& can easily determine

If we know ⊥ perpendicular dist b/w the line of action of force & moment centre then scalar is better.

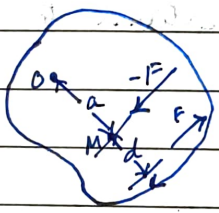
However if F & R are not perpendicular then vector is better.

## UNIT-2

## COUPLE

The moment produced by two equal, opposite and non collinear forces is called a couple.

Unique property of a couple : * Moment of the couple is same for all moment centres



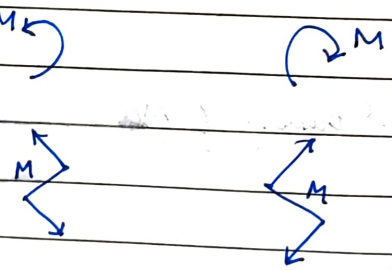

$(M_{F_L})_O = F(a+d)$

$(M_{F_M})_O = -F(a)$

$M = F(a+d) - F(a) = Fd$

* Couple is a free vector

→ Sign convention: Right hand rule




Equilibriant ?

Equilibrium: Set of forces whose resultant is 0

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~~Equilibria~~

Equilibrant will have same magnitude as resultant but opposite direction of equilibrium.

18th October, 2022

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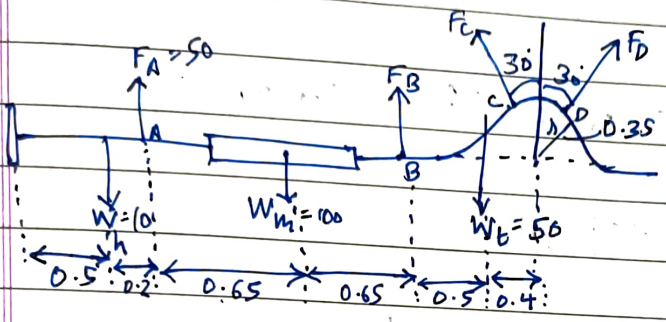
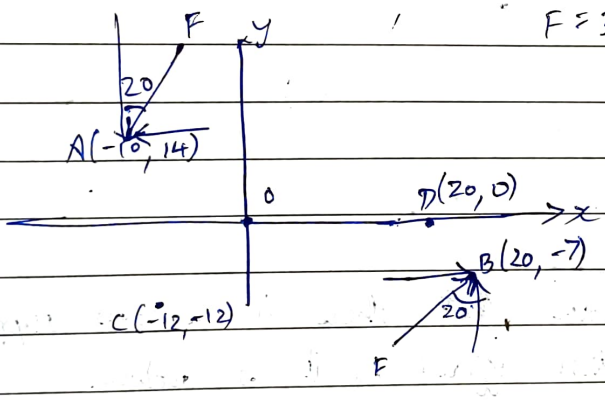



27th October, 2022

2/59. $M = 400 \times 35 \times 10^{-3}$

$= 14$ Nm c.w

2/60.




a) $M_o = +300\cos 20(10) + 300\sin 20(14) + 300\cos 20(7) + 300\sin 20(20)$

$= 300[9.39 + 4.78] + 300[18.79 + 7.39]$

~~$= 1383$ Nm CCW~~ ~~$= 81$ Nm~~ $= 10\,606.15$ Nm CCW

$=$

(OR) $d = \sqrt{(20+10)^2 + (14+7)^2} = \sqrt{900 + 441} = \sqrt{1341} = 36.61$

$M_o = Fd = 300 \times 36.61 = 10985.9$

2/98 wkt to determine M 2 methods Scalar & vector

Scalar - F x ⊥ dist

$\vec{M} = \vec{r} \times \vec{F}$

position vector which runs from moment centre to any point on the line of action of force

