Large Nc

Su Houng Lee

YONSEI UNIVERSITY

Theme:

1. Large Nc limit
2. Tetraquarks
3. Witten Veneziano formula

Ref:

E. Witten, Nucl. Phys. B 160 (1980) 57

E. Witten, Nucl. Phys. B 156 (1979) 213

G. Veneziano, Nucl. Phys B 159 (1979) 213

S. Weinberg, PRL110 (2013) 261601

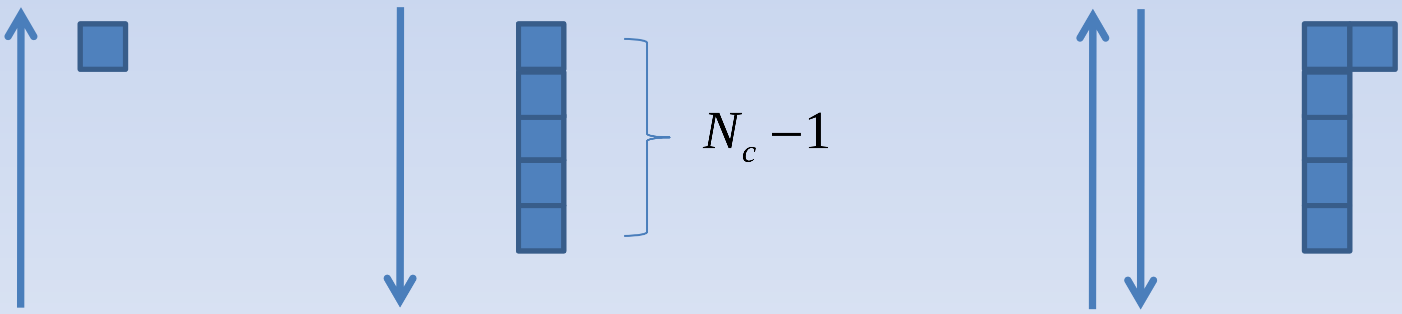
II Feynman Diagram for Large Nc -1

1. QCD is a SU(N) gauge theory with quarks in the fundamental representation

$$L = -\frac{1}{4}\mathrm{Tr}\left[\left(\partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]\right)^2\right] + \overline{\psi}\left(i\not{D} - m\right)\psi$$

$$\left(A_\mu\right)^i_j = A^a_\mu \frac{\lambda^a_{ij}}{2}, \quad \psi_j, \ \overline{\psi}^i \quad i = 1, \ldots N_c$$

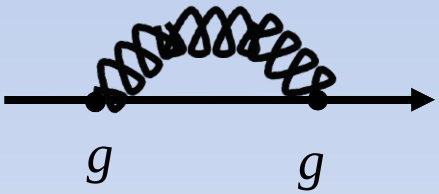
Quak, i　　　Antiquak, i　　　Gluon ij

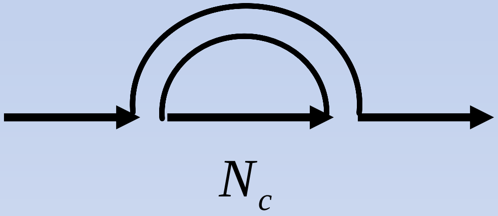


II Feynman Diagram for Large Nc-2

1. Counting of coupling g: quantum correction should have a smooth large Nc limit

$$\left(A_\mu\right)_i^j = A_\mu^a \frac{\lambda_{ij}^a}{2}, \quad \psi_j, \overline{\psi}^{\mathrm{i}} \quad i = 1, \ldots N_c$$





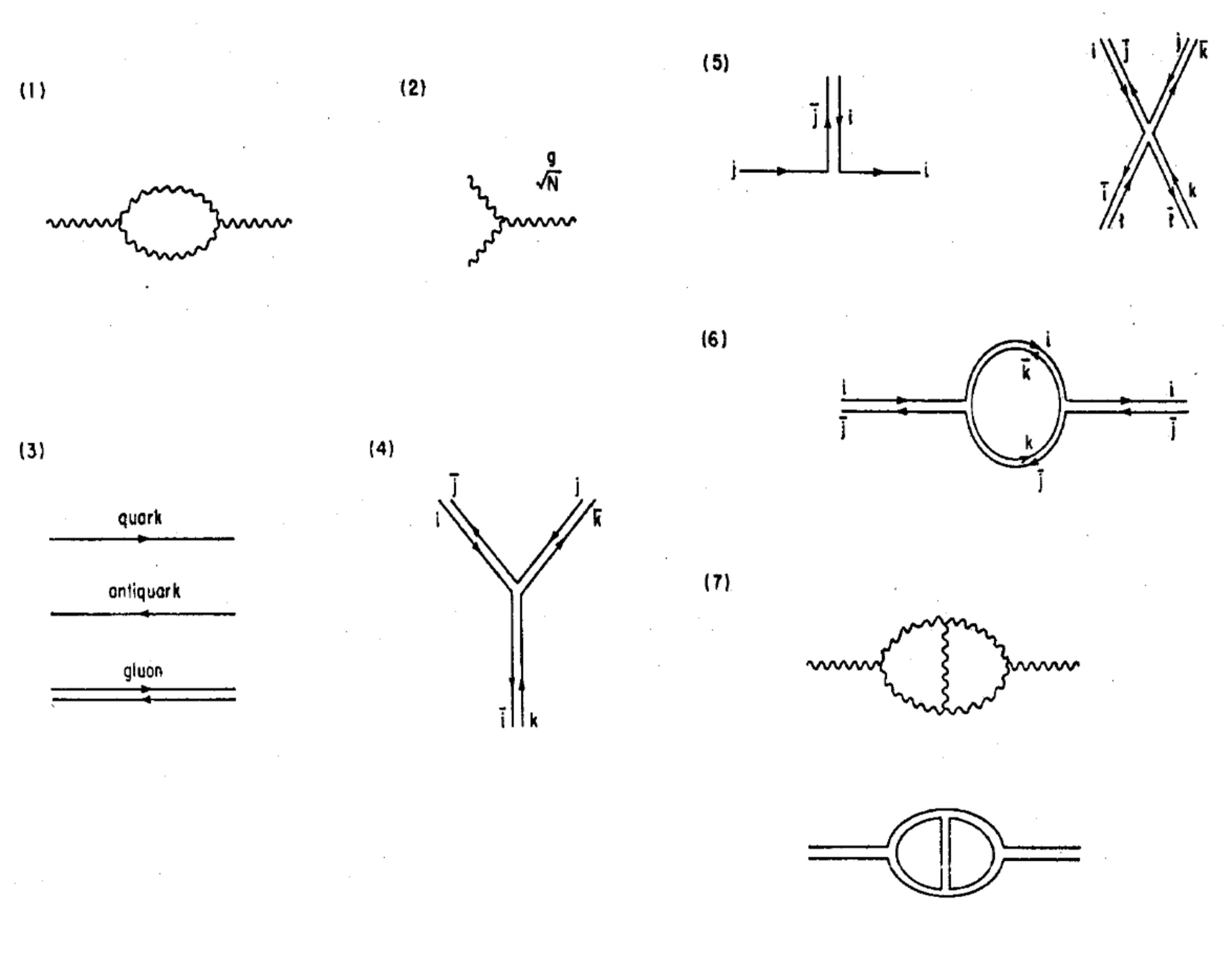
$$\text{Total counting}: \quad g^2 N_c \approx O(1)$$

