CS 341 Automata Theory
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Homework 14
Due: Tuesday, April 23

This assignment covers Sections 21.5 - 21.7

Note: We have skipped Rice's Theorem. So problems 4 - 5 are here just in case you're interested in learning about it. They are optional.

1) For each of the following languages L, state whether it is in D, SD/D or not SD. Prove your answer. Do not use Rice's Theorem. If you claim that L is not in SD, first prove that it's not in D (for practice), then prove that it's not in SD. Assume that any input of the form $\langle M \rangle$ is a description of a Turing machine.

   a) $\{\langle M \rangle$: TM M accepts exactly two strings and they are of different lengths$\}$.

      Answer. □

      Proof. □

   b) $\{\langle M, x, y \rangle : M$ accepts `xy`$\}$.

      Answer. □

      Proof. □

   c) $\{\langle M \rangle$: Turing machine M accepts all even length strings$\}$.

      Answer. □

      Proof. □

   d) $\{\langle M \rangle : M$ rejects exactly three strings that start with `a`$\}$

      Answer. □

      Proof. □

   e) $\{\langle M_a, M_b \rangle : L(M_a) - L(M_b) = \emptyset\}$.

      Answer. □

      Proof. □

2) Prove that TM_{REG} is not in SD.

   Proof. □

3) For any nonempty alphabet Σ, let L be any decidable language other than $\emptyset$ or Σ^*. Prove that $L \leq_M \neg L$.

   Proof. □

4) * Do the other half of the proof of Rice's Theorem, i.e., show that the theorem holds if $P(\emptyset) = True$. (Hint: use a reduction that is not a mapping reduction.)

Proof. □

5) * Use Rice's Theorem to prove that $\{\langle M \rangle : \text{Turing machine } M \text{ accepts at least two odd length strings}\}$ is not in D.

Proof. □