Physics of quark-gluon plasma and high-energy heavy-ion collisions

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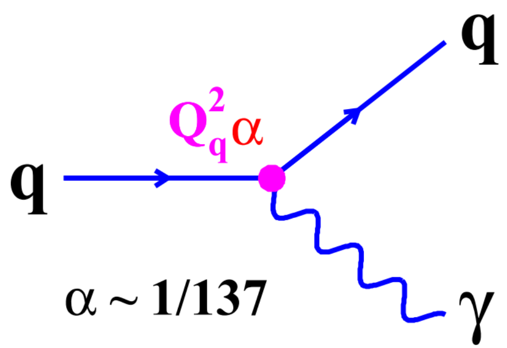
Lecture 3: Quantum chromodynamics and partons

Quantum chromodynamics (QCD)

Quantum electrodynamics (QED): theory of electromagnetic force

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}(F_{\mu\nu})^2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad D_\mu \equiv \partial_\mu + ieA_\mu(x)$$

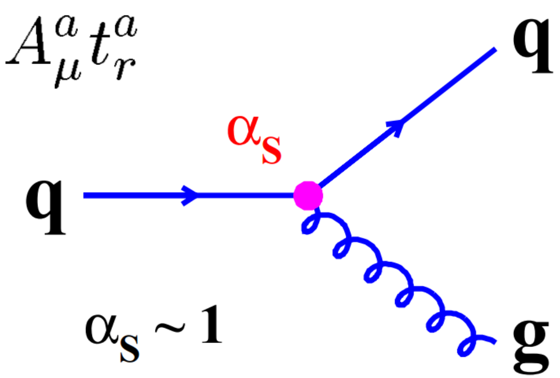


- **Abelian** U(1) gauge theory
- Charge of the force: **electric charge**
- Force mediator: **massless photons**
- Strength of interaction: $\alpha = e^2/4\pi$

Quantum chromodynamics (QCD): theory of strong force

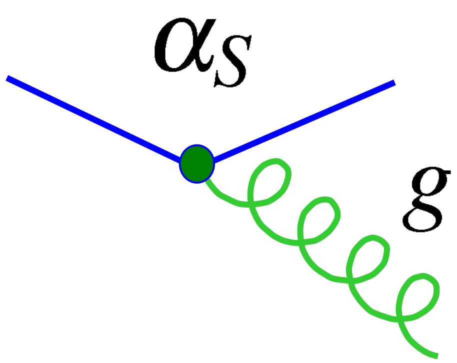
$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(i\not{D} - m)\psi$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu \qquad D_\mu = \partial_\mu - igA^a_\mu t^a_r$$



- **Non-Abelian** SU(3) gauge theory
- Charge of the force: **color charges**
- Force mediator: **massless gluons**
- Strength of interaction: $\alpha_s = g^2/4\pi$

Quantum chromodynamics (QCD)



In QCD:

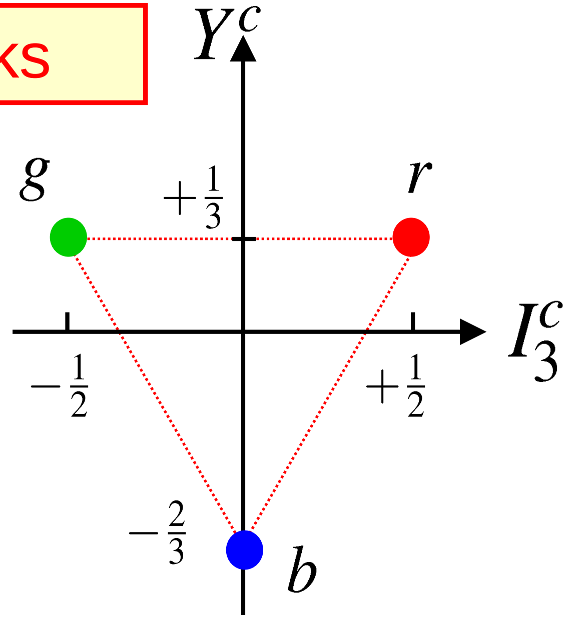
- quarks carry colour charge: r, g, b
- anti-quarks carry anti-charge: $\bar{r}, \bar{g}, \bar{b}$
- The force is mediated by massless gluons

It is the gauge SU(3) colour symmetry

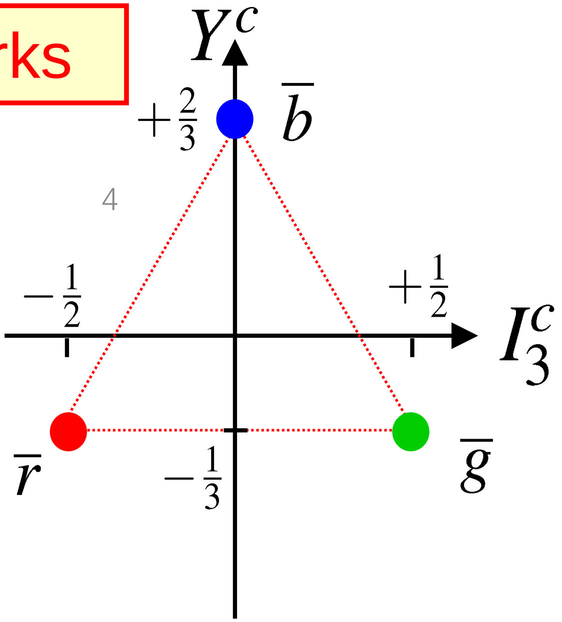
★ Colour states can be labelled by two quantum numbers, like labelling u, d, s flavour states by I_3 and Y

- I_3^c colour isospin
- Y^c colour hypercharge

quarks



anti-quarks

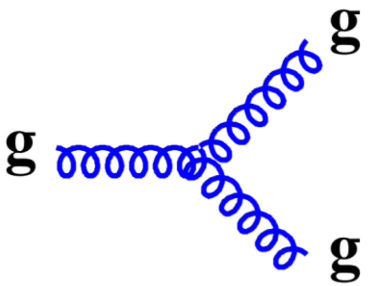


Asymptotic freedom

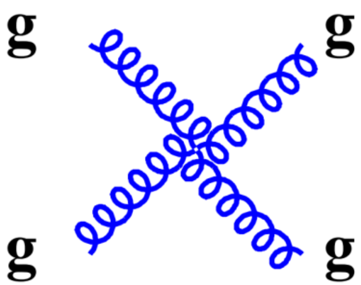
Gluon self-interaction:

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(i\not{D} - m)\psi$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

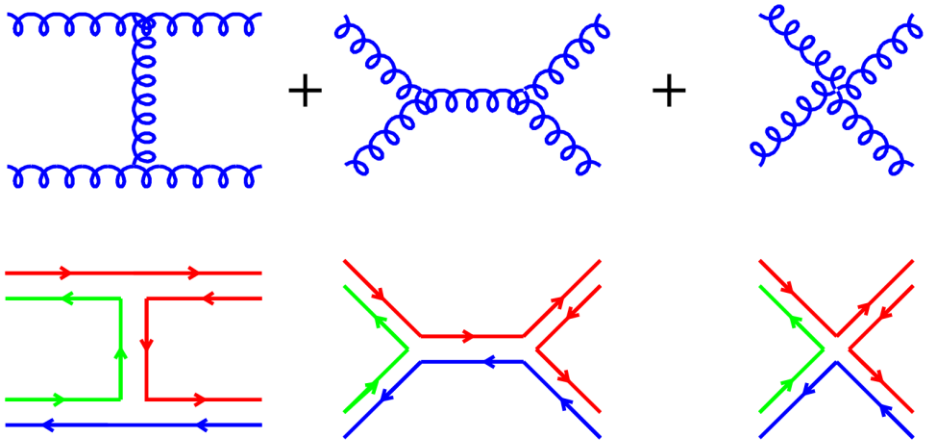


3 GLUON VERTEX



4 GLUON VERTEX

For example, gluon-gluon scattering:



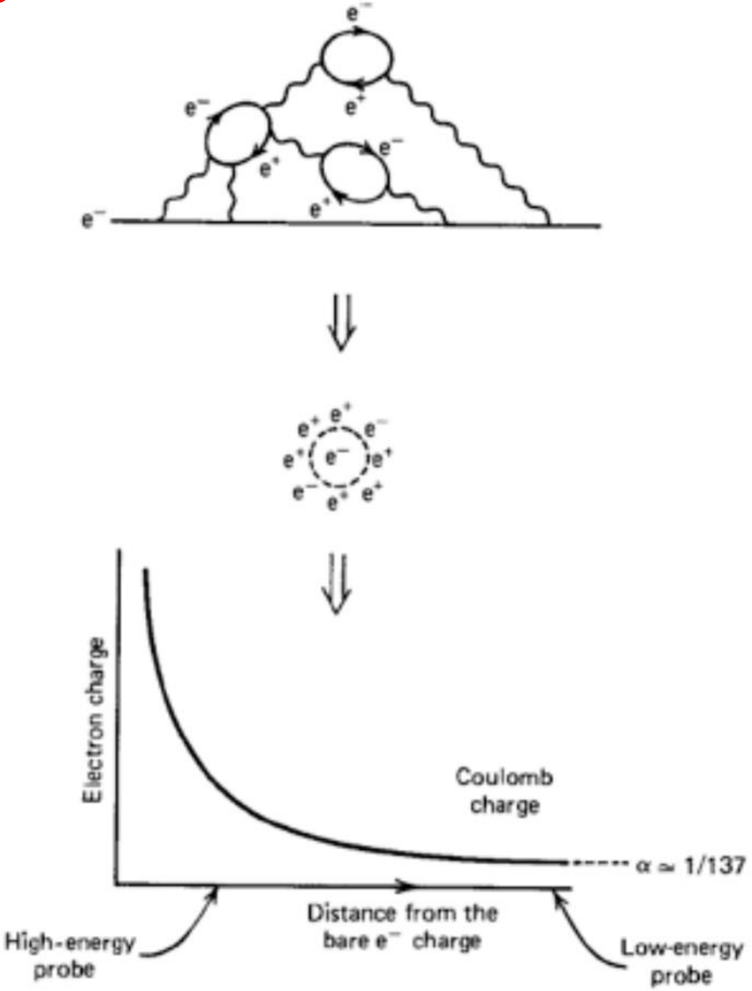
e.g. $r\,\bar{g} + g\,\bar{b} \rightarrow r\,\bar{r} + r\,\bar{b}$

This is the non-abelian feature.
It has very important consequences.

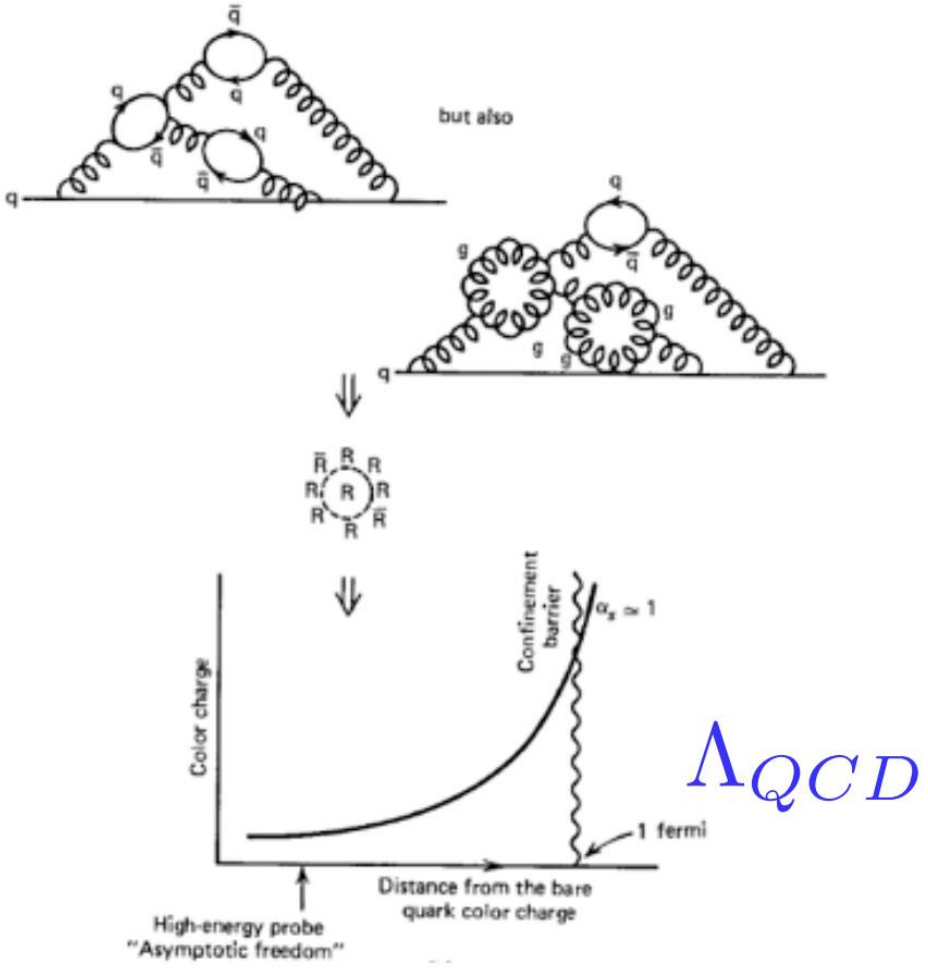
Color Confinement
Asymptotic freedom
… …

