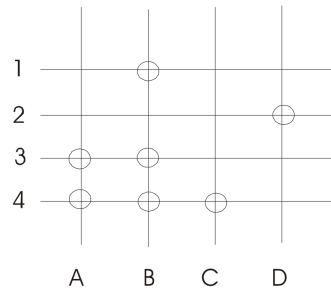


**CS 341 Automata Theory**  
**STUDENT NAME - EID**  
**Homework 13**  
**Due: Tuesday, April 17**

This assignment covers Sections 21.1 - 21.3

- 1) In Appendix E.3, we describe a straightforward use of reduction that solves a grid coloring problem by reducing it to a graph problem. Given the grid  $G$  shown here:



- a) Show the graph that corresponds to  $G$ .

*Solution.*

$$G' = \{V, E\}$$

$$V = \{A, B, C, D, 1, 2, 3, 4\}$$

$$E = \{(A, 3), (A, 4), (B, 1), (B, 3), (B, 4), (C, 4), (D, 2)\}$$

□

- b) Use the graph algorithm we describe to find a coloring of  $G$ .

*Solution.* Start with  $B$  and color each edge alternately.

$$Red = \{(B, 1), (B, 4)\}$$

$$Blue = \{(B, 3)\}$$

Now do vertex 3:

$$Red = \{(B, 1), (B, 4), (A, 3)\}$$

$$Blue = \{(B, 3)\}$$

Now vertex  $A$ :

$$Red = \{(B, 1), (B, 4), (A, 3)\}$$

$$Blue = \{(B, 3), (A, 4)\}$$

Now vertex 4: (it has one of each already, so pick arbitrarily.)

$$Red = \{(B, 1), (B, 4), (A, 3)\}$$

$$Blue = \{(B, 3), (A, 4), (C, 4)\}$$

And now the last edge:

$$\begin{aligned} Red &= \{(B, 1), (B, 4), (A, 3), (D, 2)\} \\ Blue &= \{(B, 3), (A, 4), (C, 4)\} \end{aligned}$$

□

2) In this problem, we consider the relationship between  $H$  and a very simple language  $\{a\}$ .

a) Show that  $\{a\}$  is *mapping* reducible to  $H$ .

*Solution.*

□

b) Is it possible to reduce  $H$  to  $\{a\}$ ? Prove your answer.

*Answer.*

□

*Proof.*

□

3) Show that  $H_{ALL}$  is not in  $D$  by reduction from  $H$ .

*Solution.*

□

4) For each of the following languages  $L$ , state whether or not it is in  $D$ . Prove your answer. Assume that any input of the form  $\langle M \rangle$  is a description of a Turing machine.

a)  $\{\langle M \rangle : ab \in L(M)\}$ .

*Answer.*

□

*Proof.*

□

b)  $\{\langle M, w \rangle : \text{TM } M, \text{ on input } w, \text{ begins by moving right one square onto } w. \text{ Then it never moves off } w\}$ .

*Answer.*

□

*Proof.*

□

c)  $\{\langle M \rangle : \text{there exists a string } w \text{ such that } |w| < |\langle M \rangle| \text{ and that } M \text{ accepts } w\}$ .

*Answer.*

□

*Proof.*

□

5) In Appendix J.2, we proved Theorem J.1, which tells us that the safety of even a very simple security model is undecidable, by reduction from  $H_\epsilon$ . Show an alternative proof that reduces  $A = \{\langle M, w \rangle : M \text{ is a Turing machine and } w \in L(M)\}$  to the language Safety.

*Proof.*

□