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HW 21: 4.7 - 4.11

M328K

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**4.7 Question.** Choose some relatively prime natural numbers  $a$  and  $n$  and compute the order of  $a$  modulo  $n$ . Frame a conjecture concerning how large the order of  $a$  modulo  $n$  can be, depending on  $n$ .

Answer. □

**4.8 Theorem.** Let  $a$  and  $n$  be natural numbers with  $(a, n) = 1$  and let  $k = \text{ord}_n(a)$ . Then the numbers  $a^1, a^2, \dots, a^k$  are pairwise incongruent modulo  $n$ .

Proof. □

**4.9 Theorem.** Let  $a$  and  $n$  be natural numbers with  $(a, n) = 1$  and let  $k = \text{ord}_n(a)$ . For any natural number  $m$ ,  $a^m$  is congruent modulo  $n$  to one of the numbers  $a^1, a^2, \dots, a^k$ .

Proof. □

**4.10 Theorem.** Let  $a$  and  $n$  be natural numbers with  $(a, n) = 1$ , let  $k = \text{ord}_n(a)$ , and let  $m$  be a natural number. Then  $a^m \equiv 1 \pmod{n}$  if and only if  $k|m$ .

Proof. □

**4.11 Theorem.** Let  $a$  and  $n$  be natural numbers with  $(a, n) = 1$ . Then  $\text{ord}_n(a) < n$ .

Proof. □