Asymptotic freedom

Coupling constant is not a constant: Running coupling



QED: screening

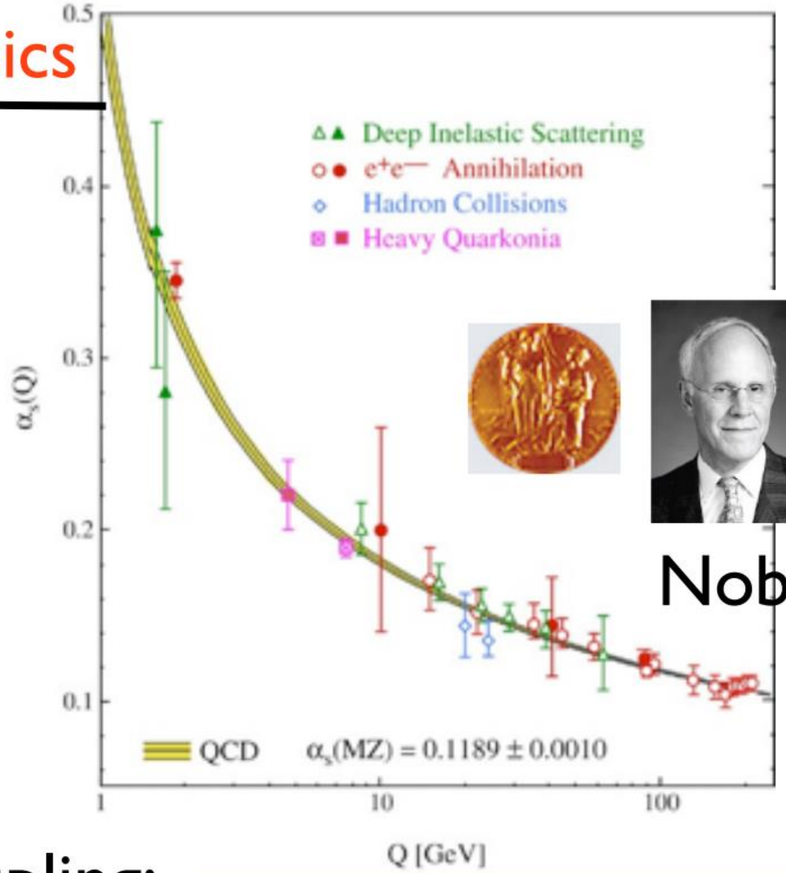


QCD: anti-screening

Asymptotic freedom

Coupling constant is not a constant: Running coupling

Nuclear Physics



Non-Abelian Gauge Theory !

Nobel Prize 2004

Running coupling:

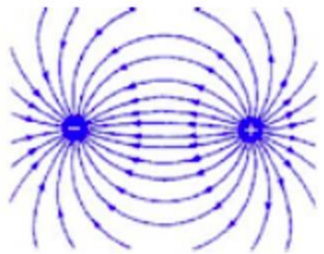
$$\alpha_s = g^2/4\pi$$

$$\alpha_s(r) = \frac{2\pi N_c}{(11N_c - 2N_f)\log(1/r\Lambda_{QCD})}$$

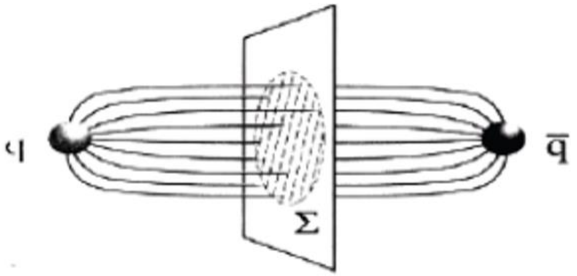
Color confinement

(not yet proven): Only colour singlet states can exist as free particles

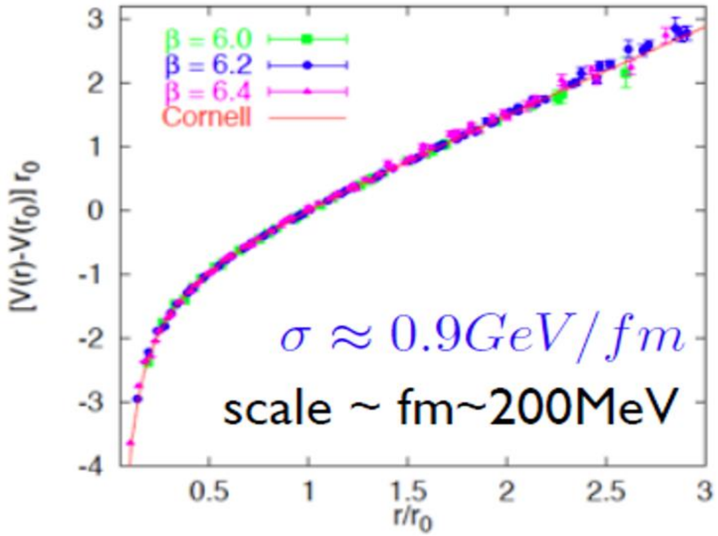
I.e., all hadrons must be "colourless" i.e. colour singlets



QED dipole field

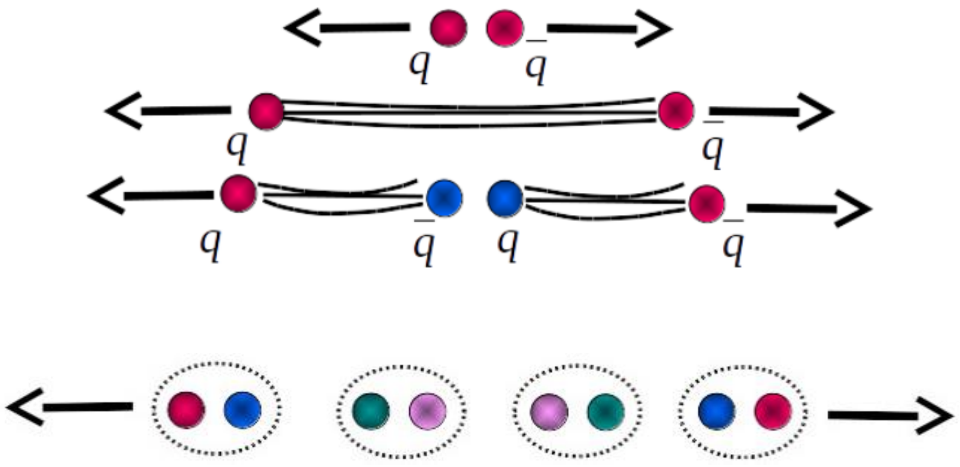


QCD dipole field



$$V(r) = -\frac{\alpha_s}{r} + \sigma r$$

As the quarks separate, the linear potential energy increases and produce new quark-antiquark pairs



Millennium Prize Problems

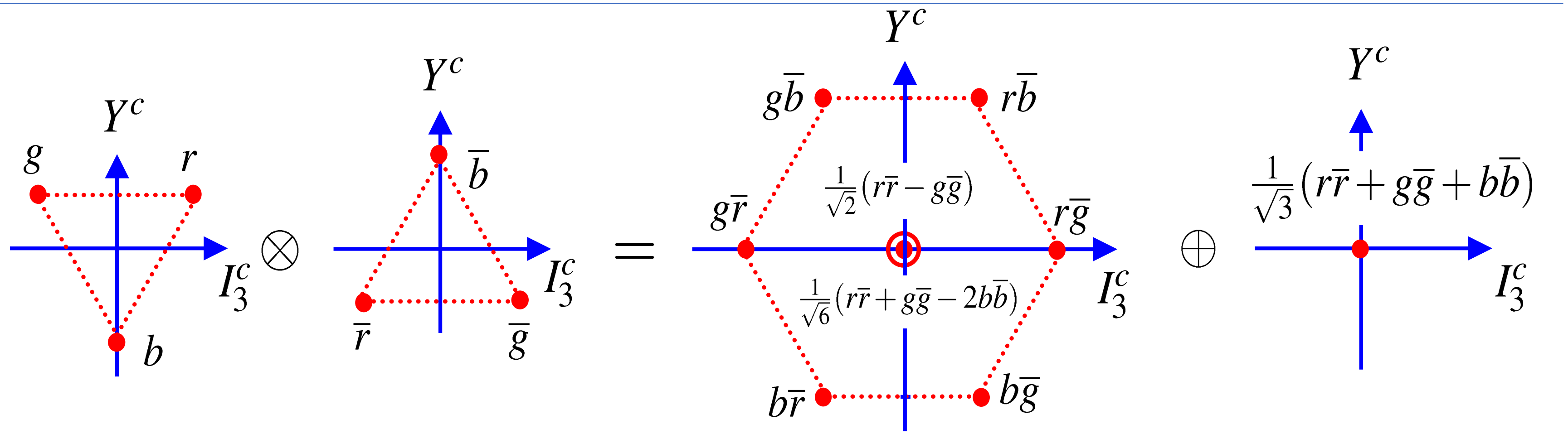
P versus NP problem
Hodge conjecture
Poincaré conjecture (solved)
Riemann hypothesis
Yang–Mills existence and mass gap
Navier–Stokes existence and smoothness
Birch and Swinnerton-Dyer conjecture

Color confinement

- **Colour confinement**

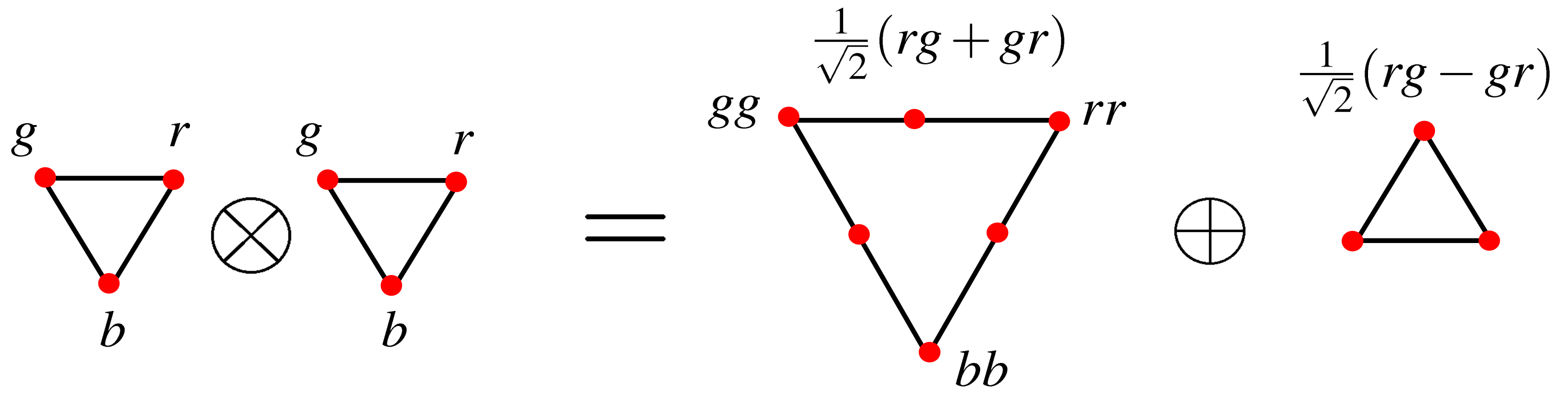
➡ **Bound states (mesons) of $\bar{q}q$ must be**

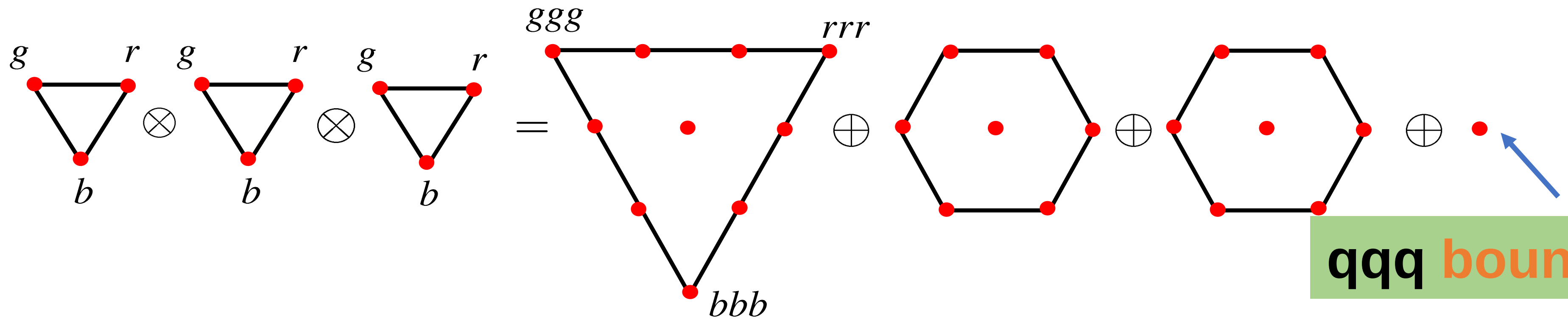
$$\psi_c^{q\bar{q}} = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$



