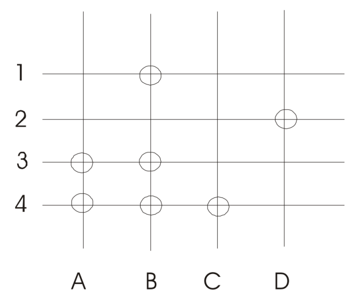
CS 341 Automata Theory
STUDENT NAME - EID
Homework 13
Due: Tuesday, April 17

This assignment covers Sections 21.1 - 21.3

1) In Appendix E.3, we describe a straightforward use of reduction that solves a grid coloring problem by reducing it to a graph problem. Given the grid G shown here:



a) Show the graph that corresponds to G.

Solution.

$$G' = \{V, E\}$$
$$V = \{A, B, C, D, 1, 2, 3, 4\}$$
$$E = \{(A, 3), (A, 4), (B, 1), (B, 3), (B, 4), (C, 4), (D, 2)\}$$

□

b) Use the graph algorithm we describe to find a coloring of G.

Solution. Start with B and color each edge alternately.

$$Red = \{(B, 1), (B, 4)\}$$
$$Blue = \{(B, 3)\}$$

Now do vertex 3:

$$Red = \{(B, 1), (B, 4), (A, 3)\}$$
$$Blue = \{(B, 3)\}$$

Now vertex A:

$$Red = \{(B, 1), (B, 4), (A, 3)\}$$
$$Blue = \{(B, 3), (A, 4)\}$$

Now vertex 4: (it has one of each already, so pick arbitrarily.)

$$Red = \{(B, 1), (B, 4), (A, 3)\}$$
$$Blue = \{(B, 3), (A, 4), (C, 4)\}$$

And now the last edge:

$$Red = \{(B,1),(B,4),(A,3),(D,2)\}$$
$$Blue = \{(B,3),(A,4),(C,4)\}$$

□

2) In this problem, we consider the relationship between H and a very simple language $\{a\}$.

   a) Show that $\{\mathtt{a}\}$ is mapping reducible to H.

   Solution. □

   b) Is it possible to reduce H to $\{\mathtt{a}\}$? Prove your answer.

   Answer. □

   Proof. □

3) Show that H_{ALL} is not in D by reduction from H.

   Solution. □

4) For each of the following languages L, state whether or not it is in D. Prove your answer. Assume that any input of the form $\langle M \rangle$ is a description of a Turing machine.

   a) $\{\langle M \rangle \ : \ \mathtt{ab} \in L(M)\}$.

   Answer. □

   Proof. □

   b) $\{\langle M, w \rangle :$ TM M, on input w, begins by moving right one square onto w. Then it never moves off $w\}$.

   Answer. □

   Proof. □

   c) $\{\langle M \rangle :$ there exists a string w such that $|w| < |\langle M \rangle|$ and that M accepts $w\}$.

   Answer. □

   Proof. □

5) In Appendix J.2, we proved Theorem J.1, which tells us that the safety of even a very simple security model is undecidable, by reduction from H_ϵ. Show an alternative proof that reduces $A = \{\langle M, w \rangle : M$ is a Turing machine and $w \in L(M)\}$ to the language Safety.

   Proof. □