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HW 13: 3.4, 3.5, 3.7
M328K
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3.4 Question. *Using paper and pencil, but no calculator, can you find the natural number* k, $0 \leq k \leq 11$, *such that* $39^{453} \equiv k \pmod{12}$.

Solution. 3. □

3.5 Exercise. *Show that* 39 *divides* $17^{48} - 5^{24}$.

Solution.

$$\begin{aligned} 17^2 &\equiv 5 \pmod{39} \\ 17^{2^{24}} &\equiv 5^{24} \pmod{39} \\ 17^{48} &\equiv 5^{24} \pmod{39} \\ 39 &\mid 17^{48} - 5^{24} \end{aligned}$$

□

3.7 Question. *Let* $f(x) = 13x^{49} - 27x^{27} + x^{14} - 6$. *Is it true that*

$$f(98) \equiv f(-100) \pmod{99}?$$

Solution. □