

All measurements—and therefore the calculations using those measurements—will have some inherent uncertainty that needs to be reported. In this class, we will work with data sets and determine the mean and standard deviation. The mean will give you the experimental value, and the standard deviation gives the uncertainty. As you progress through a scientific discipline, you will likely learn other methods for handling uncertainty.

1 Standard deviation

One way of “beating down” the uncertainty of an overall experiment is to take the results from many trials of the same experiment and use statistics. For this class, you need to perform *at least* ten trials.

Two common metrics are *mean* and *standard deviation*. A mean is simply an arithmetic average; add up each measurement and divide by the total number of measurements. In symbols:

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N}$$

where x_i is each individual measurement, and N is the total number of measurements. The standard deviation gives information about how far each datum lies from the average:

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N - 1}}$$

Squaring the difference between a data point and the average removes any plus or minus sign, and taking the square root gets us back to the appropriate units. When taking many measurements and using statistical analysis, we can report values as $x = \bar{x} \pm \sigma_x$.

For example, say you perform a projectile motion experiment where you measure the range of a projectile. You do ten trials ($N = 10$) and make the measurements:

Trial	x (cm)	Trial	x (cm)
1	50.5	6	48.0
2	49.5	7	49.0
3	51.0	8	50.0
4	55.0	9	50.5
5	48.5	10	49.5

The average, to three significant figures, is $\bar{x} = 50.2$ cm and the standard deviation is $\sigma_x = 1.94$ cm. Our final result would be reported as

$$x = 50 \pm 2 \text{ cm}$$

2 Significant figures

The uncertainty is rounded to one significant figure, and is of the same order of magnitude (decimal place) as the last significant figure of your calculated quantity. Similarly, the uncertainty on any given measurement is of the same order of magnitude as the last significant figure in the measurement.

In the previous example, the standard deviation rounded to one significant figure is 2 cm. This is in the “ones place,” so the last significant figure of the mean needed to be rounded to the ones place (50.2 cm was rounded to 50 cm). Even though our measured values were accurate to three significant figures, our final answer only has two significant figures. Perhaps if we had done more trials, our final answer could have been more precise!