- **Colour confinement**

➡ **Bound states of qq do not exist**

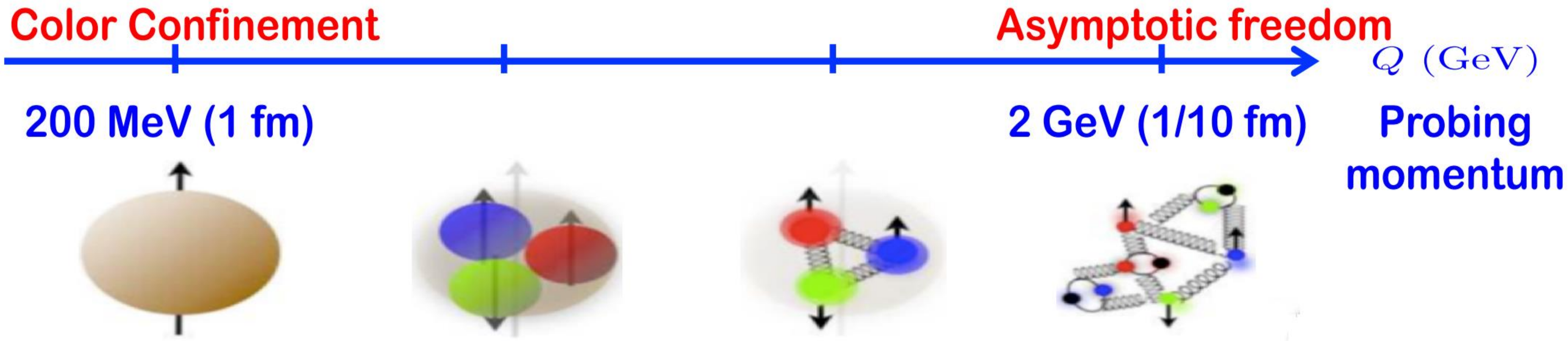




qqq bound states (baryons)

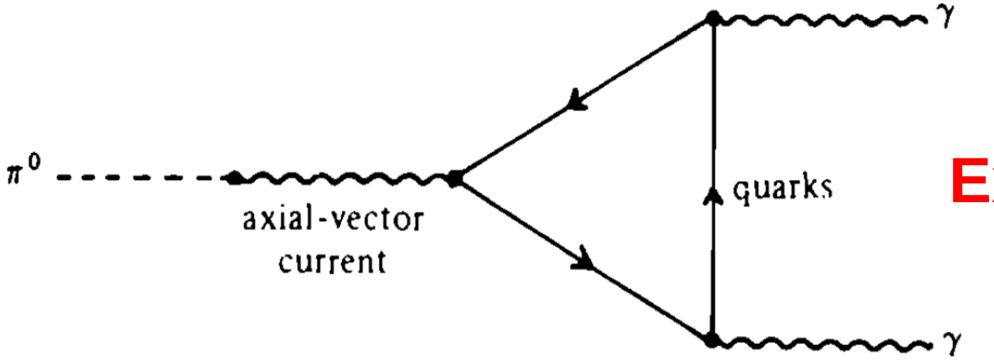
Evidence of color and quarks/gluons

How can we see quarks and gluons if they are confined.



Example 1: neutral pion decay

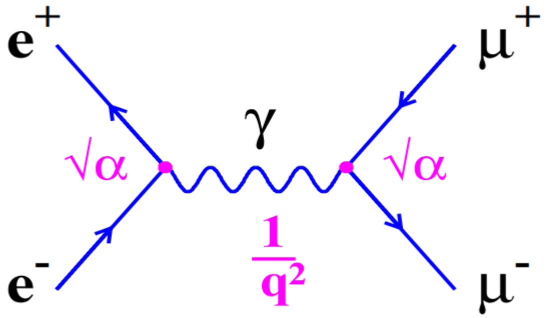
$$\Gamma(\pi^0 \to 2\gamma) = N_c^2 (Q_u^2 - Q_d^2)^2 \frac{\alpha^2 m_{\pi^0}^3}{64\pi^3 F_\pi^2} = 7.6\ \text{eV}$$



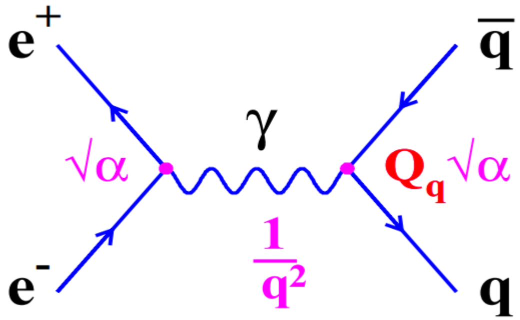
Experimental result: $\Gamma(\text{exp}) = 7.48 \pm 0.33\ \text{eV}$

Evidence of color and quarks/gluons

Example 2: electron-positron annihilations



$$e^+e^- \to \mu^+\mu^-$$



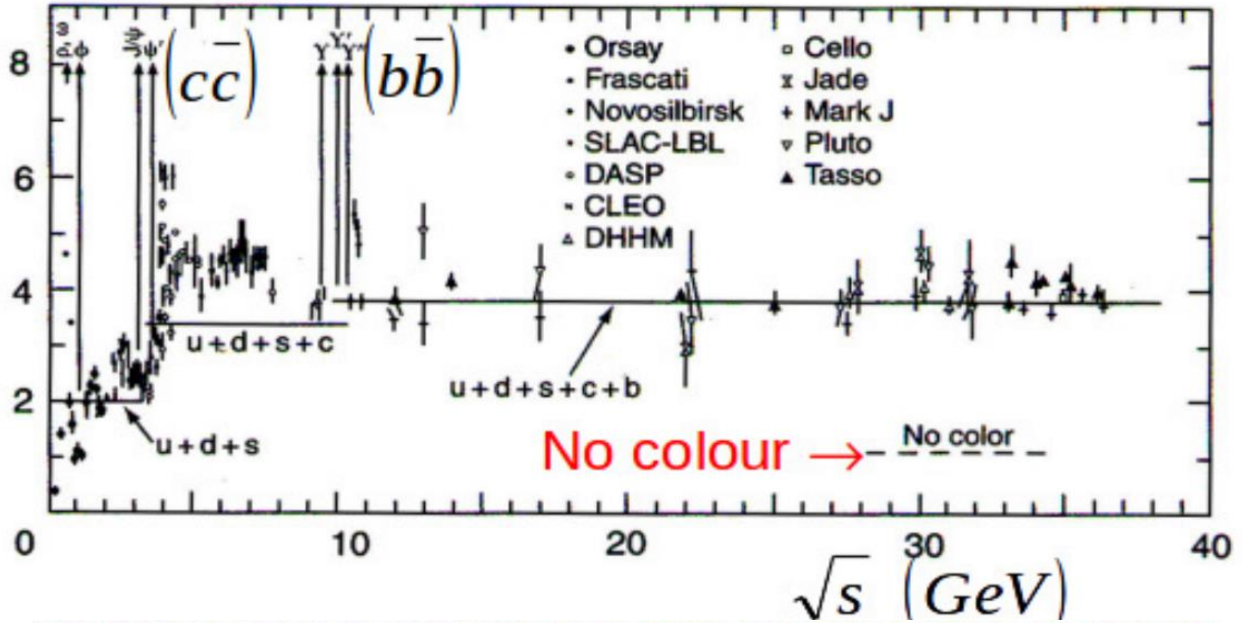
$$e^+e^- \to q\bar{q} \to hadrons$$

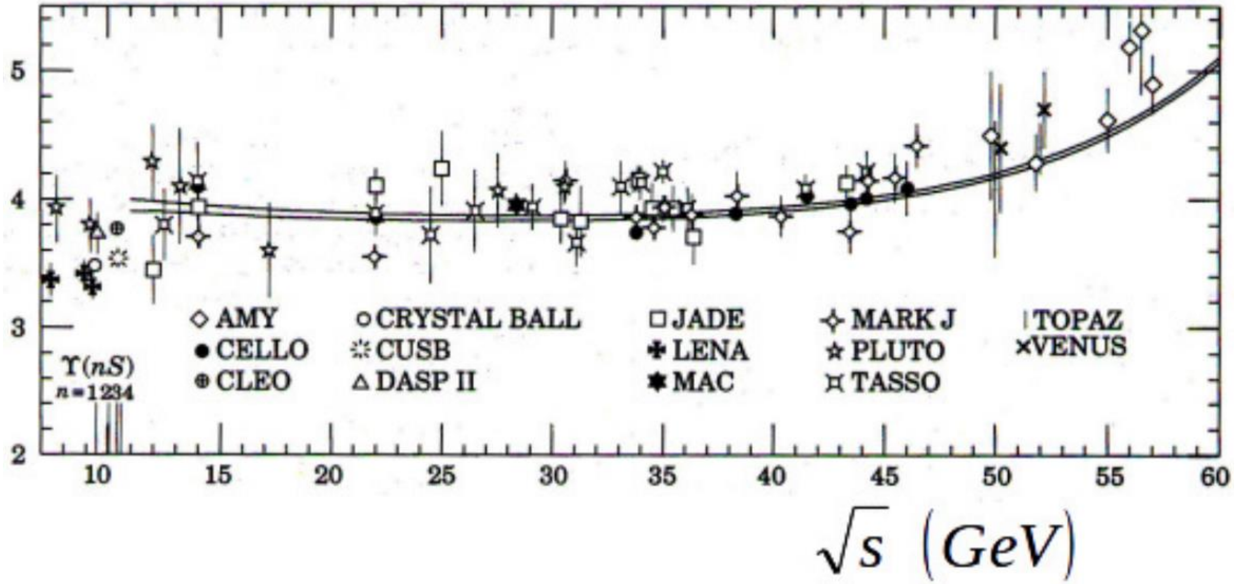
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum Q_i^2$$ (3 colours)

Energy	Expected ratio R	Energy	Expected ratio R
$\sqrt{s} > 2m_s, \quad \sim 1$ GeV	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right) = 2$ uds	$\sqrt{s} > 2m_b, \quad \sim 10$ GeV	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9}\right) = 3\frac{2}{3}$ $udscb$
$\sqrt{s} > 2m_c, \quad \sim 4$ GeV	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}\right) = 3\frac{1}{3}$ $udsc$	$\sqrt{s} > 2m_t, \quad \sim 350$ GeV	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9}+\frac{4}{9}\right) = 5$ $udscbt$

Evidence of color and quarks/gluons

Example 2: electron-positron annihilations

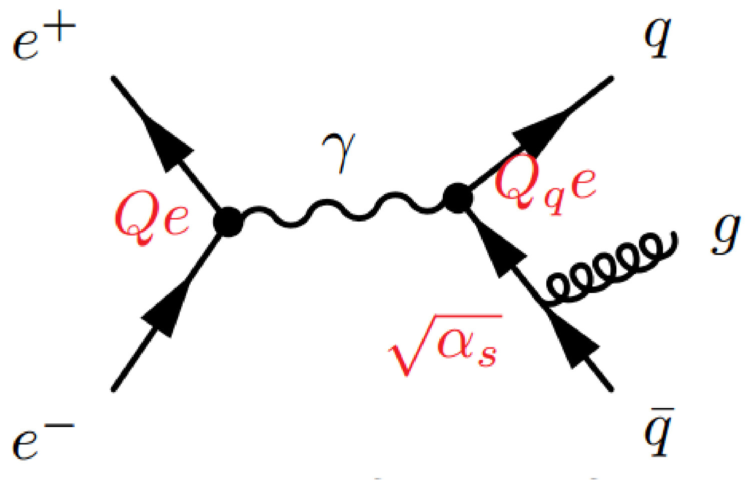




Strong evidence for color=3 and existence of quarks

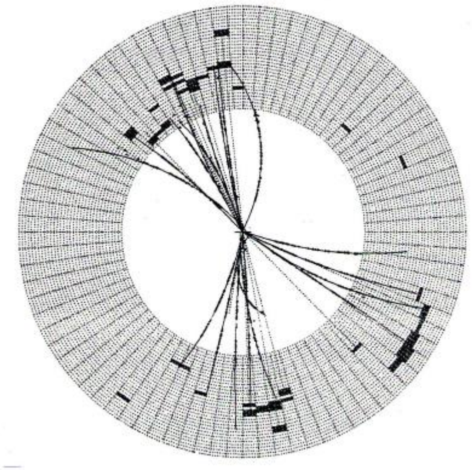
Evidence of color and quarks/gluons

Example 3: 3-jet and 4-jet events

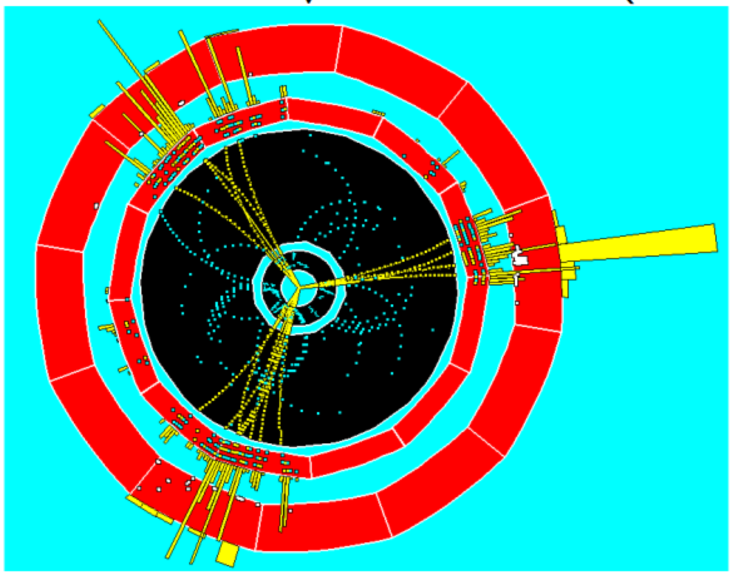


JADE event $\sqrt{s} = 31$ GeV

First direct evidence of gluons (1978)

