**Problem 1:** A laser beam is incident on the interface betweenair and some dielectric of index $n$. For small values of $\theta_i$, show that $\theta_t = \theta_i/n$. Use this and the above equation $r_\perp = -\dfrac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$ to establish that at near-normal incidence.

Solution: For $\theta_i \ll 1$, $\sin\theta_i \simeq \theta_i \ll 1$. While $n \sim 1$, we obtain

$$\sin\theta_t = \frac{\sin\theta_i}{n} \simeq \frac{\theta_i}{n} \ll 1 \tag{1}$$

Therefore

$$\sin\theta_t \simeq \theta_t \simeq \frac{\theta_i}{n} \ll 1 \tag{2}$$

The following relationships are also true:

$$\sin(\theta_i - \theta_t) \simeq \theta_i - \theta_t \ll 1 \tag{3}$$

$$\sin(\theta_i + \theta_t) \simeq \theta_i + \theta_t \ll 1 \tag{4}$$

Replacing all sine terms by angles in given equation yields

$$r_\perp(\theta_i \to 0) \simeq -\frac{\theta_i - \theta_t}{\theta_i + \theta_t} = -\frac{n-1}{n+1} \tag{5}$$

**Problem 2:** Use the above equation and the power series expansion of the sine function to establish that at near-normal incidence we can obtain a better approximation than that in above example, namely$[-r_\perp]_{\theta_i \to 0} = \left(\dfrac{n-1}{n+1}\right)\left(1 + \dfrac{\theta_i^2}{n}\right)$.

Solution: Firstly we have

$$\begin{aligned} -r_\perp &= \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \\ &= \frac{\sin\theta_i\cos\theta_t - \sin\theta_t\cos\theta_i}{\sin\theta_i\cos\theta_t + \sin\theta_t\cos\theta_i} \\ &= \frac{n\cos\theta_t - \cos\theta_i}{n\cos\theta_t + \cos\theta_i} \end{aligned} \tag{6}$$

From previous problem we know that $\theta_i \simeq n\theta_t \ll 1$. Using this relationship and the approximaion $\cos\theta \simeq 1 - \dfrac{\theta^2}{2}$ for small angles yields

$$\begin{aligned} -r_\perp &\simeq \frac{n(1-\frac{\theta_t^2}{2}) - (1-\frac{\theta_i^2}{2})}{n(1-\frac{\theta_t^2}{2}) + (1-\frac{\theta_i^2}{2})} \\ &= \frac{(n-1) + \frac{n^2-n}{2}\theta_t^2}{(n+1) - \frac{n^2+n}{2}\theta_t^2} \\ &= \frac{n-1}{n+1}\frac{1+\frac{n}{2}\theta_t^2}{1-\frac{n}{2}\theta_t^2} \\ &= \frac{n-1}{n+1}[1+\frac{n}{2}\theta_t^2][1+\frac{n}{2}\theta_t^2 + o(\theta_t^2)] \\ &= \frac{n-1}{n+1}[1+n\theta_t^2 + o(\theta_t^2)] \\ &\simeq \frac{n-1}{n+1}(1+\frac{\theta_i^2}{n}) \end{aligned} \tag{7}$$

**Problem 3.40:**

Solution: Refraction angle can be obtained from Snell's law

$$\theta_t = \arcsin\frac{\sin\theta_i}{n} = \arcsin\frac{\sin 20^\circ}{1.55} = 12.75^\circ \tag{8}$$

Related coefficients are

$$r_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} = -\frac{\sin 7.25^\circ}{\sin 32.75^\circ} = -0.23 \tag{9}$$

$$r_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} = \frac{\tan 7.25^\circ}{\tan 32.75^\circ} = 0.20 \tag{10}$$

