Geoffrey Parker - grp352
CS 341 Automata Theory
Homework 15
Due: Tuesday, April 30

This assignment covers Chapters 22 and 24.

1) Solve the linear Diophantine farmer problem presented in Section 22.1.

Solution. Let p be the number of pigs bought, c the number of cows and k the number of chickens. Then the question of how many of each was bought can be stated as the linear diophantine equations:

$$10c + 3p + 0.5k = 100$$

$$p + c + k = 100$$

□

2) Consider the following instance of the Post Correspondence problem. Does it have a solution? If so, show one.

	X	Y
1	a	bab
2	bbb	bb
3	aab	ab
4	b	a

Solution. 2, 1, 4, 3

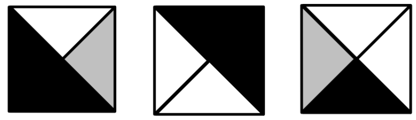
	X	Y
2	bbb	bb
1	bbba	bbbab
4	bbbab	bbbaba
3	bbbabaab	bbbabaab

□

3) Prove that, if an instance of the Post Correspondence problem has a solution, it has an infinite number of solutions. (Hint: this is really easy.)

Proof. Assume that a particular Post Correspondende problem has a solution X in n steps $x_1, x_2, \ldots, x_n$. Then $XX, XXX, XXXX, \ldots$ will also be solutions. Therefore any Post Correspondende problem which has a solution has an infinite number of solutions. □

4)) Let $TILES = \{\langle T \rangle :$ any finite surface on the plane can be tiled, according to the rules described in the book, with the tile set $T\}$. Let s be the string that encodes the following tile set:



Is $s \in TILES$? Prove your answer.

Answer. No. $\square$

Proof. $\square$

5) Is $L = \{\langle M \rangle : M$ is a PDA and $L(M) = \{x \ : \ x \in \{a,b\}^*$ and $\exists m\ (|x| = 2^m)\}\}$ decidable? Prove your answer.

Answer. Decidable. $\square$

Proof. Let $L' = \{x \ : \ x \in \{a,b\}^*$ and $\exists m\ (|x| = 2^m)\}\}$. Let $w = a^{2^k} = uvxyz$ for some u, v, x, y, and z with $|vxy| \leq k$ and $vy \neq \epsilon$. Let $w' = uv^2xy^2z$. Then $w' = a^{2^k}a^p$ for some p where $1 \leq p \leq k$. So $2^k < |w| < 2^{k+1}$. Therefore $w' \notin L'$ and by the pumping theorem for context free languages L' is not context-free. Thus there are no PDAs with the language L'. This means that $L = \emptyset$, which is regular and decideable. $\square$

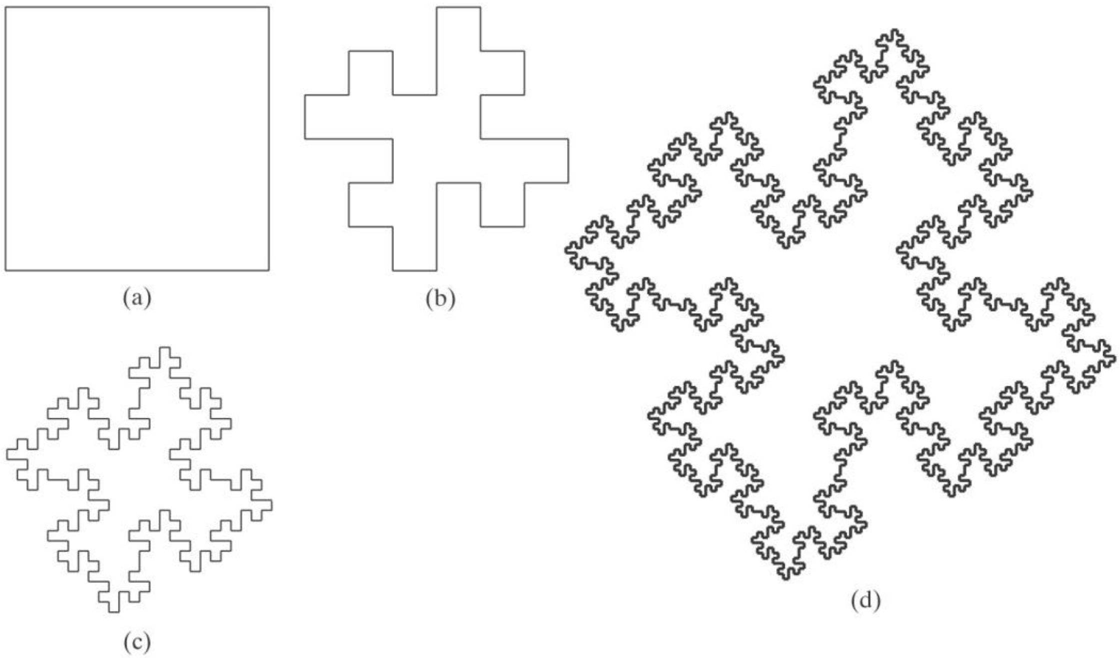
6) A language L is **D-complete** iff (1) L is in D, and (2) for every language L' in D, $L' \leq_M L$. Consider the following claim: If $L \in D$ and $L \neq \Sigma^*$ and $L \neq \emptyset$, then L is D-complete. Prove or disprove this claim.

Proof. $\square$

7) Let $\Sigma = \{1\}$. Show that there exists at least one undecidable language with alphabet Σ. (Hint: Use a counting argument.)

Proof. Let M be the Turing Machine *Trouble* from the proof that $H \not{in} D$. Let $L = \{x : x$ is 1 if M halts and ϵ if M does not halt.$\}$. Since we have shown that it is impossible to decide if M halts or not, L must not be decidable. $\square$

8) The following sequence of figures corresponds to a fractal called a *Koch island*:



These figures were drawn by interpreting strings as turtle programs, just as we did in Example 24.5 and Example 24.6. The strings were generated by an L-system G, defined with:

$$\Sigma = \{F, +, -\}.$$
$$\omega = F - F - F - F$$

To interpret the strings as turtle programs, attach meanings to the symbols in Σ as follows (assuming that some value for k has been chosen):

- F means move forward, drawing a line of length k.
- $+$ means turn left 90°.
- $-$ means turn right 90°.

Figure (a) was drawn by the first generation string ω. Figure (b) was drawn by the second generation string, and so forth. R_G contains a single rule. What is it?

Answer. $\square$