CS 341 Automata Theory
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Homework 9
Due: Thursday, March 22

This assignment covers Chapter 13.

1) For each of the following languages L, state whether L is regular, context-free but not regular, or not context-free and prove your answer.

   a) $\{(ab)^n a^n b^n : n > 0\}$.

   Answer. Not context-free. □

   Proof. □

   b) $\{xwx^R : x, w \in \{0,1\}^+\}$.

   Answer. Context-free but not regular. □

   Proof. □

   c) $\{a^i b^n$: i, $n > 0$ and $i = n$ or $i = 2n\}$.

   Answer. Context-free but not regular. □

   Proof. □

   d) $\{0^i 1^j$: $i, j \geq 0$ and $j = i^2\}$.

   Answer. Not context-free. □

   Proof. □

   e) $\{a^n b^m c^k : m \leq min(n, k)\}$.

   Answer. □

   Proof. □

   f) $\{x\#y : x, y \in \{0,1\}^*$ and when x and y are viewed as binary numbers, $y = x^2\}$. For example, the string $100\#10000 \in L$.

   Answer. Not context-free. □

   Proof. □

2) Give an example of a context-free language $L(\neq \Sigma^*)$ that contains a subset L_1 that is not context-free. Prove that L is context free. Describe L_1 and prove that it is not context-free.

   Describe L. □

   Proof. □

   Describe L_1. □

Proof. □

3) * Give an example of a context-free language L, other than one of the ones in the book, where $\neg L$ is not context-free.

Answer. □

4) Are the context-free languages closed under each of the following operations? Prove your answer.

a) $chop(L) = \{w : \exists x \in L\ (x = x_1 c x_2 \wedge x_1 \in \Sigma_L^* \wedge x_2 \in \Sigma_L^* \wedge c \in \Sigma_L \wedge |x_1| = |x_2| \wedge w = x_1 x_2)\}$.

Answer. □

Proof. □

b) Letter substitution.

Answer. □

Proof. □

5) Let $alt(L) = \{x : \exists y, n\ (y \in L, |y| = n, n > 0, y = a_1 \ldots a_n, \forall i \leq n(a_i \in \Sigma)$, and $x = a_1 a_3 a_5 \ldots a_k$, where $k =$ (if n is even then $n - 1$ else n))$\}$.

a) Consider $L = a^n b^n$. Clearly describe $L_1 = alt(L)$.

Answer. □

b) Are the context free languages closed under the function alt? Prove your answer.

Answer. □

Proof. □

6) Suppose that L is context-free and R is regular. Is $R - L$ necessarily context-free? Prove your answer.

Answer. □

Proof. □