Physics of quark-gluon plasma and high-energy heavy-ion collisions

Xu-Guang Huang

(黄旭光)

Fudan University

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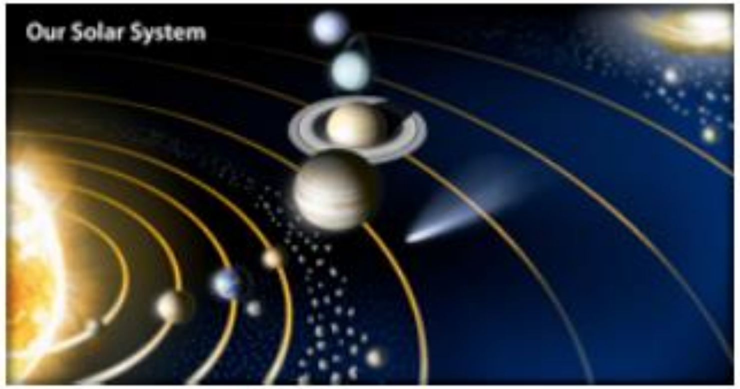
Content of the course

- Introduction to elementary particles and strong interaction
- Relativistic quantum mechanics and field theory
- Quantum chromodynamics (QCD)
- Hot and dense QCD phases
- Exploring the quark-gluon matter: heavy-ion collisions
- The properties of quark-gluon plasma
- Link to other subfields of physics: condensed matter, astrophysics, cosmology, and so on

Lecture 1: Introduction to particle physics

Matter in Our Macroscopic World

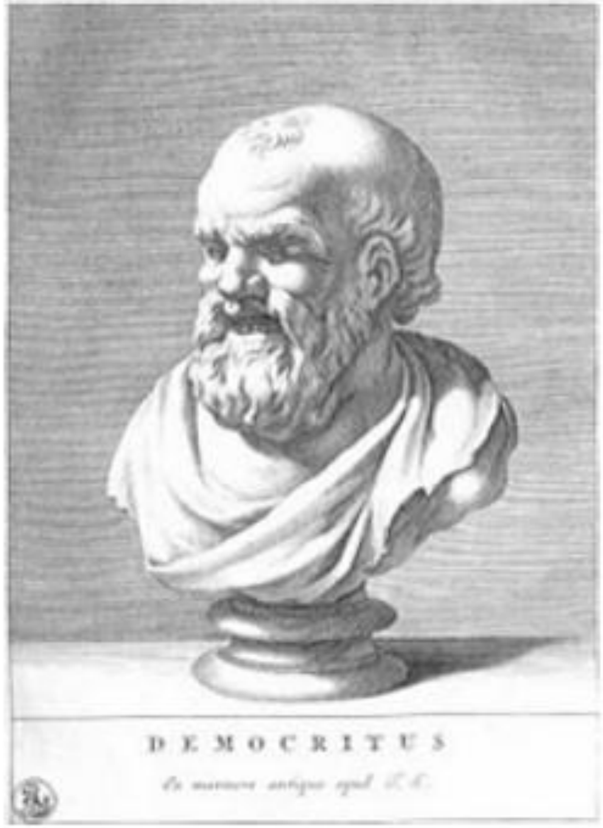
There are all forms of matter in our Nature.
Matter changes forms. But WHY?



The Ancient Philosophers: the Greeks

Empedocles:
four elements ---
fire, air, water, earth

Empedocile's



Democritus:
atomic hypothesis

Reductionism
(还原论)

