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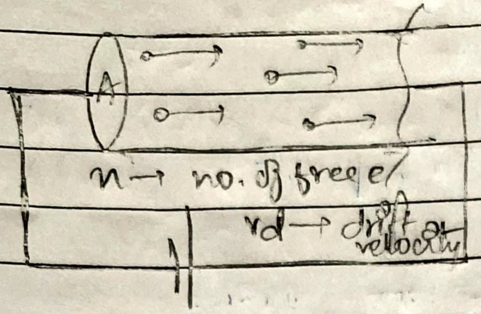
→ Expression for electrical conductivity:



In time dt, free charge travels distance 'l'.

current in volume 'Al' $= \frac{neAl}{dt}$

$V_d = l/dt \Rightarrow dt = l/V_d$
↳ drift velocity

$I = neAV_d$

for Cu wire, 1 ampere, 10 mm diameter

$V_d \simeq 10^{-5}$ m/s

$V_{Th} \simeq 10^{5}$ m/s

$V_{Th} >> V_d$

V_d → gives current, V_{Th} → gives resistance

$F = ma$

$F = \frac{m\,dv}{dt}$ $\langle V \rangle = \langle V_d \rangle + \langle V_{Th} \rangle$

$\langle V_{Th} \rangle = 0$ (avg).

$-Ee = \frac{m\,dv}{dt}$

$-Ee = \frac{m\,dV_d}{dt}$

$\int \frac{-Ee}{m} dt = \int dV_d$

$V_d = \frac{-Ee\tau}{m}$ [τ → relaxation time]

Relaxation time → time taken by e^- to go back to random motion, when electric field is switched off.

Mean collision time → time b/w two successful collisions.

$$I = neA\left(\frac{eE}{m}\right)\tau$$

$$I = \left(\frac{ne^2A\tau}{m}\right)E$$

macroscopic

$$J = \frac{I}{A} = \left(\frac{ne^2\tau}{m}\right)E$$

microscopic measurement of current

$$J = \sigma E \qquad \left[\sigma = \frac{ne^2\tau}{m}\right]$$

$\sigma \rightarrow$ conductivity

$J \propto E$

$\rho \rightarrow$ resistivity $= \frac{1}{\sigma} = \frac{m}{ne^2\tau}$

Mobility $\mu = \frac{V_d}{E}$

$$\sigma = ne\mu$$

→ 1. Temperature dependance on resistivity:

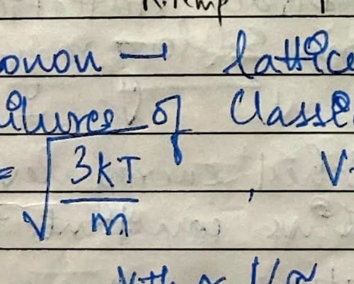
$\sigma = \frac{ne^2\tau}{m}$, e & $m \rightarrow$ constants

$\sigma \propto n$ & $\sigma \propto \tau \rightarrow$ temp. dependant

$n \rightarrow$ material dependent

$\tau \propto 1/\text{temp.} \rightarrow$ as temp ↑, vibration ↑, effective distance, time b/w two collisions ↓.

$\rho_T = \rho_0 + \rho_{phonon} \rightarrow$ Matthesson's Rule



$\rho \rightarrow$ directly proportional after room temp.

phonon → lattice vibration

→ Failures of Classical Theory of free electron:

$V_{th} = \sqrt{\frac{3kT}{m}}$, $V_{th} \propto \sqrt{T}$

$V_{th} \propto 1/\tau$, $\tau \propto 1/\sqrt{T}$

$\rho \propto 1/\tau \Rightarrow \rho \propto \sqrt{T} \rightarrow$ Theory contradicts the experimental observations

2. → conductivity & free electron concentration

	σ	n
Cu	5.9×10^{7} S/m	8×10^{23} ele/m^3
Al	3.5×10^{7} S/m	18×10^{23} ele/m^3

contradicts $\sigma = \frac{ne^2\tau}{m}$ formula

3. Specific heat of electrons:

$$C_V = \frac{dU}{dT}$$

$U = 3/2\, k_B T$ $k_B \to$ Boltzman constant $= 1.38\times10^{-23}$

$C_V = \frac{3}{2} k_B$ for one e^-.

$U = 3/2\, k_B T N_A \to$ for one mole of e^-

$C_V = \frac{3}{2} k_B N_A = \frac{3}{2} R \to$ Theory

R → Gas constant

Experimentally $C_V \simeq 10^{-4} RT$

$R = k_B N_A$

→ Quantum Free electron Theory:

1. For e^- energies are quantised (∵ e^- are quantum particles).
2. Obey Pauli's Exclusion Principle.
3. Follow Fermi-Dirac statistics
4. $e^- - e^-$ interactions & long range

→ Fermi-level → Highest energy level occupied at 0 Kelvin. Remains constant → doesn't depend on temperature.