$$g \to 1/N_c^{1/2}$$

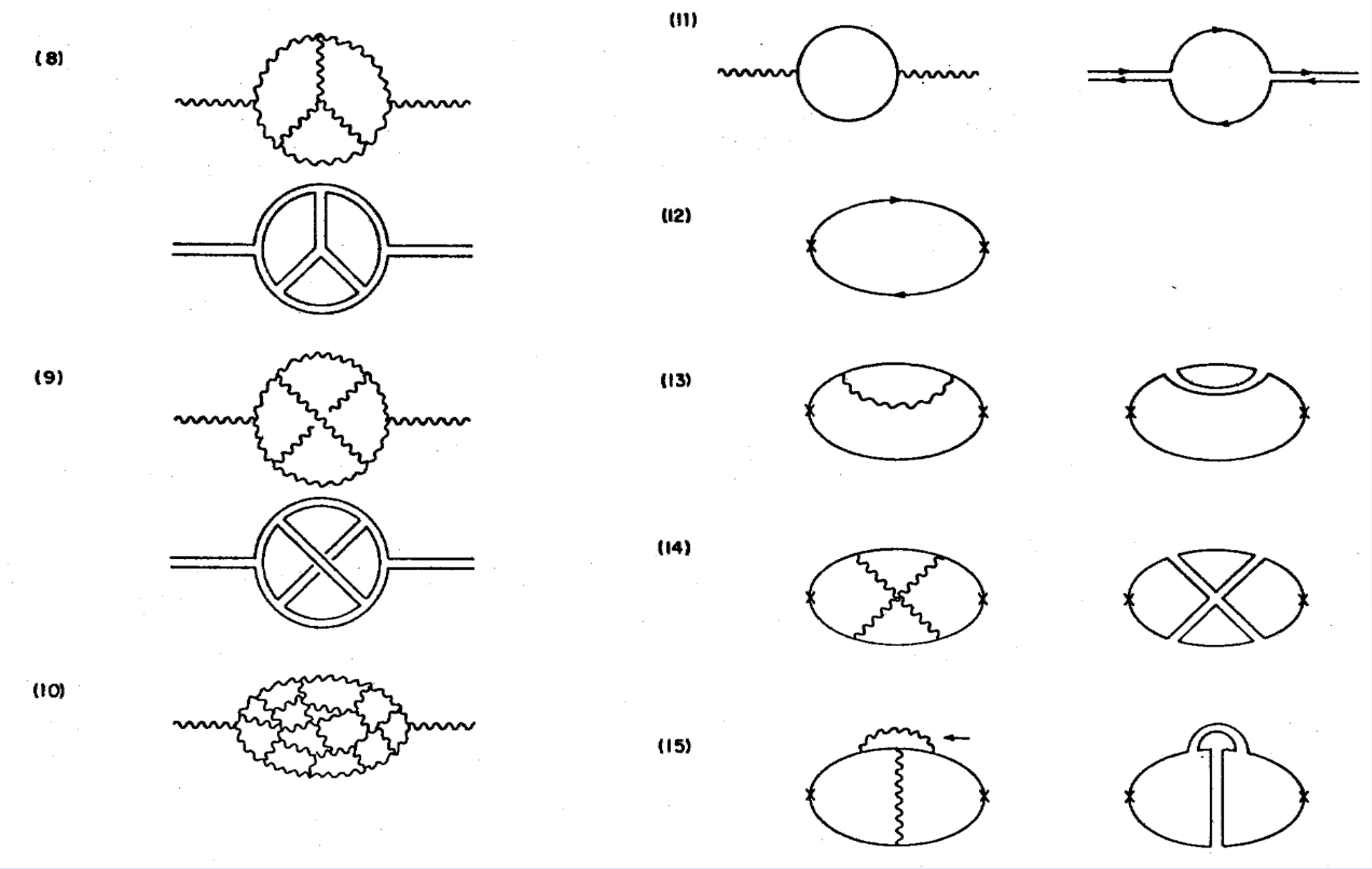
Rule 1 : count total couling n $\quad \mathrm{g}^{\mathrm{n}} \approx 1/N_c^{n/2}$

Rule 2 : additional factors can be obtained by drawing everything in terms of quark lines

