Practico 5.

Ej. 1] X e Y son Va con Rec. $\{1, 2, 3, 4\}$

$P(x,y)(n,m) = \dfrac{n+m}{80}$

a) Calcular $P(X \geq 3)$

X\Y	1	2	3	4
1	$\frac{1}{40}$	$\frac{3}{80}$	$\frac{1}{20}$	$\frac{1}{16}$
2	$\frac{3}{80}$	$\frac{1}{20}$	$\frac{1}{16}$	$\frac{3}{40}$
3	$\frac{1}{20}$	$\frac{1}{16}$	$\frac{3}{40}$	$\frac{7}{80}$
4	$\frac{1}{16}$	$\frac{3}{40}$	$\frac{7}{80}$	$\frac{1}{10}$

$P(X \geq 3) = \dfrac{1}{20} + \dfrac{1}{16} + \dfrac{3}{40} + \dfrac{7}{80} + \dfrac{1}{16} + \dfrac{3}{40} + \dfrac{7}{80} + \dfrac{1}{10} = \dfrac{3}{5}$ ✓

b) $P(X = Y)$

$P(X = Y) = \dfrac{1}{40} + \dfrac{1}{20} + \dfrac{3}{40} + \dfrac{1}{10} = \dfrac{1}{4}$ ✓

c) Calcular $P(X \cdot Y = 6)$

$P(XY = 6) = \dfrac{1}{16} + \dfrac{1}{16} = \dfrac{1}{8}$ ✓

d) $P(1 \leq X \leq 2, 2 < Y \leq 4)$

$P(1 \leq X \leq 2, 2 < Y \leq 4) = \dfrac{1}{20} + \dfrac{1}{16} + \dfrac{1}{16} + \dfrac{3}{40} = \dfrac{1}{4}$ ✓

e) Son X e Y independientes?

Si $\exists\ x, y\ /\ P_x(x) \cdot P_y(y) \neq P_{(x,y)}(x,y) \rightarrow$ no son independient...

Voy a calcular las marginales de X e Y por comprobar:

Marginales de X:

$$P_x(x) = \begin{cases} 7/40 & si & x=1 \\ 9/40 & si & x=2 \\ 11/40 & si & x=3 \\ 13/40 & si & x=4 \end{cases}$$

$P(x=1) = \frac{1}{40} + \frac{3}{80} + \frac{1}{20} + \frac{1}{16} = \frac{7}{40}$ $P(x=2) = \frac{3}{80} + \frac{1}{20} + \frac{1}{16} + \frac{3}{40} = \frac{9}{40}$

$P(x=3) = \frac{1}{20} + \frac{1}{16} + \frac{3}{40} + \frac{7}{80} = \frac{11}{40}$ $P(x=4) = \frac{1}{16} + \frac{3}{40} + \frac{7}{80} + \frac{1}{10} = \frac{13}{40}$

$$P_y(y) = \begin{cases} 7/40 & si & y=1 \\ 9/40 & si & y=2 \\ 11/40 & si & y=3 \\ 13/40 & si & y=4 \end{cases}$$

$P(y=1) = \frac{1}{40} + \frac{3}{80} + \frac{1}{20} + \frac{1}{16} = \frac{7}{40}$ $P(y=2) = \frac{3}{80} + \frac{1}{20} + \frac{1}{16} + \frac{3}{40} = \frac{9}{40}$

$P(y=3) = \frac{1}{20} + \frac{1}{16} + \frac{3}{40} + \frac{7}{80} = \frac{11}{40}$ $P(y=4) = \frac{1}{16} + \frac{3}{40} + \frac{7}{80} + \frac{1}{10} = \frac{13}{40}$

$$P_{x,y}(2,3) = \frac{1}{16} \neq P_y(3) \cdot P_x(2) = \frac{11}{40} \cdot \frac{9}{40} = \frac{99}{1600}$$

$\hookrightarrow$ X e Y no son independientes.

sdf

Ej. 2

X e Y v.a. con dist. Bernoulli de $p = \frac{1}{2}$.

S, T v.a. tal que $S = X + Y$

$\qquad\qquad\qquad T = X - Y$

a) Hallar funciones de probabilidad de masa de S, T.

