TYPE YOUR NAME HERE
HW 23: 4.18 - 4.23
M328K
April 19th, 2012

4.18 Theorem. *Let p be a prime and a be an integer. If $(a, p) = 1$, then* $\text{ord}_p(a)$ *divides $p - 1$, that is,* $\text{ord}_p(a) | p - 1$.

Proof. □

4.19 Exercise. *Compute each of the following without the aid of a calculator or computer.*

1. $512^{372} \pmod{13}$.
2. $3444^{3233} \pmod{17}$.
3. $123^{456} \pmod{23}$.

Solution.

□

4.20 Exercise. *Find the remainder upon division of* 314^{159} *by* 31.

Solution. □

4.21 Theorem. *Let n and m be natural numbers that are relatively prime, and let a be an integer. If $x \equiv a \pmod{n}$ and $x \equiv a \pmod{m}$, then $x \equiv a \pmod{nm}$.*

Proof. □

4.22 Exercise. *Find the remainder when* 4^{72} *is divided by* 91 $(= 7 \cdot 13)$.

Solution. □

4.23 Exercise. *Find the natural number $k < 117$ such that $2^{117} \equiv k \pmod{117}$. (Notice that 117 is not prime.)*

Solution. □