$R = (0.8 + 0.6 + 5\sin 30 - 4 - 3)\hat{i} + 5\cos 30\hat{j}$

$= -3.1\hat{i} - 4.33\hat{j}$ kN

$M = 3\left(\frac{140}{1000}\right) + 4\left(\frac{140+110}{1000}\right) + 0.6\left(\frac{140}{1000}\right) + 0.8\left(\frac{140+110}{1000}\right)$

$= 1.704$ kNm CCW

$R_x \times y = M_o$

$3.1 \times |y| = 1.704$

$|y| = 0.55$

$(0, -0.55)$

2/97 At E we observe that there is a rollers support so the reaction offered by it is normal

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2/60 b) $M_C = -300\cos 20(2) + 300\sin 20(26) + 300\cos 20(5) - 300\sin 20(32)$

$300[-1.88 + 8.89 + 4.69 - 10.94]$

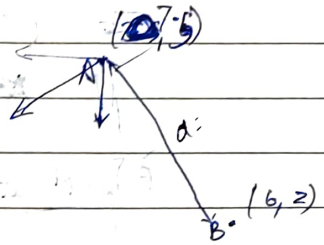
2/61 $M = F \cdot d$

$400 = 6 \times 10^3\, d$

$66.66 \times 10^{-3} = d$

$x = 66.66$ mm

$(0.066, 0)$




2/62 $F = -3.2\cos 30\,\hat{i} - 3.2\sin 30\,\hat{j}$

$= -2.77\,\hat{i} - 1.6\,\hat{j}$

$M_O = 3.2\cos 30\,(7.5)$ kNm CCW

$= 20.775$ kNm CCW

$7.5 - 6.0 = 1.5$

$d = \sqrt{(1.5)^2 + 2^2}$

$\sqrt{2.25 + 4}$

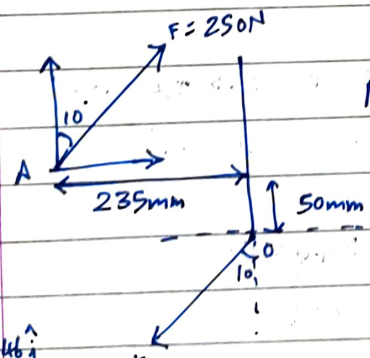
$= 2.5$

$M_B = +3.2\cos 30\,(7.5) + 3.2\sin 30\,(6)$

$= 20.78 + 9.6$

$= 24.38$ kNm CCW

2/71.




$M_O = -250\cos 10\,(0.235) - 250\sin 10\,(0.05)$

$= -57.85 - 2.17$

$= -60.02$

$= 60.02$ Nm CW ✓

F = 250N below horizontal ~~so~~ with horizontal

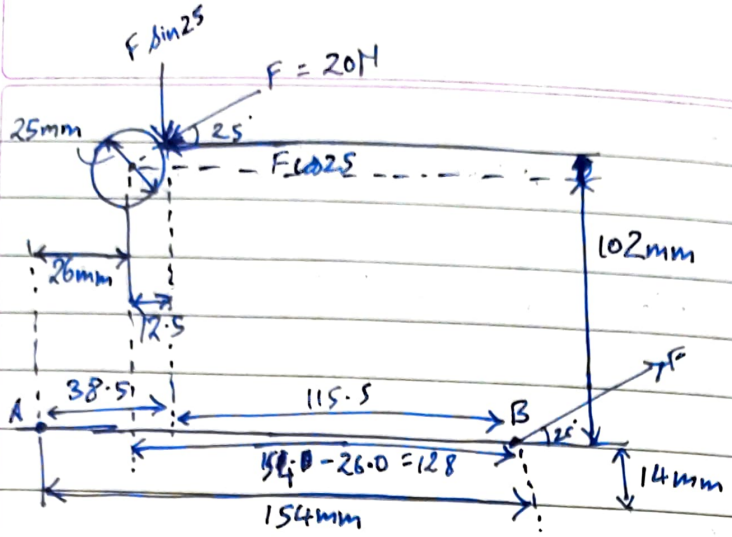
$F = 250\sin 10\,\hat{i} + 250\cos 10\,\hat{j}$

