This is a follow-up from the density lab, to give you practice working with statistics in experimental data. You will be given all measurements for the density of rubber and cork.

1 Numerical analysis

For each material, find:

- a) The mean density of the material: $\overline{\rho}$
- b) The difference between each group's results and the mean: $(\rho_i \overline{\rho})$
- c) The difference squared: $(\rho_i \overline{\rho})^2$
- d) The standard deviation:

$$\sigma_{\rho} = \sqrt{\frac{\sum_{i=1}^{N} (\rho_i - \overline{\rho})^2}{N - 1}}$$

where N is the total number of results

Include the results of each step in your report, organized in data tables.

Calculations (a)–(c) must be done by hand for the first twenty data points of each material. Yes, this is tedious. Yes, are computer programs that *could* do this for you. However, there is no better way to develop an intuition for what the equations tell you than to sit down and repeatedly do the work yourself.

When working with large data sets, where it is beneficial to apply statistics like this, the result is reported as

$$\rho = \overline{\rho} \pm \sigma_{\rho}$$

2 Visual analysis

For this part, you will be using a selection of data, for which I have calculated experimental uncertainties. Make a visual representation of the data for each material: the results on a plot with error bars given by experimental uncertainty. Draw dashed lines representing the mean, and the mean plus or minus one standard deviation. See figure 1 for an example.

One plot is to be done by hand. Yes, it is tedious. Yes, there are computer programs that could do it for you. But as with the calculations, there is no better way to develop intuition than to make the graph yourself.

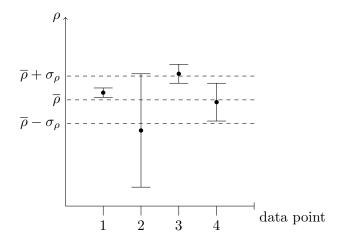


Figure 1: Plot of four results for cork, with mean and standard deviation shown. Error bars are the measurement uncertainty.

3 Putting it all together

- 1. Are there any outliers in the aggregate data set? What do they tell you, if anything?
- 2. Did you exclude any measurements from your analysis? If so, justify your decision.
- 3. What is the density of each material? Include conclusions from your measurements alone, as well as from the aggregate data set. How do the results from the aggregate data set compare to the results you found with your particular experiments? What can this tell you, if anything?
- 4. If you were asked to report on the density of rubber and cork, what would you say based on these findings? Would you be comfortable reporting a value (including uncertainty)? Would you ask for more time to perform more and/or different tests?

4 Data

4.1 All data as reported

In the following tables are the class's results from the density lab, as reported by each group—I have not edited them for any inconsistencies with significant figures, units, incorrect calculations, etc. You may decide some results are unreliable for one reason or another. If you exclude data points from your analysis you must be able to justify the exclusion.

Groups have intentionally been left anonymous. Individuals may decide to re-do calculations (including error analysis, rounding for significant figures, unit conversions, etc.) before turning in their report.