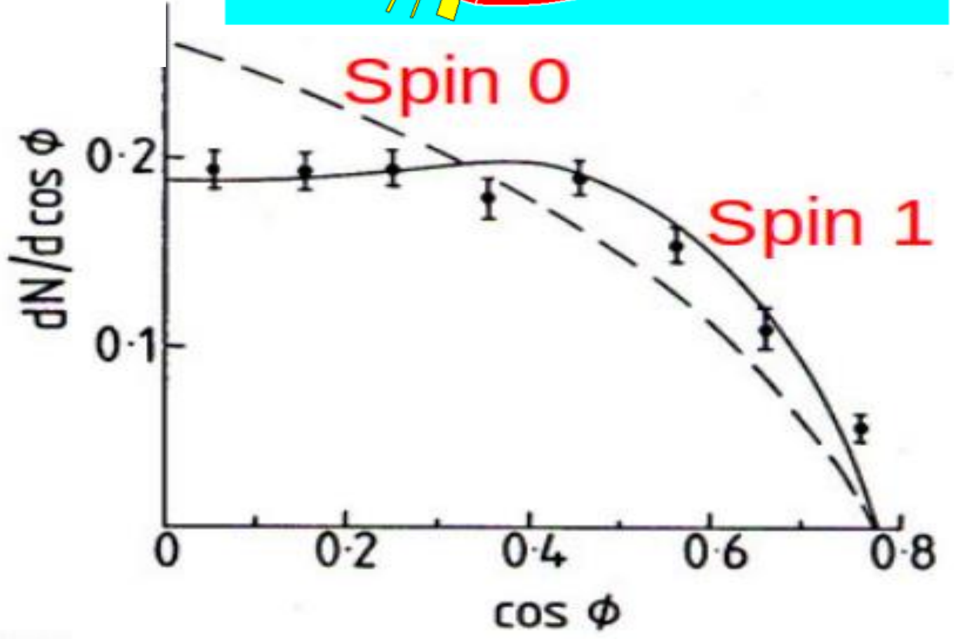


ALEPH event $\sqrt{s} = 91$ GeV (1990)



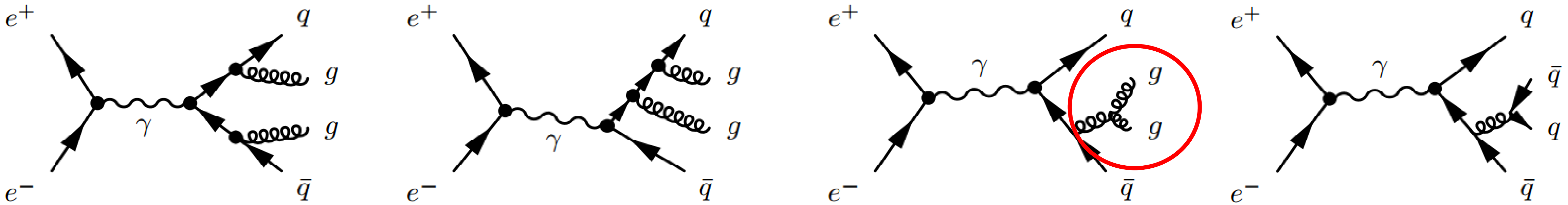
Distribution of the angle, ϕ , between the highest energy jet (assumed to be one of the quarks) relative to the flight direction of the other two (in their cm frame). ϕ distribution depends on the spin of the gluon.

$\Rightarrow$ Gluon is spin 1

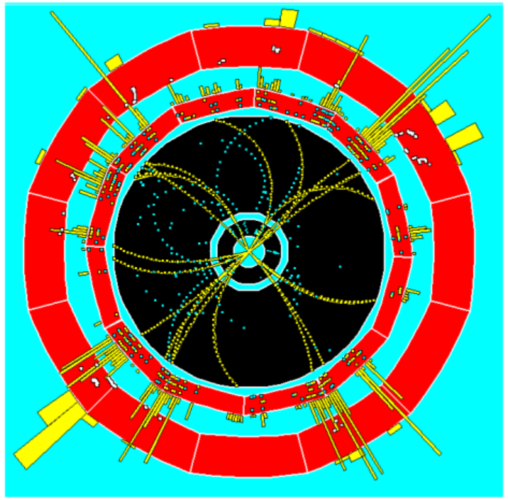


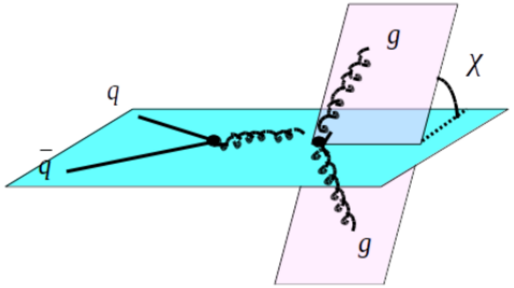
Evidence of color and quarks/gluons

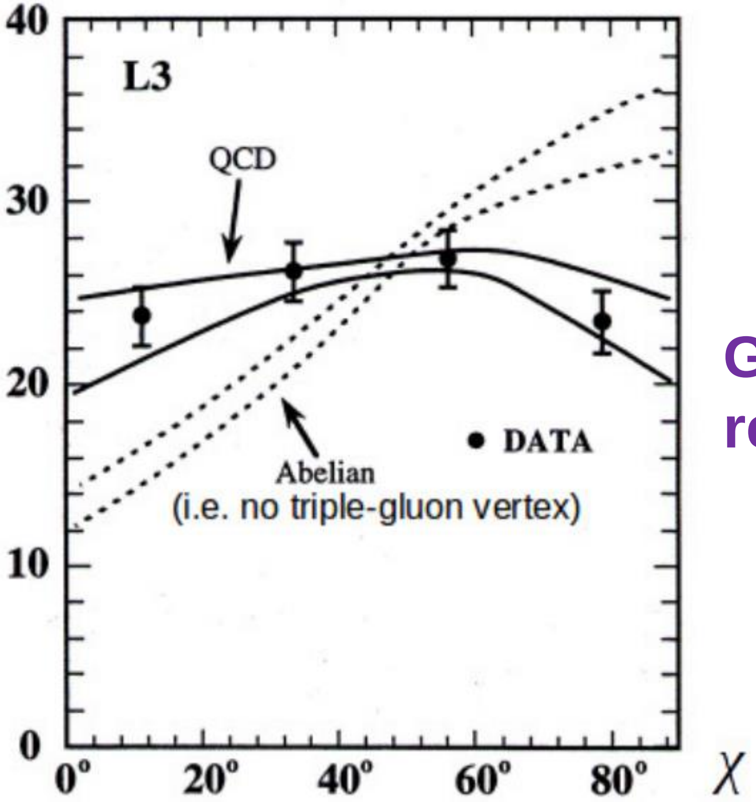
Example 3: 3-jet and 4-jet events



ALEPH 4-jet event







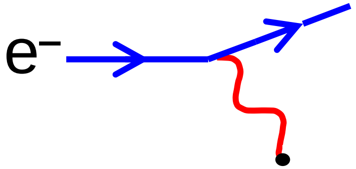
Gluon self-interaction is required to describe the data

Electron as a clean probe of nucleon

How to "see" quarks and gluons inside, e.g., a proton?

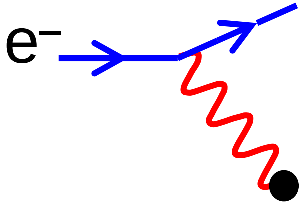
$e^-p \rightarrow e^-p$ **scattering (why electron):**

- **At very low electron energies** $\lambda \gg r_p$ **:**
  the scattering is equivalent to that from a "point-like" spin-less object



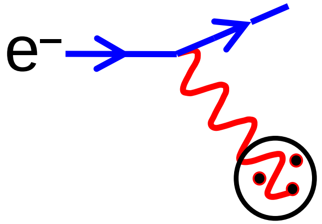
Rutherford scattering, Mott scattering

- **At low electron energies** $\lambda \sim r_p$ **:**
  the scattering is equivalent to that from a extended charged object



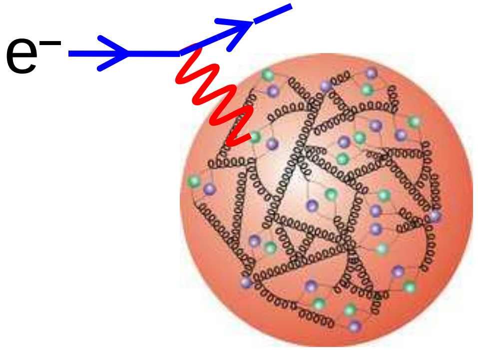
Form factor Rosenbluth formula

- **At high electron energies** $\lambda < r_p$ **:**
  the wavelength is sufficiently short to resolve sub-structure. Scattering from constituent quarks



Structure functions

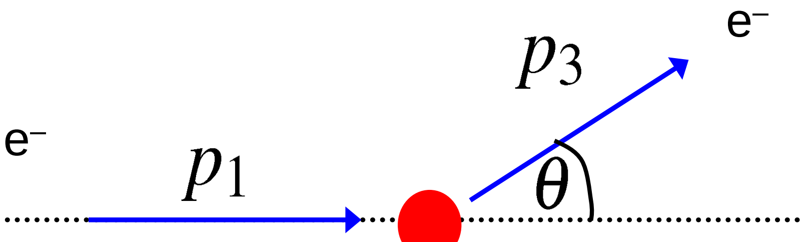
- **At very high electron energies** $\lambda \ll r_p$ **:**
  the proton appears to be a sea of quarks and gluons.



Deep-inelastic scattering, Parton distribution function

Rutherford and Mott scattering

- Elastic Coulomb scattering
- Neglect proton recoil
- Proton is a fixed spin-0 point charge
- Non-relativistic spin-0 electron: Rutherford scattering
  Ultra-relativistic spin-1/2 electron: Mott scattering



(neglect proton recoil)

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Rutherford}} = \frac{\alpha^2}{16E_K^2\sin^4\theta/2}$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} = \underbrace{\frac{\alpha^2}{4E^2\sin^4\theta/2}}_{\text{Rutherford formula with } E_K = E\ (E \gg m_e)}\ \underbrace{\cos^2\frac{\theta}{2}}_{\text{Overlap between initial/final helicity states.}}$$

$$\mathrm{d}\Omega = \mathrm{d}(\cos\theta)\mathrm{d}\phi$$

Electron kinetic energy $\quad E_K = p^2/2m_e$

$$h = \frac{\vec{s}\cdot\vec{p}}{|\vec{s}|\cdot|\vec{p}|}$$

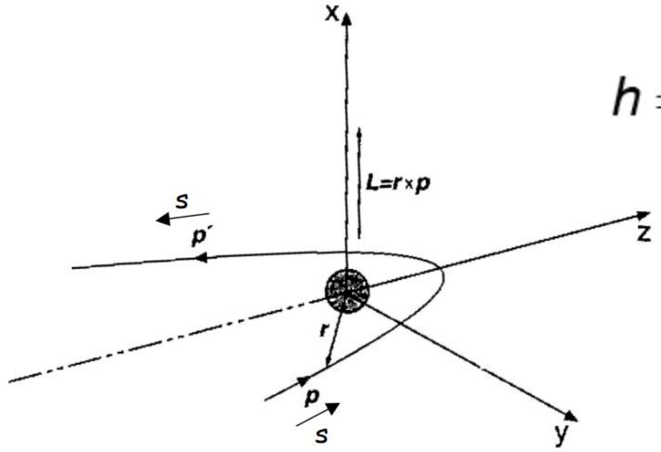
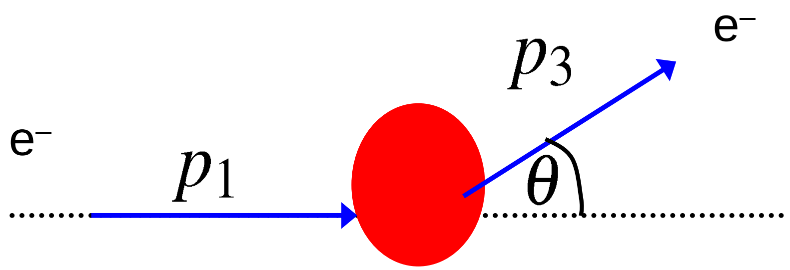


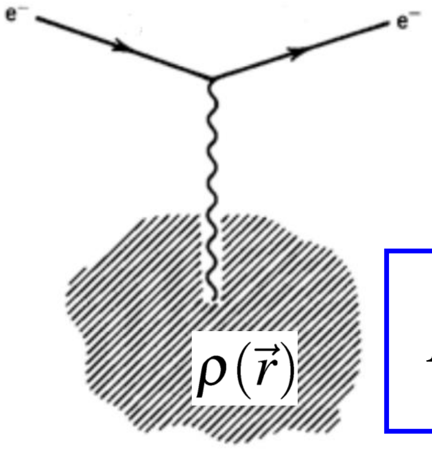
Fig. 5.3. Helicity, $h = \boldsymbol{s}\cdot\boldsymbol{p}/(|\boldsymbol{s}|\cdot|\boldsymbol{p}|)$, is conserved in the $\beta\to 1$ limit. This means that the spin projection on the z-axis would have to change its sign in scattering through 180°. This is impossible if the target is spinless, because of conservation of angular momentum.