The Ancient Philosophers: the Oriental

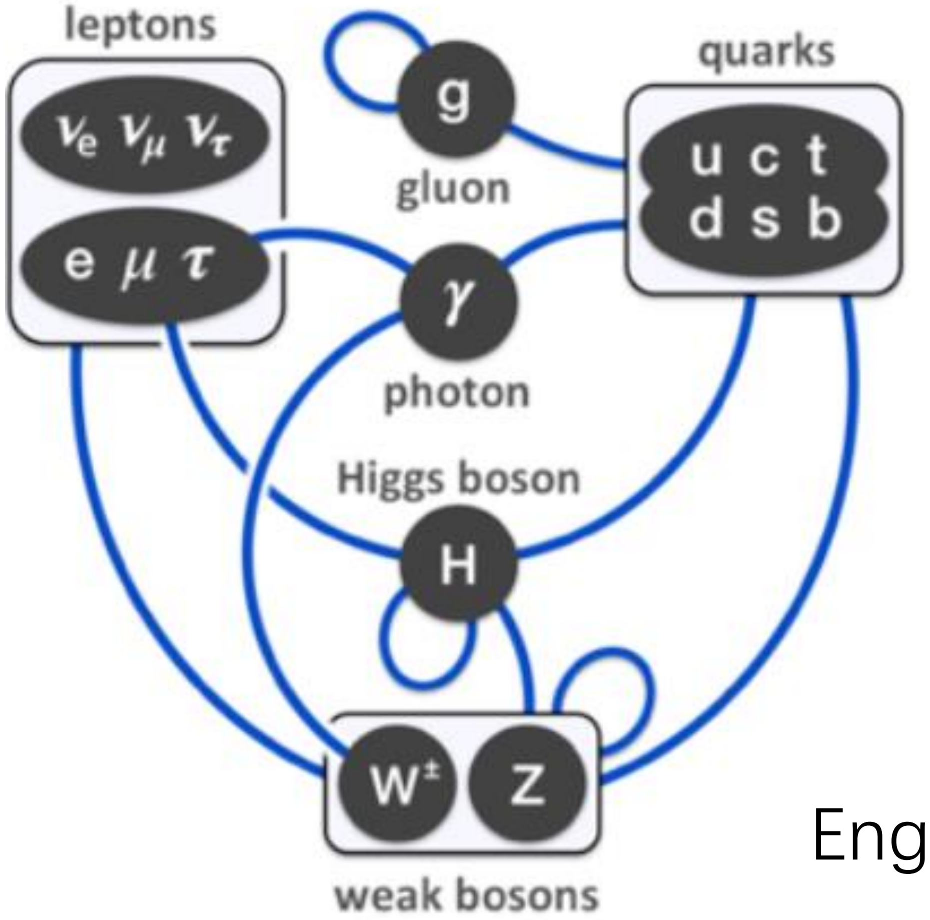
道生一
一生二
二生三
三生万物

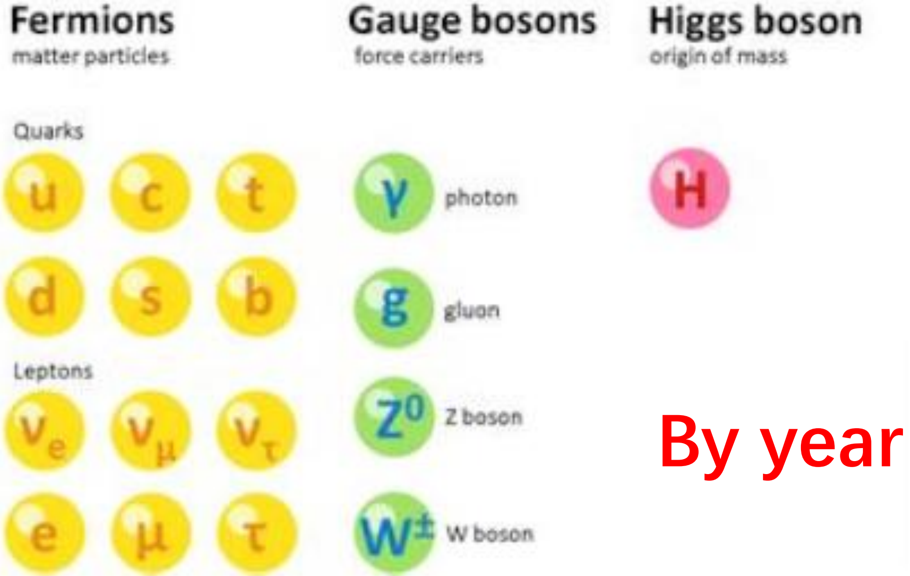
Reductionism
(还原论)

N.B.: the standard model U(1)*SU(2)*SU(3)