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→ Per degree rise in temperature: $E_{Thermal} = K_B T$,

when $T = 1K$, $E_{Thermal} = K_B T = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} \simeq 10^{-4} eV$

at $T = 300K$, $E_{Thermal} \approx 0.025 eV$.

Q- What should be temp. to raise last e^- to fermi-level?

Ans: We must raise last e^- to fermi-level & hence, total energy supplied must be equal to E_f.

$$E_{Thermal} = K_B T = E_f$$

$$\therefore T_f = \frac{E_f}{K_B}$$ T_f → fermi temperature (Hypothetical)

→ For Cu, $E_f = 7eV$

$$T_f = \frac{E_f}{K_B} = \frac{7eV \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}} \simeq 8.1 \times 10^4 K$$

$T_f \simeq 81000 K$

This shows, that it is impossible for e^- deep inside the potential well to come to fermi-level.

→ Even though, there are many free e^-, some of them cannot cross the fermi-level & the amount of conducting e^- is less.

Hence $\underset{\text{free } e^-}{N} \neq \underset{\text{conducting } e^-}{n_{eff}}$ & $N > n_{eff}$

$n_{eff} \propto E_{thermal} \propto K_B T$

→ If more free e^-, fermi-energy is higher.

$N \propto E_f$ → when e^--field absent

$$\therefore \frac{n_{eff}}{N} \propto \frac{K_B T}{E_f}$$

for Cu, $\frac{0.025 eV}{7 eV} = 0.0035 = 0.35\%$

for Cu, at RT, $E_f = 7eV$, $K_B T = 0.025 eV$

→ Kinetic energy of e^- at fermi-level

$$\frac{1}{2}mv^2 = E_f.$$

$$V_f = \sqrt{\frac{2E_f}{m}}$$

↓ fermi-velocity

For Cu, $V_f = \sqrt{\frac{2 \times 7 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$

$$V_f \simeq 10^6 \text{ m/s}$$

for Cu, $V_d \simeq 10^{-5}$ m/s

for Cu, $V_{thermal} \simeq 10^5$ m/s

→ Avg fermi-velocity, $\langle V_f \rangle = 0$, as it is random motion.

→ Maxwell - Boltzman $f(E) = g(E)e^{-\Delta E/KT}$

→ Fermi-Dirac $f(E) = \frac{1}{e^{(E-E_f)/KT} + 1}$

→ Bose-Einstein $f(E) = \frac{1}{e^{(E-E_f)/KT} - 1}$
(for particles other than e^-)

$f(E)$ → probability of occupation, which signifies the chance of occupying an energy state E by an electron.

E_f → fermi-energy

E → Any energy other than fermi-energy

Case I: at 0 Kelvin, $E < E_f$

$E - E_f$ → -ve value

$$f(E) = \frac{1}{e^{-\infty} + 1} \qquad \left[\because \frac{1}{e^{\infty}} = 0\right]$$

$$f(E) = \frac{1}{1} = 1$$

Case 2: at 0 Kelvin, $E > E_f$

$E - E_f \rightarrow +ve$

$$f(E) = \frac{1}{e^{\infty}+1} \quad [\because e^{\infty} = \infty]$$

$$f(E) = \frac{1}{\infty} = 0$$

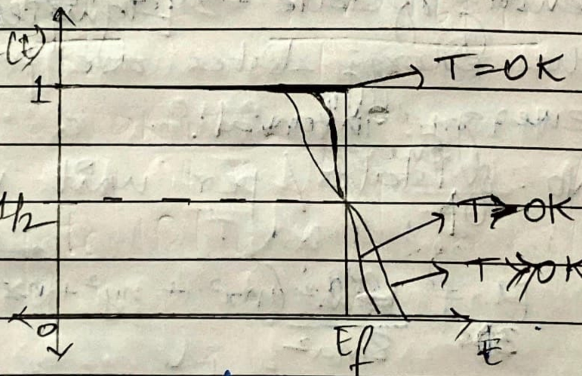
Case 3: at $T > 0$ Kelvin, $E = E_f$

$E - E_f = 0$

$$f(E) = \frac{1}{e^{0}+1}$$ ~~undetermined~~

$$f(E) = \frac{1}{2} \quad [\because e^{0} = 1]$$

→ Fermi-energy is that energy at which the occupation probability is exactly half. at $T > 0K$.