Form factor

- **Elastic Coulomb scattering**
- **Neglect proton recoil**
- **Proton is a fixed spin-0 extended charged object**
- **Consider Mott scattering**



(neglect proton recoil)

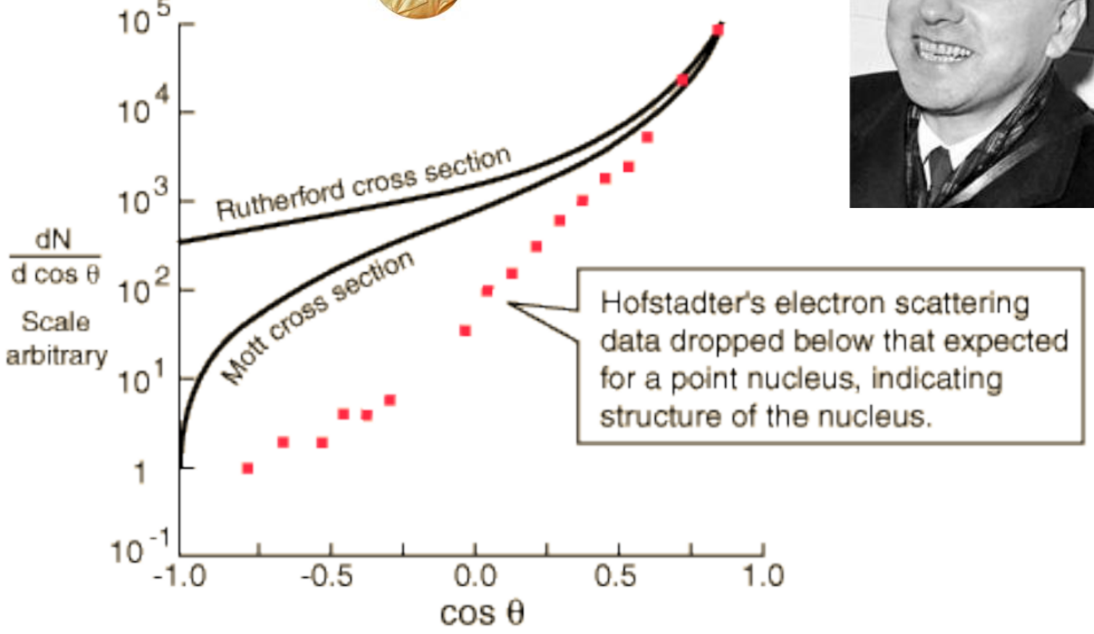


Form factor: Fourier transformation of charge distribution

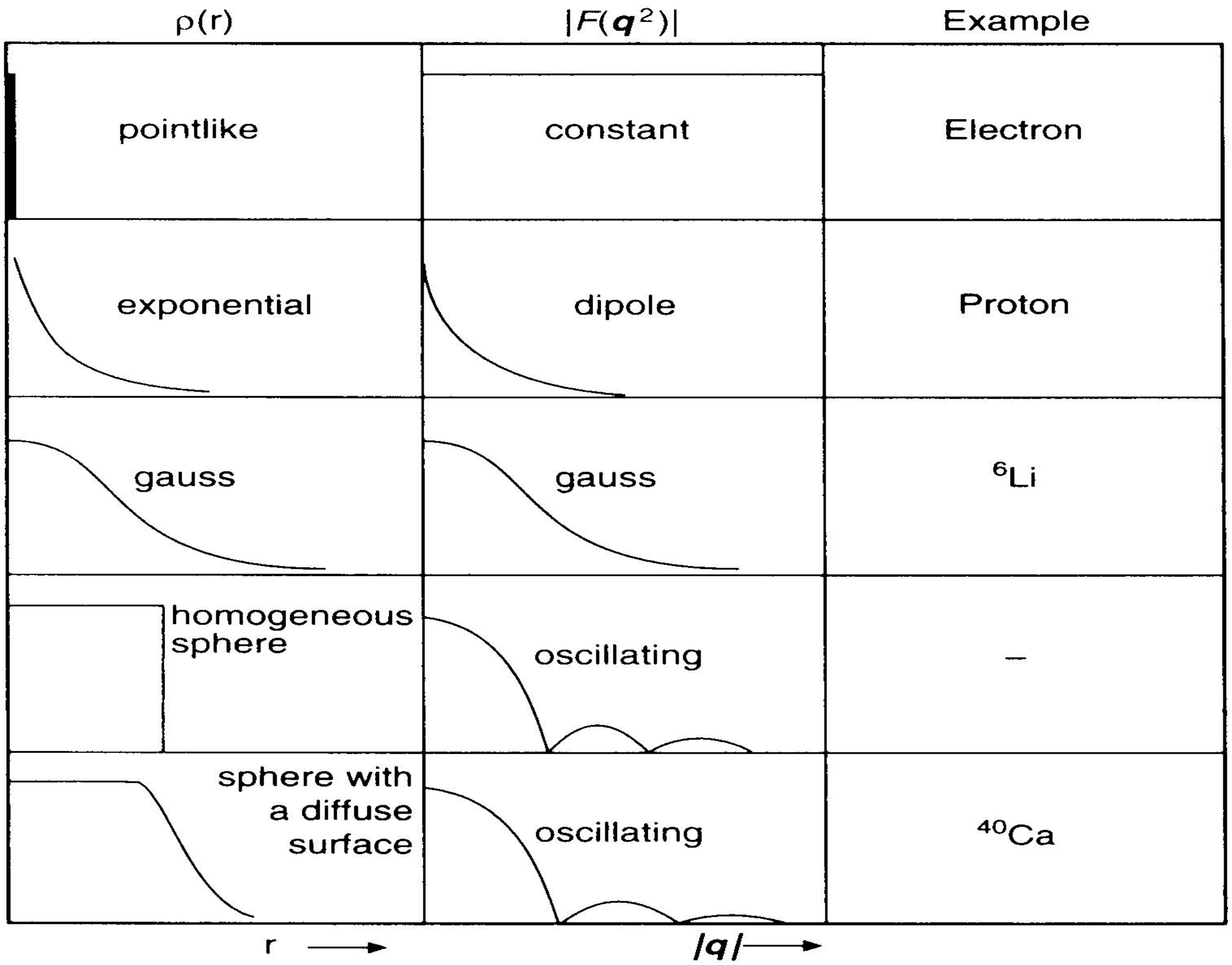
$$\boxed{F(\vec{q}^2) = \int \rho(\vec{r}) e^{i\vec{q}.\vec{r}} \mathrm{d}^3\vec{r}}$$

$$\Rightarrow \quad \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} |F(\vec{q}^2)|^2$$

1961 Hofstadter



Form factor



ρ(r)
|F(q²)|
Example
pointlike
constant
Electron
exponential
dipole
Proton
gauss
gauss
⁶Li
homogeneous sphere
oscillating
–
sphere with a diffuse surface
oscillating
⁴⁰Ca
r
|q|

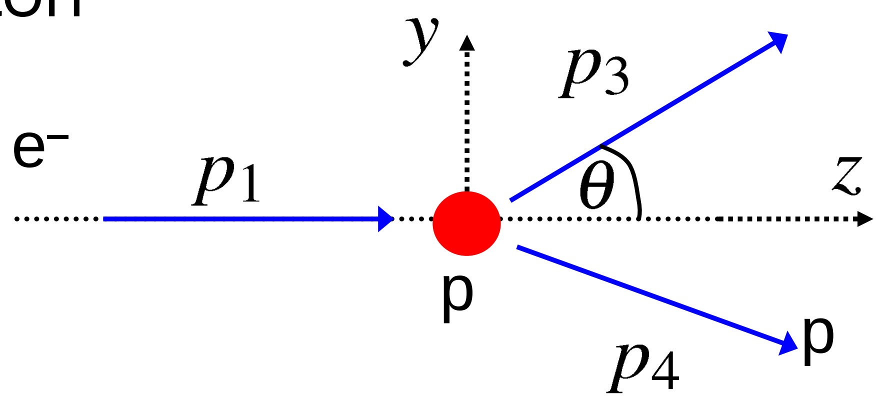
Form factor

Charge distribution $f(r)$		Form Factor $F(\boldsymbol{q}^2)$	
point	$\delta(r)/4\pi$	1	constant
exponential	$(a^3/8\pi)\cdot\exp(-ar)$	$(1+\boldsymbol{q}^2/a^2\hbar^2)^{-2}$	dipole
Gaussian	$(a^2/2\pi)^{3/2}\cdot\exp(-a^2r^2/2)$	$\exp(-\boldsymbol{q}^2/2a^2\hbar^2)$	Gaussian
homogeneous sphere	$\begin{cases} 3/4\pi R^3 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$	$3\,\alpha^{-3}(\sin\alpha - \alpha\cos\alpha)$ with $\alpha = \lvert\boldsymbol{q}\rvert R/\hbar$	oscillating

HW3: Derive the form factors listed above

Recoil of a point-like proton

- **Elastic non-Coulomb scattering**
- **Consider proton recoil**
- **Proton is a spin-1/2 point charge**
- **Electron is ultra-relativistic**



Proton recoil

Coherence of in-electron spin and out-electron spin

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{\alpha^2}{4E_1^2 \sin^4\theta/2}}_{\text{Interaction of spinless electron and spinless static proton}} \frac{E_3}{E_1}\left(\cos^2\theta/2 - \underbrace{\frac{q^2}{2M^2}\sin^2\theta/2}_{\text{Interaction of electron spin and proton spin}}\right)$$

Interaction of spinless electron and spinless static proton

Interaction of electron spin and proton spin

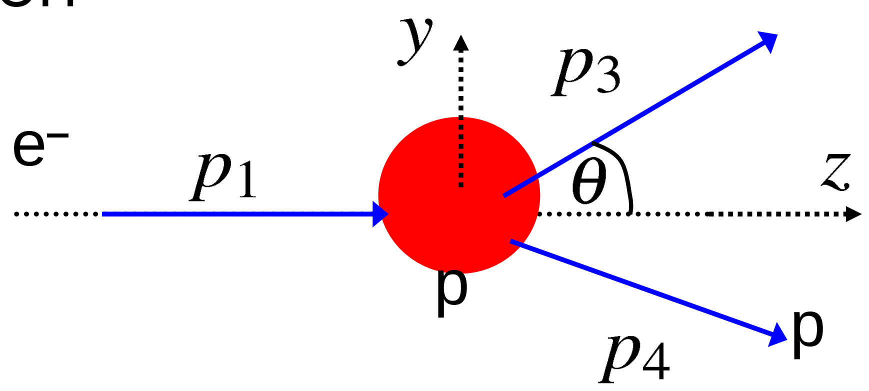
$$E_1 - E_3 = -\frac{q^2}{2M}$$

$$\frac{E_3}{E_1} = \frac{M}{M + E_1(1-\cos\theta)}$$

Depends only on one parameter

Recoil of a finite size proton

- **Elastic non-Coulomb scattering**
- **Consider proton recoil**
- **Proton is a spin-1/2 extended charged object**
- **Electron is ultra-relativistic**



In general, we need two form factors, $G_E(q^2)$, related to the charge distribution, and $G_M(q^2)$, related to the distribution of the magnetic moment.

ROSENBLUTH FORMULA.

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right)} \Leftarrow \boxed{\tau = -\frac{q^2}{4M^2} > 0}$$

Note that form factors depend on q^2 rather than $\vec{q}^2$ and cannot simply be considered in terms of the Fourier transformation of the charge and magnetic moment distributions.

But for $\tau \ll 1$ **we have** $q^2 \approx -\vec{q}^2$ **and** $G(q^2) \approx G(\vec{q}^2)$

Electric and magnetic form factors

So for $\tau = -\dfrac{q^2}{4M^2} \ll 1$

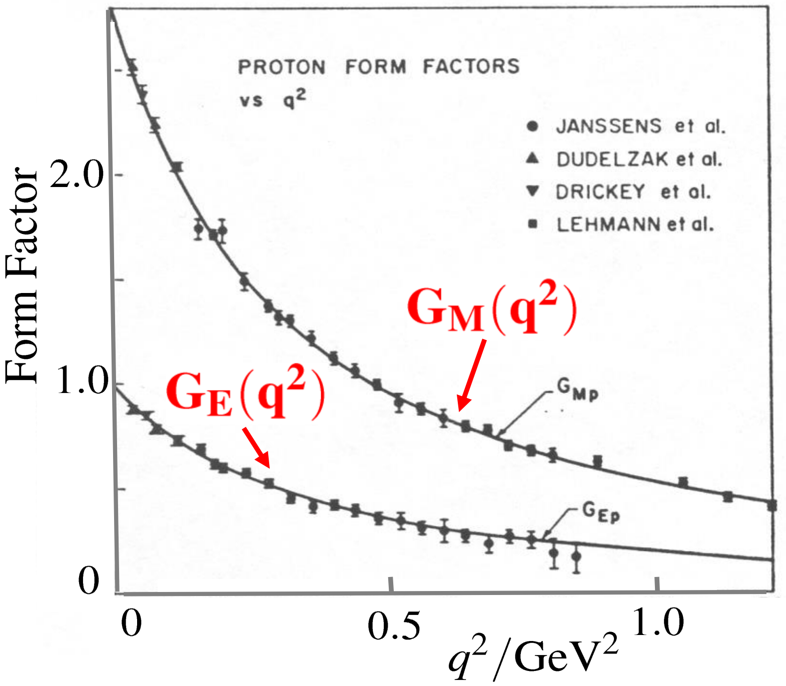
$$G_E(q^2) \approx G_E(\vec{q}^2) = \int e^{i\vec{q}.\vec{r}}\rho(\vec{r})\mathrm{d}^3\vec{r} \quad = \quad F(\vec{q}^2)$$

$$G_M(q^2) \approx G_M(\vec{q}^2) = \int e^{i\vec{q}.\vec{r}}\mu(\vec{r})\mathrm{d}^3\vec{r}$$

