CS 341 Automata Theory
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Homework 10
Due: Tuesday, March 27

This assignment reviews Chapter 13 and covers Chapter 14 and Sections 17.1 - 17.3.

1) For each of the following languages L, state whether L is regular, context-free but not regular, or not context-free and prove your answer.

   a) $\{w : w = uu^R$ or $w = ua^n$: $n = |u|$, $u \in \{a, b\}^*\}$.

   b) $\{a^n b^{2n} c^m\} \cap \{a^n b^m c^{2m}\}$.

   c) L^*, where $L = \{0 * 1^i 0 * 1^i 0* : i \geq 0\}$.

   d) $\neg L_0$, where $L_0 = \{ww : w \in \{a, b\}^*\}$.

   e) $\{x \in \{a, b\}^* : |x|$ is even and the first half of x has one more a than does the second half$\}$.

2) Give a decision procedure to answer the following question: given a context-free grammar G, does G generate any even length strings?

3) Construct a standard, one-tape Turing machine M to decide the language $L = \{x*y = z : x, y, z \in 1^+$ and, when x, y, and z are viewed as unary numbers, $xy = z\}$. For example, the string $1111*11 = 11111111 \in L$. Describe M in the macro language described in Section 17.1.5.

4) Construct a standard 1-tape Turing machine M to compute the function sub_3, which is defined as follows:

$$sub_3(n) = \begin{cases} n - 3 & \text{if } n > 2 \\ 0 & \text{if } n \leq 2. \end{cases}$$

   Specifically, compute sub_3 of a natural number represented in binary. For example, on input 10111, M should output 10100. On input 11101, M should output 11010. (Hint: you may want to define a subroutine.)