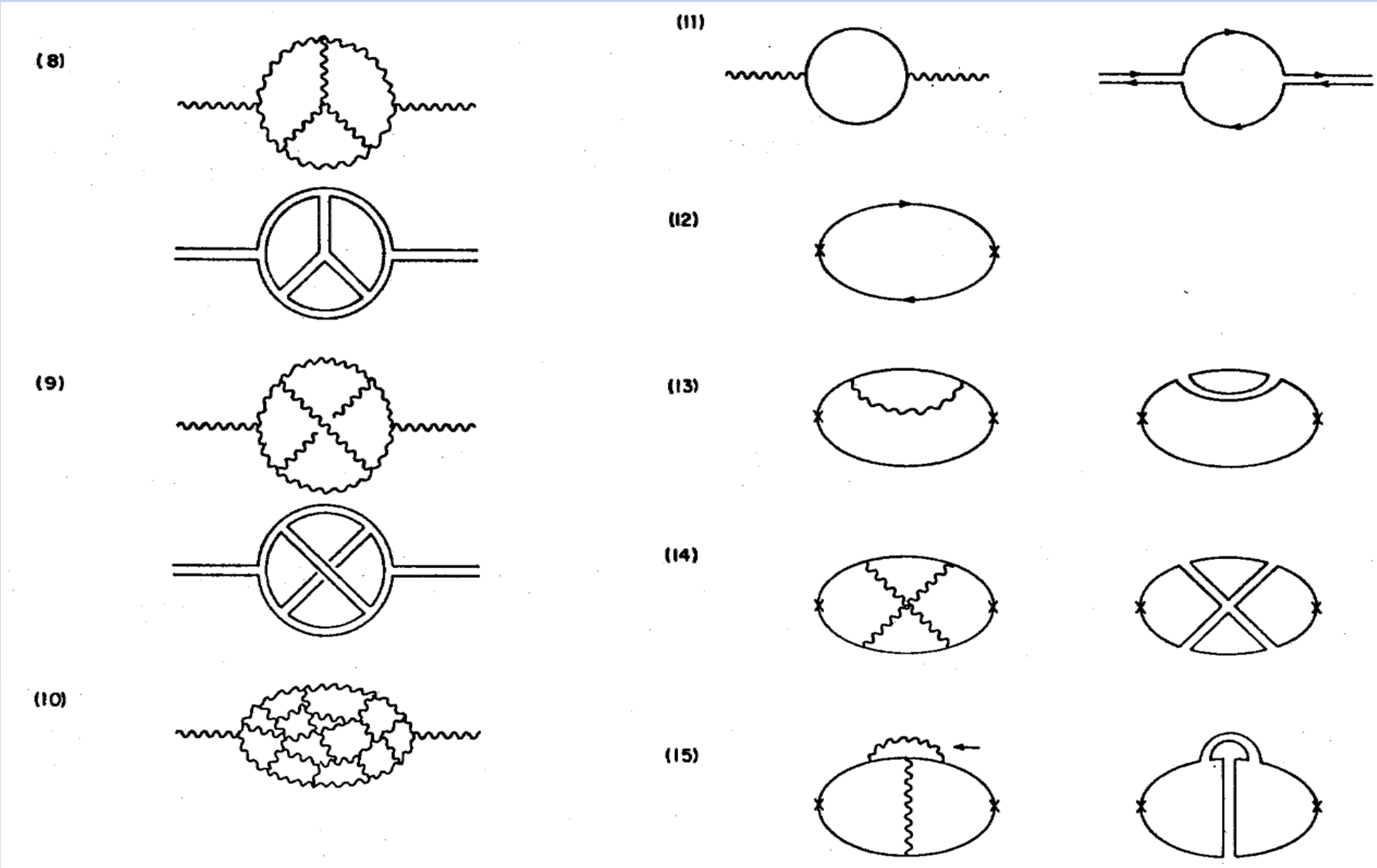
II Feynman Diagram for Large Nc-3



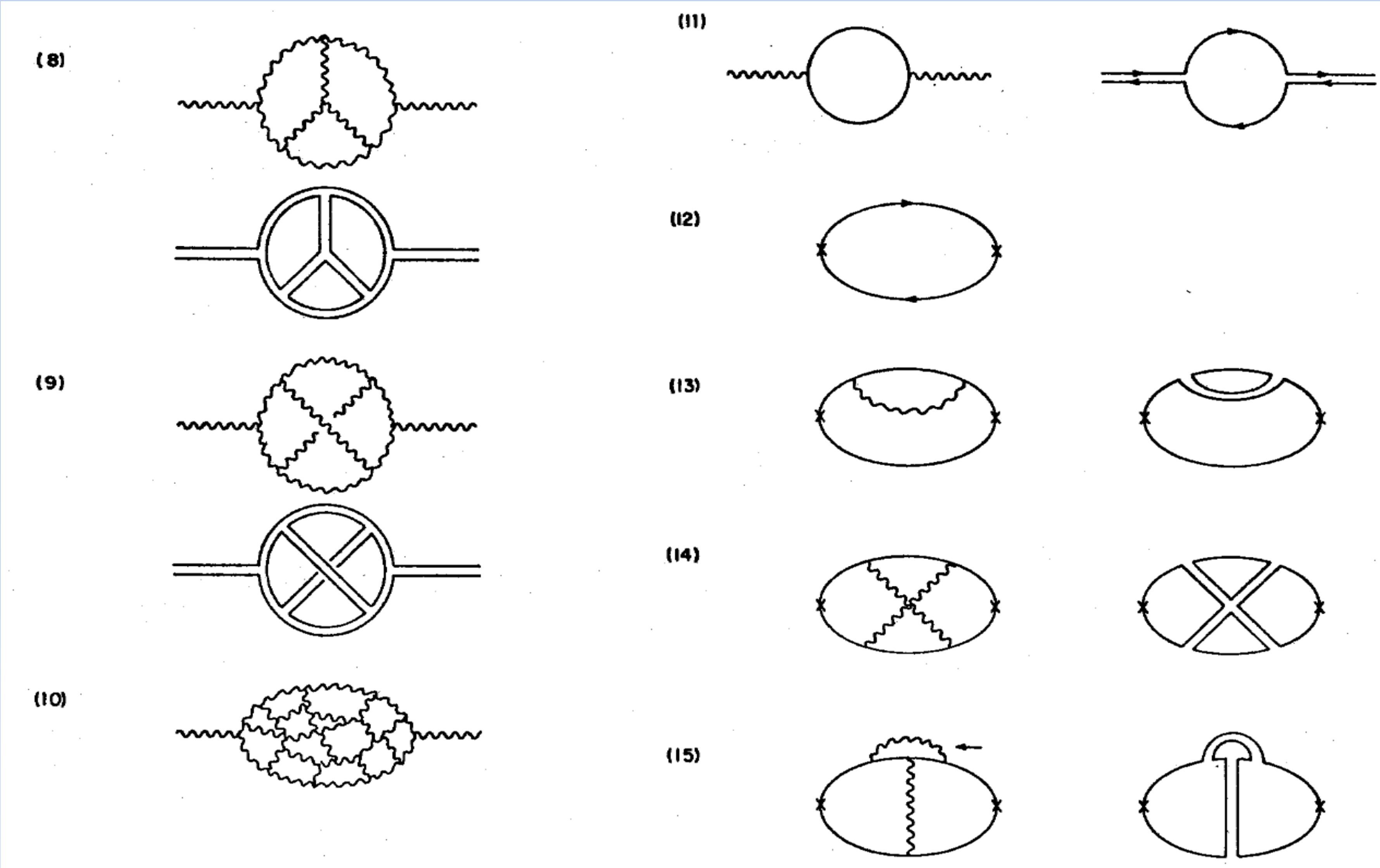
Planar diagrams survive in the large Nc limit 8→10



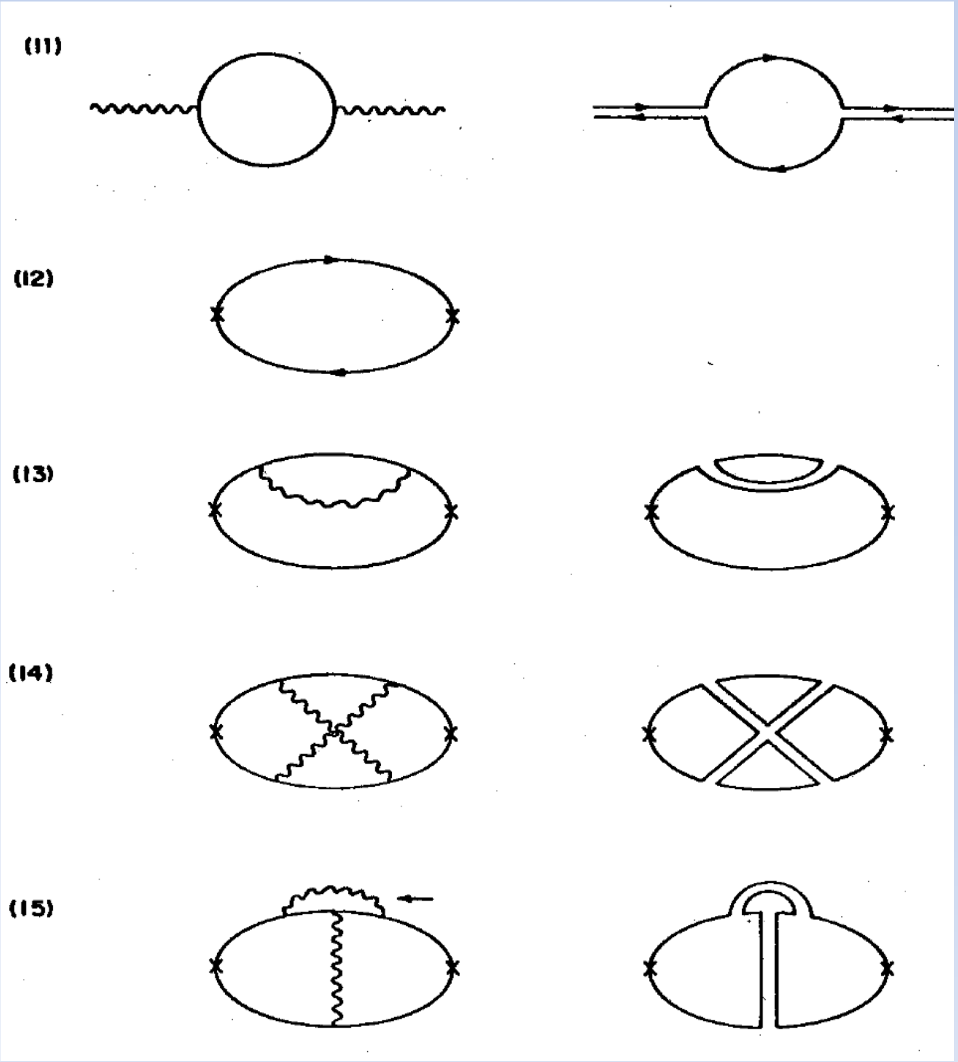
quark loops are suppress compared to gluon loop (11)

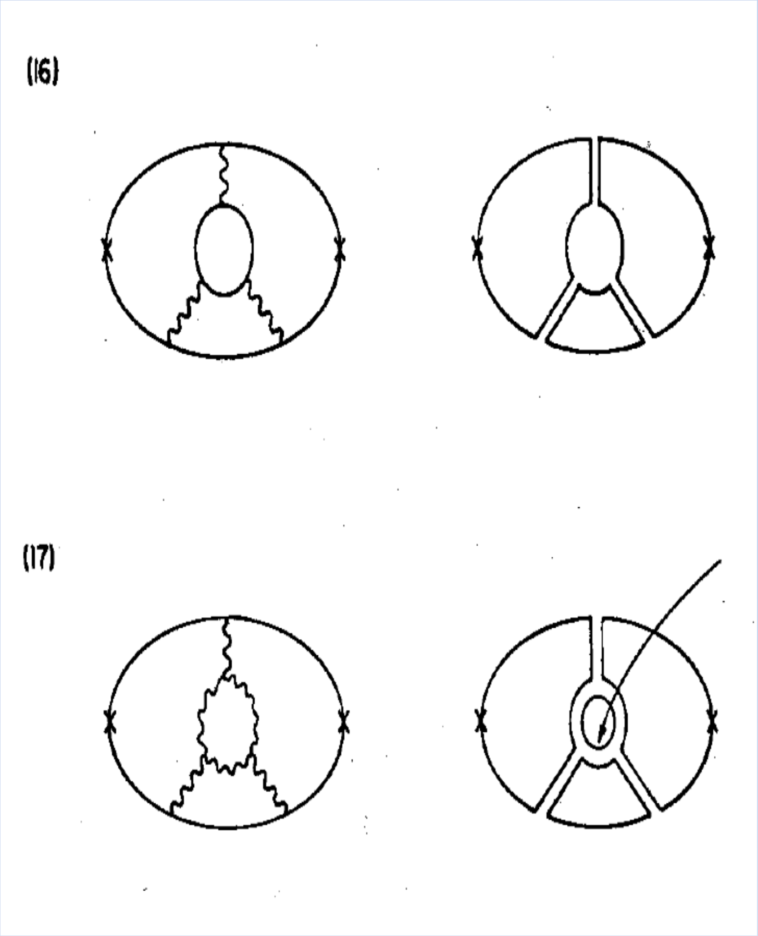


quark loops are suppress compared to gluon loop (11)



