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HW 16: 3.13-3.17
M328K
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3.13 Theorem. *Suppose $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$ is a polynomial of degree $n > 0$ with integer coefficients. Then $f(x)$ is a composite number for infinitely many integers x.*

Proof. Suppose $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$ is a polynomial of degree $n > 0$ with integer coefficients and $a_n > 0$. We will show that $f(x)$ is a composite number for infinitely many integers x. Let S be the set of integers x such that $f(x)$ is composite. Assume by way of contradiction that S is finite. Let m be the largest element of S. By theorem 2.12, there exists an integer k such that for all integers $j > k$, $f(j) > f(m)$. Now consider these two cases: $a_0 = 0$ and $a_0 \neq 0$. In the case that $a_0 \neq 0$, let $y = (k+1)\,|a_0|$. $\square$

3.14 Theorem. *Given any integer a and any natural number n, there exists a unique integer t in the set $\{0, 1, 2, \ldots, n-1\}$ such that $a \equiv t \pmod{n}$.*

Proof. Type your proof here! $\square$

3.15 Exercise. *Find three complete residue systems modulo 4: the canonical complete residue system, one containing negative numbers, and one containing no two consecutive numbers.*

Solution. Type your solution here! $\square$

3.16 Theorem. *Let n be a natural number. Every complete residue system modulo n contains n elements.*

Proof. Type your proof here! $\square$

3.17 Theorem. *Let n be a natural number. Any set of n integers $\{a_1, a_2, \ldots, a_n\}$ for which no two are congruent modulo n is a complete residue system modulo n.*

Proof. Type your proof here! $\square$