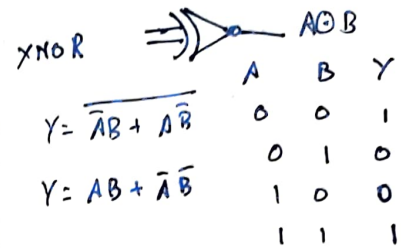


A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

\* Principle of duality : 0 with 1, 1 with 0, + with  $\cdot$ .

# LAWS

\* Distributive Law :  $A + BC = (A+B)(A+C)$

\* De Morgan's Theorem:  $\overline{A+B} = \overline{A} \cdot \overline{B}$   
 $\overline{A+B+C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$

\* Absorption Theorem:  $A + AB = A$  dual:  $A \cdot (A+B) = A$   
 $A + \overline{A}B = A + B$   
 $A(\overline{A}+B) = A \cdot B$

\* Consensus Theorem:  $\overline{A}B + \overline{A}C + B\overline{C} = \overline{A}B + \overline{A}C$

# SOP	POS
make $f=1$ and take them AND-OR $f = \sum m( )$ min-terms	make $f=0$ OR-AND $f = \prod M( )$ max terms

To convert to SOP  
 eg  $F = A + B \cdot C$   
 $F = A \cdot 1 \cdot 1 + B \cdot C \cdot 1$   
 $F = A(B+\overline{B})(C+\overline{C}) + (A+\overline{A})BC$

To convert to POS  
 eg:  $F = (x'+y)(x+z)(y+z)$   
 $F = (x'+y+0)(x+0+z)(0+y+z)$   
 $F = (x'+y+zz')(x+yy'+z)(xx'+y+z)$

# # Half Adder

## COMBINATIONAL CIRCUITS

29th November 2023

Inputs		Outputs	
X	Y	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$\text{Sum} = \bar{X} \cdot Y + X \bar{Y} = X \oplus Y \quad (X \text{ xor } Y)$$

$$\text{Carry} = X \cdot Y$$

Using NAND only

4 NAND for Sum

1 NAND for Carry

5

1 XOR for Sum

1 AND for Carry

2

# # Full Adder

Input			Output	
X	Y	Z	Carry	Sum
0	0	0	0	0
1	0	0	0	1
0	1	0	0	1
1	1	0	1	0
0	0	1	0	1
1	0	1	1	0
0	1	1	1	0
1	1	1	1	1

$$\text{Sum} = \bar{X} \bar{Y} Z + \bar{X} Y \bar{Z} + X \bar{Y} \bar{Z} + X Y Z$$

$$= X \oplus Y \oplus Z$$

$$\text{Carry} = (X \oplus Y) Z + X \cdot Y = XY + YZ + XZ$$

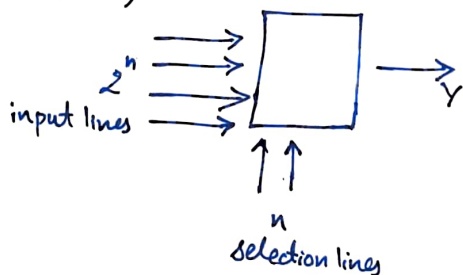
9 NAND

2 XOR

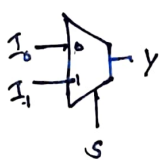
2 AND

1 OR

# # Multiplexer (MUX)



→ 2:1 MUX



S	Y
0	I <sub>0</sub>
1	I <sub>1</sub>

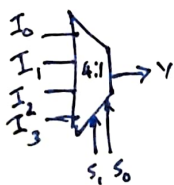
$$Y = S' I_0 + S I_1$$

using NAND gates →

$$Y = ((S' I_0 + S I_1)')'$$

$$Y = ((S' I_0)' \cdot (S I_1)')' \rightarrow 4 \text{ NAND}$$

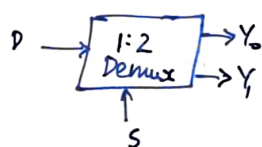
→ 4:1 MUX



S <sub>1</sub>	S <sub>0</sub>	Y
0	0	I <sub>0</sub>
0	1	I <sub>1</sub>
1	0	I <sub>2</sub>
1	1	I <sub>3</sub>

