

习题

我没有答案

高寒

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1.0

What is the partition function?

- ▶ 玻尔兹曼分布的归一化因子;
- ▶ 均匀分布的矩母函数;
- ▶ 自由能的指数 $e^{-\beta F}$;
- ▶ 能量态密度 $g(E)$ 的拉普拉斯变换;

1.1.1

Compute the partition function of a quantum harmonic oscillator with frequency ω and energy levels

$$E_n = \hbar\omega \left(n + \frac{1}{2} \right) \quad n \in \mathbf{Z}$$

Find the average energy U and entropy S as a function of temperature T .

1.1.2

Einstein constructed a simple model of a solid as N atoms, each of which vibrates with the same frequency ω . Treating these vibrations as a harmonic oscillator, show that at high temperatures, $k_B T \gg \hbar\omega$, the Einstein model correctly predicts the Dulong-Petit law for the heat capacity of a solid,

$$C_V = 3Nk_B$$

At low temperatures, the heat capacity of many solids is experimentally observed to tend to zero as $c_V \sim T^3$. Was Einstein right about this?

1.2

A particle moving in one dimension has Hamiltonian

$$H = \frac{p^2}{2m} + \lambda q^4$$

Show that the heat capacity for a gas of N such particles is $C_V = 3Nk_B/4$. Explain why the heat capacity is the same regardless of whether the particles are distinguishable or indistinguishable.

1.3

A non-relativistic particle is confined in a spherical container with radii R with potential

$$V(r) = \begin{cases} -h & , r < a \\ 0 & , a \leq r < R \\ +\infty & , r > R \end{cases}$$

Firstly predict the rough behavior of the pressure for a gas of N such particles as a function of T , then determine $P(T)$ by statistics.

2.1

Show that

1.

$$C_P - C_V = T \left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial P}{\partial T} \right)_V = -T \left(\frac{\partial V}{\partial T} \right)_P^2 \left(\frac{\partial P}{\partial V} \right)_T$$

2.

$$\left(\frac{\partial U}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V - P$$

3.

$$\left(\frac{\partial U}{\partial P} \right)_T = -T \left(\frac{\partial V}{\partial T} \right)_P - P \left(\frac{\partial V}{\partial P} \right)_T$$

2.1

4.

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T \left(\frac{\partial^2 P}{\partial T^2}\right)_V$$

5.

$$\left(\frac{\partial C_P}{\partial P}\right)_T = -T \left(\frac{\partial^2 V}{\partial T^2}\right)_P$$

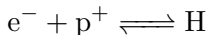
From equations above, show that for any non-ideal gas with constant C_V and C_P , the equation of state can be written as

$$(C_P - C_V)T = (P + a)(V + b)$$

where a and b are constants.

2.2

Consider the neutral gas of electrons, protons and Hydrogen. They undergo a reaction as



You know from Quantum Mechanics that the Hydrogen atom has binding energy $E = -I$ (where $I = 13.6$ eV). Let the number of Hydrogen atoms be $N_H = (1 - x)N$ and the number of electrons and protons be $N_e = N_p = xN$ with $x \in [0, 1]$.

1. What is the equilibrium condition for the gas?
2. From the condition above, show that

$$\frac{x^2}{1-x} = \frac{V}{N} \left(\frac{m_e m_p}{2\pi \hbar^2 m_H} \right)^{3/2} (k_B T)^{3/2} e^{-I/k_B T}$$

3.1

A Wigner crystal is a triangular lattice of electrons in a two dimensional plane. The longitudinal vibration modes of this crystal are bosons with dispersion relation $\omega = \sigma\sqrt{k}$. Show that, at low temperatures, these modes provide a contribution to the heat capacity that scales as $C \sim T^4$.

3.2

Consider a gas of non-interacting ultra-relativistic electrons, whose mass may be neglected. Show that $U = 3PV$. Show that at zero temperature $PV^{4/3} = \text{const.}$ Show that at high temperatures $E = 3Nk_B T$ and the equation of state coincides with that of a classical ultra-relativistic gas.

What are the results if we count the existence of positrons ($\mu_{e^+} = -\mu_{e^-}$)?

3.3

Consider the free energy $F = a(T)m^2 + b(T)m^4 + c(T)m^6$ where $b(T) < 0$ and, for stability, $c(T) > 0$ for all T . Sketch the possible behaviours of the free energy as $a(T)$ varies and, in each case, identify the ground state and metastable states. Show that the system undergoes a first order phase transition at some temperature T_c . Determine the value $a(T_c)$ and the discontinuity in m at the transition.