quark lines are only at the boundary



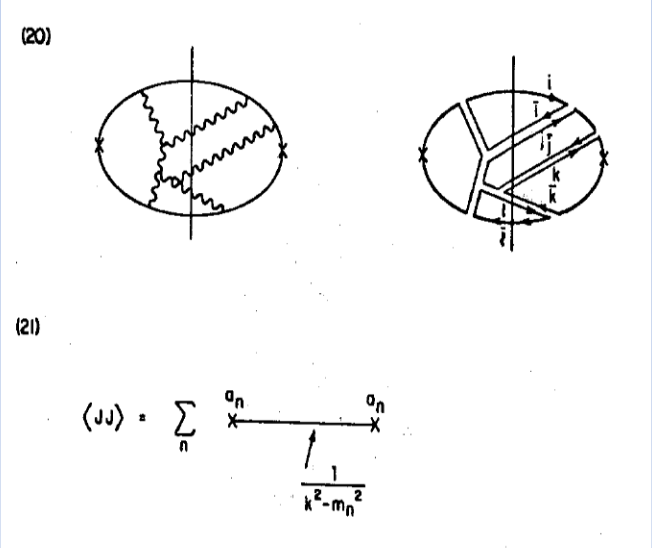


III Properties of mesons in large Nc limit

Assumption: confinement persists in the large Nc limit

- *Then can show mesons are free, stable, non interacting. Meson mass has smooth limit and the number of mesons are infinity.→ to prove this, look at two point function*

$$P(k) = \int dx e^{ikx} \langle J(x), J(0) \rangle, \qquad J = \overline{\psi}\psi$$



$$P(k) = \sum_n \frac{\left| < J \mid n > \right|^2}{k^2 - m_n^2} \xrightarrow{k \to large} \ln k^2$$

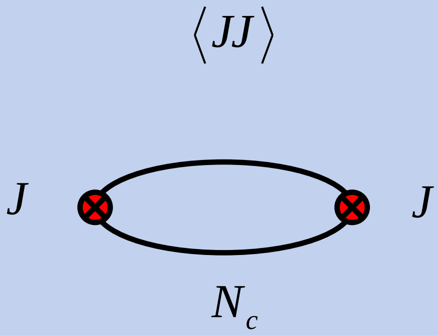
Only one meson state can contribute: $\overline{\psi}^{\,i} A_i^j A_j^k \psi_k$

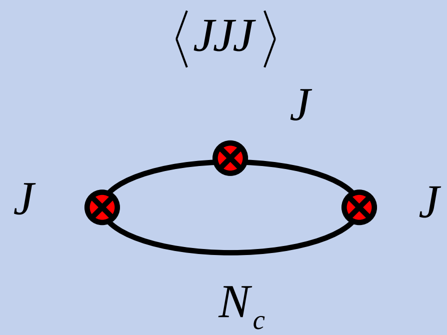
$P(k)$: should be of order Nc for any k

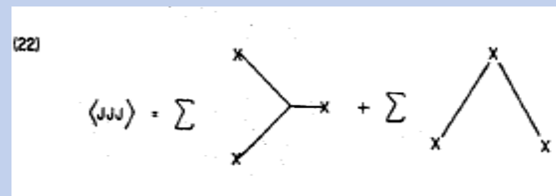
$< J \mid N >$ should be of order $N_c^{1/2}$

m_n^2 should be of order 1

There should be infinite number of mesons







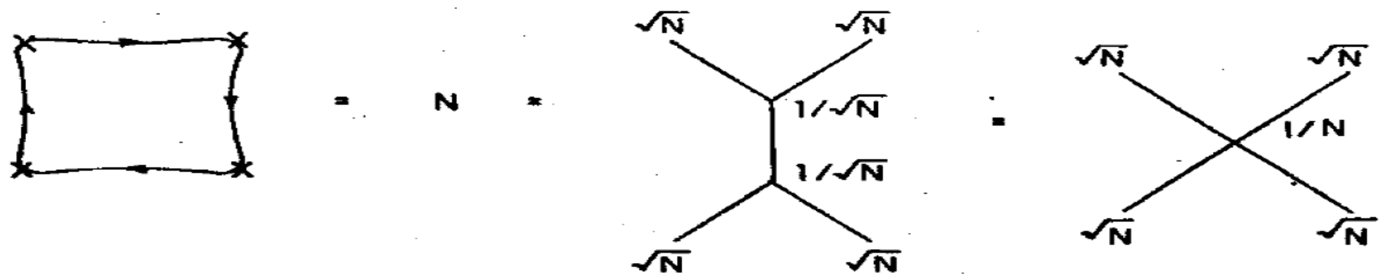
- *Using color decomposition as before, can show that only single meson states can interpolate*

$$\text{First term give : } \left| < \mathrm{J} \mid \mathrm{n} > \right|^3 \Gamma_{nnn} \rightarrow \left(N_c^{1/2}\right)^3 \frac{1}{N_c^{1/2}}$$

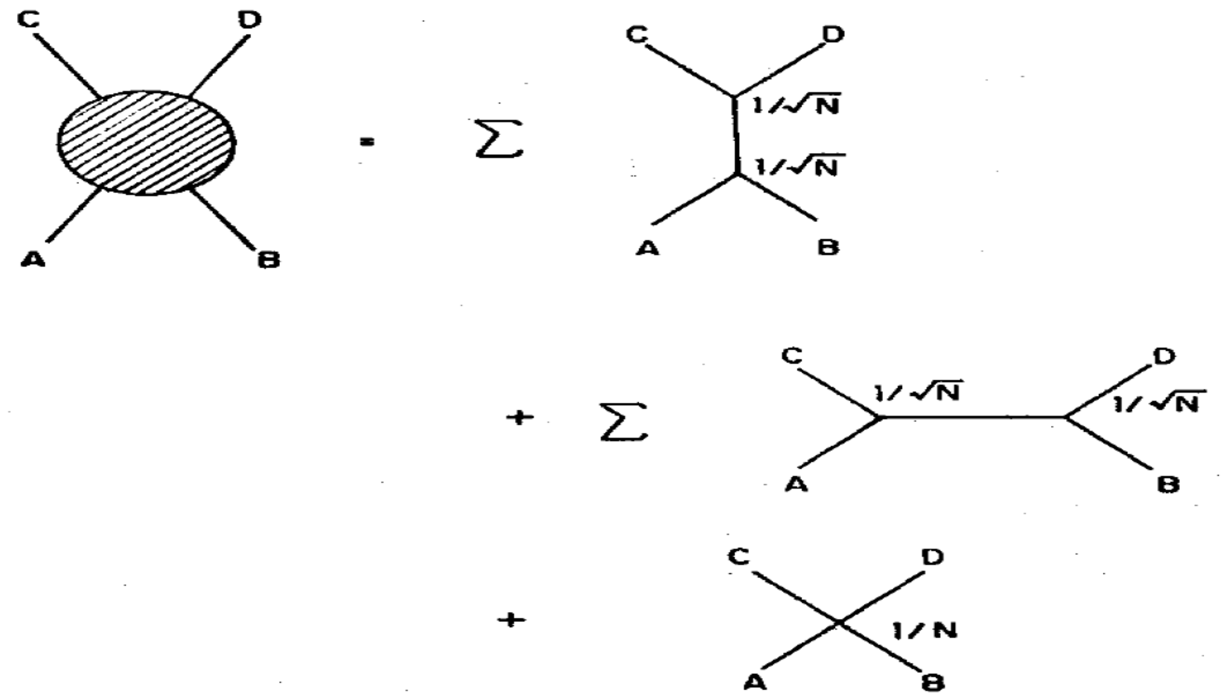
$$\text{Therefore meson decay } \Gamma_{\mathrm{A} \rightarrow \mathrm{BC}} \propto 1/\mathrm{N_c}$$

III Properties of mesons in large Nc limit: elastic scattering

(28)



