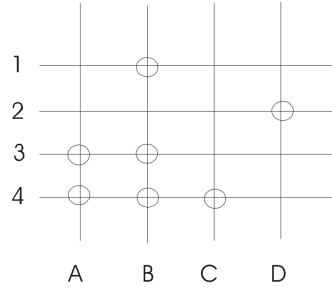


**CS 341 Automata Theory**  
**Elaine Rich**  
**Homework 13**  
**Due: Tuesday, April 17**

This assignment covers Sections 21.1 - 21.3

- 1) In Appendix E.3, we describe a straightforward use of reduction that solves a grid coloring problem by reducing it to a graph problem. Given the grid  $G$  shown here:



- a) Show the graph that corresponds to  $G$ .
  - b) Use the graph algorithm we describe to find a coloring of  $G$ .
- 2) In this problem, we consider the relationship between  $H$  and a very simple language  $\{a\}$ .
- a) Show that  $\{a\}$  is mapping reducible to  $H$ .
  - b) Is it possible to reduce  $H$  to  $\{a\}$ ? Prove your answer.
- 3) Show that  $H_{ALL}$  is not in  $D$  by reduction from  $H$ .
- 4) For each of the following languages  $L$ , state whether or not it is in  $D$ . Prove your answer. Assume that any input of the form  $\langle M \rangle$  is a description of a Turing machine.
- a)  $\{\langle M \rangle : ab \in L(M)\}$ .
  - b)  $\{\langle M, w \rangle : \text{TM } M, \text{ on input } w, \text{ begins by moving right one square onto } w. \text{ Then it never moves off } w\}$ .
  - c)  $\{\langle M \rangle : \text{there exists a string } w \text{ such that } |w| < |\langle M \rangle| \text{ and that } M \text{ accepts } w\}$ .
- 5) In Appendix J.2, we proved Theorem J.1, which tells us that the safety of even a very simple security model is undecidable, by reduction from  $H_\epsilon$ . Show an alternative proof that reduces  $A = \{\langle M, w \rangle : M \text{ is a Turing machine and } w \in L(M)\}$  to the language Safety.