$$Y = S_1' S_0' I_0 + S_1' S_0 I_1 + S_1 S_0' I_2 + S_1 S_0 I_3$$

# # Demultiplexer (DEMUX)



S	Y <sub>0</sub>	Y <sub>1</sub>
0	D	-
1	-	D

$$Y_0 = S' D \quad Y_1 = S D$$

↓

using NAND

$$Y_0 = ((S' D)')' \quad Y_1 = ((S D)')' \rightarrow 4 \text{ NAND}$$

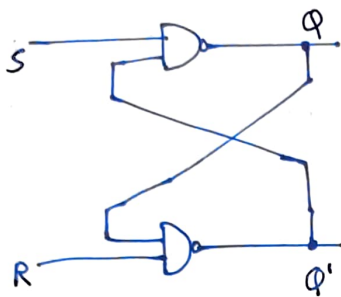
# SEQUENTIAL CIRCUITS

\* When clk = 0  
always memory state

Synchronous  
↓  
clock

Asynchronous  
↓  
no clock

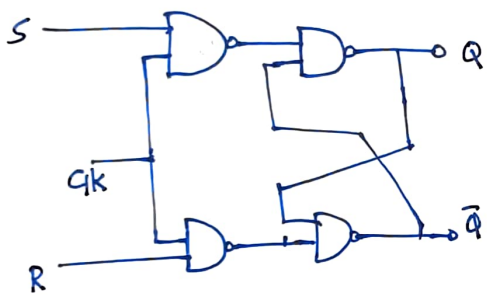
## # The R-S (Reset - Set) Latch



Input		Output
S	R	$Q_{t+1}$
1	1	$Q_{prev.}$ (memory)
0	1	1 (set)
1	0	0 (reset)
0	0	Invalid

$$Q_{t+1} = \bar{S}R + Q\bar{R}$$

## # The R-S (Reset-Set) Flip-Flop / Gated R-S latch

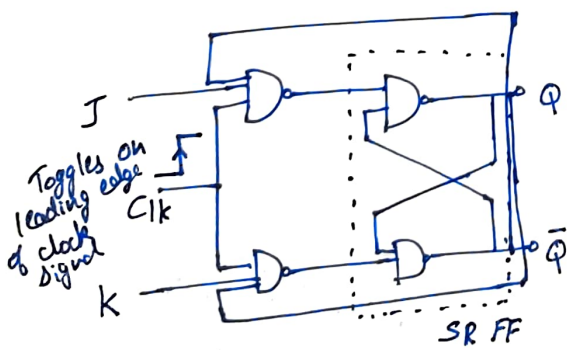


clk	$S_t$	$R_t$	$Q_t$	$Q_{t+1}$	Operation
1	0	0	0	0	No change (memory)
1	0	0	1	1	
1	0	1	0	0	Reset
1	0	1	1	0	
1	1	0	0	1	Set
1	1	0	1	1	
1	1	1	0	X	Invalid
1	1	1	1	X	
0	X	X	X	$Q_t$	No change (memory)

clk = 0  $\Rightarrow$  memory  
 $S=0, R=0$   $\Rightarrow$  memory  
 $S=0, R=1$   $\Rightarrow$  reset  
 $S=1, R=0$   $\Rightarrow$  set

$$Q_{t+1} = S + \bar{R}Q$$

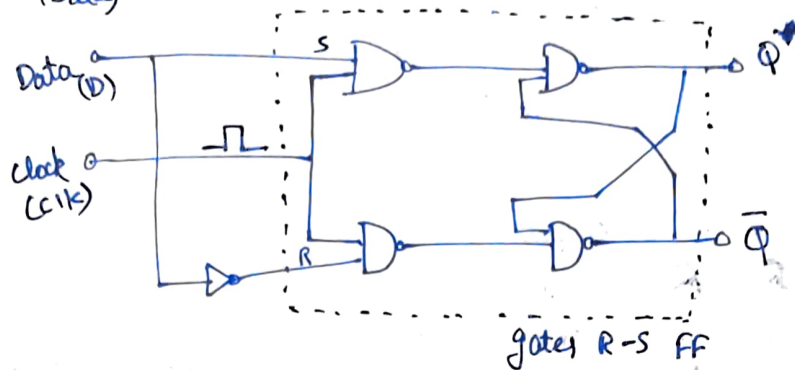
## # JK Flip-Flop (J=S, K=R)