The Final Triumph after ~2000 Years

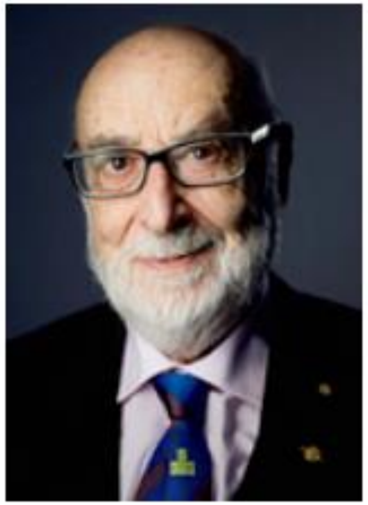
the Standard Model of
particle physics





By year 2012

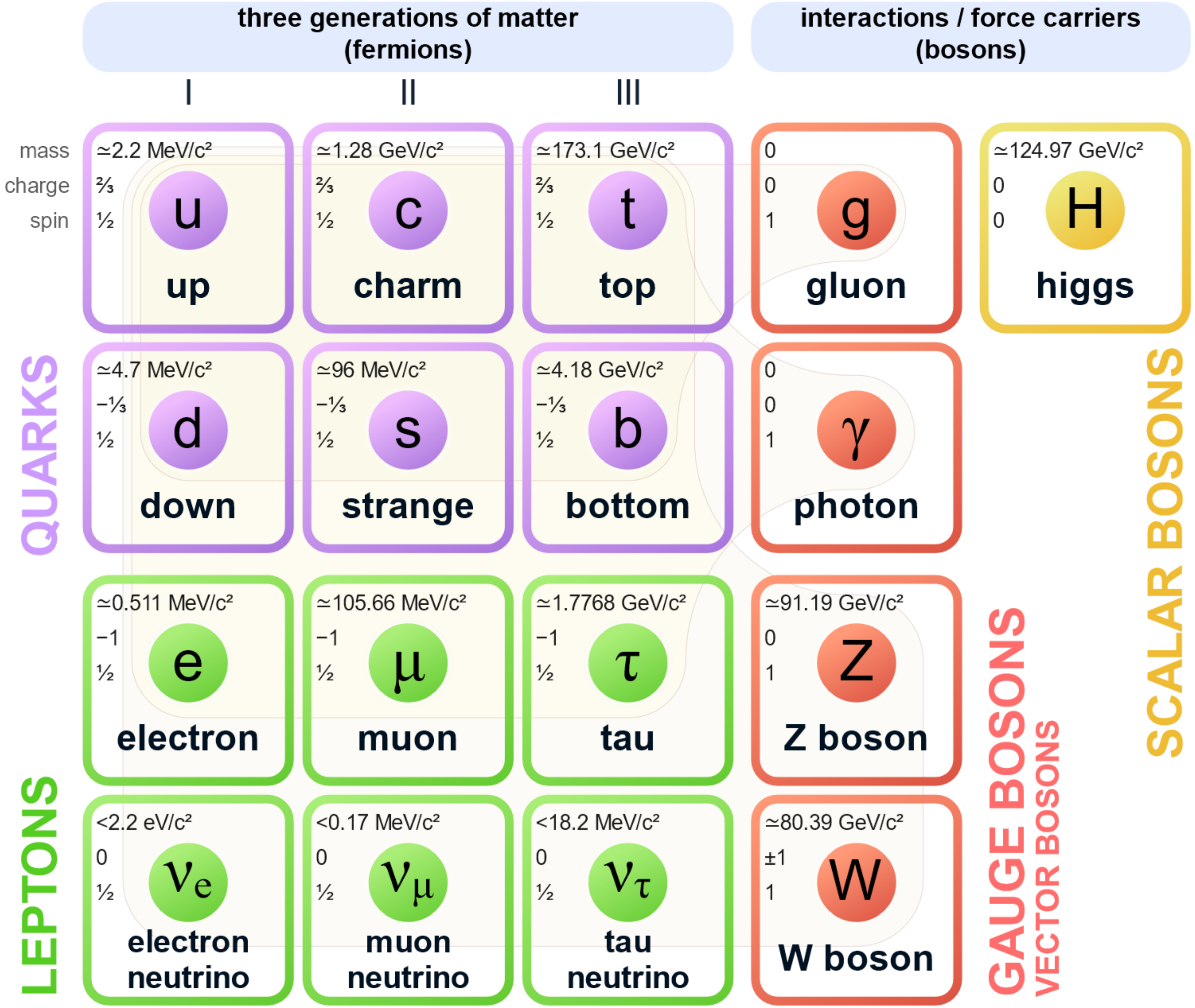
Englert



Higgs

Nobel Prize 2013

Standard Model of Elementary Particles



