This activity is designed to give you practice with measurement uncertainty, propagation of error, and significant figures.

1 Background

1.1 Measurement uncertainty

Unfortunately, no measurement is perfect. Every instrument you ever use will have some margin of error. In general, a measurement is reported as $x \pm \delta x$, though sometimes relative uncertainties (given as percents) are more useful: $x \pm {}^{\delta x}/_{x}$. Here, x represents your "best guess" at what the measurement is, and δx is your measurement uncertainty.

Part of being an honest experimenter is knowing and reporting the experimental uncertainties in your equipment. For analog instruments, a good rule of thumb is that you can estimate to halfway between the smallest increment on the device; take the smallest increment and divide by two to find the uncertainty.

> For example, on a ruler with millimeter markings, you could reasonably estimate a length to half-millimeter precision. If you are measuring something that looks to be 12.5 cm long, you would report the length as 12.50 ± 0.05 cm.

For digital instruments, you do no estimating yourself, so you cannot interpolate between two values. Because of this, your uncertainty is the smallest increment of the digital readout. For example, if a digital scale gives you precision to the tenth of a gram, the uncertainty would be ± 0.1 g.

In some cases, documentation about the instrument you are using will provide an uncertainty. Use this information if you have it, but be sure to read carefully—sometimes documentation will give different uncertainties for different situations.

1.2 Propagation of error

When you carry out a calculation using physical measurements, the uncertainty in each measurement contributes to the overall uncertainty of your calculation. In general, propagation of uncertainty depends on the rate at which a function changes with respect to change in experimental quantities. Say you have a function $F = f(x_1, x_2, \ldots, x_n)$. If the errors in measuring each variable x_i are small, random, and independent of each other, then the uncertainty in F is

$$(\delta F)^2 = \sum_{i=1}^{n} \left[\left(\frac{\partial F}{\partial x_i} \right)^2 (\delta x_i)^2 \right] \tag{1}$$

where δ denotes the experimental uncertainty.

For example, say you are doing the classic kinematics experiment where you determine the speed of a horizontally-launched projectile by measuring the height y and range x. You know the time is given by

$$t = \sqrt{\frac{2y}{g}}$$

and therefore the speed is given by

$$v = x\sqrt{\frac{g}{2y}}$$

Taking g as a precise value (this is acceptable for an introductory physics lab; the uncertainty of g is very low[a]), the uncertainty in your velocity calculation is

$$\begin{aligned}
(\delta v)^2 &= \left(\frac{\partial v}{\partial x}\right)^2 (\delta x)^2 + \left(\frac{\partial v}{\partial y}\right)^2 (\delta y)^2 \\
&= \left(\sqrt{\frac{g}{2y}}\right)^2 (\delta x)^2 + \left(-\frac{x}{2}\sqrt{\frac{g}{2y^3}}\right)^2 (\delta y)^2 \\
\therefore \delta v &= \sqrt{\frac{g}{2y}\left[(\delta x)^2 + \frac{1}{y}\left(\frac{x\delta y}{2}\right)^2\right]}
\end{aligned}$$

[a]If you do not account for latitude, the uncertainty in g is on the order of $10^{-4}\,\mathrm{m/s^2}$

I recommend reducing to simplest form to make the actual calculation more convenient: the fewer things you need to input into your calculator (or a spreadsheet formula), the easier the calculation will be.

1.3 Significant figures

In this class, uncertainty is rounded to one significant figure. The order of magnitude of the last significant figure of your measurement or calculated result must be the same order of magnitude of as the uncertainty.

In the projectile experiment from the previous example, you find unrounded values of 6.674 33 m/s for the launch velocity (v) and 0.010 14 m/s for the uncertainty in the launch velocity (δv). You would report your result as

$$v = 6.67 \pm 0.01\,\mathrm{m/s}$$

2 Tools

You have a variety of standard measuring tools, and a stopwatch.

3 Tasks

1. Use three different measurement tools with different levels of precision to measure the length of a pencil. Report each measurement with appropriate uncertainty and significant figures.

2. Release an object from rest, and time how long it takes it to reach the ground. Calculate the acceleration due to gravity. Report all data with appropriate uncertainty and significant figures. Determine the uncertainty in your calculation by applying propagation of error. Report your result with appropriate significant figures.

3. Determine the following experimental results, and the associated uncertainty. Report all values with appropriate significant figures.

   (a) You are calculating the density of an object as $\rho = {}^{m}/_{V}$. You measure the mass to be $m = 12.3 \pm 0.2\,\mathrm{kg}$ and you measure the volume directly as $V = 125 \pm 5\,\mathrm{mL}$.

   (b) You are experimentally determining acceleration due to gravity by using a projectile launcher with a launch speed known to be $7.000 \pm 0.001\,\mathrm{m/s}$. You launch the projectile horizontally, and measure the distance it traveled and the launch height. The distance traveled was $54.80 \pm 0.05\,\mathrm{cm}$ and the launch height was $150.15 \pm 0.05\,\mathrm{cm}$.

4 Grading

Do not write a full lab report for this activity. Just solve the problems presented in the Tasks section. This will be graded similarly to how I would grade a quiz.