STUDENT NAME - EID
CS 341 Automata Theory
Homework 15
Due: Tuesday, April 30

This assignment covers Chapters 22 and 24.

1) Solve the linear Diophantine farmer problem presented in Section 22.1.

Solution. □

2) Consider the following instance of the Post Correspondence problem. Does it have a solution? If so, show one.

	X	Y
1	`a`	`bab`
2	`bbb`	`bb`
3	`aab`	`ab`
4	`b`	`a`

Solution. □

3) Prove that, if an instance of the Post Correspondence problem has a solution, it has an infinite number of solutions. (Hint: this is really easy.)

Proof. □

4)) Let $TILES = \{\langle T \rangle$: any finite surface on the plane can be tiled, according to the rules described in the book, with the tile set $T\}$. Let s be the string that encodes the following tile set:

Is $s \in TILES$? Prove your answer.

Answer. □

Proof. □

5) Is $L = \{\langle M \rangle : M$ is a PDA and $L(M) = \{x : x \in \{a, b\}^*$ and $\exists m\ (|x| = 2^m)\}\}$ decidable? Prove your answer.

Answer. □

Proof. □

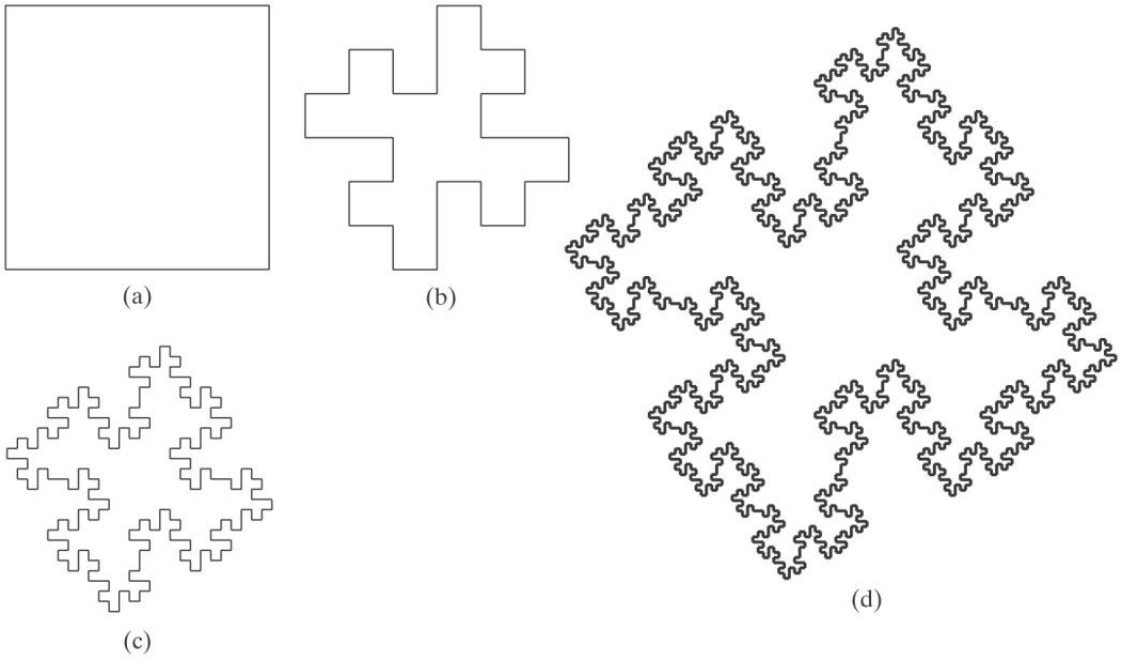
6) A language L is **D-complete** iff (1) L is in D, and (2) for every language L' in D, $L' \leq_M L$. Consider the following claim: If $L \in D$ and $L \neq \Sigma^*$ and $L \neq \emptyset$, then L is D-complete. Prove or disprove this claim.

Proof. □

7) Let $\Sigma = \{1\}$. Show that there exists at least one undecidable language with alphabet Σ. (Hint: Use a counting argument.)

Proof. □

8) The following sequence of figures corresponds to a fractal called a *Koch island*:



These figures were drawn by interpreting strings as turtle programs, just as we did in Example 24.5 and Example 24.6. The strings were generated by an L-system G, defined with:

$$\Sigma = \{F, +, -\}.$$
$$\omega = F - F - F - F$$

To interpret the strings as turtle programs, attach meanings to the symbols in Σ as follows (assuming that some value for k has been chosen):

- F means move forward, drawing a line of length k.
- $+$ means turn left 90°.
- $-$ means turn right 90°.

Figure (a) was drawn by the first generation string ω. Figure (b) was drawn by the second generation string, and so forth. R_G contains a single rule. What is it?

Answer. □