# Competitive Algorithm Design and Practice
# Longest Increasing Sub-sequence
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*http://myweb.ncku.edu.tw/~f74991073/20140319_DP.zip*

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# Longest Increasing Sub-sequence

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# LIS

- Find a sub-sequence of a given sequence in which the sub-sequence`s elements are in sorted order, lowest to highest.

- And the sub-sequence is as long as possible.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |

# LIS

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |

- An increasing sub-sequence:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 8 | **4** | 12 | 2 | 10 | 6 | **14** | 1 | 9 | 5 | 13 | 3 | 11 | 7 | **15** |

- The longest one:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 8 | 4 | 12 | **2** | 10 | **6** | 14 | 1 | **9** | 5 | **13** | 3 | 11 | 7 | **15** |

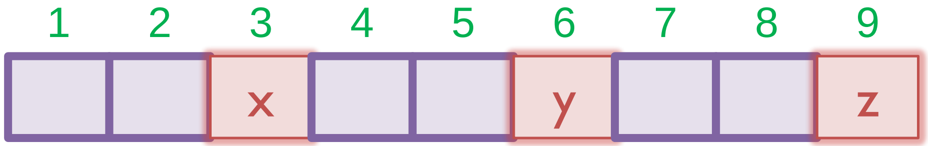
# LIS

- Naïve solutions:
  - For every sub-sequence, check if it is LIS.

- Time-complexity:
  - Every sub-sequence, $O(2^N)$.
  - For each sub-sequence checking needs $O(N)$.
  - Total: $O(N*2^N)$

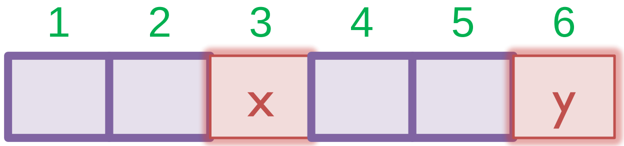
# LIS

- Observation:
- If



| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
|  |  | x |  |  | y |  |  | z |

- is LIS ended at z,
- then



| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
|  |  | x |  |  | y |

- must be LIS ended at y

# LIS

- What do we want to know?
  - The LIS ended at index k, i.e. LIS[k]

- How can we get that?
  - Find the previous number with longest LIS.
  - LIS[k] =

    max( 1, LIS[i]+1 ) for i<k & num[i] < num[k]

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# LIS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | | | | | | | | | | | | | | | | |

# LIS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | | | | | | | | | | | | | | | |

# LIS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | | | | | | | | | | | | | | | |

# LIS

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

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# LIS

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

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# LIS

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

# LIS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 1 | | | | | | | | | | | | | |

# LIS

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |

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# LIS

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |

# LIS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 1 | | | | | | | | | | | | |

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# LIS

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |

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# LIS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | | | | | | | | | | | | |

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# LIS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | | | | | | | | | | | | |

