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HW 23: 4.27, 4.31 - 4.35
M328K
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4.27 Question. *The numbers* 1, 5, 7, *and* 11 *are all the natural numbers less than or equal to* 12 *that are relatively prime to* 12, *so* $\phi(12) = 4$.

1. *What is* $\phi(7)$*?*
2. *What is* $\phi(15)$*?*
3. *What is* $\phi(21)$*?*
4. *What is* $\phi(35)$*?*

Answer. □

4.31 Theorem. *Let n be a natural number and let $x_1, x_2, \ldots, x_{\phi(n)}$ be the distinct natural numbers less than or equal to n that are relatively prime to n. Let a be a non-zero integer relatively prime to n and let i and j be different natural numbers less than or equal to $\phi(n)$. Then $ax_i \not\equiv ax_j \pmod{n}$.*

Proof. □

4.32 Theorem (Euler's Theorem)**.** *If a and n are integers with $n > 0$ and $(a, n) = 1$, then*

$$a^{\phi(n)} \equiv 1 \pmod{n}.$$

Proof. □

4.33 Corollary (Fermat's Little Theorem)**.** *If p is a prime and a is an integer relatively prime to p, then $a^{(p-1)} \equiv 1 \pmod{p}$.*

Proof. □

4.34 Exercise. *Compute each of the following without the aid of a calculator or computer.*

1. $12^{49} \pmod{15}$.

2. $139^{112} \pmod{27}$.

Solution. □

4.35 Exercise. *Find the last digit in the base* 10 *representation of the integer* 13^{474}.

Solution. □