Case 4: $T > 0K$, $E > E_f$, $f(E) = 1/2 \rightarrow 0$

Case 5: $T > 0K$, $E < E_f$, $f(E) = 1/2 \rightarrow 1$

Q- Show that the $f(E)$ of an energy level ΔE, above E_f is equal to the ~~non-occu~~ non-occupation probability of an energy level ΔE below the E_f at any given temp. T.

Ans: Above E_f:

$$E = E_f + \Delta E$$

$$f_1(E) = \frac{1}{e^{\Delta E/kT}+1} \quad \text{--- (1)}$$

Below E_f : $E = E_f - \Delta E$

$$f_2^{\$}(E) = \frac{1}{e^{-\Delta E/kT}+1}$$

$$1 - f_2^{\$}(E) = 1 - \frac{1}{e^{-\Delta E/kT}+1}$$

$$1 - f_2^{\$}(E) = \frac{1}{e^{\Delta E/kT}+1} \quad -②$$

From ① & ②, $f_1(E) = 1 - f_2^{\$}(E)$.

→ Density of Energy States:

No. of free e^- available for conductance

$2\Psi_{123}, 2\Psi_{132}, 2\Psi_{213}, 2\Psi_{231}, 2\Psi_{312}, 2\Psi_{321}$ → 6 states

Energy $= [3^2+2^2+1^2] E_0$

Energy $= 6E_0$ in this case.

Density of state → signifies how densely the energy states are packed in a given energy interval.

No. of energy states / per unit volume b/w E & $E+dE$.

$$E = \frac{h^2 (n_x^2 + n_y^2 + n_z^2)}{8ma^2} \qquad n = 1, 2, 3, \ldots$$

$E = n^2 E_0$ $\quad [n^2 = n_x^2 + n_y^2 + n_z^2]$ $\quad \frac{dE}{dn} = 2nE_0$

$n^2 = E/E_0$ $\quad dE = 2n\,dn\,E_0$

$E + dE = (n+dn)^2 E_0$ [here dn^2 → neglected]

In 3D : Volume of annualar (outer − inner) sphere

$$= \frac{4\pi}{3}(n+dn)^3 - \frac{4\pi}{3} n^3$$

$$= \cancel{\frac{4\pi n^3}{3}} + \frac{4\pi dn^3}{3} + \frac{4\pi}{3}(3n^2 dn) + \frac{4\pi}{3}(3dn^2 n) - \cancel{\frac{4\pi n^3}{3}}$$

$$= \frac{4\pi \cancel{3} n^2 dn}{\cancel{3}}$$

[$\frac{4\pi dn^3}{3}$, $\frac{4\pi}{3}(3dn^2 n)$ are very small values and are neglected]

∴ Volume of annualar sphere $= 4\pi n^2 dn$

$\frac{1}{8}^{th}$ of volume of annualar sphere $= \frac{\pi}{2} n^2 dn$

[n cannot take negative values so, 7 octants in 3D plane are neglected].

$\therefore$ Energy states available in given volume annualar sphere: $\frac{\pi}{2} n^2 dn \rightarrow \frac{\pi}{2} n (n\,dn)$ — ①

$$E = n^2 E_0$$

$$n^2 = E/E_0 \rightarrow n = \sqrt{E/E_0} \quad — ②$$

$$\frac{2n\,dn}{dE} = \frac{1}{E_0}$$

$$\Rightarrow 2n\,dn = dE/E_0$$

$$\Rightarrow n\,dn = dE/2E_0 \quad — ③$$

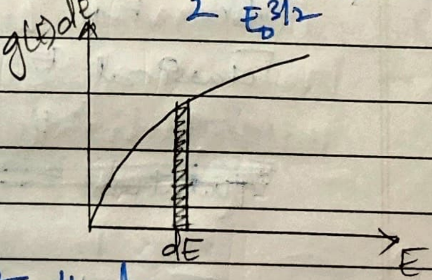
Sub. ② & ③ in ①,

$$\frac{\pi}{2} n(n\,dn) = \frac{\pi}{2}\sqrt{E/E_0} \times \frac{dE}{2E_0}$$

$$\Rightarrow \frac{\pi}{2} n(n\,dn) = \frac{\pi}{4}\frac{E^{1/2}\,dE}{E_0^{3/2}}$$

Each state can hold $2e^-$,

$$\cancel{2} \times \frac{\pi}{\cancel{4}\,2}\frac{E^{1/2}\,dE}{E_0^{3/2}} = \frac{\pi}{2}\frac{E^{1/2}}{E_0^{3/2}}dE$$



$$= \frac{\pi}{2}\frac{E^{1/2}\,dE}{\left(\frac{h^2}{8ma^2}\right)^{3/2}}$$

$$= \frac{\pi}{2}\left[\frac{8ma^2}{h^2}\right]^{3/2} E^{1/2}\,dE$$

Density of electronic states $g(E)\,dE = \dfrac{\frac{\pi}{2}\left[\frac{8ma^2}{h^2}\right]^{3/2} E^{1/2}\,dE}{a^3}$