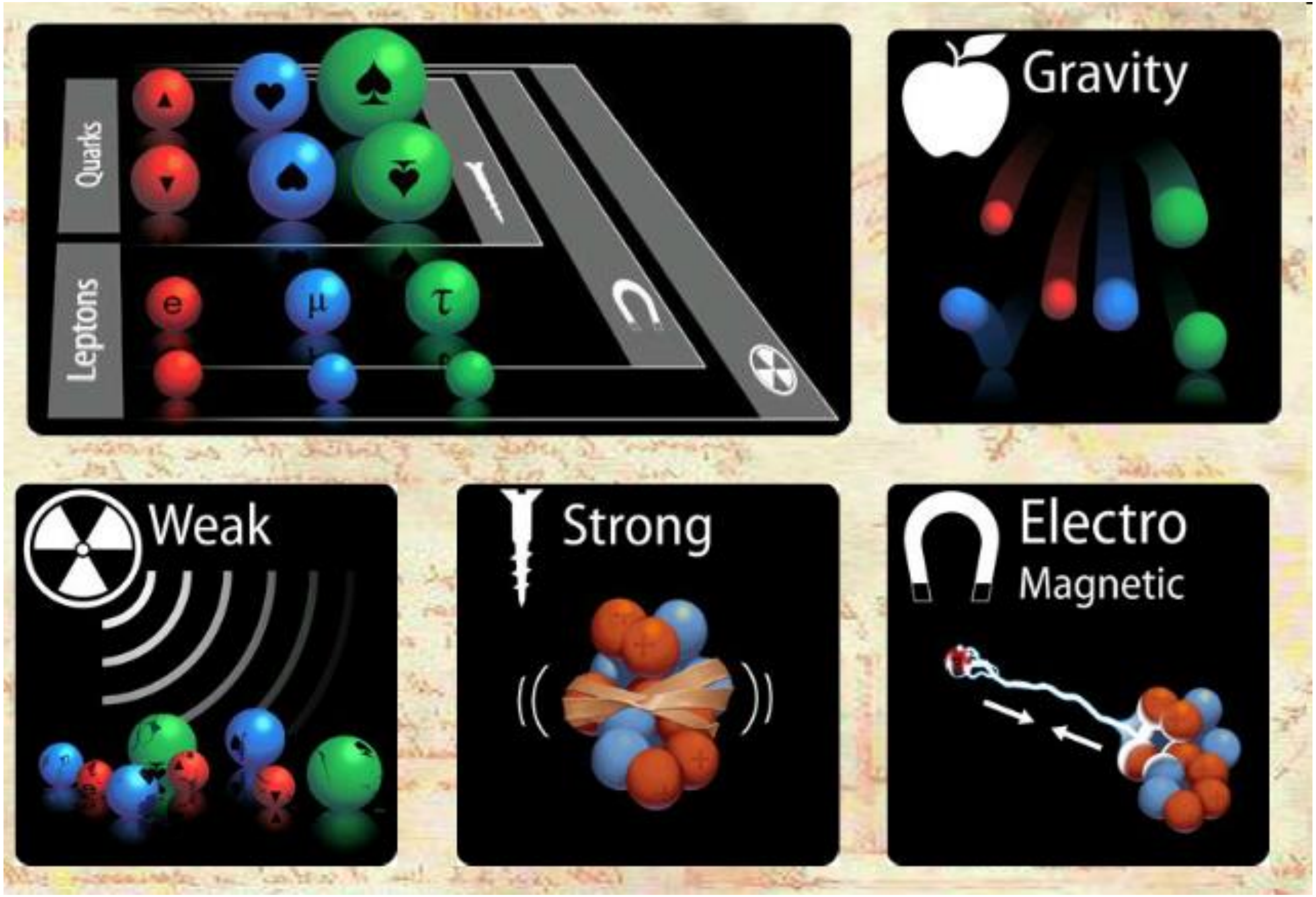
The natural units

$$\hbar = c = k_B = 1.$$

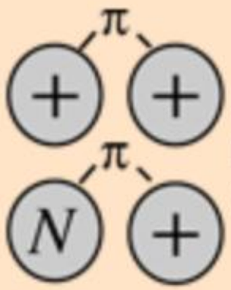
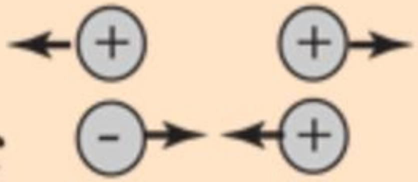
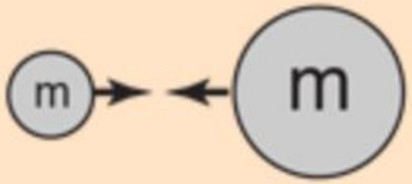
Quantum, relativity, thermodynamics