$= 43.4\,\hat{i} + 246.2\,\hat{j}$

$F = 43.4\,\hat{i} + 246\,\hat{j}$

1.395 + 1.187

2/72




$M_B = 2.93$

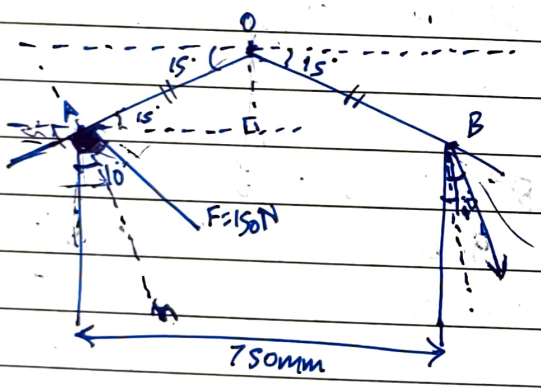
$M_B = +20\cos 25\,(0.102) + 20\sin 25\,(0.128)$

$= 1.84 + 1.08$

$= 2.919$ Nm ccw

F = 20N at 25° ccw above horizontal

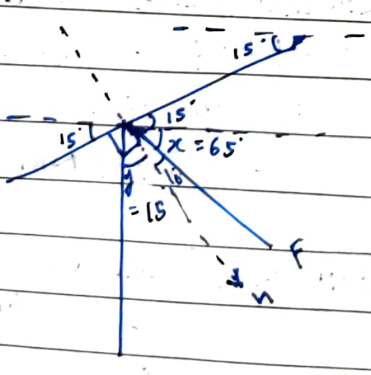
2/73





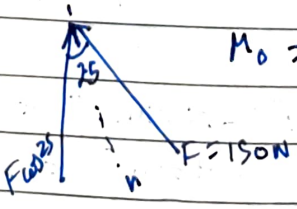
$90 + 10 + 15 + x = 180$

$x = 65$




$\Rightarrow x + 10 + y = 90$

$\Rightarrow y = 15$




$M_O = 150\cos 25\,(0.75)$

$= 101.95$ Nm CW

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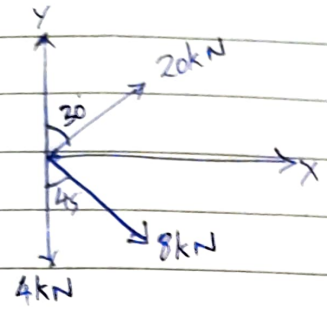


$F_x = 20\sin 30 + 8\sin 45 = 15.65 \text{ kN}$

$F_y = 20\cos 30 - 8\cos 45 - 4 = 17.32 - 5.65 - 4$

$= 7.67 \text{kN}$

$\vec{R} = 15.65\hat{i} + 7.67\hat{j} \text{ kN}$

$\theta = \tan^{-1}\left(\frac{7.67}{15.65}\right) = 26.10°$

$|R| = \sqrt{(15.65)^2 + (7.67)^2} = 17.34 \text{ kN}$

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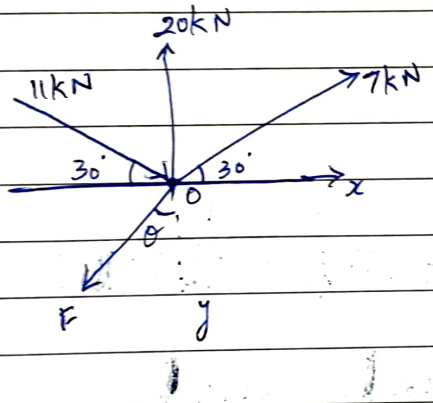



$R_x = 9\text{kN}$ (given directed towards right)