$\therefore$ Density of electronic states $g(E)\,dE = \frac{\pi}{2}\left[\frac{8m}{h^2}\right]^{3/2} E^{1/2}\,dE$

No. density of electrons occupying E & E+dE : $N(E)\,dE = g(E)\,dE \times f(E)$

$$N(E)\,dE = \cancel{g(E)dE}\ \frac{\pi}{2}\left[\frac{8m}{h^2}\right]^{3/2} E^{1/2}\,dE \times \frac{1}{e^{(E-E_f)/kT}+1}$$

for 0K & $E<E_f$, $f(E)=1$

no. of free e^- / unit volume

$$n = \int_0^{E_f} N(E)\,dE = \int_0^{E_f} \frac{\pi}{2}\left[\frac{8m}{h^2}\right]^{3/2} E^{1/2}$$

$$n = \frac{\pi}{2}\left[\frac{8m}{h^2}\right]^{3/2} \int_0^{E_f} E^{1/2}\,dE$$

$$n = \frac{\pi}{\cancel{2}}\left[\frac{8m}{h^2}\right]^{3/2} E_f^{3/2} \times \frac{\cancel{2}}{3}$$

$$n = \frac{\pi}{3}\left[\frac{8m}{h^2}\right]^{3/2} E_f^{3/2}$$

→ Merits of Quantum free e^- Theory:

→ Temp. variation of 'ρ':

Accd to Classical free e^- theory:

$$\rho \propto \sqrt{T}$$

experimentally : $\rho \propto T$

$$\sigma_{classical} = \frac{ne^2\tau}{m}$$

$$\cancel{\sigma_{quantum} = \frac{n_{eff}e^2 \times \lambda}{m\,v_f}}$$

$$\sigma_{quantum} = \frac{n_{eff}\,e^2\,\lambda}{m\,v_f}$$

λ → mean free path

v_f → fermi velocity

$$\lambda / v_f = \tau$$

$$\sigma_{quantum} = \frac{n_{eff}\,e^2}{m}\,\frac{\lambda}{v_f}$$

here, e^2/m → constant

n_{eff}/v_f → material dependent

λ → temperature dependent

when Temp↑, frequency of vibration ↑, so larger area covered in ~~[illegible]~~ same time, so $\lambda \downarrow$.

$\text{Temp.} \propto \frac{1}{\lambda}$, $\sigma \propto \frac{1}{\text{Temp.}} \propto \sigma$,

$\sigma \propto \frac{1}{\text{Temp.}}$

→ Concentration dependence on 'σ':

Acc^d to Classical free e^- theory: $\sigma \propto n$.

But acc^d to experimental observations:

	n	
Cu	$8.45 \times 10^{22}\ cm^{-3}$	even though
Al	$18.06 \times 10^{22}\ cm^{-3}$	$\sigma_{Cu} > \sigma_{Al}$

Acc^d to Quantum free e^- theory: $\sigma \propto \frac{n_{eff}}{V_f}$

for Cu → $V_f = 1.57 \times 10^8\ cm/s$

for Al → $V_f = 2.02 \times 10^8\ cm/s$

since $\frac{1}{V_{f_{Cu}}} > \frac{1}{V_{f_{Al}}}$, $\sigma_{Cu} > \sigma_{Al}$.

→ Contribution of Electronic Specific Heat:

Acc^d to classical free e^- theory: $C_V = \frac{3}{2}R = \frac{3}{2}N_A k_B$

Acc^d to experimental observations:

$C_V \simeq 10^{-4} RT$

$U = \frac{3}{2} k_B T N_A$.

Acc^d to Quantum free e^- theory:

$U = \frac{3}{2} k_B T n_{eff}$

$n_{eff} = \frac{N k_B T}{E_f}$

$U = \frac{3 k_B T}{2} \frac{N_A k_B T}{E_f}$

$N = N_A$ for monovalent atoms of 1 mole.

