

**CS 341 Automata Theory**  
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**Homework 8**  
**Due: Tuesday, March 6th**

This assignment covers Chapter 12.

- 1) Build a PDA to accept each of the following languages  $L$ :
  - a)  $\{a^n b^m : m \leq n \leq 2m\}$ .
  - b)  $\{w \in \{a, b\}^* : \text{every prefix of } w \text{ has at least as many } a\text{'s as } b\text{'s}\}$ .
  - c)  $\{a^n b^m : m \geq n, m - n \text{ is even}\}$ .
- 2) Let  $L = \{ba^{m_1}ba^{m_2}ba^{m_3} \dots ba^{m_n} : n \geq 2, m_1, m_2, \dots, m_n \geq 0, \text{ and } m_i \neq m_j \text{ for some } i, j\}$ .
  - a) Show a PDA that accepts  $L$ .
  - b) Show a context-free grammar that generates  $L$ .
  - c) Prove that  $L$  is not regular.
- 3) Consider the language  $L = L_1 \cap L_2$ , where  $L_1 = \{ww^R : w \in \{a, b\}^*\}$  and  $L_2 = \{a^n b^* a^n : n \geq 0\}$ .
  - a) List the first four strings in the lexicographic enumeration of  $L$ .
  - b) Write a context-free grammar to generate  $L$ .
  - c) Show a natural PDA for  $L$ . (In other words, don't just build it from the grammar using one of the two-state constructions presented in the book.)
  - d) Prove that  $L$  is not regular.
- 4) \* Let  $L = \{w \in \{a, b\}^* : \text{the first, middle, and last characters of } w \text{ are identical}\}$ .
  - a) Show a context-free grammar for  $L$ .
  - b) Show a natural PDA that accepts  $L$ .
  - c) Prove that  $L$  is not regular.