**Geoffrey Parker - grp352**
**HW 13: 3.1-3.3**
**M328K**
**March 6th, 2012**

**3.1 Exercise.** *Show that 41 divides $2^{20}-1$ by following these steps. Explain why each step is true.*

1. $2^5 \equiv -9 \pmod{41}$.
2. $(2^5)^4 \equiv (-9)^4 \pmod{41}$.
3. $2^{20} \equiv 81^2 \pmod{41} \equiv (-1)^2 \pmod{41}$.
4. $2^{20} - 1 \equiv 0 \pmod{41}$.

*Solution.*

1. $2^5 \equiv -9 \pmod{41}$. Because $2^5 - (-9) = 41$ and $41 \mid 41$.
2. $(2^5)^4 \equiv (-9)^4 \pmod{41}$. By step 1 and thm 1.18.
3. $2^{20} \equiv 81^2 \pmod{41} \equiv (-1)^2 \pmod{41}$. By step 2 and thm 1.11.
4. $2^{20} - 1 \equiv 0 \pmod{41}$. Because $1 \equiv 1 \pmod{41}$ and thm 1.12.

□

**3.2 Question.** *In your head, can you find the natural number $k$, $0 \le k \le 11$, such that $k \equiv 37^{453} \pmod{12}$?*

*Solution.*

$$1 \equiv 37 \pmod{12}$$
$$1^k \equiv 37^k \pmod{12}$$
$$1 \equiv 37^{453} \pmod{12}$$

□

**3.3 Question.** *In your head or using paper and pencil, but no calculator, can you find the natural number $k$, $0 \le k \le 6$, such that $2^{50} \equiv k \pmod{7}$.*

*Solution.* Well, $50 \equiv 2 \pmod{3}$, so $2^{50} \equiv 4 \pmod{7}$. □