Unit	Metric value	Derivation
1 eV^{-1} of length	1.97×10^{-7} m	$= \frac{\hbar c}{1\,\mathrm{eV}}$
1 eV of mass	1.78×10^{-36} kg	$= \frac{1\,\mathrm{eV}}{c^2}$
1 eV^{-1} of time	6.58×10^{-16} s	$= \frac{\hbar}{1\,\mathrm{eV}}$
1 eV of temperature	1.16×10^{4} K	$= \frac{1\,\mathrm{eV}}{k_\mathrm{B}}$

If gravity, G=1


Quarks
Leptons
e
μ
τ
Gravity
Weak
Strong
Electro
Magnetic

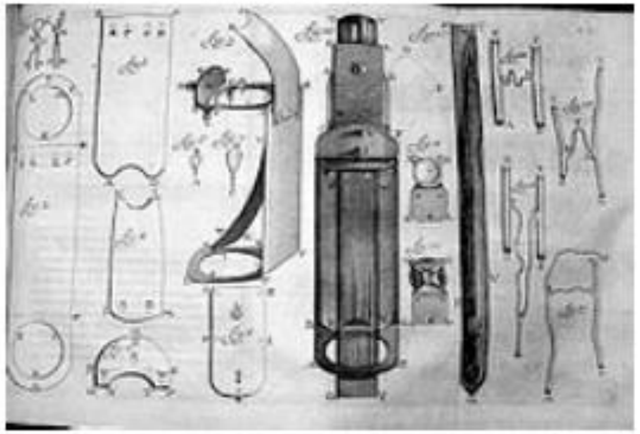
Fundamental Forces

Force		Strength	Range (m)	Particle
Strong	 Force which holds nucleus together	1	10^{-15} (diameter of a medium sized nucleus)	gluons, π(nucleons)
Electro-magnetic		$\frac{1}{137}$	Infinite	photon mass = 0 spin = 1
Weak	 neutrino interaction induces beta decay	10^{-6}	10^{-18} (0.1% of the diameter of a proton)	Intermediate vector bosons W^+, W^-, Z_0, mass > 80 GeV spin =1
Gravity		6×10^{-39}	Infinite	graviton ? mass = 0 spin = 2

Toward the "Invisible" Microscopic World

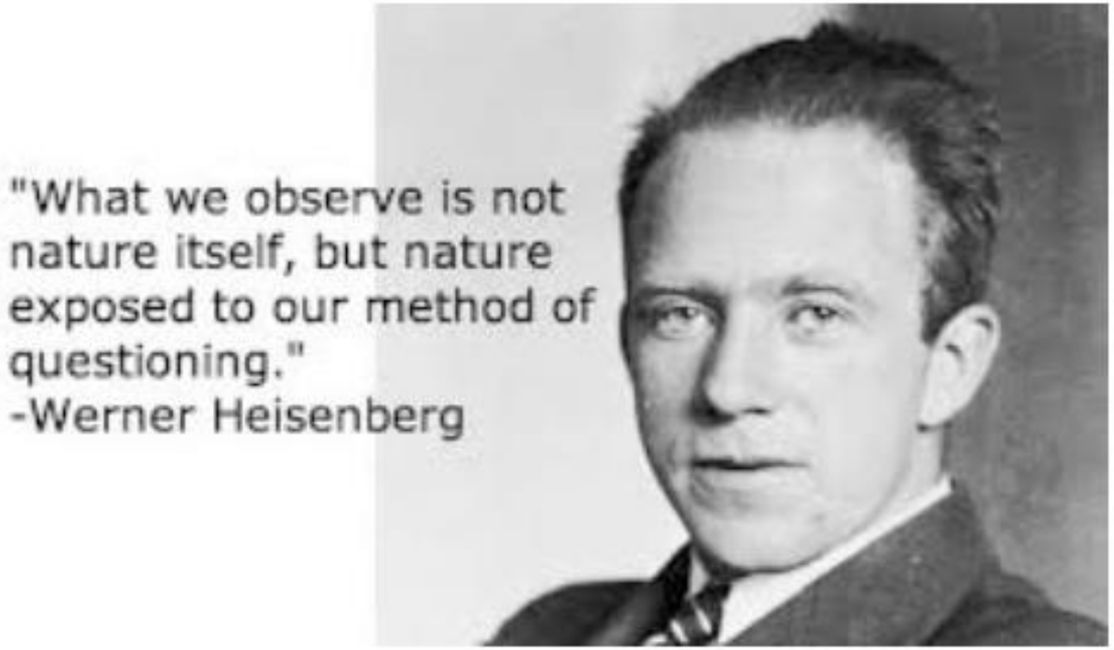
Leeuwenhoek (~1670):
using improved microscope,
he discovered a whole new world,
e.g. bacteria

