Nuclear models

2 The Fermi Gas Model

Nucleus has many properties that can be rather well understood in terms of independent particle behavior. The simplest of this is the degenerate Fermi Gas (T=0 Fermi Gas)

1) Definition:

The nucleons are assumed to move freely except for effects of the exclusion principle throughout a sphere of radius $R = R_0 A^{1/3}, \quad R_0 \cong 1.2 \text{ fm}$

Due to Pauli principle, the nucleon energy will increase as we pack more and more nucleons on top of each other. The maximum nucleon energy level is called the Fermi Level.

2) Energy level and number of states.

Consider a 3 dimensional sphere in the phase space (p, x space). The number of states is the total phase space volume divided by Planck constant h.

$$n = \frac{1}{h^3}\int d^3x d^3p \times 2 = \frac{V\frac{4}{3}\pi p_F^3}{h^3}2 = \frac{Vp_F^3}{6\pi^2\hbar^3}2 = \frac{\frac{4}{3}\pi R_0^3 A p_F^3}{6\pi^2\hbar^3}2 = \frac{4A}{9\pi}\left(\frac{R_0 p_F}{\hbar}\right)^3$$

Where the factor of two at the end is due to the degeneracy coming from the spin=1/2 of the nucleon.

Now, one can apply this formula to both the neutron and proton separately and their respective Fermi momentum and obtain

$$P_N = \frac{\hbar}{R_0}\left(\frac{9\pi N}{4A}\right)^{1/3}, \quad P_Z = \frac{\hbar}{R_0}\left(\frac{9\pi Z}{4A}\right)^{1/3} \qquad \text{-------------(2.1)}$$

3) Energies

a) Average Fermi Energy

Note (2.1) is almost constant for all nuclei because N=Z, Then

$$P_F \approx 270 \text{ MeV} \quad \text{and} \quad E_F = \frac{p_F^2}{2m} \approx 40 \text{ MeV (maxium energy)}$$

b) Average kinetic energy per ncleon

$$\langle E\rangle = \frac{\int_0^{p_F}\frac{p^2}{2m}d^3p}{\int_0^{p_F}d^3p} = \frac{3}{5}\left(\frac{p_F^2}{2m}\right) \approx 24 \text{ MeV} \quad \text{justifies non relativistic limit}$$

c) Total energy

$$\left\langle E(Z,N)\right\rangle = N\left\langle E_N\right\rangle + Z\left\langle E_Z\right\rangle = \frac{3}{10m}\left(NP_N^2 + ZP_Z^2\right) = \frac{3}{10m}\cdot\frac{\hbar^2}{R_0^2}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N^{5/3}+Z^{5/3}}{A^{2/3}}$$

As before, we can consider the total energy to be a function of two variables. Let us fix A and take N=A-Z Then, the minimum occurs at Z=A/2. Therefore, let us express it in terms of two new variables

$\varepsilon = Z - N$, $A = N + Z$, such that $Z = \frac{1}{2}A(1+\varepsilon/A)$, $N = \frac{1}{2}A(1-\varepsilon/A)$,

Using $(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots$, and assuming $(\varepsilon/A) \ll 1$

$$\left\langle E(Z,N)\right\rangle = \frac{3}{10m}\cdot\frac{\hbar^2}{R_0^2}\left(\frac{9\pi}{8}\right)^{2/3}\left(A + \frac{5}{9}\frac{(Z-N)^2}{A} + ..\right)$$

The second terms gives $11\ \mathrm{MeV}\ \frac{(Z-N)^2}{A}$, which h is only half as big as we need in experiment. The differences comes from potential effects.