Nuclear models

It is difficult to understand the properties of the nucleus from the two body nuclear force. Therefore, one introduces simple ideas to capture important properties. These models can be categorized into two sets.

1) Independent particle model (IPM)

Nucleons are assumed to move independently in a common nuclear potential

→ Fermi Gas Model, Shell Model.

2) Strong interaction models (SIM) (Collective)

Nucleons are strongly coupled to each other.

→ Liquid drop model, Collective models

1 The Liquid Drop Model

1) The striking feature of Nucleus is that the charge density is constant up to R (radius of nucleus) and independent of A. This is the feature of a liquid → Liquid drop model.

2) Binding energy is defined as

$$B/c^2 = Z \times m_P + N \times m_n - m_{nucleus} \approx Z \times m_H + N \times m_n - m(Z,N)$$

Here Z = Proton number, N = neutron number , $A = N + Z$, $m_H, m(Z,N)$ are the mass of the corresponding atoms

Sometimes the mass excess is quoted in terms of the atomic mass unit

1 u= mass of ^{12}C (inlcuding electron)/12=931.481 MeV/c^2 $\approx 1.66043\times10^{-24}\,g$

$\Delta = m(Z,N)c^2 - Auc^2$

3) Average Binding Energy B/A

The B/A of ground state Nuclei has the following A dependence has the following characteristics

a) B/A saturates to about 8 to 9 MeV → such behavior is not true for coulomb type of interaction

b) There is a maximum when A = 60

c) Stable nuclei have the properties that Z≈N

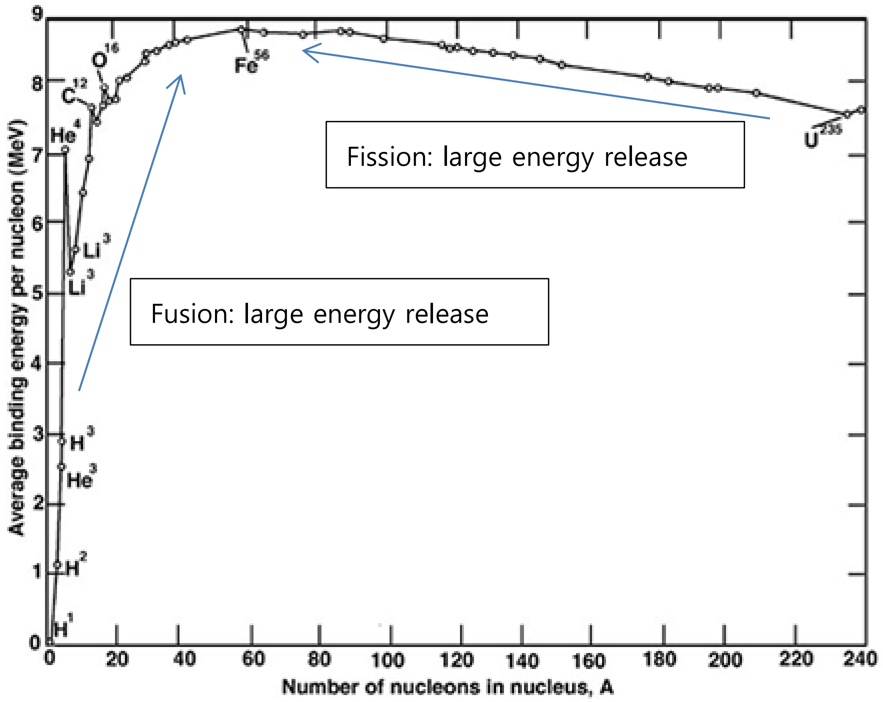


Figure 1 Binding energy per nucleon

4) Bohr and Weizsacker

Let us construct a formula that well represents the characteristics of the binding energy curve above

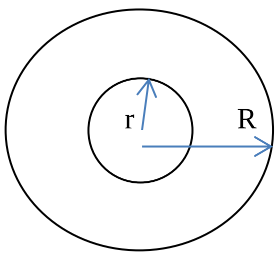
a) Volume Energy

$$E_V = a_v^{'} A u c^2, \qquad B_V = \left(uc^2 - a_v^{'}\right) A = a_v A$$

b) Surface Energy

$B_S = -a_s A^{2/3} \longrightarrow$ Area of surface

c) Coulomb energy



uniformly distributed Z charge with the radius of the nuclei $R = R_0 A^{1/3}$

charge density is given by $\rho \times \frac{4}{3}\pi R^3 = Z \quad \rightarrow \quad \rho = \frac{3Z}{4\pi R^3}$

The potential at position r is given as $V(r) = \frac{1}{4\pi\varepsilon_0} \frac{\frac{4}{3}\pi r^3 \rho}{r}$

Therefore, the Coulomb energy needed to bring the charges together to a sphere of Radius R with uniform charge density would be the following