Table 1: Rubber

Density	Units	Density	Units	Density	Units	Density	Units
1.35	g/ml	1.42	$\rm g/cm^3$	1.3	$\rm g/cm^3$	1.45	g/cm^3
1.39	g/ml	1.48	$\rm g/cm^3$	1.4	$\rm g/cm^3$	1.39	$\rm g/cm^3$
1.16	g/ml	1.43	$\rm g/cm^3$	1.3	$\rm g/cm^3$	1.28	$\rm g/cm^3$
1.33	g/ml	1.39	$ m g/cm^3$	1.3	$\mathrm{g/cm^3}$	1.11	$\mathrm{g/cm^3}$
1.32	g/ml	1.45	$ m g/cm^3$	0.0014	${ m g/mm^3}$	1.2	$\mathrm{g/cm^3}$
1.35	g/ml	1.42	${ m g/cm^3}$	0.0014	${ m g/mm^3}$	1.5	$\rm g/cm^3$
1.30	g/ml	1.48	$\rm g/cm^3$	0.0014	$\rm g/mm^3$	1.4	$\rm g/cm^3$
1.16	g/ml	1.43	$\mathrm{g/cm^3}$	0.0013	${ m g/mm^3}$	1.3	$\rm g/cm^3$
1.33	g/ml	1.39	$\mathrm{g/cm^3}$	0.0013	${ m g/mm^3}$	1.3	$\rm g/cm^3$
1.32	g/ml	1.45	$ m g/cm^3$	1.37	g/ml	1.37	g/ml
1.36	$\mathrm{g/cm^3}$	1.3	g/ml	1.25	g/ml	1.3	g/ml
1.31	$\mathrm{g/cm^3}$	1.3	g/ml	1.39	g/ml	1.37	g/ml
1.35	$\rm g/cm^3$	1.31	g/ml	1.24	g/ml	1.3	g/ml
1.66	$\rm g/cm^3$	1.32	g/ml	1.4	g/ml	1.58	g/ml
1.35	$\rm g/cm^3$	1.37	g/ml	1.3	$\rm g/cm^3$	1.27	$\rm g/cm^3$
1.1986	not reported	0.001366	not reported	1.2	$\rm g/cm^3$	1.24	$\rm g/cm^3$
0.0159	not reported	0.001393	not reported	1.2	$\rm g/cm^3$	1.28	$\rm g/cm^3$
1.386	not reported	0.001366	not reported	1.3	$\rm g/cm^3$	1.29	$\rm g/cm^3$
1.2183	not reported	0.001301	not reported	1.3	$\rm g/cm^3$	1.27	$\rm g/cm^3$
1.2608	not reported	0.001283	not reported	1.15	ml/g	1.08	$\mathrm{cm/g}$
1.3	$\rm g/cm^3$	1.32	$\mathrm{g/cm^3}$	1.14	ml/g	1.05	$\mathrm{cm/g}$
1.6	$\rm g/cm^3$	1.35	$\rm g/cm^3$	1.19	ml/g	1.09	$\mathrm{cm/g}$
1.3	$\rm g/cm^3$	1.32	$\rm g/cm^3$	1.16	ml/g	1.04	$\mathrm{cm/g}$
1.3	g/cm^3	1.32	g/cm^3	1.52	ml/g	1.51	cm/g

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Table 1: Rubber

Density	Units	Density	Units	Density	Units	Density	Units
1.3	g/cm^3	1.3	g/cm^3	1.1	not reported		not reported
1.76	$\rm g/mm^3$	11.29	m g/ml	1.3	not reported	1.3	not reported
1.76	$\rm g/mm^3$	11.43	g/ml	1.3	not reported	1.3	not reported
1.85	$ m g/mm^3$	13.17	g/ml	1.3	not reported	1.3	not reported
1.97	$ m g/mm^3$	13.5	g/ml	1.4	not reported	1.3	not reported
1.76	$ m g/mm^3$	11.57	g/ml				
1.4	g/cm^3	1.28	g/cm^3				

