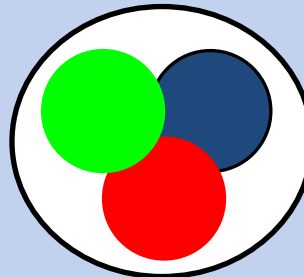
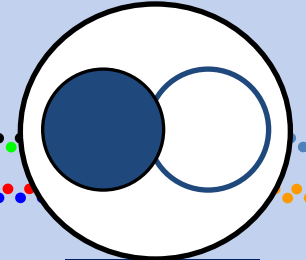
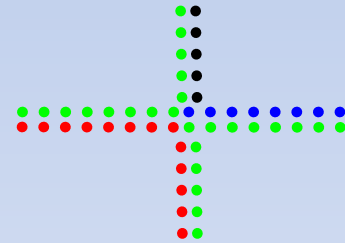
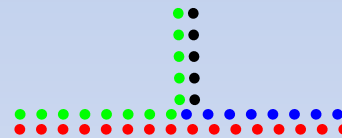


QED**vs****QCD****Charge****Boson****Boson
interaction** **β function**

$$\beta(e) = \frac{e^3}{12\pi^2}$$

**Baryon****Meson**

$$\beta(g) = -\left(11 - \frac{2n_f}{3}\right) \frac{g^3}{16\pi^2}$$

- QCD Lagrangian density

☞ Lagrangian density for QCD

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \quad A_\mu = \frac{\lambda^a}{2} A_\mu^a = T^a A_\mu^a$$

$$L = -\frac{1}{4} \text{Tr} [F_{\mu\nu} F_{\mu\nu}] + \bar{\psi} (iD_\mu \gamma^\mu - m) \psi$$

$$D_\mu \psi = (\partial_\mu - igA_\mu) \psi$$

$$F_{\mu\nu} = \frac{1}{-ig} [D_\mu, D_\nu] = T^a (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c)$$

- Appendix : Dirac equation

👉 Dirac equation: Field equation for spin 1/2 particle

$$(i\partial_\mu \gamma^\mu - m)\psi = 0$$

$$E^2 = p^2 + m^2$$

$$\psi = \begin{pmatrix} \psi_{+\uparrow} \\ \psi_{+\downarrow} \\ \psi_{-\uparrow} \\ \psi_{-\downarrow} \end{pmatrix}$$

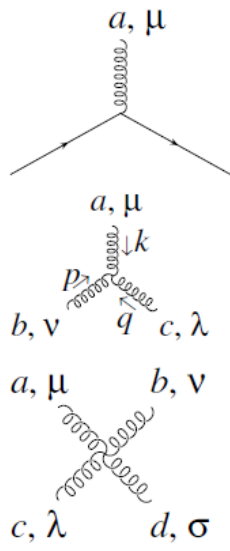
$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

- QCD Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi}^j (i \not{D}_j^k - m \delta_j^k) \psi_k = -\frac{1}{2} \text{Tr} F^2 + \bar{\psi} (i \not{D} - m) \psi .$$

$$\mathcal{L} = \mathcal{L}_0 + g A_\mu^a \bar{\psi} \gamma^\mu T^a \psi - g f^{abc} (\partial_\mu A_\nu^a) A^{b\mu} A^{c\nu} - g^2 f^{eab} f^{ecd} A_\mu^a A_\nu^b A^{c\mu} A^{d\nu} .$$

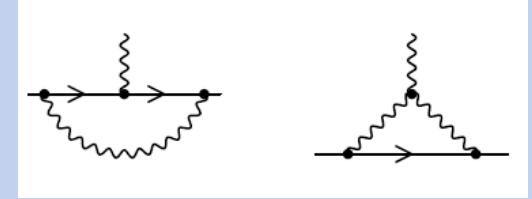
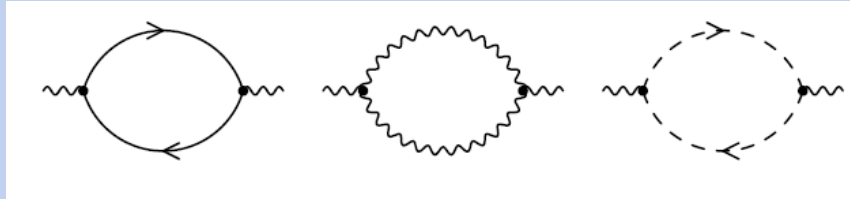
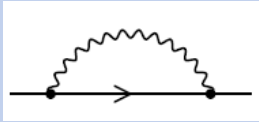


The image shows three Feynman diagrams representing QCD interactions. The first diagram is a three-point vertex with two fermion lines (solid lines with arrows) and one gluon line (wavy line). The second diagram is a four-point vertex with two fermion lines and two gluon lines. The third diagram is a four-gluon vertex with four gluon lines. Each diagram is equated to a mathematical expression.

$$\begin{aligned}
 & \text{Diagram 1: } = ig\gamma^\mu T^a \\
 & \text{Diagram 2: } = gf^{abc}[g^{\mu\nu}(k-p)^\lambda + g^{\nu\lambda}(p-q)^\mu + g^{\lambda\mu}(q-k)^\nu] \\
 & \text{Diagram 3: } = -ig^2[f^{abe}f^{cde}(g^{\mu\lambda}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\lambda}) + \\
 & \quad f^{ace}f^{bde}(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\sigma}g^{\nu\lambda}) + \\
 & \quad f^{ade}f^{bce}(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\sigma\nu})]
 \end{aligned}$$

- QCD Running coupling constant

$$\mu \frac{d}{d\mu} F = \left(\mu \frac{\partial}{\partial \mu} + \beta(g_s) \frac{\partial}{\partial g_s} \right) F(E, \mu, g_s) = 0.$$



Asymptotic freedom

$$\frac{d \log \alpha_s(\mu)}{d \log \mu} = -2\beta(\alpha_s(\mu))$$

$$\beta(\alpha_s) = \beta_0 \frac{\alpha_s}{4\pi} + \dots \quad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$$

$$\text{Feynman diagram} = T_F \text{ wavy line}$$

$$\text{Tr } t^a t^b = T_F \delta^{ab}$$

$$\text{Feynman diagram} = C_A \text{ wavy line} \quad \text{or} \quad i f^{acd} i f^{bdc} = C_A \delta^{ab}$$

$$C_A = 2T_F N_c$$

$$\text{Feynman diagram} = C_F \text{ fermion line} \quad \text{or} \quad t^a t^a = C_F$$

$$C_F = T_F \left(N_c - \frac{1}{N_c} \right)$$

