TYPE YOUR NAME HERE
HW 18: 3.23-3.25
M328K
March 29th, 2012

3.23 Question. *Let a, b, and n be integers with $n > 0$. How many solutions are there to the linear congruence $ax \equiv b \pmod{n}$ in the canonical complete residue system modulo n? Can you describe a technique to find them?*

Solution. Type your solution here! □

3.24 Theorem. *Let a, b, and n be integers with $n > 0$. Then,*

1. *The congruence $ax \equiv b \pmod{n}$ is solvable in integers if and only if $(a, n)|b$.*

2. *If x_0 is a solution to the congruence $ax \equiv b \pmod{n}$, then all solutions are given by*
$$x_0 + \left(\frac{n}{(a, n)} \cdot m\right) \pmod{n}$$
for $m = 0,\ 1,\ 2,\ \ldots\ ,\ (a, n) - 1$.

3. *If $ax \equiv b \pmod{n}$ has a solution, then there are exactly (a, n) solutions in the canonical complete residue system modulo n.*

Proof. Type your proof here! □

3.25 Exercise. *A band of 17 pirates stole a sack of gold coins. When they tried to divide the fortune into equal portions, 3 coins remained. In the ensuing brawl over who should get the extra coins, one pirate was killed. The coins were redistributed, but this time an equal division left 10 coins. Again they fought about who should get the remaining coins and another pirate was killed. Now, fortunately, the coins could be divided evenly among the surviving 15 pirates. What was the fewest number of coins that could have been in the sack?*

Solution. Type your solution here! □