Homework of Week 9

Deadline: 9:00am, December 24 (Thursday), 2020

1. Prove that, for every integer n, there exists a way to 2-color the edges of K_x so that there is no monochromatic clique of size k when $x = n - \binom{n}{k}2^{1-\binom{k}{2}}$. Note that K_x stands for the x-vertex complete graph. (Hint, start by 2-coloring the edges of K_n and fix things up.)

2. For every integer n, there exists a coloring of the edges of the complete graph K_n by two colors so that the total number of monochromatic copies of K_4 is at most $\binom{n}{4}2^{-5}$. Design a deterministic, efficient algorithm to find such a coloring.

3. Given an n-vertex undirected graph $G = (V, E)$ and a permutation σ on V, define $S(\sigma) \subseteq V$ as follows: for any $i \in V$, $i \in S(\sigma)$ if and only if i has no neighbor in G that precedes i in the permutation σ. We know that $S(\sigma)$ is an independent set of G. Design a deterministic, efficient algorithm to produce a permutation σ such that the cardinality of $S(\sigma)$ is at least $\sum_{i=1}^{n} \frac{1}{d_i+1}$, where d_i is the degree of vertex i in G.

4. (Bonus: 5 points) Page 23 of Lecture 9 presents a randomized algorithm to find a graph with big chromatic number and big girth. Can you derive an efficient deterministic algorithm from it?

5. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string $s_1 s_2 ... s_i ... s_{20}$, where s_i is 1 if the i^{th} trial gets Head, and otherwise is 0.