clk	J	K	$Q_t$	$Q_{t+1}$	Operation
1	0	0	0	0	memory
1	0	0	1	1	
1	0	1	0	0	Reset $Q=0$
1	0	1	1	0	
1	1	0	0	1	Set $Q=1$
1	1	0	1	1	
1	1	1	0	1	Toggle
1	1	1	1	0	
0	X	X	X	0	Toggle action
0	X	X	X	1	

$$Q_{t+1} = J\bar{Q} + \bar{K}Q$$

## # D-Flip-Flop (Data)



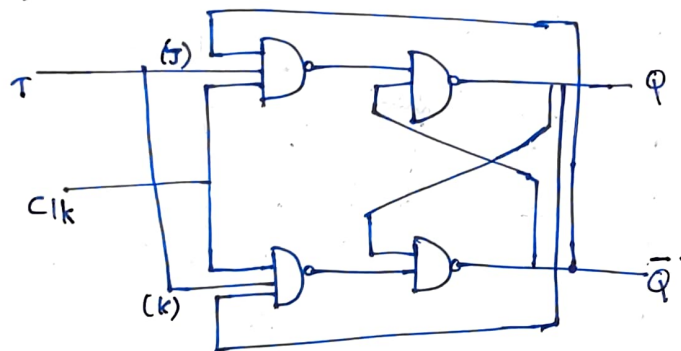
clk	D	$Q^+$
0	X (doesn't matter)	$Q$ (memory)
1	0	0
1	1	1

} o/p follows i/p

Input D	present state $Q_t$	next state $Q_{t+1}$
0	0	0
0	1	0
1	0	1
1	1	1

$$Q_{t+1} = DQ$$

## # T-Flip-Flop (Toggle)



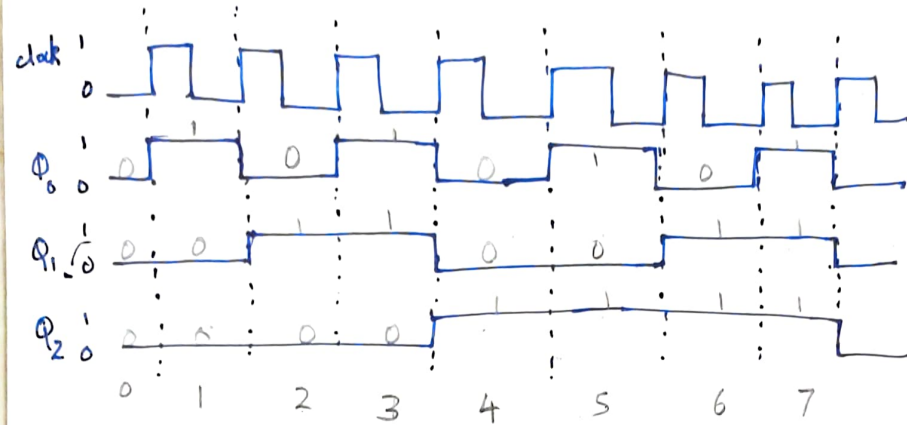
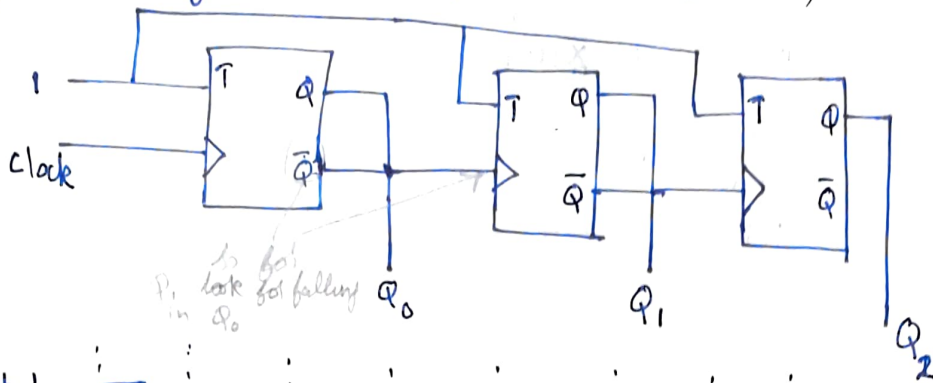
clk	T	$Q^+$
0	X	$Q$ (memory)
1	0	$Q$ (memory)
1	1	Toggles

T	Q	$Q^+$
0	0	0
0	1	1
1	0	1
1	1	0

$$Q_{t+1} = T\bar{Q} + \bar{T}Q$$

$$= T \oplus Q$$

## 3-bit asynchronous ~~UP~~ counter (uses TFF)



no. of negative edge of clock.

