

STUDENT NAME - EID
CS 341 Automata Theory
Homework 15
Due: Tuesday, April 30

This assignment covers Chapters 22 and 24.

- 1) Solve the linear Diophantine farmer problem presented in Section 22.1.

Solution.

□

- 2) Consider the following instance of the Post Correspondence problem. Does it have a solution? If so, show one.

//TODO: Table width

	X	Y
1	a	bab
2	bbb	bb
3	aab	ab
4	b	a

Solution.

□

- 3) Prove that, if an instance of the Post Correspondence problem has a solution, it has an infinite number of solutions. (Hint: this is really easy.)

Proof.

□

- 4)) Let $TILES = \{\langle T \rangle : \text{any finite surface on the plane can be tiled, according to the rules described in the book, with the tile set } T\}$. Let s be the string that encodes the following tile set:

//TODO: include graphics

Is $s \in TILES$? Prove your answer.

Answer.

□

Proof.

□

- 5) Is $L = \{\langle M \rangle : M \text{ is a PDA and } L(M) = \{x : x \in \{a, b\}^* \text{ and } \exists m (|x| = 2^m)\}\}$ decidable? Prove your answer.

Answer.

□

Proof. □

- 6) A language L is **D-complete** iff (1) L is in D , and (2) for every language L' in D , $L' \leq_M L$. Consider the following claim: If $L \in D$ and $L \neq \Sigma^*$ and $L \neq \emptyset$, then L is D-complete. Prove or disprove this claim.

Proof. □

- 7) Let $\Sigma = \{1\}$. Show that there exists at least one undecidable language with alphabet Σ . (Hint: Use a counting argument.)

Proof. □

- 8) The following sequence of figures corresponds to a fractal called a *Koch island*:

These figures were drawn by interpreting strings as turtle programs, just as we did in Example 24.5 and Example 24.6. The strings were generated by an L-system G , defined with:

$$\begin{aligned}\Sigma &= \{F, +, -\}. \\ \omega &= F - F - F - F\end{aligned}$$

To interpret the strings as turtle programs, attach meanings to the symbols in Σ as follows (assuming that some value for k has been chosen):

- F means move forward, drawing a line of length k .
- $+$ means turn left 90° .
- $-$ means turn right 90° .

Figure (a) was drawn by the first generation string ω . Figure (b) was drawn by the second generation string, and so forth. R_G contains a single rule. What is it?

Answer. □