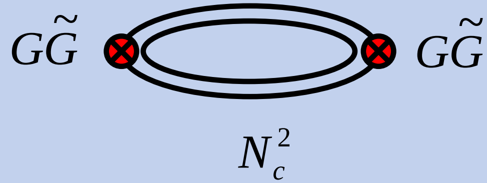
(29)



III Properties of mesons in large Nc limit: Glueball

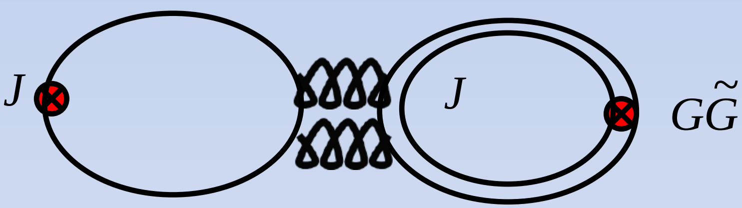
Assumption: composed of two gluons $G_{\mu\nu}G^{\mu\nu}$

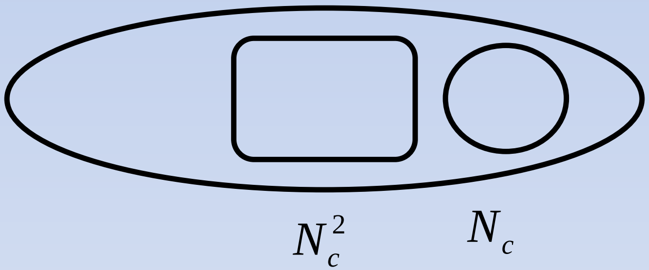
$$\langle G^2, G^2 \rangle = \sum \frac{\left| < G^2 \,|\, n > \right|^2}{k^2 - m_G^2}$$



- *Then can show glueballs are free, stable and non interacting and infinite in number.* →

Amplitude for glue to mix with meson : $1/N_c^{1/2}$





$$< J \,|\, n > C_{nG} < G \,|\, J_G > \propto N_c^{1/2} C_{nG} N_c = \left(1/N_c^{1/2}\right)^4 N_c^3 \quad \rightarrow C_{nG} = 1/N_c^{1/2}$$

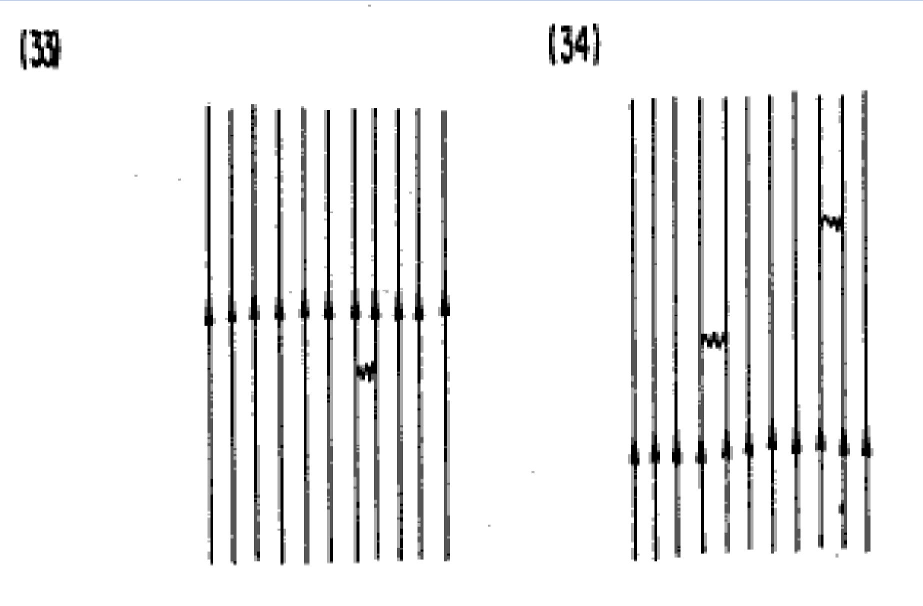
Amplitude for k mesons and l glue state: $N_c^{-l-\frac{k}{2}+1}$

IV Attractiveness of large Nc limit

- *Mesons are qq states qqqq such as quark sea are suppressed*
- *Zweig rule suppression : decay from qq creation is suppressed*
- *Octet singlet degeneracy: contribution from disconnected diagrams are suppressed*

V Baryon in the large Nc limit

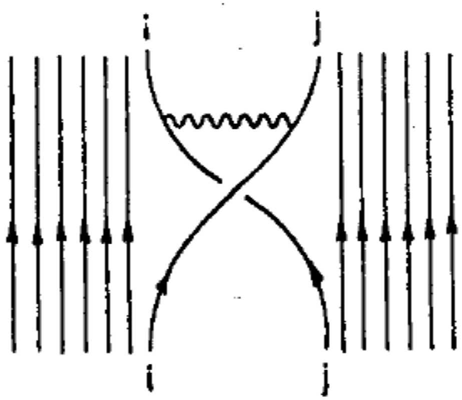
- *Baryon mass is of order Nc*



$$N_c^2 \times \frac{1}{N_c}$$

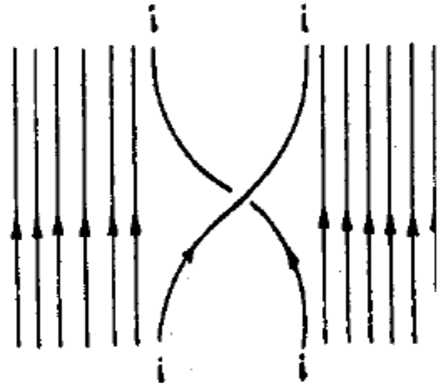
- *Baryon Baryon interaction*

(35)



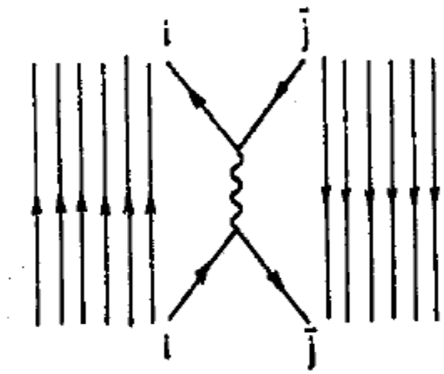
$$N_c \times \frac{1}{N_c} \times N_c$$

(36)



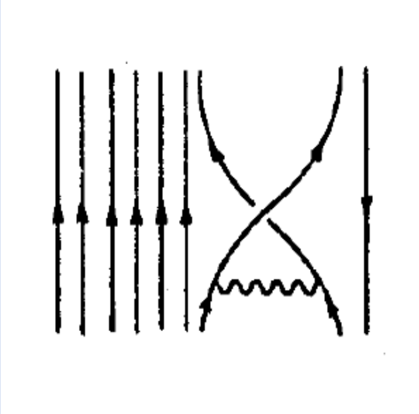
$$N_c$$

(37)



VI Baryon-Meson Scattering

- *Baryon-Meson*



$$N_c \times \frac{1}{N_c} \times 1$$

Tetraquarks in Large Nc

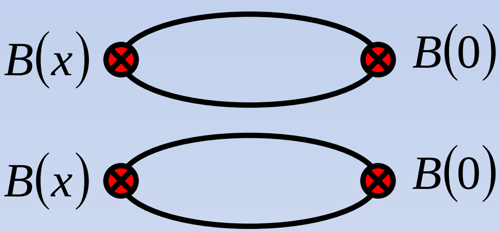
S. Weinberg, PRL110 (2013) 261601

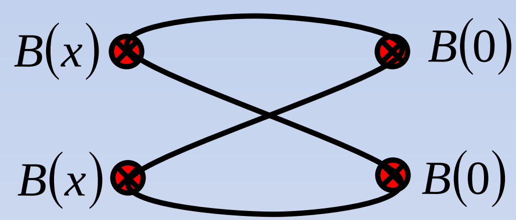
Original argument why there is no Tetraquark