$U = \frac{3 N_A k_B^2 T^2}{2 E_f}$

$$U = \frac{3}{2} K_B N_A \frac{K_B T^2}{E_f}$$

$$U = \frac{3}{2}\frac{R K_B T^2}{E_f}$$

$$C_v = \frac{\partial U}{\partial T}$$

$$C_v = \frac{3 R K_B}{\cancel{2} E_f}(\cancel{2} T)$$

$$C_v = \frac{3RTK_B}{E_f}$$

Here, $\frac{K_B}{E_f} \simeq 10^{-4}$ $\left[\begin{array}{l} K_B \approx 10^{-23} \\ E_f \approx 10^{-19} \text{ (while converting)} \end{array} \right.$

$C_{v\,\text{Quantum}} \simeq 3RT \times 10^{-4}$

→ Ratio of thermal conductivity to electrical conductivity
(Weidmann – Franz's Law)

$$\frac{K}{\sigma} = LT$$

L → Lorenz constant

Calculated $L = 1.12 \times 10^{-8}\ W\Omega/K^2$

Observed $L = 2.26 \times 10^{-8}\ W\Omega/K^2$

from classical free e^- theory:

Thermal conductivity $k = \frac{1}{3}\frac{C_v}{V} v_{th} \lambda$

V → volume v_{th} → thermal velocity

C_v → specific heat λ → mean free path

$$K = \frac{\frac{1}{\cancel{3}}\left(\frac{\cancel{3} N_a K_B}{2}\right)}{V} \times \sqrt{\frac{3K_BT}{m}} \times \lambda$$

$$K = \frac{1}{2}\frac{N_A}{V} K_B \times \sqrt{\frac{3K_BT}{m}} \times \lambda$$

$$K = \frac{1}{2} n K_B \sqrt{\frac{3K_BT}{m}}\,\lambda$$

n = no. of e^- per volume

$$\sigma = \frac{ne^2}{m}\frac{\lambda}{v_{th}} = \frac{ne^2}{m}\frac{\lambda}{\sqrt{\frac{3K_BT}{m}}}$$

$$\frac{K}{\sigma} = \frac{\frac{1}{2}\cancel{n}K_B \times \sqrt{\frac{3K_BT}{m}} \times \cancel{\lambda}}{\frac{\cancel{n}e^2}{m} \times \frac{\cancel{\lambda}}{\sqrt{\frac{3K_BT}{m}}}}$$

$$\frac{K}{\sigma} = \frac{1}{2}\frac{K_B}{e^2} \times \cancel{m} \times \frac{3K_BT}{\cancel{m}}$$

$$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{K_B}{e}\right)^2 T$$

$$\frac{K}{\sigma} = LT \quad , \quad L = \frac{3}{2}\left(\frac{K_B}{e}\right)^2$$

From Quantum free e⁻ theory:

$$C_V = \frac{\pi^2}{2} N \frac{K_B^2 T}{E_f}$$

$$K = \frac{1}{3}\frac{C_V}{V} v_f \lambda = \frac{1}{3}\frac{\pi^2}{2}\frac{N}{V}\frac{K_B^2 T}{E_f} v_f \lambda$$

$$K = \frac{1}{3}\frac{\pi^2}{2}\frac{N}{V}\frac{K_B^2 T}{E_f} v_f v_f \tau$$

$$[\lambda = v_f \tau]$$

$$K = \frac{\pi^2}{6} \times n \times \frac{K_B^2 T}{E_f} \times v_f^2 \tau \times \frac{m_e}{m_e}$$

$$K = \frac{\pi^2}{3} \times n \times \frac{K_B^2 T}{\cancel{E_f}} \times \cancel{\frac{1}{2} m_e v_f^2} \times \frac{\tau}{m_e}$$

$$K = \frac{\pi^2}{3}\frac{n K_B^2 T \tau}{m_e}$$

$$\sigma = \frac{ne^2\tau}{m}$$

$$\frac{K}{\sigma} = \frac{\frac{\pi^2 \cancel{n} K_B^2 T \cancel{\tau}}{3 \cancel{m}}}{\frac{\cancel{n}e^2\cancel{\tau}}{\cancel{m}}} = \frac{\pi^2}{3}\left(\frac{K_B}{e}\right)^2 T$$

$$\frac{K}{\sigma} = LT$$

$$L_{calculated} = 2.45 \times 10^{-8}\ W\text{-}\Omega/K^2 \qquad L = \frac{\pi^2}{3}\left(\frac{K_B}{e}\right)^2$$

Phonon → Quanta of lattice vibration

→ Drawback of Quantum free e^- theory:

→ ~~Electron is quantised~~

Assumptions of QFT
- → Energy → quantised
- → Follows Pauli - exclusion principle
- → Follows Fermi - Dirac statistics
- → $e^- - e^-$ & longrange phonon-e^- interactions → neglected

→ It cannot distinguish b/w conductors, semi-conductors & insulators. Acc to this theory, every material has e^- available for conduction. Hence, every material is a conductor acc to QFT.

