CS 341 Automata Theory
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Homework 6
Due: Tuesday, February 21

This assignment covers Chapter 9 and a review of regular languages.

Chapter 9

1) Define a decision procedure for each of the following questions. Argue that each of your decision procedures gives the correct answer and terminates.

   a) Given two DFSMs M_1 and M_2, is $L(M_1) = L(M_2)^R$?

      Procedure. First, take M_1 and construct a new machine M_1' as follows. Create a new start state and provide epsilon transitions to every accepting state of M_1. Then mark these states as non accepting and mark the old start state as accepting. Now take every state transition and reverse it. Now minimize M_1' and M_2 and compare them. If they are the same machine, then $L(M_1) = L(M_2)^R$ so return $True$. Otherwise return $False$. □

      Proof. □

   b) Given an FSM M and a regular expression α, is it true that $L(M)$ and $L(\alpha)$ are both finite and M accepts exactly two more strings than α generates.

      Procedure. First, determine if $L(M)$ and $L(\alpha)$ are both finite. For M, $L(M)$ is finite iff M has no loops, and for α, $L(\alpha)$ is finite iff α contains neither $*$ nor $+$. Now, if both of them are finite, count all the strings each generates. If $|L(M)| = 2 + |L(\alpha)|$ then return $True$, else return $False$. □

      Proof. □

Review

2) For each of the following languages L, state whether or not L is regular. Prove your answer.

   a) $\{w \in \{0, 1, \#\}^* \ : \ w = x\#y$, where $x, y \in \{0, 1\}^*$ and $|x|\cdot|y| \equiv_5 0\}$. (Let $\cdot$ mean integer multiplication).

      Answer. □

      Proof. □

   b) $\{w \in \{1\}^* : w$ is, for some $n \geq 1$, the unary encoding of $10^n\}$. (So $L = \{1111111111,\ 1^{100},\ 1^{1000},\ \ldots\}$.)

      Answer. □

      Proof. □

3) Define a **color word** to be an English word that is the name of a color. So some example color words are `red`, `fuschia`, and `ochre`. Define an **animal word** to be an English word that is the common name of an animal. So some example animal words are `cow`, `cats`, and `hippopotamus`. Let $L = \{w : w$ is a sentence with legal English syntax and the number of color words in w equals the number of animal words in $w\}$. As examples, observe that `red cats like catnip` $\in L$, but `red cats like blue green balls` $\notin L$. Note that, to be in L, w must satisfy the syntactic rules of English. It is not necessary for w to make sense. So, for example, `red red blue red cats like dogs and dogs and dogs` $\in L$. Prove that L is not regular. (Be particularly careful if you use the Pumping Theorem. You must choose a w that is actually in L.)

Proof. $\square$

4) In this problem, we consider a very restricted subset of Boolean expressions. Define an **operator** to be one of the four symbols: $\neg$, $\wedge$, $\vee$, and $\rightarrow$. Define a variable to be one of the five symbols: `P, Q, R, S` and `T`. Let $L = \{w : w$ is a syntactically legal Boolean expression *without parentheses* and the number of operators in w is exactly equal to the number of variables in $w\}$. Examples:

$\neg P \rightarrow Q$	is in L.
$P \wedge R \wedge \neg S \rightarrow R$	is in L.
$P \rightarrow Q$	is not in L.
$\neg\neg P$	is not in L.

Is L regular? Prove your answer.

Answer. $\square$

Proof. $\square$

5) For each of the following claims, state whether it is $True$ or $False$. Prove your answer.

a) If $L = L_1 L_2$ and L is regular then L_1 and L_2 must be regular.

Answer. $\square$

Proof. $\square$

b) $(\neg(\neg L)$ is regular$) \rightarrow (L$ is regular$)$.

Answer. $\square$

Proof. $\square$

c) $(L_1 - L_2$ is regular$) \rightarrow (L_1$ is regular$)$.

Answer. $\square$

Proof. $\square$

d) $(L^R$ is regular$) \rightarrow (L$ is regular$)$.

Answer. $\square$

Proof. $\square$

e) For any language L, $L \cup \{a^n b^n \ : \ n \geq 0\}$ must not regular.

Answer. $\square$

Proof. $\square$

f) Given any language L, it cannot be true that $L - \{a^n b^n \ : \ n \geq 0\}$ is regular.

Answer. $\square$

Proof. $\square$

g) The finite languages are closed under Kleene star.

Answer. □

Proof. □