$X : Rec = \{0, 1\}$ 	$P(X = 1) = \frac{1}{2}$

$\qquad\qquad\qquad\qquad P(X = 0) = \frac{1}{2}$

$Y = Rec \{0, 1\}$ 	$P(Y = 1) = \frac{1}{2}$

$\qquad\qquad\qquad\qquad P(Y = 0) = \frac{1}{2}$

$$S : \begin{array}{l} 0 + 0 = \frac{1}{4} \\ 0 + 1 \\ 1 + 0 \end{array} \Big\} \; 2/4 = \frac{1}{2} \qquad S = \begin{bmatrix} P(S=0) = \frac{1}{4} \quad \checkmark \\ P(S=1) = \frac{1}{2} \\ P(S=2) = \frac{1}{4} \end{bmatrix}$$

$$\qquad\qquad 1 + 1 = \frac{1}{4}$$

$$T : \begin{array}{l} 0 - 0 = 0 \; \leftarrow \\ 0 - 1 = -1 \quad \frac{1}{4} \\ 1 - 0 = 1 \quad \frac{1}{4} \\ 1 - 1 = 0 \; \leftarrow \end{array} \Big\} 2/4 = \frac{1}{2} \qquad T \begin{bmatrix} P(T=0) = \frac{1}{2} \\ P(T=-1) = \frac{1}{4} \quad \checkmark \\ P(T=1) = \frac{1}{4} \end{bmatrix}$$

b) Hallar función de probabilidad de masa conjunta de S, T

T \ S	0	1	2	f(T)
0	1/4	0	1/4	1/2
1	0	1/4	0	1/4
-1	0	1/4	0	1/4
f(T)	1/4	1/2	1/4	

No son independientes porque

$P(0,0) = 1/4 \neq P(T=0) \, P(S=0) = 1/2 \cdot 1/2 = 1/8$

T	0	1	-1
f(T)	1/4	1/2	1/4

$E(T) = 0(1/4) + 1/2 \cdot 1 + 1/2 \cdot (-1)$

$E(T) = 0 + 1/2 - 1/2 \Rightarrow E(T) = 0$

sdf

$P(S=0, T=0)$

$P(S=0)$
$\quad x=0 \quad 7=0$
$P(T=0)$
$\quad x=0 \quad 7=0$
$\quad x=1 \quad x=1$

$\left.\begin{array}{l} \end{array}\right\} P(x=0) \cdot P(y=0) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(S=0, T=1)$

$P(S=0)$
$\quad x=0 \quad 7=0$
$P(T=1)$
$\quad x=1 \quad 7=0$

$\left.\begin{array}{l} \end{array}\right\}$ cero coincidencias.

$P(S=0, T=-1)$

$P(S=0)$
$\quad x=0 \quad 7=0$
$P(T=-1)$
$\quad x=0 \quad 7=-1$

$\left.\begin{array}{l} \end{array}\right\}$ cero

$P(S=1, T=0)$

$P(S=1)$
$\quad x=1 \quad 7=0$
$\quad x=0 \quad 7=1$
$P(T=0)$
$\quad x=0 \quad 7=0$
$\quad x=1 \quad 7=1$

$\left.\begin{array}{l} \end{array}\right\}$ cero

$P(S=1, T=1)$

$P(S=1)$
$\quad x=1 \quad 7=0$
$\quad x=0 \quad 7=1$
$P(T=1)$
$\quad x=1 \quad 7=0$

$\left.\begin{array}{l} \end{array}\right\} P(x=1) \cdot P(y=0) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(S=1, T=-1)$

$P(S=1)$
$\quad x=1 \quad 7=0$
$\quad x=0 \quad 7=1$
$P(T=-1)$
$\quad x=0 \quad 7=1$

$\left.\begin{array}{l} \end{array}\right\} P(x=0) \cdot P(y=1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(S=2, T=0)$

$P(S=2)$
$\quad x=1 \quad 7=1$
$P(T=0)$
$\quad x=0 \quad 7=0$
$\quad x=1 \quad 7=1$

$\left.\begin{array}{l} \end{array}\right\} P(x=1) P(y=1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(S=2, T=1)$

$P(S=2) \quad x=1 \quad 7=1$
$P(T=1) \quad x=1 \quad 7=0$

$\left.\begin{array}{l} \end{array}\right\}$ cero.

$P(S=2, T=-1)$

$P(S=2) \quad x=1 \quad x=1$
$P(T=1) \quad x=0 \quad 7=0$

$\left.\begin{array}{l} \end{array}\right\}$ cero.

S	0	1	2
$F(s)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

$E(S) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4}$

$E(S) = 0 + \frac{1}{4} + \frac{1}{2} = \frac{1}{8}$

$E(T \cdot S) = 0 \cdot \frac{1}{8} = 0$

$E(TS) = 0$ ✓

sdf