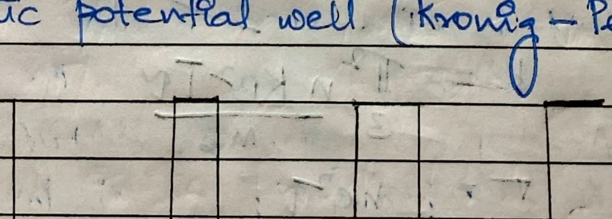
$$R_H = \frac{1}{nq} = \frac{-1}{ne} \text{ (for } e^-\text{)}.$$

R_H → Hall ~~constant~~ coefficient

Material with +ve R_H → e^--deficient (hole)

→ Assumption that longrange photon-e^- interactions are neglected is wrong. As, we are neglecting individual potentials & assuming general potential which is wrong.

→ Periodic potential well (Kronig - Penny Model).



Applying Schrodinger's eq^n:

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi(x) = 0$$

$$V(x+a) = V(x)$$

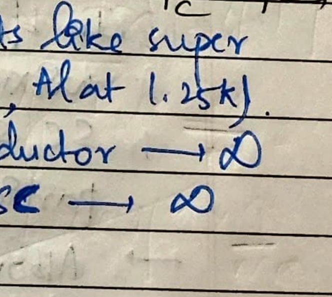
$$\psi(x) = Ae^{ikx}$$
$$\psi(x+a) = Ae^{ik(x+a)} = Ae^{ikx} \cdot e^{ika}$$
$$k = 2\pi/a$$
$$\psi(x+a) = Ae^{ik(x+a)} = Ae^{ikx} \cdot e^{i2\pi}$$
$$\psi(x+a) = \psi(x)\left[e^{i2\pi}\right]$$
$$\psi(x+a) = \psi(x) \cdot u_k(x)$$

[$u_k(x) \rightarrow$ constant]
↓
modulates the wave

↓
Bloch's Theorem
↓
Periodic potential exists in metal, not constant potential.

→ Superconductivity:
- → Resistivity due to lattice vibrations at higher temperature.
- → Even at absolute zero, there is some residual resistivity (ρ_0).

$$\rho = \rho(T) + \rho_0$$

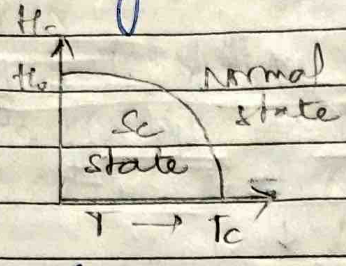


$T_c \rightarrow$ ~~Transition~~ critical temp.
below which material acts like super conductor. (Hg at 4.2K, Al at 1.25K).
- → Conductivity for super conductor $\rightarrow \infty$
- → Mean free path (λ) for SC $\rightarrow \infty$
- → ρ for Sc $\rightarrow 0$

→ Meissner effect:
Superconducting materials excludes magnetic lines of force from its interior → like perfect diamagnetic material.

→ Critical field:
Presence of strong external fields → drives SC material to normal conducting state. Magnetic field at which material loses its superconducting phase → critical field.

→ Critical field strength is temp. dependent & given by $H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]$



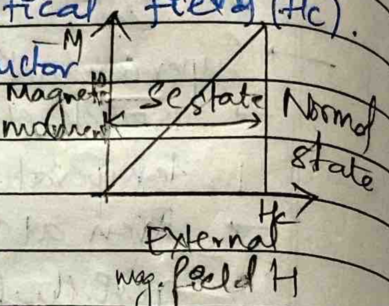
→ Types of superconductor:

→ Type 1 (Soft SC):

→ Exhibit complete Meissner Effect

→ Consists of one critical field (H_c).
Below it → superconductor
above → normal

<u>Ex:</u> Al, Pb, In, etc.

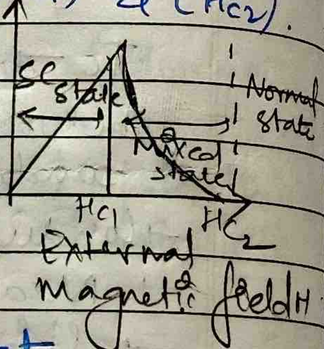


→ Type 2 (Hard SC):

→ Partially obey Meissner effect

→ Has two critical field (H_{c1}) & (H_{c2}).

→ Below H_{c1} → SC state
In b/w H_{c1} & H_{c2} →
mixed (vortex) state
SC state along with
partial flux penetration.



→ Above H_{c2} → normal state

<u>Ex:</u> Transition metals, ~~[illegible]~~ Si, V, Nb

→ Observations:

→ Current in SC persists for long time.

→ Not observed in monovalent metals.

→ Exhibited by metals for which valence e^- no. is b/w 2 & 8.

→ Observed in metals having ↑ ρ at normal temp.

