Theory of probability and random graphs – HW6

Q1. Solution:

针对第 i 位，其哈希值为 0 的概率分别为 $\Pr\left(F(X)_i\right) = \left(1 - \frac{1}{m}\right)^{kn}$

当 X 中的第 i 位不同于 Y 的第 i 位，则概率为

$$\Pr\left(F(X)_i \neq F(Y)_i\right) = \Pr\left(F(X \cap Y)_i = 0\right)\Pr\left(F(X - X \cap Y)_i \neq F(Y - X \cap Y)_i\right)$$
$$= \Pr(F(X \cap Y)_i = 0)\left[\Pr\left(F(X - X \cap Y)_i = 0, \Pr\left(F(X - X \cap Y)_i = 1\right)\right.\right.$$
$$\left.\left.+\Pr\left(F(X - X \cap Y)_i = 1, \Pr\left(F(X - X \cap Y)_i = 0\right)\right]\right.$$
$$= 2\left(1 - \frac{1}{m}\right)^{kn}\left[1 - \left(1 - \frac{1}{m}\right)^{k(n-|X\cap Y|)}\right]$$

$$\mathrm{E}(F(X)_i \neq F(Y)_i) = m\Pr(F(X)_i \neq F(Y)_i) = 2m\left(1 - \frac{1}{m}\right)^{kn}\left[1 - \left(1 - \frac{1}{m}\right)^{k(n-|X\cap Y|)}\right]$$

the rest will be easy

Q2. Solution:

对于随机图 $\mathcal{G}_n$，其生成为一个特定随机图 G 的概率为 [n 结点无向完全图子图数目为 C_n^2]

$$\Pr\left(\mathcal{G}_n = G\right) = \left(\frac{1}{2}\right)^{C_n^2}$$

对于随机图 $\mathcal{G}_{n,\frac{1}{2}}$，其生成为一个 m 条边特定随机图 G 的概率为 [二项分布]

$$\Pr\left(\mathcal{G}_{n,\frac{1}{2}} = G\right) = \left(\frac{1}{2}\right)^{C_n^2 - m}\left(1 - \frac{1}{2}\right)^m = \left(\frac{1}{2}\right)^{C_n^2}$$

由于 $\Pr(\mathcal{G}_n = G) = \Pr\left(\mathcal{G}_{n,\frac{1}{2}} = G\right)$，所以 $\mathcal{G}_n$ 和 $\mathcal{G}_{n,\frac{1}{2}}$ 同分布