CS 341 Automata Theory
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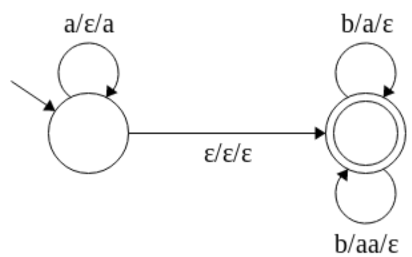
Homework 8
Due Tuesday, March 6

This assignment covers Chapter 12.

1) Build a PDA to accept each of the following languages L:

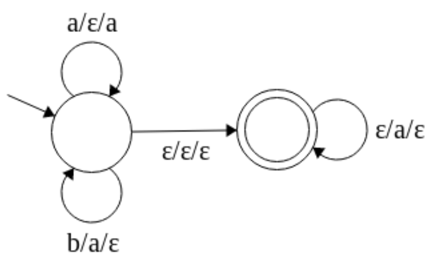
a) $\{\mathtt{a}^n\mathtt{b}^m : m \le n \le 2m\}$.



Solution: □

b) $\{w \in \{\mathtt{a},\mathtt{b}\}^* :$ every prefix of w has at least as many $\mathtt{a}$'s as $\mathtt{b}$'s$\}$.

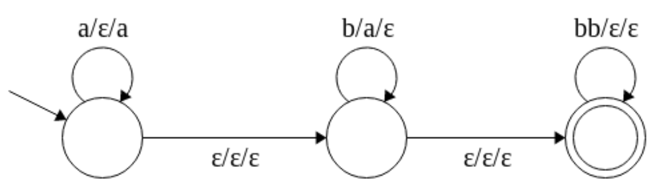
Solution:



□

c) $\{\mathtt{a}^n\mathtt{b}^m : m \ge n, m - n \text{ is even}\}$.

Solution:

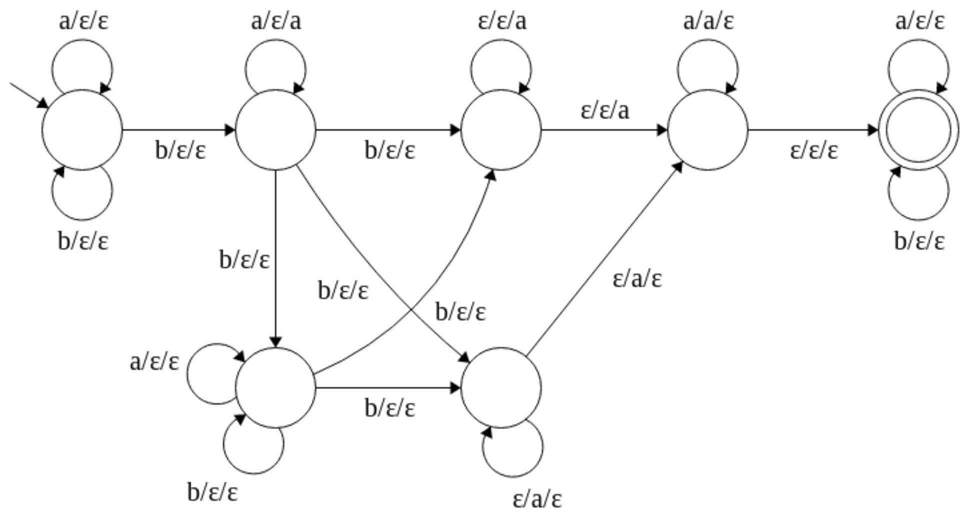


□

2) Let $L = \{\mathtt{ba}^{m_1}\mathtt{ba}^{m_2}\mathtt{ba}^{m_3}\ldots\mathtt{ba}^{m_n} : n \geq 2,\ m_1, m_2, \ldots, m_n \geq 0, \text{ and } m_i \neq m_j \text{ for some } i,\ j\}$.

a) Show a PDA that accepts L.

Solution:



□

b) Show a context-free grammar that generates L.

Solution:

$$
\begin{aligned}
S &\to GS \mid SG \mid Y \\
Y &\to bL \mid bR \\
L &\to aLa \mid aL \mid aXb \\
R &\to aRa \mid Ra \mid Xba \\
X &\to GX \mid \epsilon \\
G &\to Ga \mid b
\end{aligned}
$$

□

c) Prove that L is not regular.

Proof: If L were regular, then L^R would be regular. If L^R were regular, then $\neg(L^R)$ would be regular. If $\neg(L^R)$ were regular, then $L' = \neg(L^R) \cap a^*ba^*b$ would be regular. $L' = a^nba^nb$. Let $w = a^kba^kb$. Then y must be a^p for some $p \geq 1$, with y in the first region of a's. Pumping out generates a new string $w' = a^{n-p}ba^nb$ which is not in L'. So by the pumping theorem L' is not regular. Therefore L is not regular. □

3) Consider the language $L = L_1 \cap L_2$, where $L_1 = \{ww^R : w \in \{\mathtt{a}, \mathtt{b}\}^*\}$ and $L_2 = \{\mathtt{a}^n\mathtt{b}^*\mathtt{a}^n : n \geq 0\}$.

a) List the first four strings in the lexicographic enumeration of L.

Solution: ϵ, `aa`, `bb`, `aaaa` □

b) Write a context-free grammar to generate L.

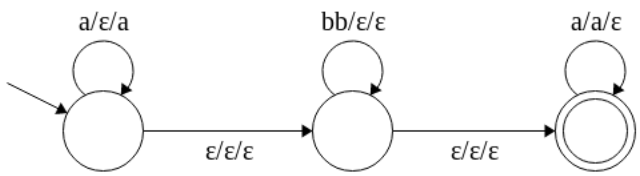
Solution:

$$S \to aSa \mid B$$
$$B \to bBb \mid \epsilon$$

□

c) Show a natural PDA for L. (In other words, dont just build it from the grammar using one of the two-state constructions presented in the book.)

Solution:



□

d) Prove that L is not regular.

Proof: Let $w = a^k bba^k$. Then y must be a^p for some $p \geq 1$ in the first group of a's. Now pump out generates a new string $w' = a^{k-p}bba^k$ which is not in L. Therefore by the pumping theorem L is not regular. □

4) * Let $L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* :$ the first, middle, and last characters of w are identical$\}$.

a) Show a context-free grammar for L.

Solution: □

b) Show a natural PDA that accepts L.

Solution: □

c) Prove that L is not regular.

Proof: □