→ Destroyed by applying ↑ mag. fields

mean free path → conductor → high
↓
Super conductor → ∞

or excessive currents.

→ ferro & anti-ferro materials → not SC.

→ BCS Theory of Superconductors.

→ Application of Super Conductors:

→ SQUID → Superconducting Quantum Interference Device → capable of detecting weak magnetic field. Used as sensors in MRI scanning.

→ Used for magnetic levitation.

→ Super conducting systems are potential candidate for QUBITS which are fundamental to creating quantum computers.

Q- Calculate the mobility of e^- in Cu, assuming that each e^- contributes 1 free e^- for conduction. Given: $\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega m$, $W_{Cu} = 63.54$, density $= 8.96 \times 10^3\ kg/m^3$, $N_a = 6.022 \times 10^{23}$

$$\sigma = ne\mu$$

$$\mu = \frac{\sigma}{ne} = \frac{1}{\rho ne}$$

$$n = \frac{\text{no. of free } e^-/\text{atom} \times N_a \times \text{Density}}{\text{Atomic weight} \times 10^{-3}}$$

$$n = \frac{1 \times 6.022 \times 10^{23} \times 8.96 \times 10^3}{63.54 \times 10^{-3}}$$

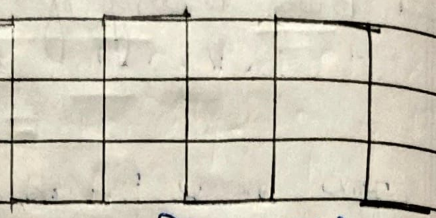
$$n = 8.49 \times 10^{28} / m^3$$

$$\mu = \frac{1}{1.76 \times 10^{-8} \times 1.6 \times 10^{-19} \times 8.49 \times 10^{28}}$$

$$\mu = 4.18 \times 10^{-3}$$

→ Kronig - Penny model :
Approximated the periodic potential as long chain of coupled finite square wells.

For region 1,
with $V = 0$,



$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E\psi = 0 \quad \cancel{\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi_1 = 0}$$

For region 2, $V = V_0$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$$

wave function of e^- in modulated wave given by Bloch function $\psi_k(x) = V_k(x+a)$

Using Bloch Theorem & all boundary conditions for continuity of wave function, soln:

$$\cos(ka) = \frac{ma V_0 c}{\hbar^2} \frac{\sin(ka)}{ka} + \cos(ka)$$

Range from -1 to 1

A transcendental eqn with approx. solns.
Decreasing amplitude with ↑ in ka.

Periodic potential inside solid results in discontinuities in the energy level.

$k = n\pi/a$; $n = \pm 1, \pm 2, \ldots$
↳ soln & energy exists.

In limit $\frac{ma V_0 c}{\hbar^2} \to \infty$, $\sin(ka) = 0$ which implies $k = \frac{n\pi}{a}$ & hence, $E_n = \frac{\hbar^2 \pi^2}{2ma^2}$

In limit $\frac{ma V_0 c}{\hbar^2} \to 0$, $\cos(ka) = 0$ which implies $k = k$, $E_n = \frac{\hbar^2 k^2}{2m}$

→ Case for conductors:

→ Fermi level → uppermost (conduction) band

→ All states below Fermi-level are filled & all levels above are empty.

→ conductors (metals) → partially filled conduction band.

→ Case for semi-conductors,

→ completely filled valence & completely empty conduction band

→ Energy gap 3-5 eV.

→ Case for insulators

→ Energy gap > 5eV.

→ e^- conduction impossible.

Q- $\rho = 1.54 \times 10^{-8}\ \Omega m$, $n = 5.8 \times 10^{28}/m^3$, find τ.

Ans:

$$\sigma = \frac{ne^2\tau}{m}$$

$$\tau = \frac{\sigma m}{ne^2}$$

$$\tau = \frac{0.64 \times 10^{8} \times 9.1 \times 10^{-31}}{5.8 \times 10^{28} \times 1.6 \times 1.6 \times 10^{-38}}$$

$$\tau = 3.92 \times 10^{-14}\ s$$

Q- $\rho = 1.54 \times 10^{-8}\ \Omega m$, Efield of 1V/s, find v_d, μ, τ. Assuming $n = 5.8 \times 10^{28}/m^3$

Ans:

$$\tau = \frac{\sigma m}{ne^2}$$

$$\tau = \frac{0.64 \times 10^{8} \times 9.1 \times 10^{-31}}{5.8 \times 10^{28} \times 1.6 \times 1.6 \times 10^{-38}}$$

$$\tau = 3.92 \times 10^{-14}\ s$$

$$J = \sigma E$$

~~J = 0.64 × 10⁸ × 1~~

$nev_d = \sigma E$

$5.8 \times 10^{28} \times 1.6 \times 10^{-19} \times V_d = 0.64 \times 10^{8} \times 1$

$\therefore V_d = 0.69 \text{ m/s}$

Q- Fermi-level of Ag = 5.5 eV at 0K. Calculate no. of free e^- & probability of occupation for e^- with energy 5.6 eV at same temp.

