**CS 341 Automata Theory**
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**Homework 15**
**Due: Tuesday, April 30**

This assignment covers Chapters 22 and 24.

1) Solve the linear Diophantine farmer problem presented in Section 22.1.

2) Consider the following instance of the Post Correspondence problem. Does it have a solution? If so, show one.

| | X | Y |
|---|---|---|
| 1 | a | bab |
| 2 | bbb | bb |
| 3 | aab | ab |
| 4 | b | a |

3) Prove that, if an instance of the Post Correspondence problem has a solution, it has an infinite number of solutions. (Hint: this is really easy.)

4) ) Let $TILES = \{\langle T \rangle$ : any finite surface on the plane can be tiled, according to the rules described in the book, with the tile set $T\}$. Let $s$ be the string that encodes the following tile set:

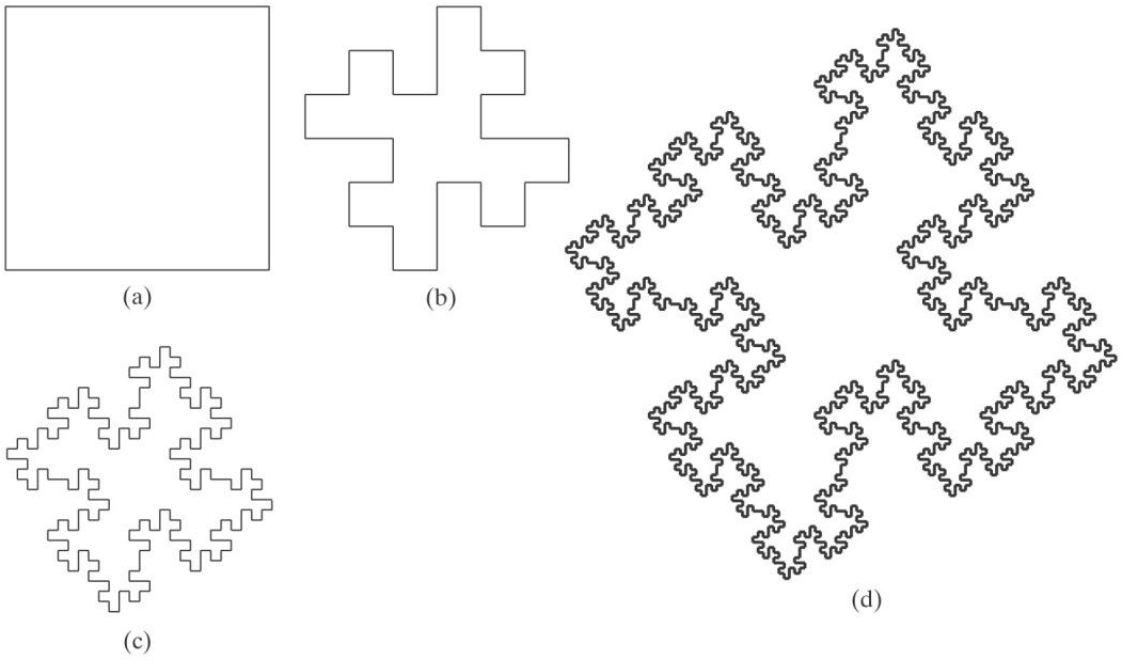
Is $s \in TILES$? Prove your answer.

5) Is $L = \{\langle M \rangle : M$ is a PDA and $L(M) = \{x : x \in \{a, b\}^*$ and $\exists m\ (|x| = 2^m)\}\}$ decidable? Prove your answer.

6) A language $L$ is **D-complete** iff (1) $L$ is in $D$, and (2) for every language $L'$ in $D$, $L' \leq_M L$. Consider the following claim: If $L \in D$ and $L \neq \Sigma^*$ and $L \neq \emptyset$, then $L$ is D-complete. Prove or disprove this claim.

7) Let $\Sigma = \{1\}$. Show that there exists at least one undecidable language with alphabet $\Sigma$. (Hint: Use a counting argument.)

8) The following sequence of figures corresponds to a fractal called a *Koch island*:




These figures were drawn by interpreting strings as turtle programs, just as we did in Example 24.5 and Example 24.6. The strings were generated by an L-system $G$, defined with:

$$\Sigma = \{F, +, -\}.$$
$$\omega = F - F - F - F$$

To interpret the strings as turtle programs, attach meanings to the symbols in $\Sigma$ as follows (assuming that some value for $k$ has been chosen):

- $F$ means move forward, drawing a line of length $k$.
- $+$ means turn left $90°$.
- $-$ means turn right $90°$.

Figure (a) was drawn by the first generation string $\omega$. Figure (b) was drawn by the second generation string, and so forth. $R_G$ contains a single rule. What is it?