Therefore

$$E_{\perp,r} = |10\ \mathrm{V/m} \times -0.23| = 2.3\ \mathrm{V/m} \tag{11}$$

$$E_{\parallel,r} = 20\ \mathrm{V/m} \times 0.20 = 2.0\ \mathrm{V/m} \tag{12}$$

**Problem 3.44:**

Solution: For normal incidence, $\theta_i \simeq n\theta_t \ll 1$. Therefore

$$t \equiv t_\perp \simeq t_\parallel \simeq \frac{2\theta_t}{\theta_i + \theta_t} \simeq \frac{2}{n+1} \tag{13}$$

where $n = \dfrac{n_C}{n_w} = 1.035$. For irradiance $I \propto v\epsilon E^2 \simeq nE^2$, transmitted irradiance is

$$I = \frac{n_C E_t^2}{n_w E_i^2} I_0 = nt^2 I_0 \simeq 399.88\ \mathrm{W/m^2} \tag{14}$$

**Problem 3.63:**

Solution: When $\theta_i = \pi/2 - \theta_t$, reflected coefficients are

$$\begin{aligned} r_\perp &= -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \\ &= -\frac{\sin(\pi/2 - 2\theta_t)}{\sin\pi/2} = -\cos 2\theta_t \\ r_\parallel &= \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \\ &= \frac{\tan(\pi/2 - 2\theta_t)}{\tan\pi/2} = 0 \end{aligned} \tag{15}$$

$r_\parallel = 0$ indicates that there is no polarization on the direction parallel to the POI.

**Problem 3.68:**

Solution: For any $\vec{E}$ field, we have a decompositon as

$$\vec{E} = E_\perp \vec{e}_\perp + E_\parallel \vec{e}_\parallel \tag{16}$$

The azimuthal angle $\gamma$ is defined as $\gamma = \arctan \frac{E_\perp}{E_\parallel}$. For incidented field $\vec{E}_i$, reflected field $\vec{E}_r$ and transmitted field $\vec{E}_t$, according to the definition of all coefficients,

$$\vec{E}_r = r_\perp E_{i,\perp} \vec{e}_\perp + r_\parallel E_{i,\parallel} \vec{e}_\parallel \tag{17}$$

$$\vec{E}_t = t_\perp E_{i,\perp} \vec{e}_\perp + t_\parallel E_{i,\parallel} \vec{e}_\parallel \tag{18}$$

$R$ is defined as

$$\begin{aligned} R &= \frac{\vec{E}_r^2}{\vec{E}_i^2} \\ &= \frac{r_\perp^2 E_{i,\perp}^2 + r_\parallel^2 E_{i,\parallel} \vec{e}_\parallel}{E_{i,\perp}^2 + E_{i,\parallel}^2} \\ &= \frac{r_\perp^2 \sin^2 \gamma_i + r_\parallel^2 \cos^2 \gamma_i}{\sin^2 \gamma_i + \cos^2 \gamma_i} \\ &= R_\perp \sin^2 \gamma_i + R_\parallel \cos^2 \gamma_i \end{aligned} \tag{19}$$

where $R_\perp = r_\perp^2$ and $R_\parallel = r_\parallel^2$.

$T$ can be obtained similarly

$$\begin{aligned} T &= n \frac{\cos \theta_t}{\cos \theta_i} \frac{\vec{E}_t^2}{\vec{E}_i^2} \\ &= n \frac{\cos \theta_t}{\cos \theta_i} \frac{t_\perp^2 E_{i,\perp}^2 + t_\parallel^2 E_{i,\parallel} \vec{e}_\parallel}{E_{i,\perp}^2 + E_{i,\parallel}^2} \\ &= n \frac{\cos \theta_t}{\cos \theta_i} (t_\perp^2 \sin^2 \gamma_i + t_\parallel^2 \cos^2 \gamma_i) \\ &= T_\perp \sin^2 \gamma_i + T_\parallel \cos^2 \gamma_i \end{aligned} \tag{20}$$

where $T_\perp = n \frac{\cos \theta_t}{\cos \theta_i} t_\perp^2$ and $T_\parallel = n \frac{\cos \theta_t}{\cos \theta_i} t_\parallel^2$.