# LIS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | | | | | | | | | | | | |

# LIS

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

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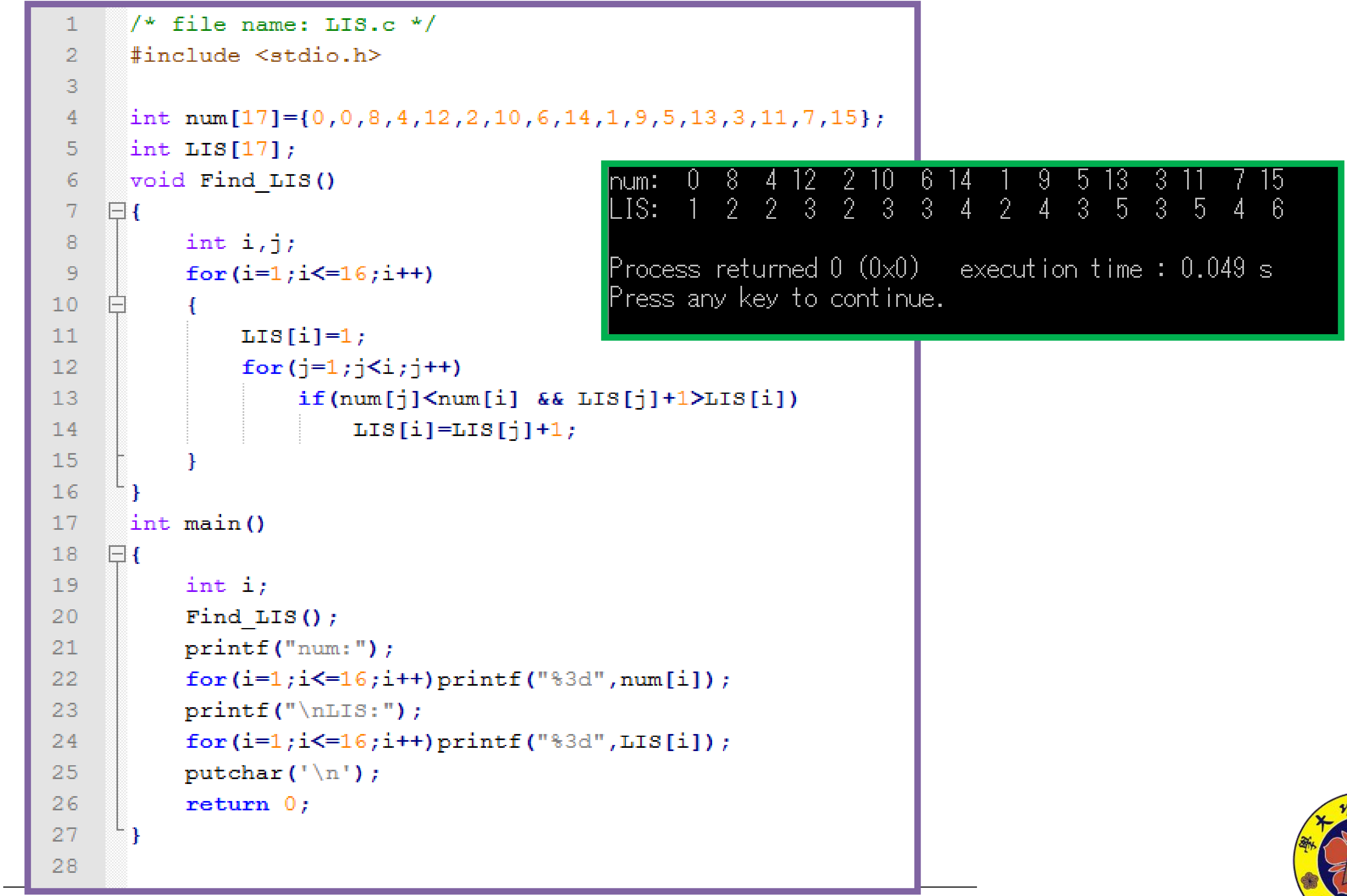
# LIS

- Time-complexity:
  - For each element find the previous longest LIS.
  - Every element, O(N)
  - Each element, check previous elements, O(N)
  - Total: O(N*N) => $O(N^2)$

- Space-complexity:
  - Additional LIS array, O(N)

# LIS



```c
/* file name: LIS.c */
#include <stdio.h>

int num[17]={0,0,8,4,12,2,10,6,14,1,9,5,13,3,11,7,15};
int LIS[17];
void Find_LIS()
{
    int i,j;
    for(i=1;i<=16;i++)
    {
        LIS[i]=1;
        for(j=1;j<i;j++)
            if(num[j]<num[i] && LIS[j]+1>LIS[i])
                LIS[i]=LIS[j]+1;
    }
}
int main()
{
    int i;
    Find_LIS();
    printf("num:");
    for(i=1;i<=16;i++)printf("%3d",num[i]);
    printf("\nLIS:");
    for(i=1;i<=16;i++)printf("%3d",LIS[i]);
    putchar('\n');
    return 0;
}

```

```
num:  0  8  4 12  2 10  6 14  1  9  5 13  3 11  7 15
LIS:  1  2  2  3  2  3  3  4  2  4  3  5  3  5  4  6

Process returned 0 (0x0)   execution time : 0.049 s
Press any key to continue.
```

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# POJ 2533

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# Learn more!

- Sometimes we need to output a solution too, but how?

