CS 341 Automata Theory
Elaine Rich
Homework 12
Due: Tuesday, April 10

This assignment covers Chapter 20.

1) * Let $L_1, L_2, \ldots, L_k$ be a collection of languages over some alphabet Σ such that:

   - For all $i \neq j$, $L_i \cap L_j = \emptyset$.
   - $L_1 \cup L_2 \cup \cdots \cup L_k = \Sigma^*$.
   - $\forall i$ (L_i is in SD).

   Prove that each of the languages L_1 through L_k is in D.

2) If L_1 and L_3 are in D and $L_1 \subseteq L_2 \subseteq L_3$, what can we say about whether L_2 is in D?

3) Let M be a Turing machine that lexicographically enumerates the language L. Prove that there exists a Turing machine M' that decides L^R.

4) Construct a standard one-tape Turing machine M to enumerate the language A^nB^n. Assume that M starts with its tape equal to $\square$. Also assume the existence of the printing subroutine P, defined in Section 20.5.1.

5) Recall the function *mix*, defined in Example 8.23. Neither the regular languages nor the context-free languages are closed under *mix*. Are the decidable languages closed under *mix*? Prove your answer.

6) Let $\Sigma = \{a, b\}$. Consider the set of all languages over Σ that contain only even length strings.

   a) How many such languages are there?

   b) How many of them are semidecidable?