- *Tetraquark in quark-antiquark basis*

$$Q = c_1(\bar{q}\Gamma q)(\bar{q}\Gamma q) + c_2(\bar{q}\Gamma\lambda^a q)(\bar{q}\Gamma\lambda^a q)$$

$$\langle Q(x), Q(0)\rangle = \underbrace{\langle B(x), B(0)\rangle\langle B(x), B(0)\rangle}_{N_c^2} + \underbrace{\langle B(x)B(x), B(0)B(0)\rangle_{connected}}_{N_c}$$

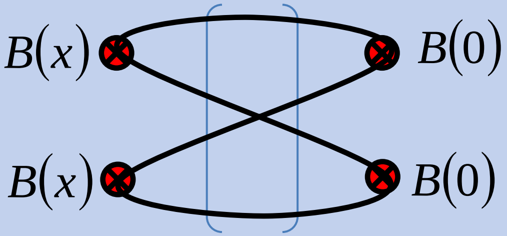




Tetraquark lives here

So what? Tetraquark – current coupling

- *Tetraquark current coupling* $\langle 0 | Q(x) | T \rangle \propto N_c^{1/2}$ $\quad Q(x) = B(x)B(x)$

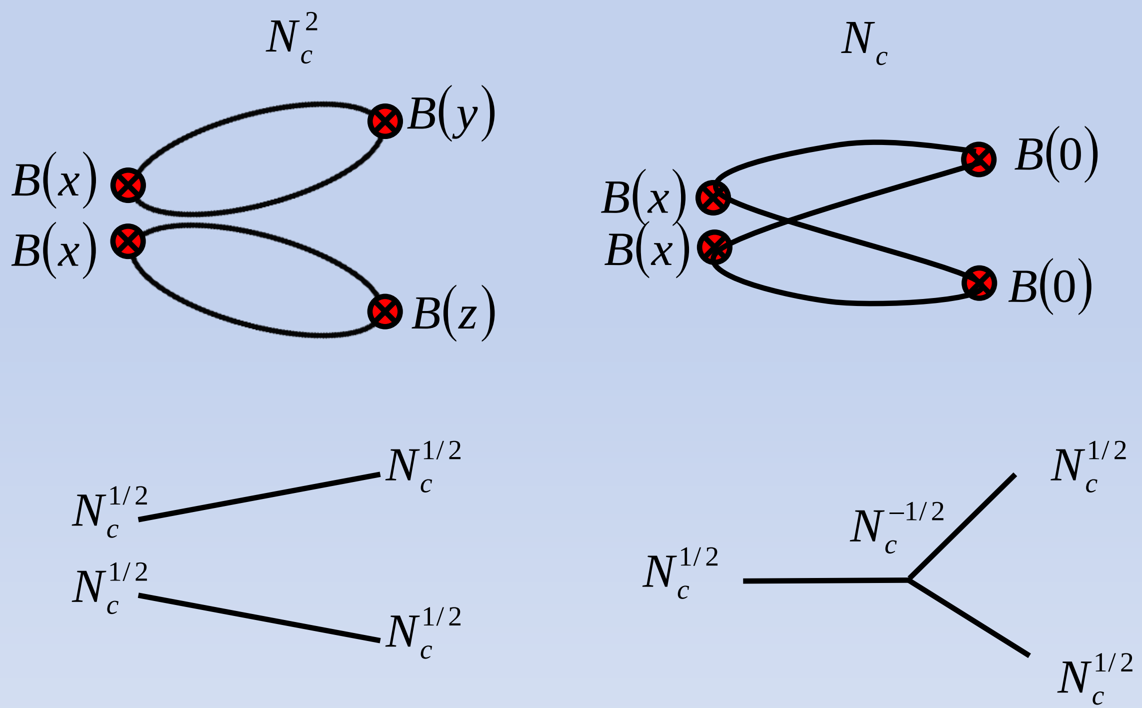


Tetraquark

Tetraquark – meson+meson decay width

- *Tetraquark current coupling* $Q(x) = B(x)B(x)$

$$\langle 0 | Q(x)B(y)B(z) | 0 \rangle = \langle 0 | B(x)B(y) | 0 \rangle \langle 0 | B(x)B(z) | 0 \rangle + \langle 0 | Q(x)B(y)B(z) | 0 \rangle_{connected}$$



Hence Tetraquark-meson-meson coupling Nc $^{-1/2}$

→ *Decay width is Nc* $^{-1}$

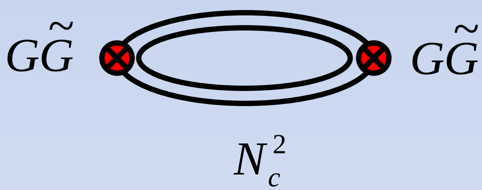
η' mass

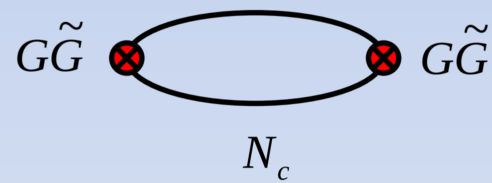
E. Witten, Nucl. Phys. B 156 (1979) 213

G. Veneziano, Nucl. Phys B 159 (1979) 213

η' mass? Witten-Veneziano formula - I

- Correlation function $P(k) = -i\int dx e^{ikx}\langle G\tilde{G}(x), G\tilde{G}(0)\rangle$
- Contributions from glue only $P_0(k=0) \neq 0$ from low energy theorem
- When massless quarks are added $P(k) = -i\int dx e^{ikx}\langle \partial^\mu j_\mu^5(x), \partial^\mu j_\mu^5(0)\rangle \propto k^\mu k^\nu P_{\mu\nu} \xrightarrow{k=0} 0$
- Large Nc argument $$P(k) = \sum_{glueballs}\frac{\langle 0|G\tilde{G}|glueball\rangle^2}{k^2 - m_n^2} + \sum_{mesons}\frac{\langle 0|G\tilde{G}|meson\rangle^2}{k^2 - m_n^2}$$





- Need η' meson $$+\frac{\langle 0|G\tilde{G}|\eta'\rangle^2}{k^2 - m_{\eta'}^2} \quad \text{with} \quad m_{\eta'}^2 \approx O\left(\frac{1}{N_c}\right)$$

$$\rightarrow \quad P(k=0) = \boxed{P_0(0)} - \frac{\boxed{\langle 0|G\tilde{G}|\eta'\rangle^2}}{m_{\eta'}^2} = 0$$

Witten-Veneziano formula – II

- η' meson $$\frac{\left\langle 0 | G\tilde{G} | \eta' \right\rangle^2}{m_{\eta'}^2} = P_0(0)$$

$$\frac{\left(\frac{4\pi}{\alpha}\right)^2 \left(\frac{1}{N_F}\sqrt{N_F}\, m_{\eta'}^2 f_{\eta'}\right)^2}{m_{\eta'}^2} = \left(\frac{4\pi}{3\alpha}\right)^2 \frac{8}{11N/3} \left\langle \frac{\alpha}{\pi} G^2 \right\rangle$$

Lee, Zahed (01)

$$m_{\eta'}^2 f_{\eta'}{}^2 = \frac{8}{11N} \left\langle \frac{\alpha}{\pi} G^2 \right\rangle \rightarrow \quad 250\,\mathrm{MeV} < \; m_{\eta'} < 432\;\mathrm{MeV}$$ at m → 0 limit

Should be related to $m_{\eta'}(958) - m_\eta(547) = 411\,\mathrm{MeV}$

Few Formula in Large Nc

- *Meson*

$$m(1),\quad \Gamma(1/N_c),\quad g_{mmm}(1/N_c^{1/2}),\quad \langle 0|\bar{q}\Gamma q|m\rangle(N_c^{1/2}),\quad \langle 0|GG|m\rangle(N_c^{1/2})$$

- *Glueball*

$$m(1),\quad \Gamma(1/N_c^2),\quad g_{ggg}(1/N_c),\quad \langle 0|\bar{q}\Gamma q|m\rangle(N_c),\quad \langle 0|GG|g\rangle(N_c)$$

- *Baryon*

$$m(N_c),\quad g_{mBB}(N_c^{1/2}),\quad \langle B|\bar{q}\Gamma q|B\rangle(N_c),\quad \langle B|GG|B\rangle(N_c)$$