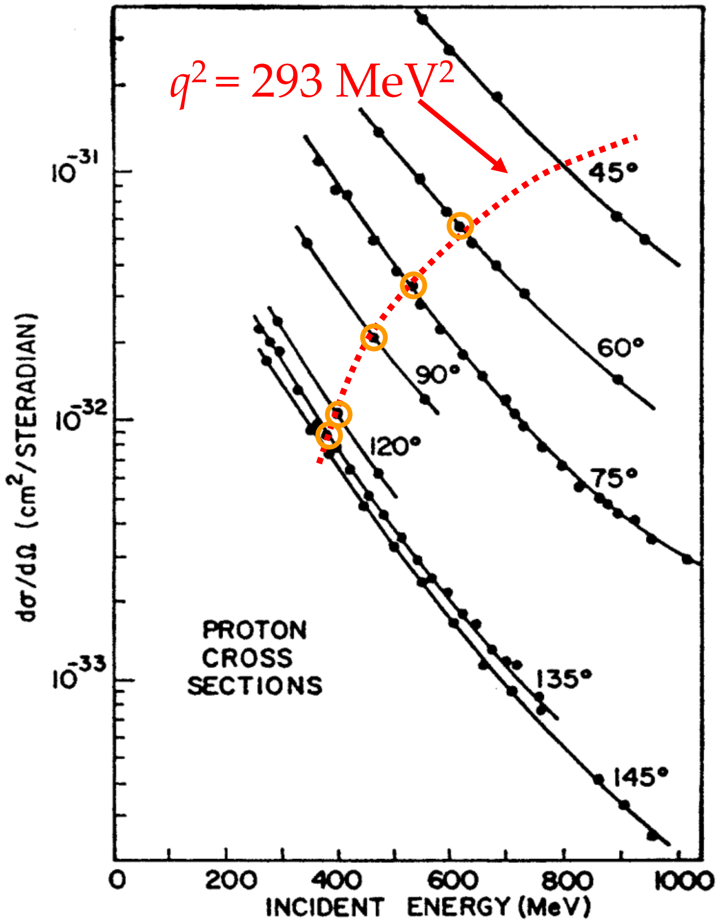
- **EXAMPLE:** $e^-p \to e^-p$ **at** $\boldsymbol{E_{\mathrm{beam}}}$ **= 529.5 MeV**

Experimentally find $G_M(q^2) = 2.79G_E(q^2)$, i.e. the electric and and magnetic form factors have same distribution

Note that $\mu_p = +2.79$



E.B.Hughes et al., Phys. Rev. 139 (1965) B45



Electric and magnetic form factors

So for $\tau = -\frac{q^2}{4M^2} \ll 1$

$$G_E(q^2) \approx G_E(\vec{q}^2) = \int e^{i\vec{q}.\vec{r}} \rho(\vec{r}) \mathrm{d}^3\vec{r} \quad = \quad F(\vec{q}^2)$$

$$G_M(q^2) \approx G_M(\vec{q}^2) = \int e^{i\vec{q}.\vec{r}} \mu(\vec{r}) \mathrm{d}^3\vec{r}$$

- **EXAMPLE:** $e^-p \to e^-p$ **at** E_{beam} **= 529.5 MeV**

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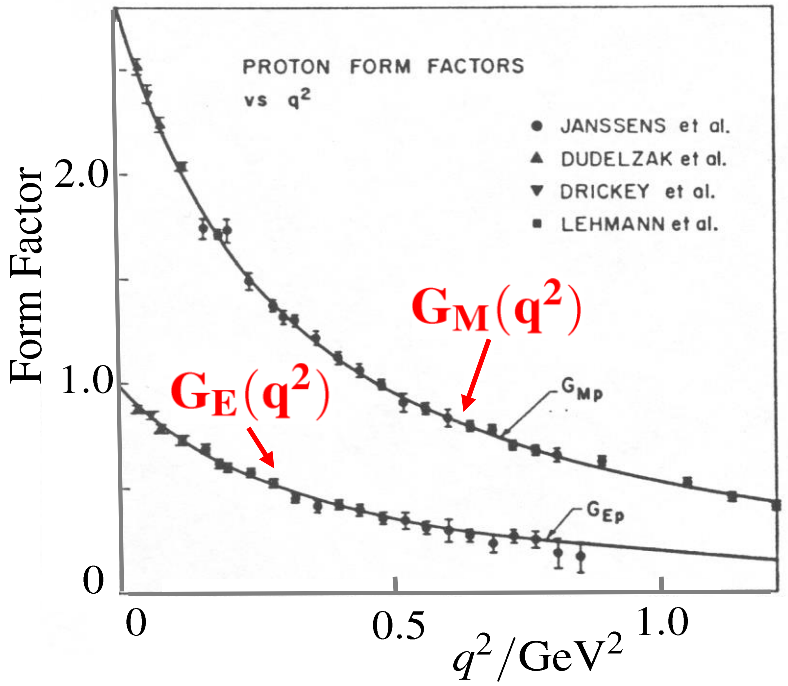


HW4: Prove this.

At small q^2,

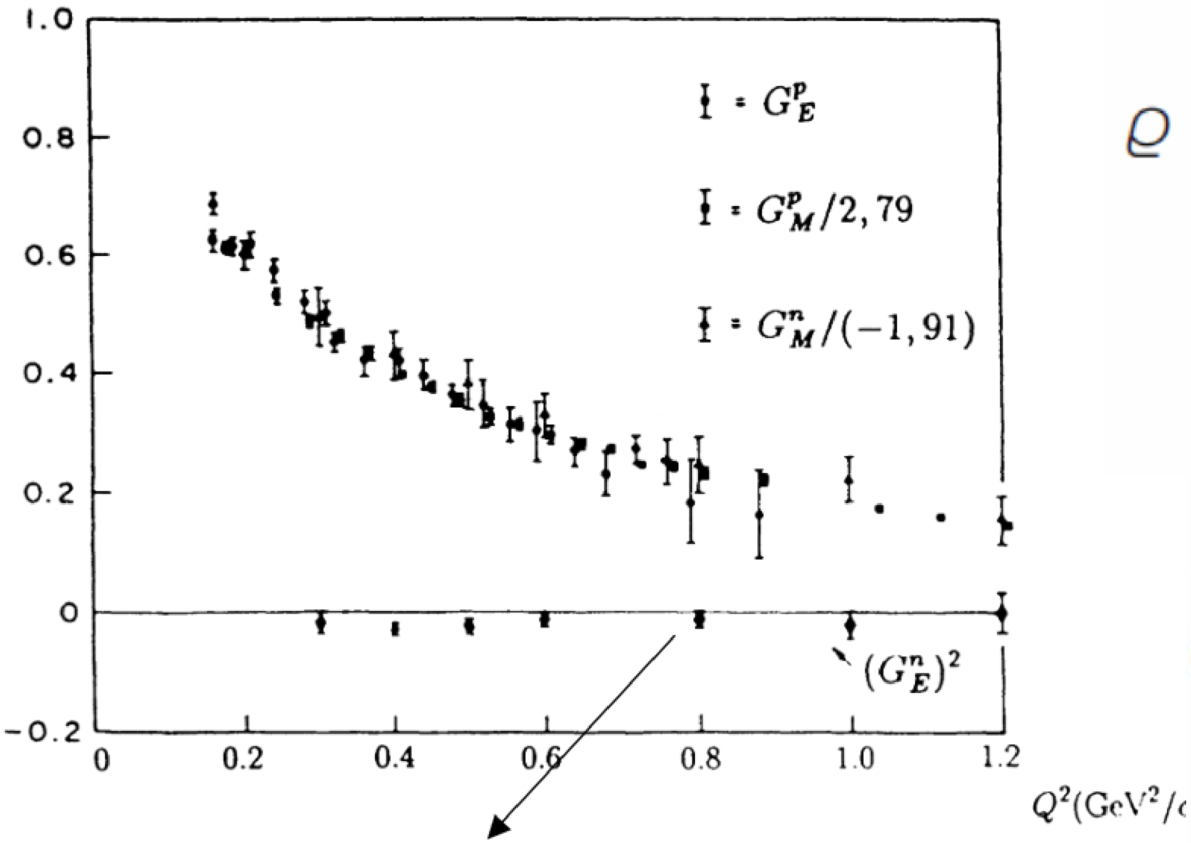
$G_E(q^2) = 1 - q^2 \langle r^2 \rangle / 6$,

