 2.0$$

$$Mg = \frac{ge}{2me} \text{ eti. } [g_e = 2]$$

$$M_s = \mu_j^0 = \frac{\text{eti}}{m_0}$$

$$\Gamma M_j^0 = M_s + M_f \\ \text{but } M_f = 0$$

$\therefore M_s > M_f$, $M_f \rightarrow \text{negligible (gets quenched after sometime)}$.

→ Precession of magnetic moment:

Dot product : $\vec{u} \cdot \vec{B} = MB \cos \theta = E_{mag}$
 Cross product : $\vec{M} \times \vec{B} = MB \sin \theta = \Gamma_{mag}$

$$MB \sin \theta = \Gamma L \text{ (Batho)}$$

$$\vec{\Gamma} = \frac{e}{2me} \vec{B} \sin \theta$$

$$\vec{\Gamma} = \left(\frac{eB}{2me} \right) L \sin \theta \quad ; \quad \Gamma = \frac{eB}{2me} L \sin \theta$$

$$\Gamma = \frac{eB}{2me} \omega_L \sin \theta$$

$$\omega_L = \frac{eB}{2me} \rightarrow \text{Larmor frequency}$$

$$\omega_L = 8.79 \times 10^{10} \text{ Hz}$$

$$\omega_L/B = 8.79 \times 10^{10} \text{ Hz/T} \\ = 87.9 \text{ GHz/T}$$

$$\omega_L = \frac{eB}{2mp}$$

$$\omega_L = 4.78 \times 10^7 \text{ Hz} = 47.8 \text{ MHz}$$

- Precession of total angular momentum:
- Permanent magnetic moment of an atom can't precess around applied field
- Restricted to finite set of orientations
- for given g , allowed orientations are $2g + 1$
- There are 2 conflicting forces:
 - magnetic moment (constructive)
 - thermal energy (kT) (destructive)

$$\langle A \rangle = \frac{\sum g_A e^{-E_A/kT}}{\sum e^{-E_A/kT}}$$

Arg. of any quantised quantity

Magnetic energy $E_g = \vec{H} \cdot \vec{B}$

Dipole moment $\mu = g \mu_B m_g$

$$E_g = g \mu_B m_g B = g \mu_B H m_g$$

$$M = N \times \langle \mu \rangle$$

$$M = N \frac{\sum m_g}{\sum m_g} = g \mu_B \exp\left(\frac{m_g \mu_B H}{k_B T}\right)$$

$$\sum m_g = g \exp\left(\frac{m_g \mu_B H}{k_B T}\right)$$

Arg.

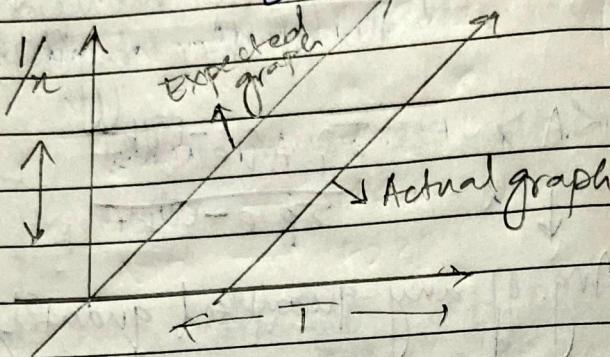
magnetisation

of paramagnetic material
consisting of N atoms per
unit volume

→ Susceptibility of paramagnetic materials
low magnetization

$$M = \frac{Ng^2 \mu_0 M_B^2}{3k_B T} \frac{g(g+1)H}{3} = \chi H$$

$$\chi = \frac{Ng^2 \mu_0 M_B^2 g(g+1)}{3k_B T} = \frac{C}{T}$$



$$\chi = \frac{C}{T - T_c} \quad \text{Curie - Weiss Law}$$

Total field : $H_T = H + H_\theta$

Magnetic Susceptibility :

$$\chi = M/H_T = \frac{M}{H + \lambda M} = \frac{C}{T + T_c}$$

$$\text{Curie - Weiss law } \chi = \frac{C}{T - T_c}$$

Below Curie temperature → Ferromagnetic

Above Curie Temperature → Paramagnetic

→ Diamagnetic materials:

No unpaired e^- in orbitals

Atoms / molecules → 0 magnetic moment
hence, no permanent magnetism.

Ex: Zn → $[Ar] 3d^{10} 4s^2$.

They have negative susceptibility

In presence of ext. field, interaction b/w field E & e^- . This effect similar to that due to Lenz's law, in which, induced mag. fields tend to oppose change which created them.

Diamagnetic \rightarrow negative susceptibility.

$$\text{Magnetic susceptibility: } \chi_{dm} = \frac{N e^2 M_0}{6\pi r^3}$$

$\chi \rightarrow$ independent of temperature.

$r \rightarrow$ radius of nucleus to outermost shell

All materials \rightarrow some degree of diamagnetism at low fields.

→ Paramagnetic materials:

Atoms have non-zero mag. moment due to unpaired e^- . Ex: Al \rightarrow [Ne] $3s^2 3p^1$
They have \uparrow small positive susceptibility

→ Ferromagnetic materials:

large +ve susceptibility values.

Internal dipole moments.

Mag. dipole moments of atoms arising from spin of electrons. Ex: Fe \rightarrow [Ar] $3d^6 4s^2$

Magnetic domains \rightarrow regions where the dipoles are aligned.