Ans:- $f(E) = \dfrac{1}{e^{E-E_f/k_BT} + 1}$

$E_f = 5.5 \times 1.6 \times 10^{-19} J = 8.8 \times 10^{-19}$

$E = 5.6 \times 1.6 \times 10^{-19} J = 8.96 \times 10^{-19} J$

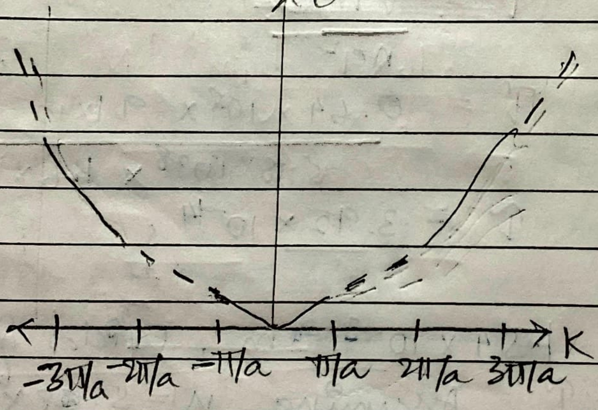
$n = \dfrac{\pi}{3}\left(\dfrac{8m}{h^2}\right)^{3/2} E_f^{3/2}$

$n = 7.067 \times 10^{55} \times 8.255 \times 10^{-28} / m^3$

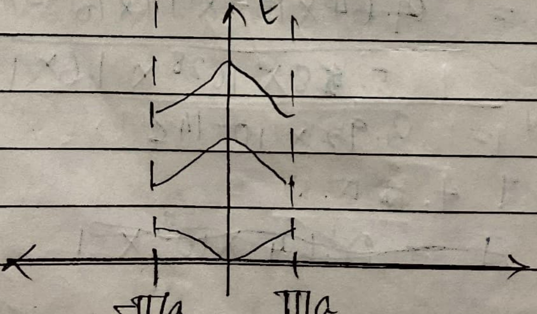
$n = 5.83 \times 10^{28} / m^3$

$\dfrac{E - E_f}{k_BT} = \dfrac{(5.6 - 5.5) \times 1.6 \times 10^{19}}{1.38 \times 10^{-23} \times 300}$

→ Extended Zone Scheme



→ Reduced Zone scheme



$E = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}$ $[\because p = \hbar k]$

$\frac{dE}{dk} = \frac{\hbar^2 k}{m}$ ← velocity

$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m}$

~~$\frac{dE}{dp}$ $E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$ $\frac{dE}{dk}$ $\frac{d^2E}{dp^2}$ $dE/dk = \hbar^2/m$~~

$m = \frac{\hbar^2}{\frac{d^2E}{dk^2}} = \left[\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right]^{-1}$

m^* → apparent/effective mass → it's the ~~effective~~ apparent mass an e^- would gain or lose while moving in the energy band.

$m^* > m_0$ → e^- gained mass
$m^* < m_0$ → e^- lost mass
where m_0 = mass of e^- $= 9.1 \times 10^{-31}$ kg

Q- Calculate critical magnetic field to destroy super conductivity at temp. 0.2K below critical temp. when $H_0 = 5T$. $T_c = 4.2K$

Ans:

$H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]$

$H_c = 5\left[1 - \left(\frac{4.0}{4.2}\right)^2\right]$

~~$H_c = 5(0.997)$~~ $H_c = 5[0.092]$

~~$H_c = 4.98T$~~ $H_c = 0.464\,T$

Q- Calculate temp. at which there is 2% chance that an energy level 0.2eV below fermi-level is occupied.

Ans:

$f(E) = \frac{1}{e^{E-E_f/k_BT} + 1}$

$\frac{2}{100} = \frac{1}{e^{-0.2/k_BT} + 1}$

$$e^{+0.2/k_BT} + 1 = 50$$

$$e^{+0.2/1.38\times10^{23}\times T} + 1 = 50$$

$$\frac{+0.2}{1.38\times10^{-23}\times T} = \ln(49)$$

$$\frac{+0.2\times1.6\times10^{-19}}{1.38\times10^{23}\times T} = 3.8918$$

$$T = \frac{0.2\times1.6\times10^{-19}}{1.38\times10^{-23}\times3.8918}$$

$$T = 595.75\ K$$