Homework of Week 8

Deadline: 9:00am, December 17 (Thursday), 2020

1. We mentioned a probabilistic proof of Turán theorem in the lecture notes. Recall the random process generating an independent set S. Let p be the probability that the independent set S has size at least $\frac{|V|}{D+1}$. Show that $p \geq \frac{1}{2D|V|^2}$.

2. Given an n-vertex undirected graph $G = (V, E)$, consider the following method of generating an independent set. Given a permutation σ of the vertices, define a subset $S(\sigma)$ of the vertices as follows: for each vertex i, $i \in S(\sigma)$ if and only if no neighbor j of i precedes i in the permutation σ.

   - Show that each $S(\sigma)$ is an independent set in G.
   - Suggest a natural randomized algorithm to produce σ for which you can show that the expected cardinality of $S(\sigma)$ is $\sum_{i=1}^{n} \frac{1}{d_i+1}$, where d_i is the degree of vertex i.

3. Prove the following claims.

   - For every integer n, there exists a coloring of the edges of the complete graph K_n by two colors so that the total number of monochromatic copies of K_4 is at most $\binom{n}{4}2^{-5}$.
   - Give a randomized algorithm for finding a coloring with at most $\binom{n}{4}2^{-5}$ monochromatic copies of K_4 that runs in expected time polynomial in n.

4. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string $s_1s_2...s_i...s_{20}$, where s_i is 1 if the i^{th} trial gets Head, and otherwise is 0.