→ Hysteresis:

Mag. field density (B) vs applied field (H) forms hysteresis loop.

Saturation magnetisation \rightarrow dipole alignment in direction of applied field.

Retentivity: $B \neq 0$ when $H=0$.

Coresivity: $B=0$ when $H \neq 0$.

Flipping of magnetisation in opp. directions.

→ Anti-ferrimagnetic materials:

Exchange energy $\rightarrow -ve$.

Equal dipole moment & antiparallel to one another.

Magnetisation on each sub-lattice will add to zero - net magnetisation

Net magnetisation is zero below certain temp. called Neel temperature T_N .

Above $\rightarrow T_N \rightarrow$ materials are paramagnets.

Material T_N (K)

FeO \rightarrow 198

MnO \rightarrow 116

NiO \rightarrow 523.

→ Ferrimagnetic materials:

Dif. magnetic moments \rightarrow unequal & antiparallel cations of two/more types

Net magnetisation $\neq 0$

Exhibit spontaneous magnetisation

Exhibit hysteresis

→ Magnetic Susceptibility v/s Temperature

~~Graph~~

→ Dielectrics →

- Conductors & semi-conductors at appropriate temp. conduct electrically.
- Insulators don't conduct electrically.
- Dielectrics → Insulators that have very few or no conduction electrons.

→ Dipoles:

- Very high electrical resistivities ($\sim 10^{14} \Omega \cdot \text{cm}$)
- Electric field (E) creates dipoles leading to polarisation.
- Net dipole moment / unit volume \mathbf{P} polarization P .
- At very high E , due to ionisation of free e^- starts conducting current (few kV/mv).

→ Polarisation:

In parallel plate capacitor, $E_0 = \frac{q_0}{A\epsilon_0}$, where

$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$ → permittivity of free space

If $E = 0$, no net electric dipole moment.
Dielectric placed b/w capacitor plates will induce surface charge (q') $E = \frac{q - q'}{A\epsilon_0} = \frac{q}{A\epsilon_r\epsilon_0}$

$$\text{Capacitance} = \frac{q}{V} = \frac{A\epsilon_0\epsilon_r}{d}$$

$$E_r = \frac{C}{\epsilon_0}$$

→ before dielectric applied.

$$E = \frac{q - q'}{A\epsilon_0\epsilon_r} \Rightarrow E_r \epsilon_0 E = \frac{q}{A} - \frac{q'}{A}$$

$$\frac{q}{A} = \epsilon_0 E_r E + \frac{q'}{A}$$

$$\epsilon_0 E + P = D$$

$$P = D - \epsilon_0 E$$

$$\frac{P}{\epsilon_0} = (\epsilon_r E - \epsilon_0 E) \rightarrow$$

$$\frac{P}{\epsilon_0} = \epsilon_0 (\epsilon_r - 1) E$$

$$\frac{P}{\epsilon_0} = \epsilon_0 \chi_r E = N \epsilon_0 E$$

↑
Dielectric susceptibility

$\chi \rightarrow$ polarisability of individual dipoles.

$$\chi = \frac{N \epsilon}{\epsilon_0} = \epsilon_r - 1$$

Relates macroscopic dielectric susceptibility with macroscopic polarisability.

→ Clausius - Mossotti Relation:

$$\frac{N \epsilon}{3 \epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

Relates macroscopic dielectric constant to microscopic polarisability of material.

→ Types of polarisation:

Separation of charge → polarisation.

→ Space - charge polarisation:

1. If the molecular dipoles in a 10^{-3} m radius water drop are all pointed in the same direction, calculate polarization if water drop. Given dipole moment of water is $6 \times 10^{-30} \text{ Cm}$.

Solu: Polarisation = Magnetisation = Dipoles/volume

$$P = \frac{6 \times 10^{-3}}{4\pi \times (10^{-3})^3}$$

$$P = 1.4323 \times 10^{-21} \text{ C/m}^3$$

\propto (polarisability)

2. Assuming P of KIr atom as $2.18 \times 10^{-40} \text{ F/m}^2$ calculate ϵ_r dielectric constant. KIr has 2.7×10^{25} atoms/volume at NTP.

$$\frac{N_A}{\epsilon_0} = \frac{2.18 \times 10^{-40}}{2.7 \times 10^{25} \times 2.18 \times 10^{-40}}$$

$$\frac{N_A}{\epsilon_0} = \frac{2.7 \times 10^{25} \times 2.18 \times 10^{-40}}{8.854 \times 10^{-12}} = 2.215 \times 10^{-4}$$

$$2.215 \times 10^{-4} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$(2.215 \times 10^{-4})\epsilon_r + 4.43 \times 10^{-4} = \epsilon_r - 1$$

$$(2.215 \times 10^{-4})\epsilon_r - \epsilon_r = -1 - 4.43 \times 10^{-4}$$

$$\epsilon_r (2.215 \times 10^{-4} - 1) = -4.43 \times 10^{-4}$$

$$\epsilon_r = 1.0006$$

3. An elemental dielectric has $\epsilon_r = 12$ & contain 5×10^{29} atoms/m³. Calculate dielectric polarisability

$$\frac{N_A}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\frac{5 \times 10^{29} \times 12}{3 \times 8.854 \times 10^{-12}} = \frac{11}{14}$$

$$\alpha = 4.17 \times 10^{-41} \text{ F/m}^2$$

$$[\alpha_p = 4\pi \epsilon_0 R^3]$$