CS 341 Automata Theory
Geoffrey Parker - grp352
Homework 12
Due: Tuesday, April 10

This assignment covers Chapter 20.

1) * Let $L_1, L_2, \ldots, L_k$ be a collection of languages over some alphabet Σ such that:

   - For all $i \neq j$, $L_i \cap L_j = \emptyset$.
   - $L_1 \cup L_2 \cup \cdots \cup L_k = \Sigma^*$.
   - $\forall i$ (L_i is in SD).

   Prove that each of the languages L_1 through L_k is in D.

   Proof. □

2) If L_1 and L_3 are in D and $L_1 \subseteq L_2 \subseteq L_3$, what can we say about whether L_2 is in D?

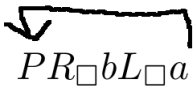
   Answer. Let L_1 be $\emptyset$ and L_3 be the language of all turing machines. If L_2 is $\emptyset$, then it's decidable, if L_2 is H, then it's semidecidable but not decidable, and if L_2 is $\neg H$ then it's not even semidecidable. In all three cases $L_1 \subseteq L_2 \subseteq L_3$, so we can't say anything at all about L_2. □

3) Let M be a Turing machine that lexicographically enumerates the language L. Prove that there exists a Turing machine M' that decides L^R.

   Proof. Construct M', a turing machine with input w. M' will use M to start generating a lexicographic enumeration of L. If it encounters w^R, it will halt and return $true$. If it encounters a string longer than w, it will halt and return $false$. Thus M' decides L^R. □

4) Construct a standard one-tape Turing machine M to enumerate the language $A^n B^n$. Assume that M starts with its tape equal to □. Also assume the existence of the printing subroutine P, defined in Section 20.5.1.

   Solution. $PR_{\square}bL_{\square}a$ then loop.



   □

5) Recall the function mix, defined in Example 8.23. Neither the regular languages nor the context-free languages are closed under mix. Are the decidable languages closed under mix? Prove your answer.

Answer. Yes. □

Proof. Let L be a decidable language and M be a machine that decides L. Then we can construct M', a machine that decides $mix(L)$ by first using the subroutine X to mix the input string, then passing control to M to decide if the string is in L or not.

The subroutine X works as follows:

1. Find the midpoint of the input string.
2. Move left to right over the second half of the string, overwriting it with blanks and copying it right to left onto tape 2. This generates the reverse of the second half of the string on the second tape.
3. Copy the contents of tape two back onto the end of the remaining string on tape 1.

□

6) Let $\Sigma = \{a, b\}$. Consider the set of all languages over Σ that contain only even length strings.

a) How many such languages are there?

Answer. Uncountably infinitely many. It's the powerset of even length strings over Σ. □

b) How many of them are semidecidable?

Answer. Countably infinitely many. Shown in theorem 20.3 in the book. □