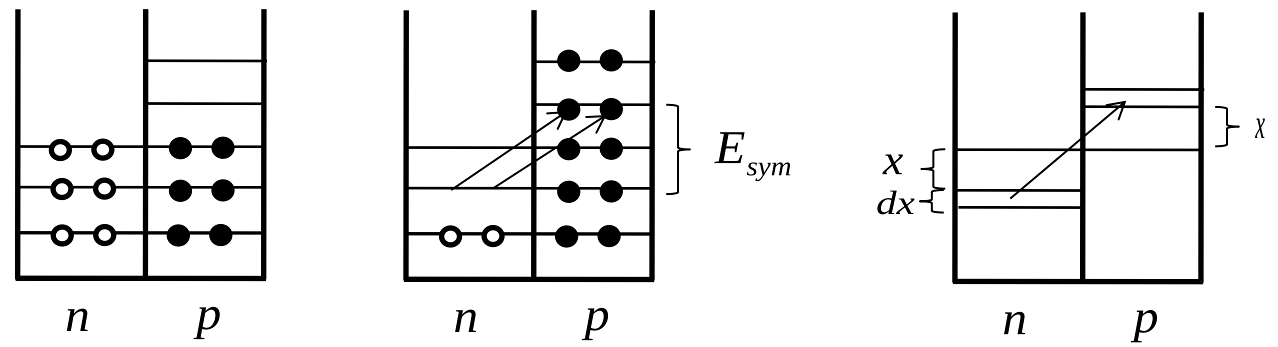
$$E_C = \int_0^R V(r) \times \rho 4\pi r^2 dr = \int_0^R \frac{1}{4\pi\varepsilon_0} \frac{\frac{4}{3}\pi r^3 \rho}{r} \times \rho 4\pi r^2 dr = \frac{4\pi}{3\varepsilon_0}\rho^2 \int_0^R r^4 dr = \frac{4\pi}{15\varepsilon_0}\rho^2 R^5$$

→ $E_C \propto \frac{Z^2}{R^6} R^5 \propto Z^2 A^{-1/3} \quad \rightarrow \quad B_C = -a_c Z^2 A^{-1/3}$

d) Symmetry Energy

Nuclei are more stable when Z and N are similar→ this is a consequence of Pauli Principle and not a property of liquid drop.

This can be formally derived in the shell model, but the important property can be understood in a dimple model based on energy levels of protons and neutrons and Pauli principle.



Suppose the energy spacing between energy levels e is constant. Then the symmetry energy can be obtained as in the right figure.

$$E_S = \int_0^{\frac{1}{2}(Z-N)e} 2x \times dx = \frac{1}{4} e^2 (Z-N)^2$$

That is, the symmetry energy is proportional to $(Z-N)^2$

Realistic case, e is not constant and one finds

$$B_S = -a_{sym}\frac{(Z-N)^2}{A}$$

e) Total

$$B = a_V A - a_S A^{2/3} - a_{sym}\frac{(Z-N)^2}{A} - a_c Z^2 A^{-1/3}$$

$$\frac{B}{A} = a_V - a_S A^{-1/3} - a_{sym} \frac{(Z-N)^2}{A^2} - a_c Z^2 A^{-4/3} \qquad ----(1)$$

From fit to the experimental observation, one finds

$a_V = 15.6$ MeV, $a_S = 16.8$ MeV, $a_{sym} = 23.3$ MeV, $a_c = 0.72$ MeV

Note (1) is a function of two variables, Z, N, with A=Z+N. That is, it is a two dimensional function. However, to plot Figure 1, we have to plot it as a function of A. We therefore have to express eq(1) in terms of A and Z, or A and N. Then, for given A, we determine Z from the following condition.

$$\frac{\partial}{\partial Z}\left(\frac{B}{A}\right)_{A=const} = 0 \qquad ---(2)$$

To understand what Eq.(1) means, let us consider the simplified case where we neglect the last term in Eq.(1). Then Eq.(2) can be expressed as

$$\frac{\partial}{\partial Z}\left[a_V - a_S A^{-1/3} - a_{sym} \frac{(2Z-A)^2}{A^2}\right] = -a_{sym} 4 \frac{(2Z-A)}{A^2} = 0 \;\rightarrow\; Z = \frac{A}{2}$$

One then puts Z=A/2 back into eq.(1) and plots it aa function of A. In a full calculation, we should not neglect the last term in Eq.(1), but the procedure would be the same.

To get a rough idea of how the curve would look like, let us take Z=N=A/2 and substitute it into Eq.(1), then we get

$$\frac{B}{A} = a_V - a_S A^{-1/3} - a_c \frac{1}{4} A^{2/3}$$

When A is small, the second term dominates and explains the initial increase.

At intermediate A, the first terms dominates and explains the constant behvior.

At large A, the last terms dominates and explains the decrease and the instability for a large nucleus.