Table 2: Cork

Density	Units	Density	Units	Density	Units	Density	Units
0.25	g/ml	0.25	$\rm g/cm^3$	0.20	$\rm g/cm^3$	0.18	g/cm^3
0.14	g/ml	0.17	$\rm g/cm^3$	0.33	g/cm^3	0.33	$\rm g/cm^3$
0.20	g/ml	0.31	$\rm g/cm^3$	0.29	$\rm g/cm^3$	0.23	$\rm g/cm^3$
0.26	g/ml	0.18	$\rm g/cm^3$	0.24	$\mathrm{g/cm^3}$	0.18	$\rm g/cm^3$
0.13	g/ml	0.22	$\rm g/cm^3$	0.00017	${ m g/mm^3}$	0.16	$\rm g/cm^3$
0.25	g/ml	0.25	$\rm g/cm^3$	0.00015	${ m g/mm^3}$	0.17	$\rm g/cm^3$
0.14	g/ml	0.17	$\rm g/cm^3$	0.00020	${ m g/mm^3}$	0.2	$\rm g/cm^3$
0.2	g/ml	0.31	$\rm g/cm^3$	0.00017	${ m g/mm^3}$	0.17	$\rm g/cm^3$
0.26	g/ml	0.18	$\rm g/cm^3$	0.00024	${ m g/mm^3}$	0.24	$\rm g/cm^3$
0.13	g/ml	0.22	$\rm g/cm^3$	0.19	g/ml	0.22	g/ml
0.2	$\mathrm{g/cm^3}$	0.19	g/ml	0.13	g/ml	0.11	g/ml
0.2	$\mathrm{g/cm^3}$	0.22	g/ml	0.22	g/ml	0.26	g/ml
0.31	$\mathrm{g/cm^3}$	0.3	g/ml	0.16	g/ml	0.16	g/ml
0.15	$\rm g/cm^3$	0.16	g/ml	0.2	g/ml	0.2	g/ml
0.2	$\mathrm{g/cm^3}$	0.19	g/ml	0.2	$\rm g/cm^3$	0.2	$\rm g/cm^3$
0.1571	not reported	0.00017	not reported	0.2	$\rm g/cm^3$	0.19	$\mathrm{g/cm^3}$
0.1746	not reported	0.000141	not reported	0.2	$\rm g/cm^3$	0.19	$\mathrm{g/cm^3}$
0.195	not reported	0.000183	not reported	0.2	$\rm g/cm^3$	0.14	$\mathrm{g/cm^3}$
0.1723	not reported	0.000167	not reported	0.2	$\rm g/cm^3$	0.2	$\rm g/cm^3$
0.241	not reported	0.00024	not reported	0.28	ml/g	0.24	$\mathrm{cm/g}$
0.17	$\mathrm{g/cm^3}$	0.23	$\rm g/cm^3$	0.32	ml/g	0.25	$\mathrm{cm/g}$
0.23	g/cm^3	0.49	g/cm^3	0.22	ml/g	0.24	cm/g

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Table 2: Cork

Density	Units	Density	Units	Density	Units	Density	Units
0.26	$\rm g/cm^3$	0.49	g/cm^3	0.32	ml/g	0.25	cm/g
0.26	$\rm g/cm^3$	0.22	$\rm g/cm^3$	0.23	ml/g	0.25	$\mathrm{cm/g}$
0.23	$\rm g/cm^3$	0.21	$\rm g/cm^3$	0.24	not reported	0.27	not reported
0.29	g/mm^3	2.6	g/ml	0.4	not reported	0.2	not reported
1.0	g/mm^3	2.17	g/ml	0.48	not reported	0.23	not reported
0.18	g/mm^3	2.2	g/ml	0.23	not reported	0.23	not reported
0.2	g/mm^3	2.57	g/ml	0.29	not reported	0.29	not reported
0.26	g/mm^3	2.0	g/ml				
0.29	$\rm g/cm^3$	0.27	g/cm^3				

4.2 Sample of data with uncertainty

For the following tables, I chose a sample of the class's data, and calculated experimental uncertainties $(\delta\rho)$ based on the measurement technique and instruments used. For this set of data, I have given all results with significant figures appropriate to the measurement techniques and instruments used. However, I have not edited the data for incorrect readings of the measurement instrument.

Table 3: Rubber

 ρ (g/ml) $\delta \rho \; (g/ml)$ $\delta \rho \, (g/ml)$ ρ (g/ml) 1.4 0.11.1 0.10.340.01 1.4 0.11.2 0.10.340.01 1.3 0.11.3 0.110 10 10 10 1.3 0.20.40.21.4 0.11.4 0.11.4 0.11.6 0.21.3 0.11.2 0.11.2 0.11.5 0.21.2 0.10.11.4

Table 4: Cork

ρ (g/ml)	$\delta\rho$ (g/ml)	ρ (g/ml)	$\delta \rho \; (\mathrm{g/ml})$
0.25	0.03	0.14	0.02
0.050	0.004	0.19	0.02
0.16	0.01	0.042	0.004
0.18	0.02	0.23	0.03
3	3	3	2
0.27	0.04	0.18	0.03
0.17	0.02	0.24	0.02
0.22	0.03	0.11	0.02
0.20	0.03	0.18	0.02
0.28	0.03	0.32	0.04
0.27	0.02	0.20	0.02