$\Rightarrow$ $r_{rms} \approx 0.8$ fm

The charge radius of proton

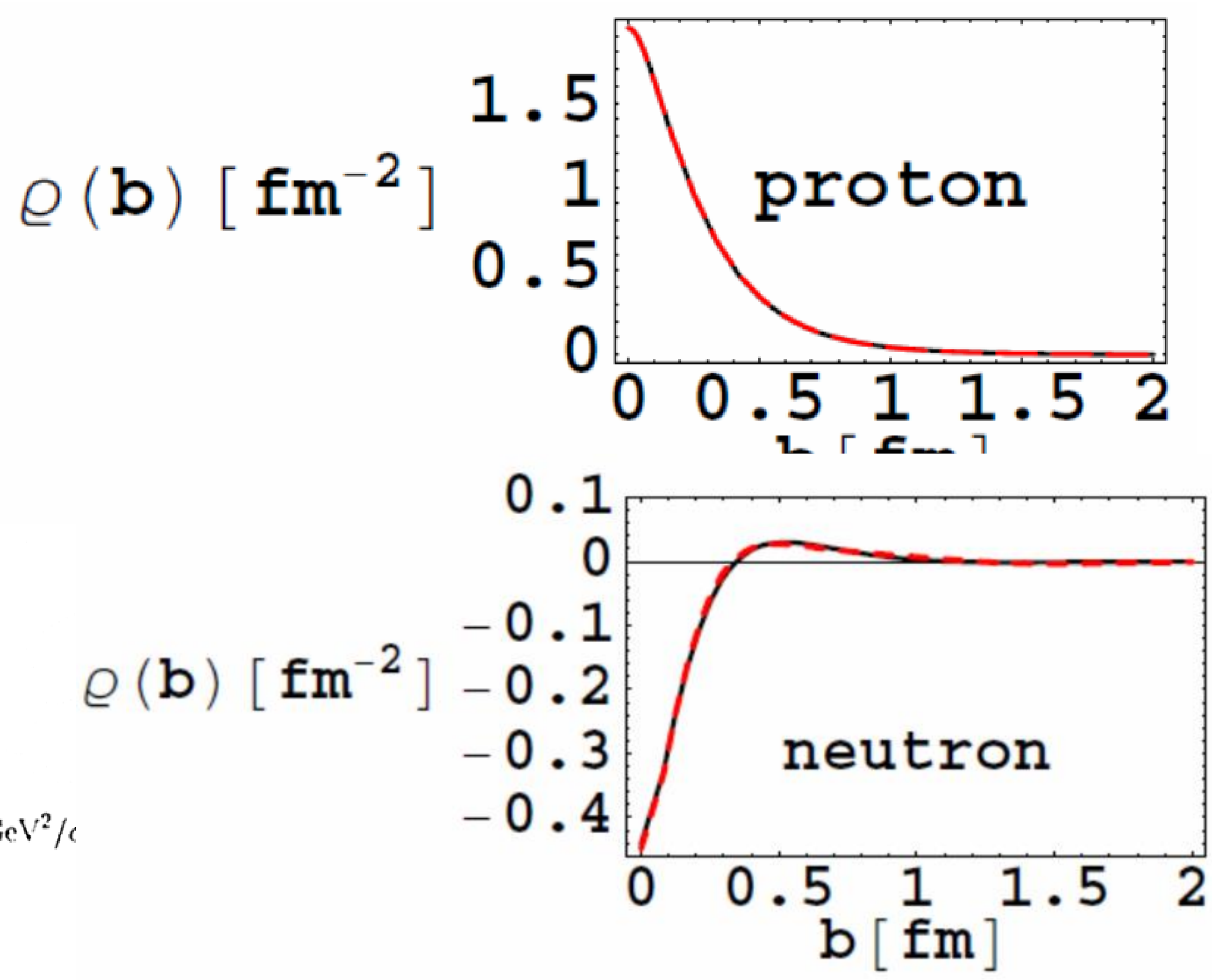
Electric and magnetic form factors

For neutron



Obtained from scattering experiments of e⁻ on d

$$\sqrt{\langle r^2 \rangle_n} = 0.10 \pm 0.01\, fm$$

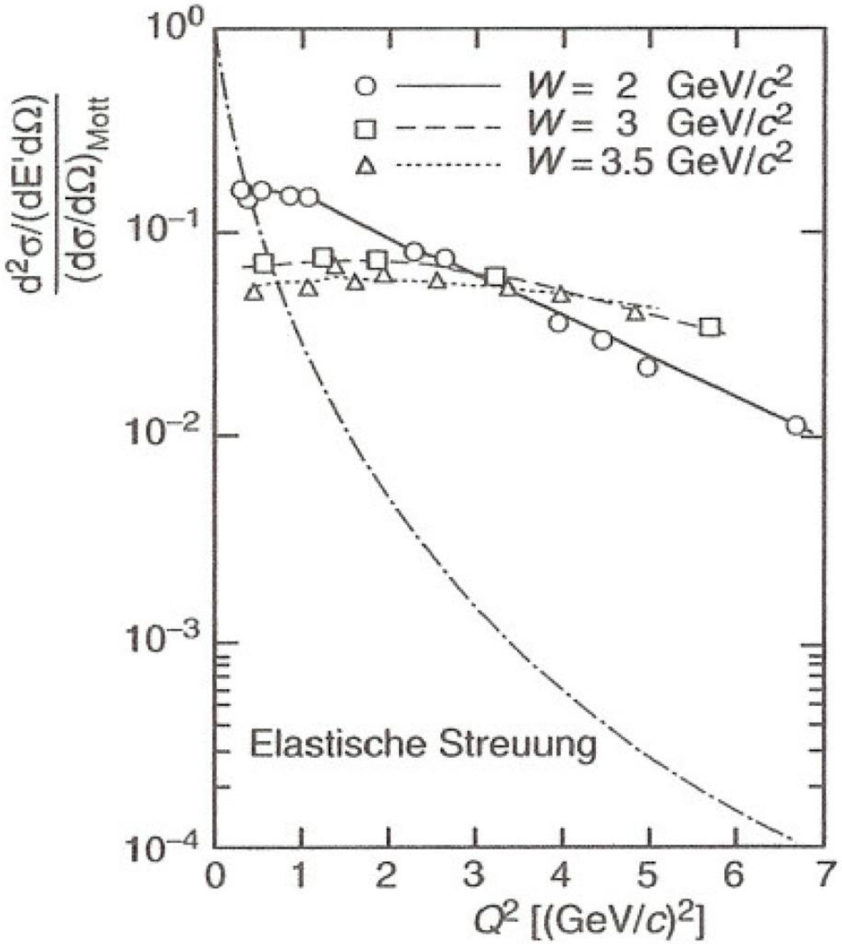


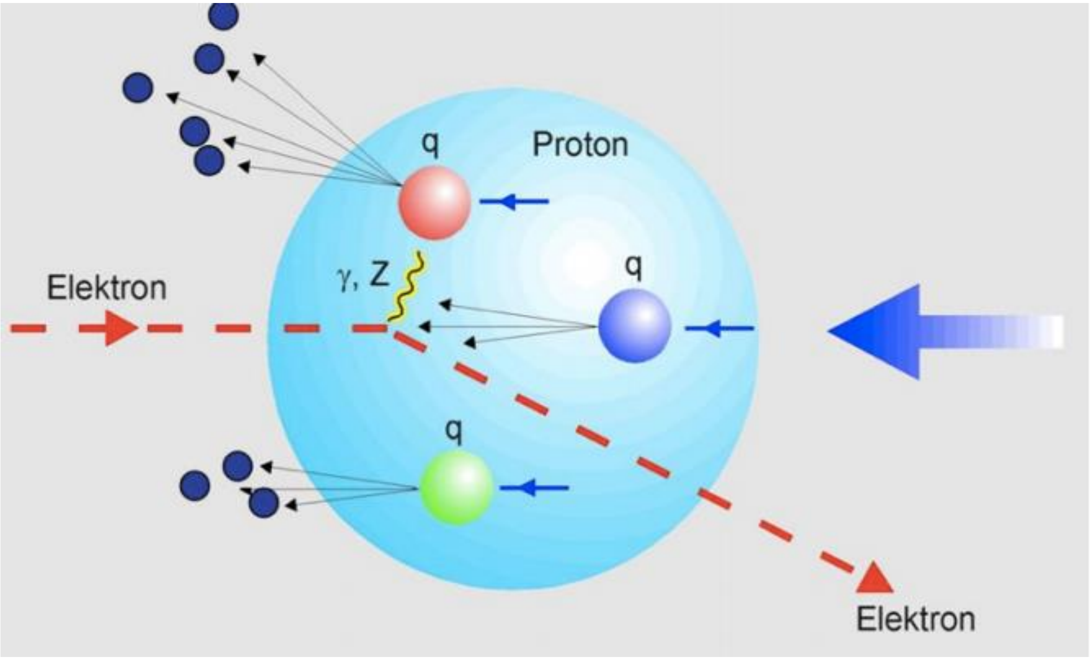
Miller, PRL 99, 112001 (2007)

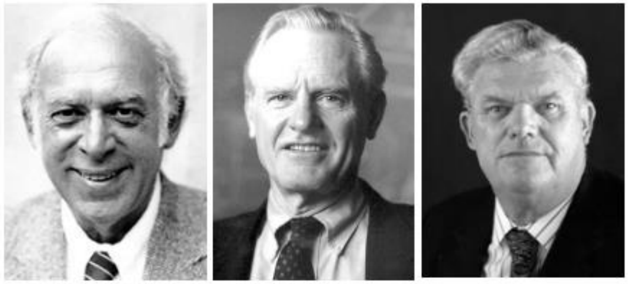
Deep Inelastic Scattering (DIS)

Modern Rutherford experiment --- DIS

Provides a way to "directly see" quarks in a proton





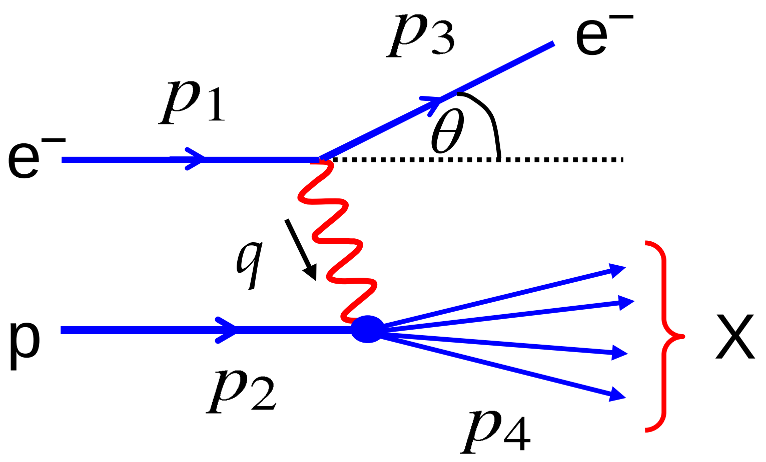


Friedman, Kendall, Taylor

1990

Deep Inelastic Scattering (DIS)

DIS kinematics x, y, ν, Q^2



•In the Lab. Frame:

$$Q^2 = 4E_1E_3\sin^2\theta/2$$

$$x = \frac{Q^2}{2M(E_1 - E_3)}$$

$$y = \frac{M(E_1 - E_3)}{ME_1} = 1 - \frac{E_3}{E_1}$$

$$\nu = E_1 - E_3$$

$$\boxed{Q^2 \equiv -q^2}$$

Measure of resolution power

$$\boxed{x \equiv \frac{Q^2}{2p_2 \cdot q}}$$

Bjorken variable: Measure of momentum fraction of struck quark

$$\boxed{y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}}$$

Measure of inelasticity

$$\boxed{\nu \equiv \frac{p_2 \cdot q}{M}}$$

Measure of Energy transfer

Only two are independent
(For elastic scattering only one independent)

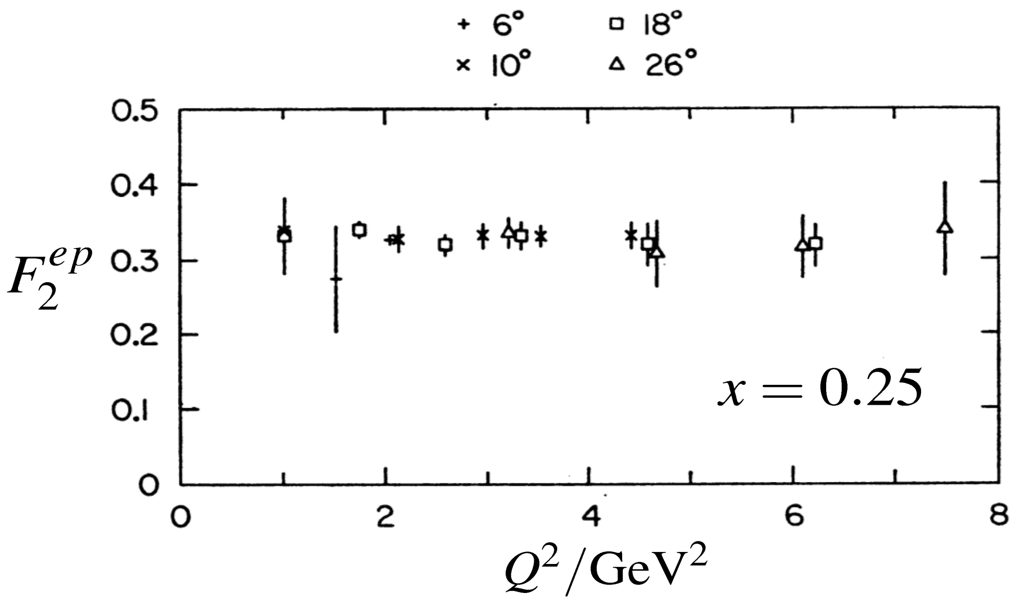
Deep Inelastic Scattering (DIS)

- **In the Lab. frame,**

$$\boxed{\frac{d^2\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\theta/2}\left[\frac{1}{\nu}F_2(x,Q^2)\cos^2\frac{\theta}{2} + \frac{2}{M}F_1(x,Q^2)\sin^2\frac{\theta}{2}\right]}$$

Electromagnetic Structure Function

Pure Magnetic Structure Function



J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203

Bjorken Scaling, indepdence on Q:

$$\boxed{F_1(x,Q^2) \to F_1(x) \qquad F_2(x,Q^2) \to F_2(x)}$$

Point particle inside proton is detected

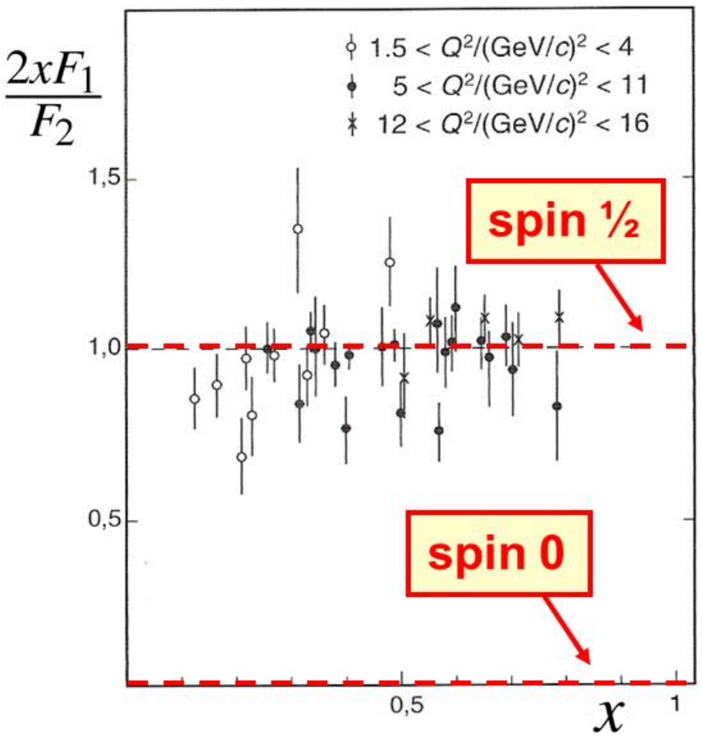
Deep Inelastic Scattering (DIS)

- **In the Lab. frame,**

$$\frac{d^2\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\theta/2}\left[\frac{1}{\nu}F_2(x,Q^2)\cos^2\frac{\theta}{2} + \frac{2}{M}F_1(x,Q^2)\sin^2\frac{\theta}{2}\right]$$

Electromagnetic Structure Function

Pure Magnetic Structure Function



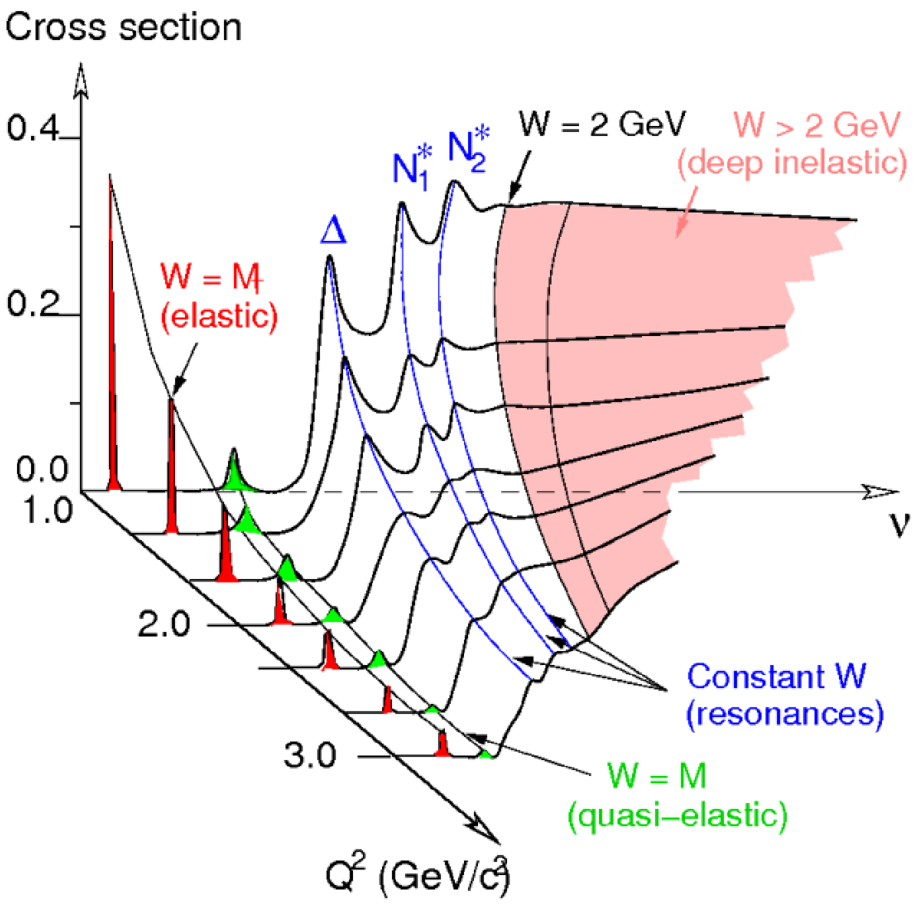
Callan-Gross relation:

