1 Overview

A necessary condition for equilibrium is that the sum of the forces acting on a body is zero. Note that this means the sum of the components of the forces must individually equal zero:

$$\sum F_x = 0 \qquad \text{and} \qquad \sum F_y = 0 \tag{1}$$

However this alone is not sufficient to establish equilibrium in a rigid body which has forces acting on it which are not concurrent, i.e. all acting through the same point.

An example of this is a bar, which is free to rotate and has two operating rocket engines mounted on the ends as shown in figure 1. Even if F_1 and F_2 should have a vector sum of zero the bar would still have an angular velocity, which would increase with time and the bar would therefore not be in equilibrium.

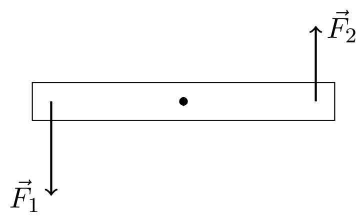

Figure 1: A bar rotated by two rockets.

The second condition for equilibrium is that the sum of the torques acting on a rigid body be equal to zero:

$$\sum \tau = 0 \tag{2}$$

A force that is applied "off center" produces a torque. The perpendicular distance from the line along which the force vector lies to the pivot point on any system is called the lever arm distance (also known as the moment arm distance). A torque is caused by a force that tends to cause a rotation pivot point. The torque exerted on the body depicted in figure 2 by a force $\vec{F}$ about the axis shown is defined by

$$\tau = rF\sin\theta \tag{3}$$

We see from figure 3 and a little trigonometry that $r\sin\theta$ is the perpendicular distance from the rotation axis to the line of action of the force. The lever arm for any point of rotation is the perpendicular distance from the point to the line along which the force acts. Alternately, we can just take the component of $\vec{F}$ (acting at that point) that is perpendicular to r. This component would be $F\sin\theta$.

Torques which tend to produce counterclockwise rotation we will define as positive, and torques which tend to produce clockwise rotation will correspondingly be negative.

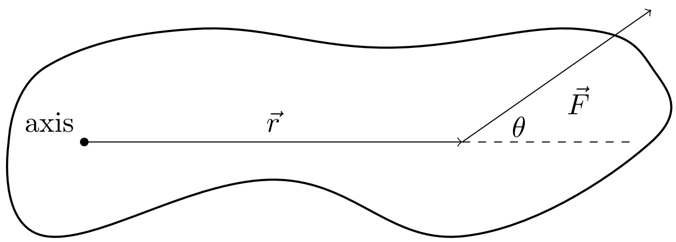

Figure 2: A force applied "off-center" to an object that is allowed to rotate around an axis.

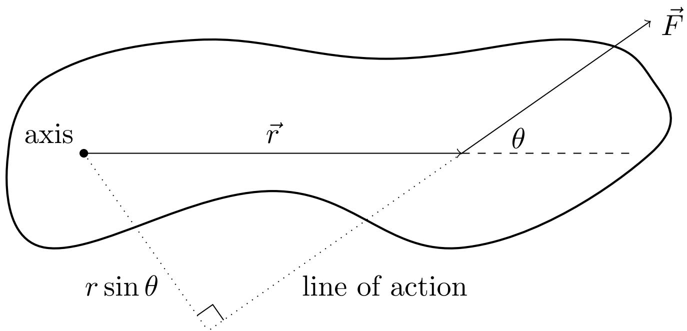

Figure 3: More detailed geometry of the situation in figure 2.

2 Materials

- Cenco-Moe apparatus
- Science string
- Masses

3 Tasks

1. Set up a condition of stable equilibrium, with the Cenco-Moe apparatus level, as shown in figure 4. Your apparatus has four protractors rather than the three illustrated here. The 4^{th} protractor can be used to aid in determining if the apparatus is level. It is not at the location of the center of mass, and the center of mass is not necessarily at the center of the apparatus.

2. Determine the magnitude and direction of each force acting on the system. Do not neglect the force of gravity acting on the apparatus.

3. Find the sum of the x and y components of the forces acting on the apparatus. (This is somewhat of a reality check; what do you expect the net force to be?)

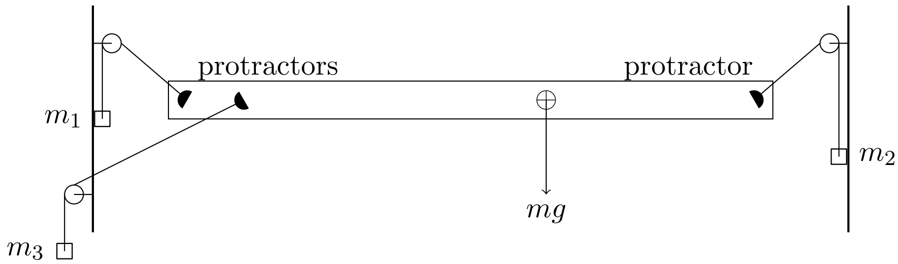

Figure 4: Cenco-Moe apparatus.

4. Determine the torque exerted on the apparatus by each of the three hanging masses. Clearly state what your reference point is for the torque measurements.
5. Determine the location of the center of mass of the apparatus, relative to the same reference point you used for torque calculations.

4 Questions

1. Would you expect the torques to have the same values individually if a different point were chosen as your axis? Would you expect the sum to differ? Explain.
2. Both work and torque are products of force and distance. Explain the difference(s) between work and torque.

5 Grading

You will not write a formal lab report. Instead, report all your data, calculations, and answers to the questions. Include a diagram of your setup, with all masses and angles labeled with appropriate values from your particular experiment. Indicate where the center of mass of the apparatus is on your diagram.