- *η' meson*

$$\underbrace{\frac{\overbrace{\langle 0|G\tilde{G}|\eta'\rangle^2}^{O(N_c^1)}}{\underbrace{m_{\eta'}^2}_{O(1/N_c^1)}}} = P_0(0) = \left(\frac{4\pi}{3\alpha}\right)^2 \frac{8}{11N/3}\underbrace{\left\langle\frac{\alpha}{\pi}G^2\right\rangle}_{O(N_c^2)}$$

η' mass is a large $1/N_c$ correction

- *glueball*

$$\frac{\overbrace{\langle 0|GG|g\rangle^2}^{O(N_c^2)}}{\underbrace{m_g^2}_{O(1)}} = S_0(0) = \left(\frac{4\pi}{3\alpha}\right)^2 \frac{18}{11N/3}\underbrace{\left\langle\frac{\alpha}{\pi}G^2\right\rangle}_{O(N_c^2)}$$

Witten-Veneziano formula – IV

- *Low energy theorem is a Non-perturbative effect*

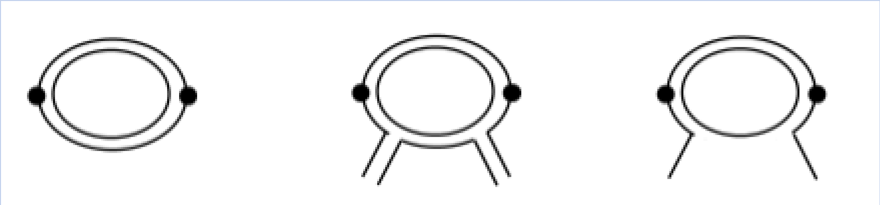
$$S(q) = i\int dx e^{iqx}\left\langle \frac{3\alpha}{4\pi}G^2(x)\ \frac{3\alpha}{4\pi}G^2(0)\right\rangle = \frac{18}{11}\left\langle \frac{\alpha}{\pi}G^2\right\rangle$$

$$P(q) = i\int dx e^{iqx}\left\langle \frac{3\alpha}{4\pi}G\tilde{G}(x)\ \frac{3\alpha}{4\pi}G\tilde{G}(0)\right\rangle = -\frac{8}{11}\left\langle \frac{\alpha}{\pi}G^2\right\rangle$$

η' mass is a large $1/N_c$ correction

Witten-Veneziano formula at finite T (Kwon, Morita, Wolf, Lee: PRD 12)

- *Large N_c counting* $$P(k) = i\int dx e^{ikx} \left\langle G\tilde{G}(x), G\tilde{G}(0) \right\rangle_m$$



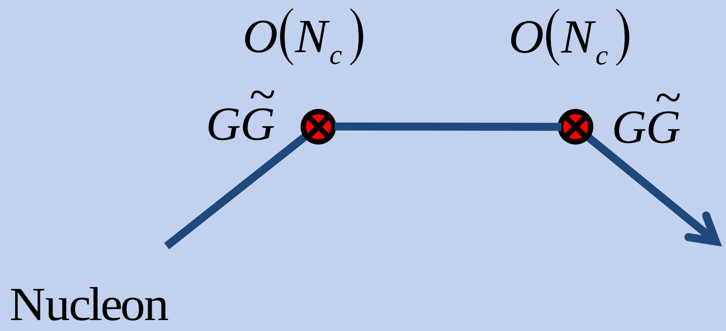
N_c^2 N_c^2 N_c

- *At finite temperature, only gluonic effect is important*

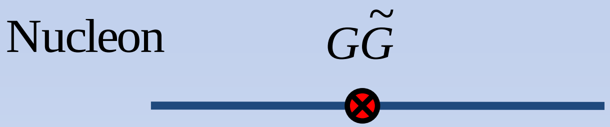
$$P(k) = \underbrace{\sum_{glueballs} \frac{\left\langle 0 | G\tilde{G} | glueball \right\rangle^2}{k^2 - m_n^2}}_{\text{Glue } N_c^2} + \underbrace{\sum_{mesons} \frac{\left\langle 0 | G\tilde{G} | meson \right\rangle^2}{k^2 - m_n^2}}_{\text{Quark } N_c} + \underbrace{\text{Scattering Term}}_{\text{Quark } N_c^2 \ ?}$$

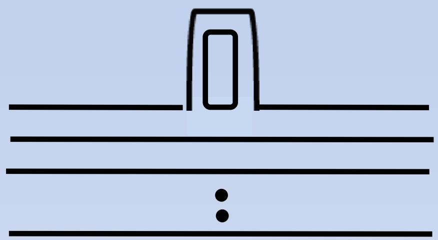
$$\rightarrow \quad P(k=0) = P_0(0) - \frac{\left\langle 0 | G\tilde{G} | \eta' \right\rangle_c^2}{m_{\eta'}^2} + \text{scattering ?}$$

- *Large Nc argument for Nucleon Scattering Term*



Witten



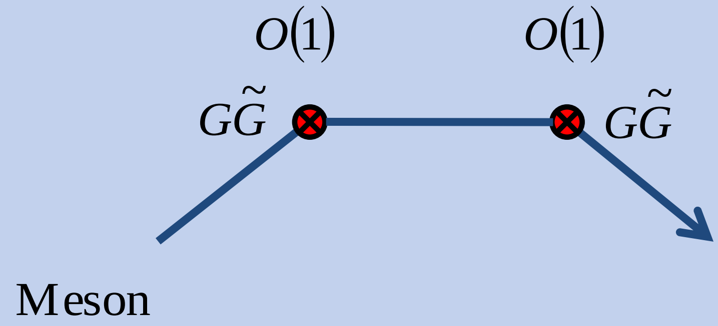


$$N_c\left(\frac{1}{N_c^{1/2}}\right)^2 N_c = N_c$$

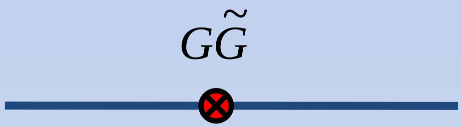
That is, scattering terms are of order N_c and can be safely neglected

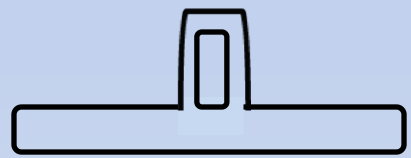
$$\rightarrow \quad \frac{\langle n|G\tilde{G}|n\rangle^2}{m_N}\rho_{density} \qquad N_c^2\left(\frac{1}{N_c}\right) = N_c$$

- *Large Nc argument for Meson Scattering Term*



Witten





$$N_c^2\left(\frac{1}{N_c^{1/2}}\right)^2\left(\frac{1}{N_c^{1/2}}\right)^2=1$$

That is, scattering terms are of order 1 and can be safely neglected

$$\rightarrow \quad P_0(0)=\frac{\left\langle 0\,|\,G\tilde{G}\,|\,\eta'\right\rangle^2}{m_{\eta'}^2}$$

WV relation remains the same

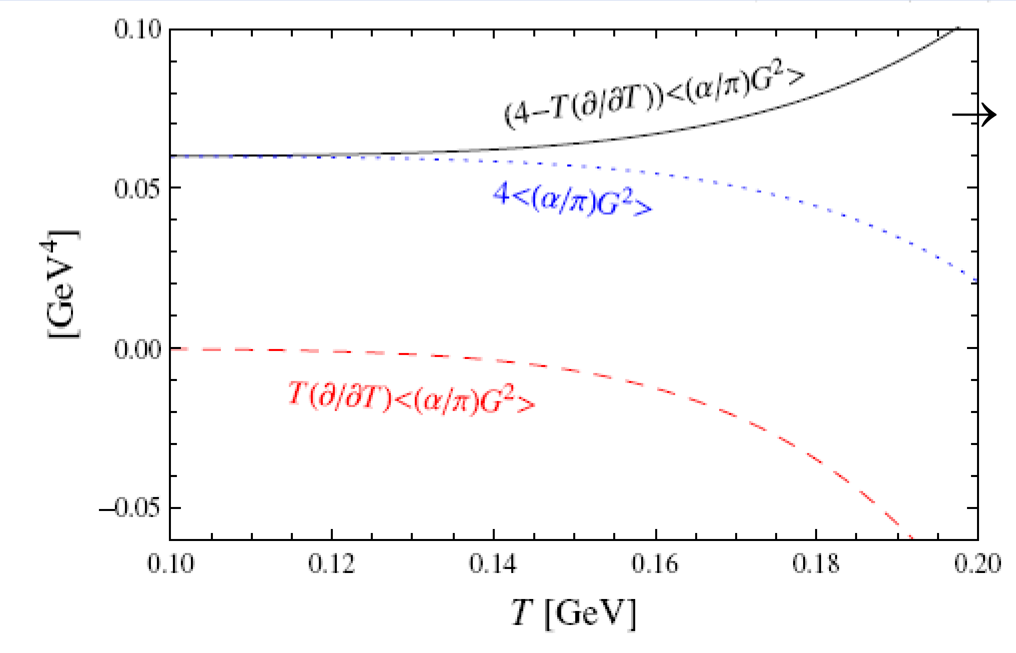
- *LET (Novikov, Shifman, Vainshtein, Zhakarov) at finite temperature : Ellis, Kapusta, Tang (98)*