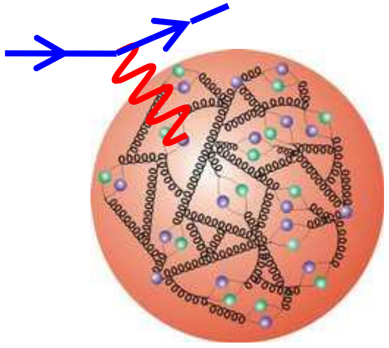
$$\boxed{F_2(x) = 2xF_1(x)}$$

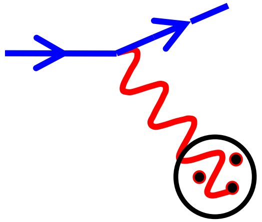
$\Rightarrow$

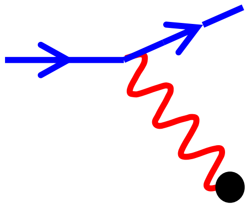
Such point particle is spin-1/2

Deep Inelastic Scattering (DIS)

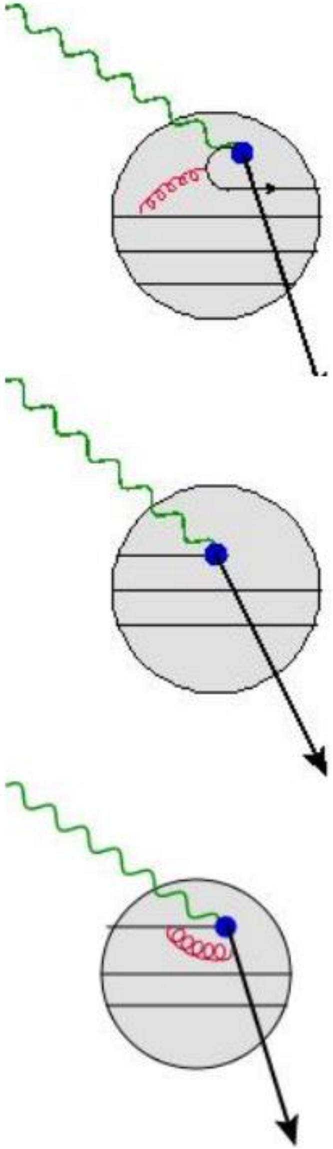


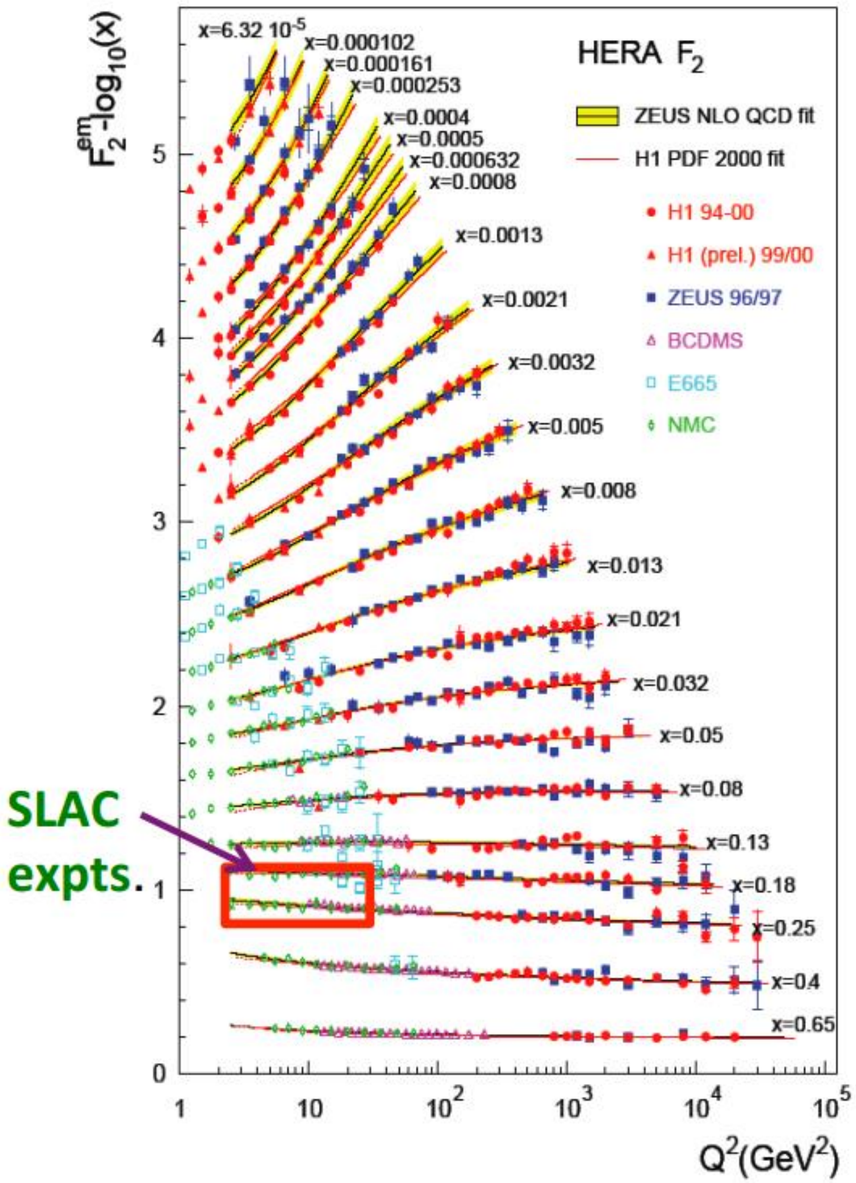






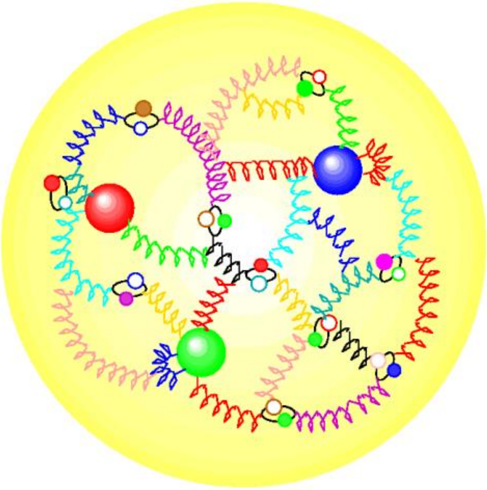
Structure function



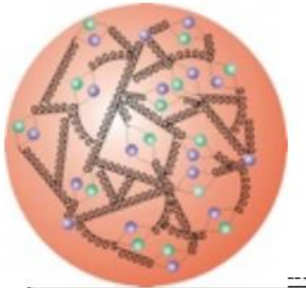


Violation of Bjorken scaling

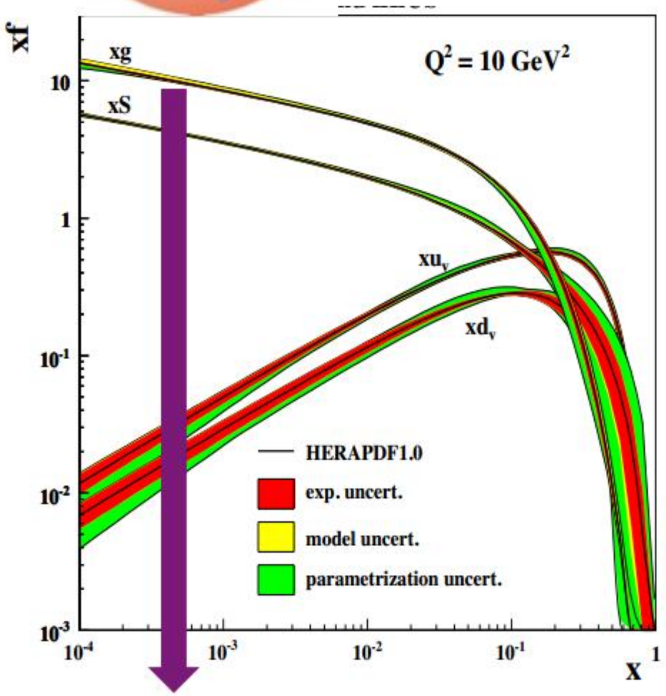
Small x: Gluons and sea quarks



Parton distribution function

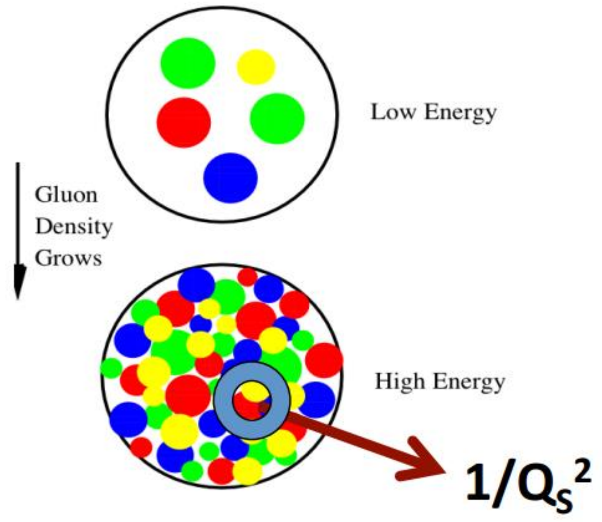


Gluons and
“sea” quarks

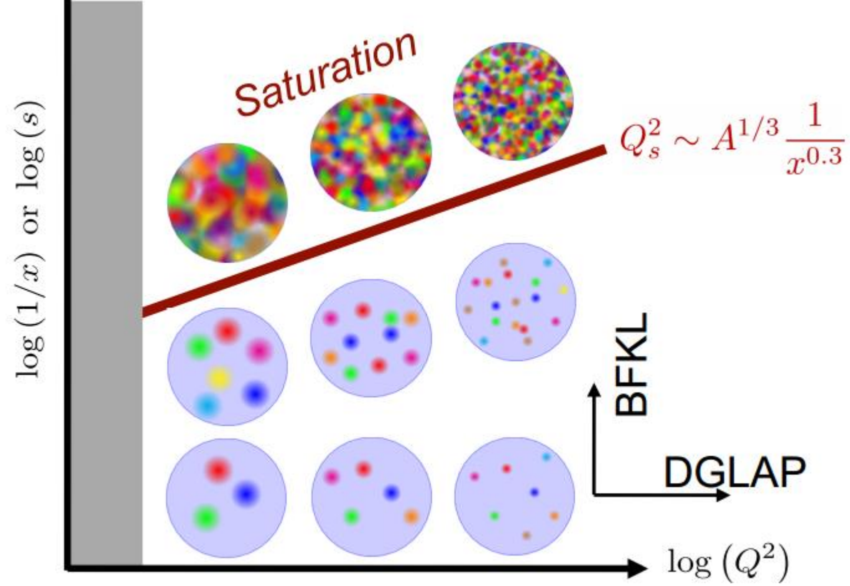


The proton at high energies
(small x) is dominated
by glue!

Emergent dynamical saturation scale grows with energy

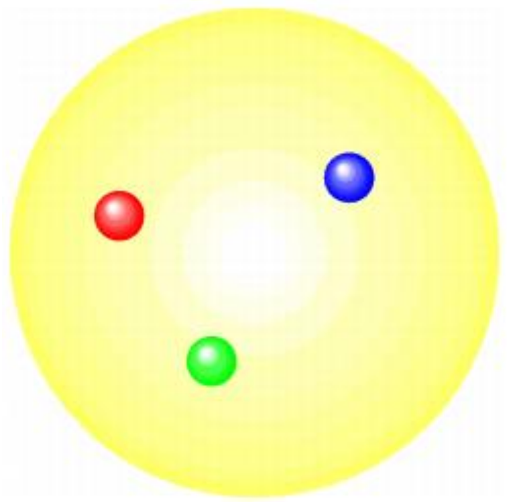


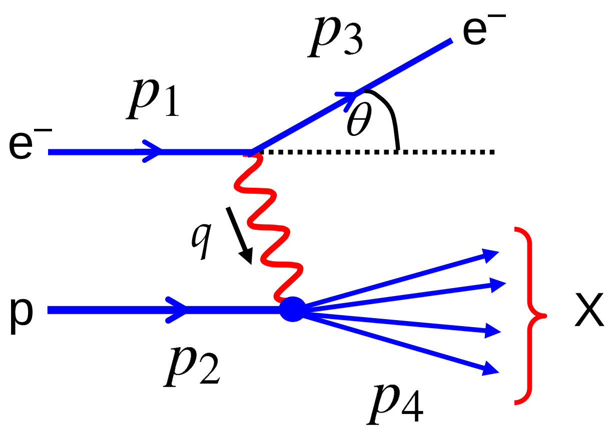
Powerful ``Wilsonian” RG equations describe evolution in this landscape

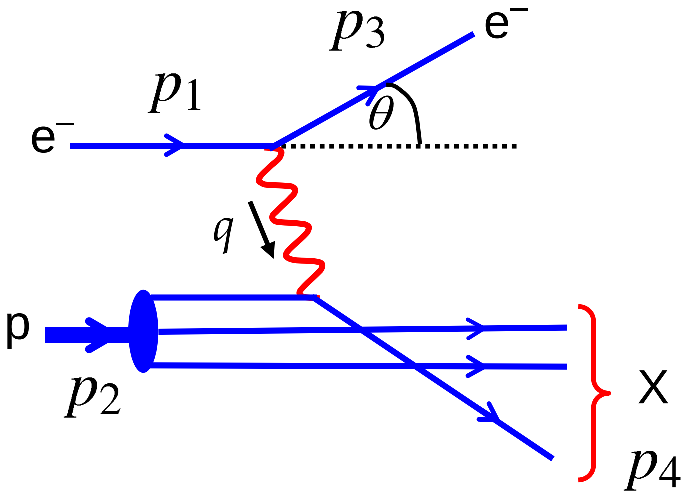


Saturation and color glass condensate

Parton model







Parton model

Assume: Proton made of partons, electron-parton elastic scattering