- 1. Find the number backwards.
  - Assume ans=length of LIS
  - Find k such that LIS[k]=ans.
  - Then from index 1~k find i such that LIS[i] = LIS[k]-1 && num[i] < num[k], and so on
- 2. Additional array pre[].
  - pre[k] = i such that LIS[i] = LIS[k]-1 && num[i] < num[k]

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# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |
| pre | 1 | 1 | 1 | 2 | 1 | 2 | 5 | 4 | 11 | 7 | 3 | 10 | 5 | 10 | 7 | 14 |

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# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |
| pre | 1 | 1 | 1 | 2 | 1 | 2 | 5 | 4 | 11 | 7 | 3 | 10 | 5 | 10 | 7 | 14 |

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# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |
| pre | 1 | 1 | 1 | 2 | 1 | 2 | 5 | 4 | 11 | 7 | 3 | 10 | 5 | 10 | 7 | 14 |

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |
| pre | 1 | 1 | 1 | 2 | 1 | 2 | 5 | 4 | 11 | 7 | 3 | 10 | 5 | 10 | 7 | 14 |

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |
| pre | 1 | 1 | 1 | 2 | 1 | 2 | 5 | 4 | 11 | 7 | 3 | 10 | 5 | 10 | 7 | 14 |

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |
| pre | 1 | 1 | 1 | 2 | 1 | 2 | 5 | 4 | 11 | 7 | 3 | 10 | 5 | 10 | 7 | 14 |

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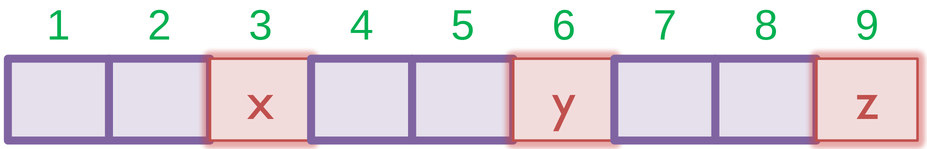
# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |
| pre | 1 | 1 | 1 | 2 | 1 | 2 | 5 | 4 | 11 | 7 | 3 | 10 | 5 | 10 | 7 | 14 |

# Learn more!

- Sometimes $O(N^2)$ is too slow……



- If LIS[x] = LIS[y](mean-while x>=y)
- For some z that LIS[z] = LIS[x]+1

  We can append z after either x or y to form LIS, by the way y might be smaller!

- That is, for LIS[] = k we can memorize just one number, the smaller one, y.

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

LIS =1 | 0 |

# Learn more!

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

|  |  |
|---|---|
| LIS =2 | 8 |
| LIS =1 | 0 |

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =2 | 4 |
| LIS =1 | 0 |

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# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =3 | 12 |
| LIS =2 | 4 |
| LIS =1 | 0 |

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# Learn more!

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

|  |  |
|---|---|
| LIS =3 | 12 |
| LIS =2 | 2 |
| LIS =1 | 0 |

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# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =3 | 10 |
| LIS =2 | 2 |
| LIS =1 | 0 |

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# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =3 | 6 |
| LIS =2 | 2 |
| LIS =1 | 0 |

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# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =4 | 14 |
| LIS =3 | 6 |
| LIS =2 | 2 |
| LIS =1 | 0 |

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# Learn more!

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

|  |  |
|---|---|
| LIS =4 | 14 |
| LIS =3 | 6 |
| LIS =2 | 1 |
| LIS =1 | 0 |

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =4 | 9 |
| LIS =3 | 6 |
| LIS =2 | 1 |
| LIS =1 | 0 |

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# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =4 | 9 |
| LIS =3 | 5 |
| LIS =2 | 1 |
| LIS =1 | 0 |

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =5 | 13 |
| LIS =4 | 9 |
| LIS =3 | 5 |
| LIS =2 | 1 |
| LIS =1 | 0 |

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =5 | 13 |
| LIS =4 | 9 |
| LIS =3 | 3 |
| LIS =2 | 1 |
| LIS =1 | 0 |

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =5 | 11 |
| LIS =4 | 9 |
| LIS =3 | 3 |
| LIS =2 | 1 |
| LIS =1 | 0 |

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# Learn more!

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =5 | 11 |
| LIS =4 | 7 |
| LIS =3 | 3 |
| LIS =2 | 1 |
| LIS =1 | 0 |

***Made By mike199250***

# Learn more!

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| num | 0 | 8 | 4 | 12 | 2 | 10 | 6 | 14 | 1 | 9 | 5 | 13 | 3 | 11 | 7 | 15 |
| LIS | 1 | 2 | 2 | 3 | 2 | 3 | 3 | 4 | 2 | 4 | 3 | 5 | 3 | 5 | 4 | 6 |

| | |
|---|---|
| LIS =6 | 15 |
| LIS =5 | 11 |
| LIS =4 | 7 |
| LIS =3 | 3 |
| LIS =2 | 1 |
| LIS =1 | 0 |

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# Learn more!

- There will be some ordering, so we can use binary-search to find LIS[k], then update the array.

- Find the best place it can be, and update
- It`s greedy!