## TYPE YOUR NAME HERE

HW 20: 4.1 - 4.6 M328K April 5th, 2012

**4.1 Exercise.** For i = 0, 1, 2, 3, 4, 5, and 6, find the number in the canonical complete residue system to which  $2^i$  is congruent modulo 7. In other words, compute  $2^0 \pmod{7}, 2^1 \pmod{7}, 2^2 \pmod{7}, \ldots, 2^6 \pmod{7}$ .

Solution. Type your solution here! $\Box$
<b>4.2 Theorem.</b> Let a and n be natural numbers with $(a, n) = 1$ . Then $(a^j, n) = 1$ for any natural number $j$ .
Proof. Type your proof here!
<b>4.3 Theorem.</b> Let $a$ , $b$ , and $n$ be integers with $n > 0$ and $(a, n) = 1$ . If $a \equiv b \pmod{n}$ , then $(b, n) = 1$ .
Proof. Type your proof here!
<b>4.4 Theorem.</b> Let a and n be natural numbers. Then there exist natural numbers if and j, with $i \neq j$ , such that $a^i \equiv a^j \pmod{n}$ .
Proof. Type your proof here!
<b>4.5 Theorem.</b> Let $a, b, c,$ and $n$ be integers with $n > 0$ . If $ac \equiv bc \pmod{n}$ and $(c, n) = 1$ , then $a \equiv b \pmod{n}$ .
Proof. Type your proof here!
<b>4.6 Theorem.</b> Let a and n be natural numbers with $(a, n) = 1$ . Then there exists a

Proof. Type your proof here! □

natural number k such that  $a^k \equiv 1 \pmod{n}$ .