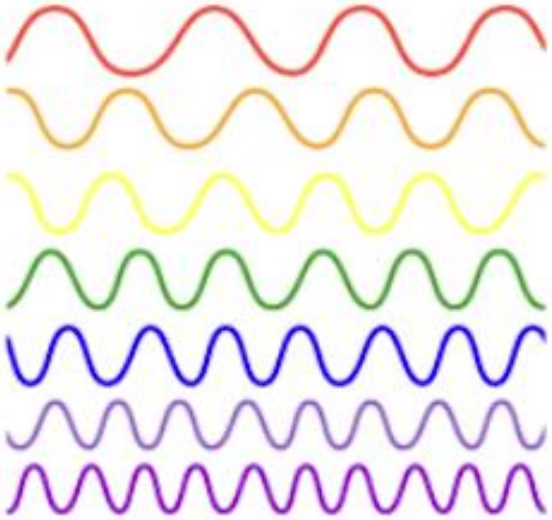
庄子：
一尺之捶，
日取其半，
万世不竭。



Detection tool is crucial!

The Scale of Probing Tools



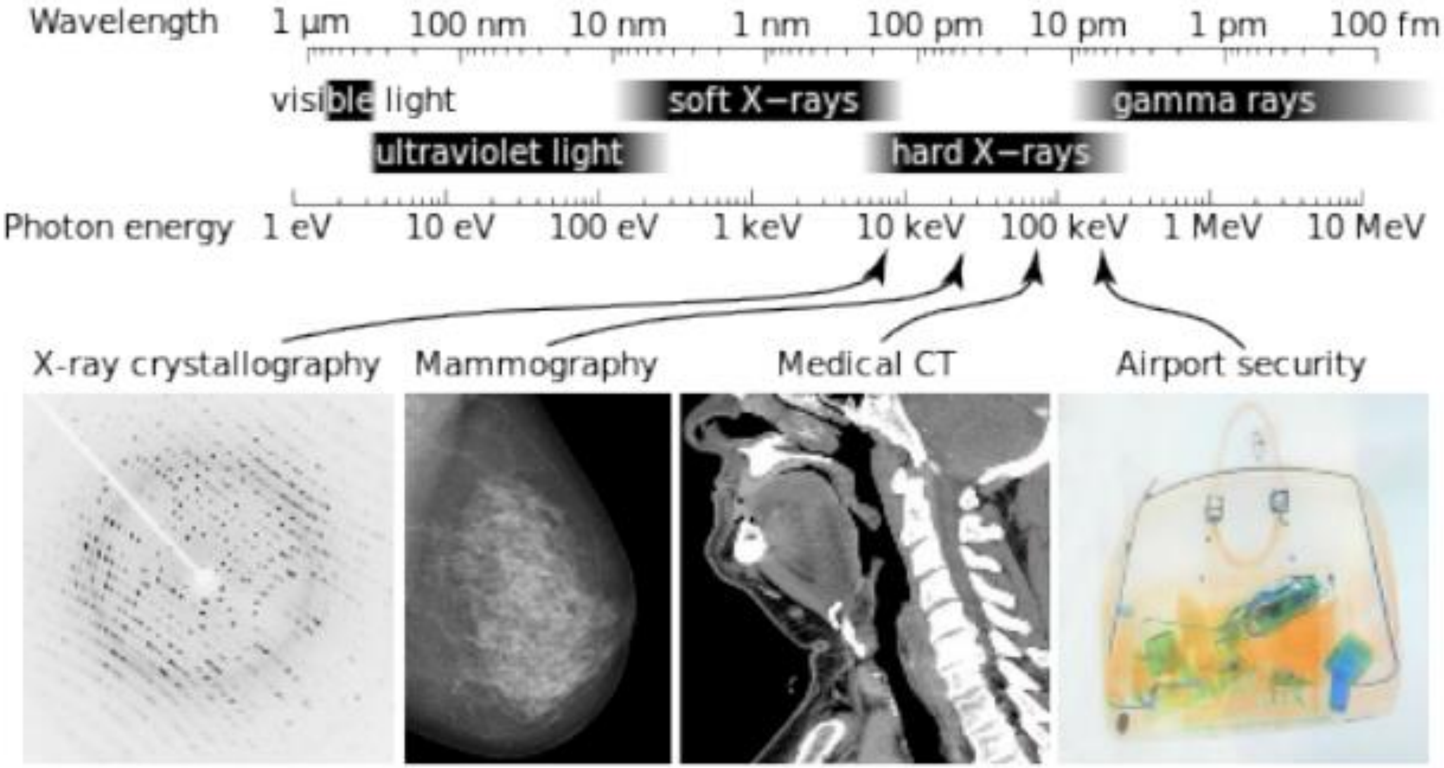


For visible light (optical probe): what scale does it probe?

How can we probe even smaller scale,