$$\frac{d}{d\left(-1/4g_0^2\right)}\langle Op\rangle = -i\int dx e^{ikx}\langle Op(x), g_0^2 GG(0)\rangle$$

$$\langle Op\rangle_T = const\left[M_0 \exp\left(-\frac{8\pi^2}{bg_0^2}\right)\right]^d + c'T^d = \langle Op\rangle_{T0} + c'T^d$$

$$\frac{d}{d\left(-1/4g_0^2\right)}\langle Op\rangle_T = \frac{32\pi^2}{b}\left(d - T\frac{\partial}{\partial T}\right)\langle Op\rangle_T = \frac{32\pi^2}{b}\left(d - T\frac{\partial}{\partial T}\right)\langle Op\rangle_{T0}$$

- *Lee, Zahed (2001)* $P_0(0) = -\frac{2}{b}\left(d - T\frac{\partial}{\partial T}\right)\left\langle \frac{\alpha}{\pi} G^2\right\rangle$



→ Morita et al. (2012)

Weak T dependence even near T_c

$$\rightarrow \quad P_0(0) = \frac{\langle 0|G\tilde{G}|\eta'\rangle_c^2}{m_{\eta'}^2}$$

- $\langle 0 | G\tilde{G} | \eta' \rangle$ at finite temperature

$$P(k) = \int d^4x e^{ikx} \left[\langle G\tilde{G}(x), G\tilde{G}(0) \rangle \right] = \frac{\langle 0 | G\tilde{G} | \eta' \rangle^2}{k^2 - m_{\eta'}^2} + \dots$$

$$= k^\mu k^\nu \int d^4x e^{ikx} \left(\frac{4\pi}{\alpha N_F} \right)^2 \left[\langle \bar{q}(x) i\gamma_\mu \gamma^5 q(x), \bar{q}(0) i\gamma_\nu \gamma^5 q(0) \rangle - \langle \bar{q}(x) i\gamma_\mu q(x), \bar{q}(0) i\gamma_\nu q(0) \rangle \right]$$

$$\xrightarrow{\text{chiral sym restored phase}}$$

$$= k^\mu k^\nu \int d^4x e^{ikx} \left(\frac{4\pi}{\alpha N_F} \right)^2 \left[\langle \bar{q}(x) i\gamma_\mu \gamma^5 \tau^a q(x), \bar{q}(0) i\gamma_\nu \gamma^5 \tau^a q(0) \rangle - \langle \bar{q}(x) i\gamma_\mu \tau^a q(x), \bar{q}(0) i\gamma_\nu \tau^a q(0) \rangle \right]$$

$= 0$ for any k^μ, when Chiral symmetry is restored

Therefore, $\langle 0 | G\tilde{G} | \eta' \rangle \to 0$ *when chiral symmetry gets restored*

- *W-V formula at finite temperature:*

$$\langle \bar{q}q \rangle^2$$

$$\uparrow$$

$$\frac{\langle 0 | G\tilde{G} | \eta' \rangle^2}{m_{\eta'}^2} = P_0(0) \longrightarrow \left(\frac{4\pi}{3\alpha}\right)^2 \frac{2}{11}\left(d - T\frac{\partial}{\partial T}\right)\left\langle \frac{\alpha}{\pi} G^2 \right\rangle$$

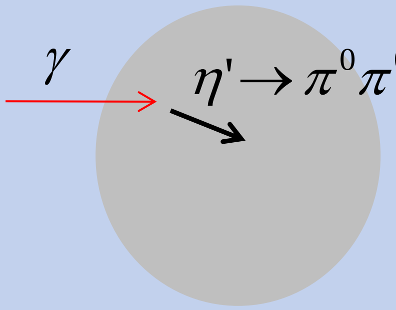
Smooth temperature dependence even near Tc

Therefore , $m_{\eta'} - m_{\eta} \propto \langle \bar{q}q \rangle$

→ *eta' mass should decrease at finite temperature*

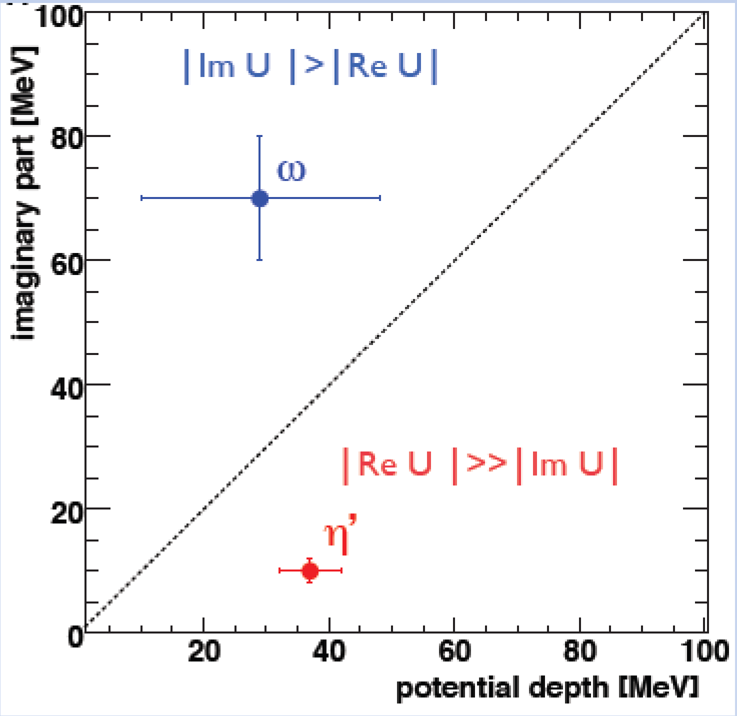
Experimental evidence of property change of η' in matter ?

CBELSA/TAPS coll



$$\eta' \to \pi^0 \pi^0 \eta \to 6\gamma$$

$$V = -(37 \pm 10 \pm 10)\ \mathrm{MeV} + i(10 \pm 2.5)\ \mathrm{MeV}$$



10 % reduction of mass from around 400 MeV from chiral symmetry breaking

Summary

1. η' correlation functions should exhibit symmetry breaking from N-point function in SU(N) flavor even when chiral symmetry is restored.

→ For SU(2), UA(1) effect will be broken in the two point function

2. In W-V formula η' mass is related to quark condensate and thus should reduce at finite temperature independent of flavor due to chiral symmetry restoration

→

a) Could serve as signature of chiral symmetry restoration

b) Dilepton in Heavy Ion collision

c) Measurements from nuclear targets seems to support it ?

Summary

1. Chiral symmetry breaking in Correlator

$$\langle \bar{q}(0)q(0)\rangle \text{ or } \langle VV - AA\rangle \xrightarrow{m\to 0} \langle \psi^{+}_{\lambda=0}\psi_{\lambda=0} \times \text{form factor}\rangle$$

2. $U_A(1)$ breaking effects in Correlators

$$\langle \eta'\eta' - \sigma\sigma\rangle \xrightarrow{m\to 0} \langle \psi^{+}_{\lambda=0}\psi_{\lambda=0} \times \text{form factor}\rangle + m^{N_f-2}(\text{zero mode})$$

→ Restored in SU(3) and real world

3. WV formula suggest mass of η ' reduces in medium and at finite temperature: due to chiral symmetry restoration

4. Renewed interest in Theory and Experiments both for nuclear matter and at may be at finite T