$F_x = 7\cos 30 - F\sin\theta + 11\cos 30 = 9$

$F_y = 20 - F\cos\theta - 11\sin 30 = 0$

$= F\cos\theta$

$9 = 6.06 - F\sin\theta + 9.52$

$F\sin\theta = 6.586$

$\tan\theta = \frac{6.586}{18.9}$

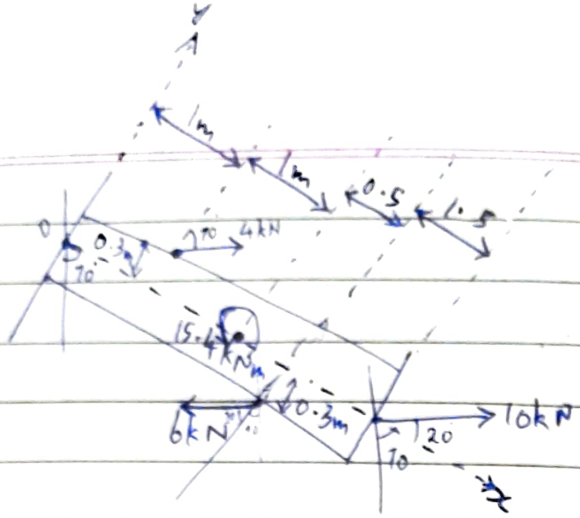
$\theta = 20.09°$

$F = 19.3 \text{ kN}$

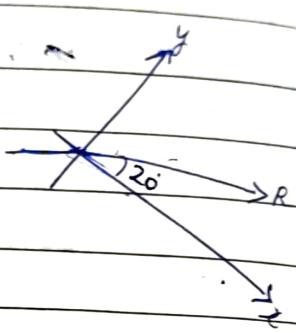
$1.25^2 = 1.56$

$1.8^2 = 3.24$

2/81




$R = 10 + 4 - 6 = 8\text{kN}$.




$R = 8\cos 20\,\hat{i} + 8\sin 20\,\hat{j}$

$= 7.52\,\hat{i} + 2.74\,\hat{j}$ kN

$M_o = 15 + 4\cos 70(1) - 6\cos 70(2.5) - 6\sin 70(0.3)$
$- 4\sin 70(0.3) + 10\sin 20(4)$

$= 15 + 1.368 - 5.13 - 1.69 - 1.27 + 13.68$

$= 22.10$ kNm CCW

Eqn: of line of action

$\vec{r} \times \vec{F} = M_o$

$(x\hat{i} + y\hat{j}) \times (7.52\hat{i} + 2.74\hat{j}) = 22.10\hat{k}$

$2.74x - 7.52y = 22.10$

$2.74x - 22.10 = 7.52y$

$y = 0.034x - 2.93$

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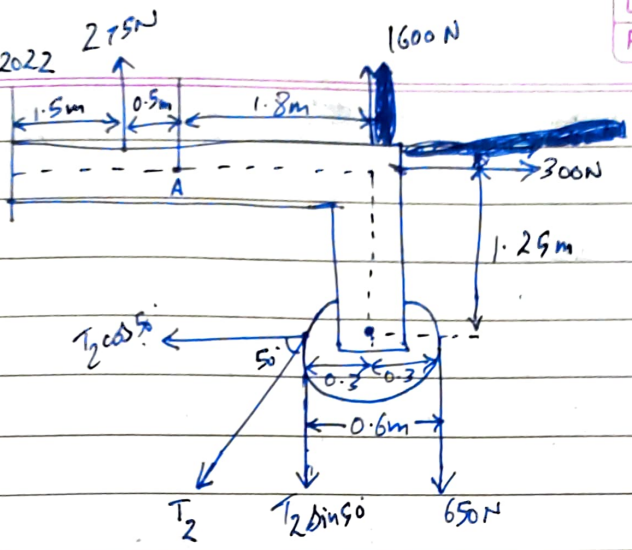


$\Sigma M_A = 0$

$-275(0.5) + 1600(1.8) - 650(1.8+0.3) - T_2\cos 50(1.25 - 0.3) - T_2\sin 50(1.8 - 0.3) = 0$