$Q_0$  (LSB)

$Q_1$

$Q_2$  (MSB)

0  
1  
2  
3  
4  
5  
6  
7

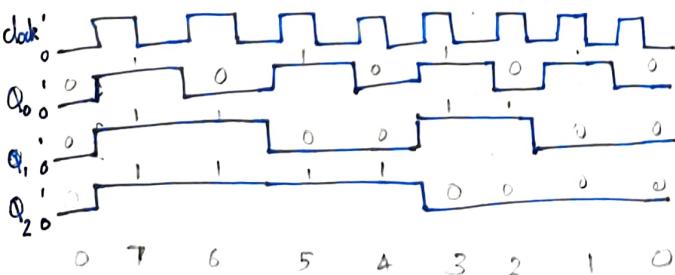
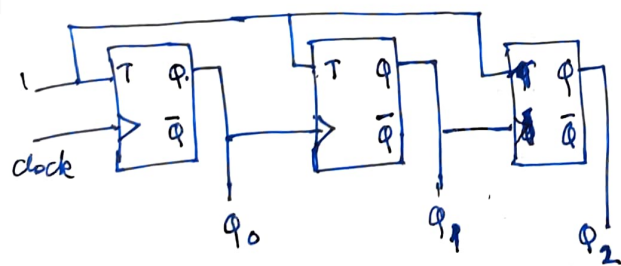
0  
1  
0  
1  
0  
1  
0  
1

0  
0  
1  
1  
0  
0  
1  
1

0  
0  
0  
0  
1  
1  
1  
1

SISO register uses D FF

## # 3-bit asynchronous DOWN counter (uses TFF)



no. of negative edge of clock

$Q_0$  (LSB)

$Q_1$

$Q_2$  (MSB)

0  
1  
2  
3  
4  
5  
6  
7

0  
1  
0  
1  
0  
1  
0  
1

0  
1  
1  
0  
0  
1  
1  
0

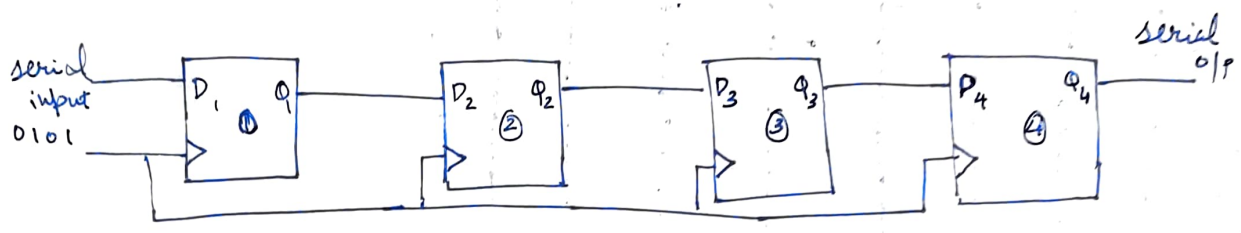
0  
1  
1  
1  
1  
0  
0  
0



\* NAND & NOR  $\rightarrow$  universal gates

	NOT	AND	OR	NOR	XOR	XNOR
NAND $F = \overline{X \cdot Y}$	1	2	3	4	4	5
NOR $F = \overline{X + Y}$	1	3	2	4	5	4

# # 4 bit SISO shift register



clk		Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>
0	0	0	0	0	0
clk 1	1	0	0	0	0
clk 2	0	1	0	0	0
clk 3	1	0	1	0	0
clk 4	0	1	0	1	0
clk 5	0	0	1	0	1
clk 6	0	0	0	1	0

clk		Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>
0	0	0	0	0	0
clk 1	1	1	0	0	0
clk 2	1	0	1	0	0
clk 3	1	1	0	1	0
clk 4	1	0	1	0	1
clk 5	1	0	0	1	0
clk 6	1	0	0	0	1
clk 7	1	0	0	0	0