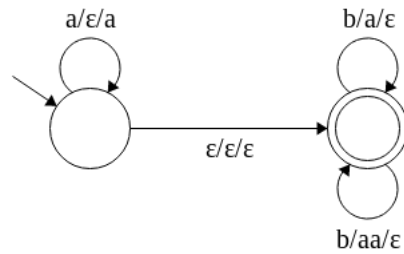**CS 341 Automata Theory**
Geoffrey Parker - grp352
Homework 8
Due Tuesday, March 6

This assignment covers Chapter 12.

1) Build a PDA to accept each of the following languages L:
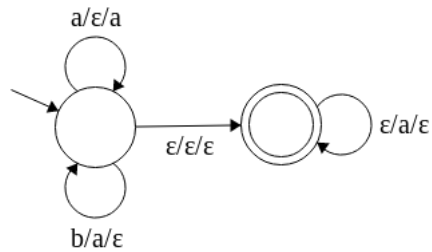
    a) $\{a^n b^m : m \leq n \leq 2m\}$.

a/ε/a   b/a/ε

ε/ε/ε

b/aa/ε

    Solution: ☐

    b) $\{w \in \{a, b\}^* : \text{every prefix of } w \text{ has at least as many a's as b's}\}$.

    Solution:

a/ε/a

ε/ε/ε   ε/a/ε

b/a/ε

    ☐

    c) $\{a^n b^m : m \geq n, m - n \text{ is even}\}$.

    Solution:

a/ε/a   b/a/ε   bb/ε/ε

ε/ε/ε   ε/ε/ε

    ☐
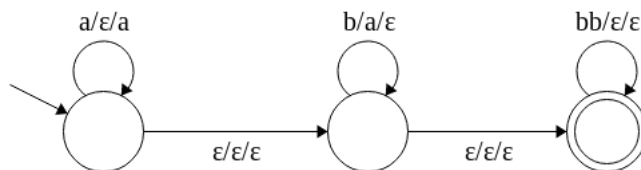
2) Let $L = \{\mathtt{ba}^{m_1}\mathtt{ba}^{m_2}\mathtt{ba}^{m_3}\ldots\mathtt{ba}^{m_n} : n \geq 2, m_1, m_2, \ldots, m_n \geq 0,$ and $m_i \neq m_j$ for some $i, j\}$.

a) Show a PDA that accepts L.
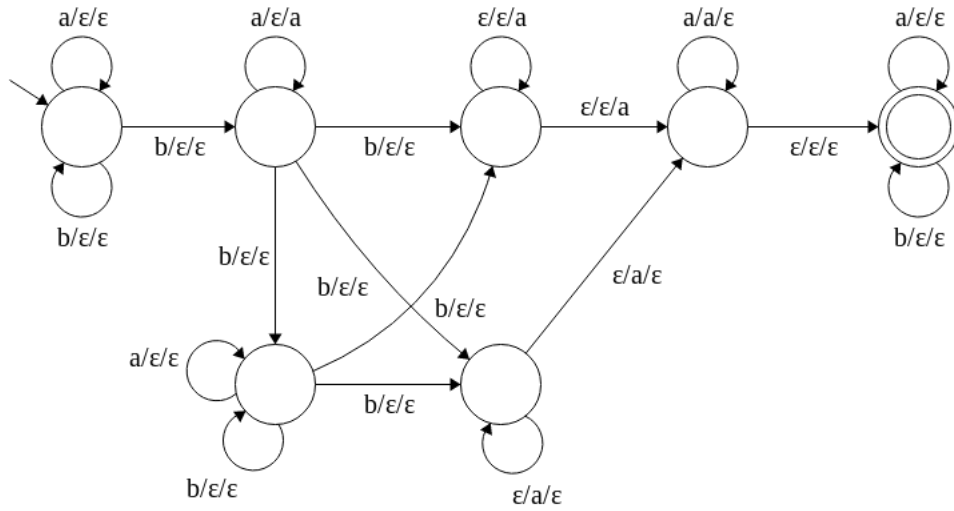
Solution:

b) Show a context-free grammar that generates L.

Solution:

$$S \rightarrow GS \mid SG \mid Y$$
$$Y \rightarrow bL \mid bR$$
$$L \rightarrow aLa \mid aL \mid aXb$$
$$R \rightarrow aRa \mid Ra \mid Xba$$
$$X \rightarrow GX \mid \epsilon$$
$$G \rightarrow Ga \mid b$$

c) Prove that L is not regular.

Proof:

3) Consider the language $L = L_1 \cap L_2$, where $L_1 = \{ww^R : w \in \{\mathtt{a}, \mathtt{b}\}^*\}$ and $L_2 = \{\mathtt{a}^n\mathtt{b}^*\mathtt{a}^n : n \geq 0\}$.

a) List the first four strings in the lexicographic enumeration of L.

Solution: ϵ, `aa`, `aaaa`, `abba`

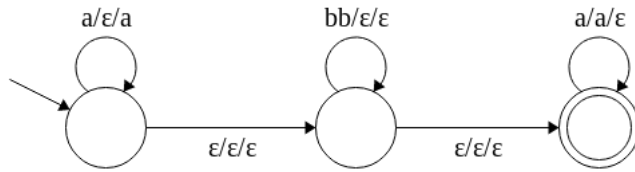b) Write a context-free grammar to generate L.

Solution:

$$S \rightarrow aSa \mid B$$
$$B \rightarrow bBb \mid \epsilon$$

2

c) Show a natural PDA for L. (In other words, dont just build it from the grammar using one of the two-state constructions presented in the book.)

Solution:

□

d) Prove that L is not regular.

Proof: Let $w = a^k bba^k$. Then y must be a^p for some p in the first group of a's. Now pump out and the new string is not in the language because the a's are no longer balanced. □

4) * Let $L = \{w \in \{a, b\}^* : \text{the first, middle, and last characters of } w \text{ are identical}\}$.

a) Show a context-free grammar for L.

Solution: □

b) Show a natural PDA that accepts L.

Solution: □

c) Prove that L is not regular.

Proof: □