$-137.5 + 2880 - 1365 - T_2(0.81) - T_2(1.149) = 0$

$1377.5 = T_2(1.959)$

$T_2 = 706.77\,N$ ✗ 783 ✗

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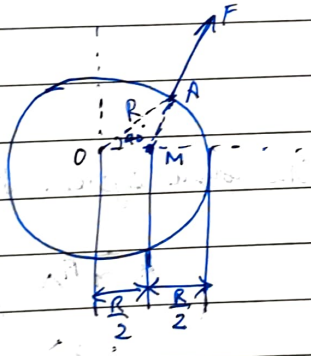
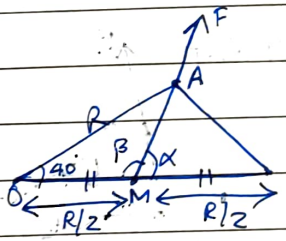


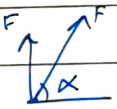

equivalent force-couple system at O




$AM^2 = R^2 + \left(\frac{R}{2}\right)^2 - 2R\left(\frac{R}{2}\right)\cos 40$

$= 0.484R^2$

$AM = 0.696R$




$\frac{\sin\beta}{R} = \frac{\sin 40}{0.696R}$ $\quad \beta = 112.5$

$\Rightarrow \alpha = 180 - 112.5 = 67.5$

$M_O = (F\sin\alpha)\frac{R}{2} = 0.462FR$ CCW

2/76 $\Sigma M_O = -4\cos 15\,(40) + 4\sin 15\,(10) + 300 = 155.8$ N·mm CCW

$F_x = 4\cos 15 = 3.863$

$F_x\, y = 155.8$

$y = \frac{155.8}{3.863} = 40.331$ below O

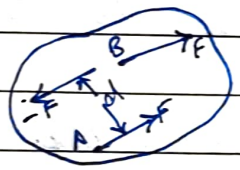
$\Rightarrow (0, -40.3)$ ✓

$F\cos 20\,(x\cos 25 + 100\cos 10)$

$F\sin 20\,(x\sin 25 + 100\sin 10)$

14th November, 2022

pdf pg 66 fig 2/12 # Force Couple System

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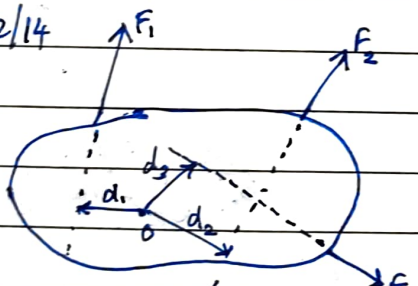
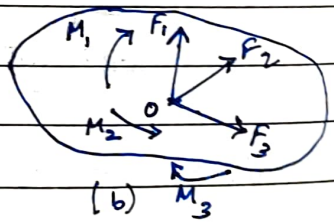
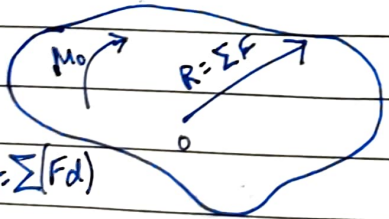



(a) Force system

Force couple system at O of A

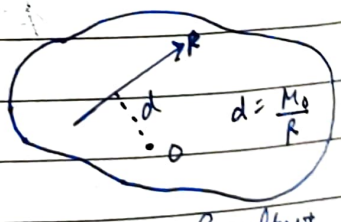



(b)




$M_O = \Sigma(Fd)$

(c) Resultant




(d) Resultant shifted to a distance d

Difference b/w 2/12 and 2/14

Ans: In 2/12 there is only 1 force, in 2/14 there are two forces.

Define Force - Couple system

Ans: As given in 2/12 we are finding the effect of the force which is acting at A but the effect is on B. ~~That means~~ To do that we draw two forces at B i.e +F & -F. the +F at B is same as +F at A and the moment M is equal to F x perpendicular distance b/w moment centre B and line of action of force at A.