hw1

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1 Problem 1

1. Memoryless implies

$$\forall n, k \in \mathbb{Z}^+ : \mathsf{P}(X = n + k | X > k) = \mathsf{P}(X = n)$$

or

$$\forall n, k \in \mathbb{Z}^+ : \frac{\mathsf{P}(X = n + k \cap X > k)}{\mathsf{P}(X > k)} = \frac{\mathsf{P}(X = n + k)}{\mathsf{P}(X > k)} = \mathsf{P}(X = n)$$

or

$$\forall n, k \in \mathbb{Z}^+ : \mathsf{P}(X = n + k) = \mathsf{P}(X > k)\mathsf{P}(X = n)$$

Let $k = 1$, $\mathsf{P}(X = 1) = p$, then

$$\forall n, k \in \mathbb{Z}^+ : \mathsf{P}(X = n + 1) = (1 - p)\mathsf{P}(X = n)$$

Using mathematical induction to prove that

$$\forall n \in \mathbb{Z}^+ : \mathsf{P}(X = n) = (1 - p)^{n-1}p$$

2 Problem 2

Recall *Coupon collector's problem*, there are 2 coupons in this case

$$\mathsf{E}(X) = \mathsf{E}(X_1) + \mathsf{E}(X_2) = 1 + \frac{1}{0.5} = 3$$

where X_i represents the number of boxes bought while you have $i - 1$ types of coupons until you get the i_{th} type