Linear Algebra

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Chapter 1

Vector Spaces

1.1 Introduction

Note. Experiments show that if two like quantities act together, their effect is predictable. In this case, the vectors used to represent these quantities can be combined to form a resultant vector that represents the combined effects of the original quantities. This resultant vector is called the *sum* of the original vectors, and the rule for their combination is called the *parallelogram law*.

Axiom 1.1.1 (Parallelogram Law for Vector Addition)**.** The sum of two vectors x and y that act at the same point P is the vector beginning at P that is represented by the diagonal of parallelogram having x and y as adjacent sides.

Note. Since a vector beginning at the origin is completely determined by its endpoint, we sometimes refer to *the point* x rather than *the endpoint of the vector* x if x is a vector emanating from the origin.

Note. Besides the operation of vector addition, there is another natural operation that can be performed on vectors — the length of a vector may be magnified or contracted. This operation, called *scalar multiplication*, consists of multiplying the vector by a real number. If the vector x is represented by an arrow, then for any real number t, the vector tx is represented by an arrow in the same direction if $t \geq 0$ and in the opposite direction if $t < 0$. The length of the arrow tx is $|t|$ times the length of the arrow x. Two nonzero vectors x and y are called **parallel** if $y = tx$ for some nonzero real number t. (Thus nonzero vectors having the same or opposite directions are parallel.)