e.g. toward SUB-ATOMIC scale?

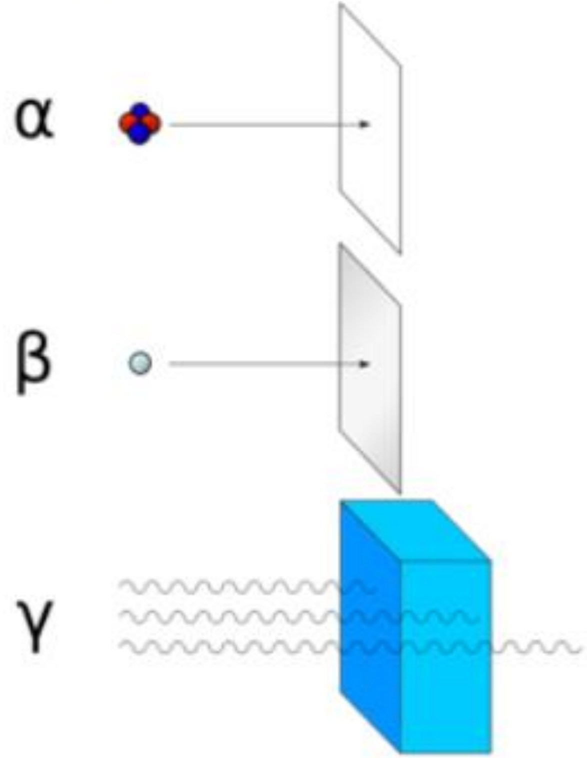
At the Turn of the 20th Century

Wilhelm Röntgen



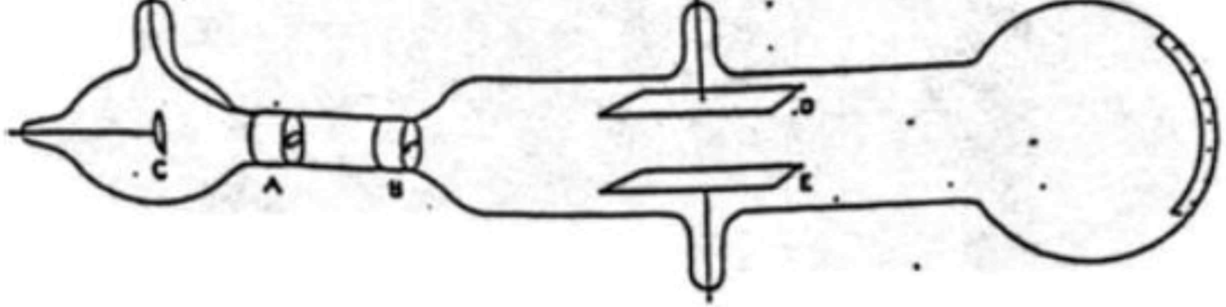
X-ray was later used to probe and reveal the crystal structure of solid.

Alpha, Beta, Gamma, ...



Rutherford

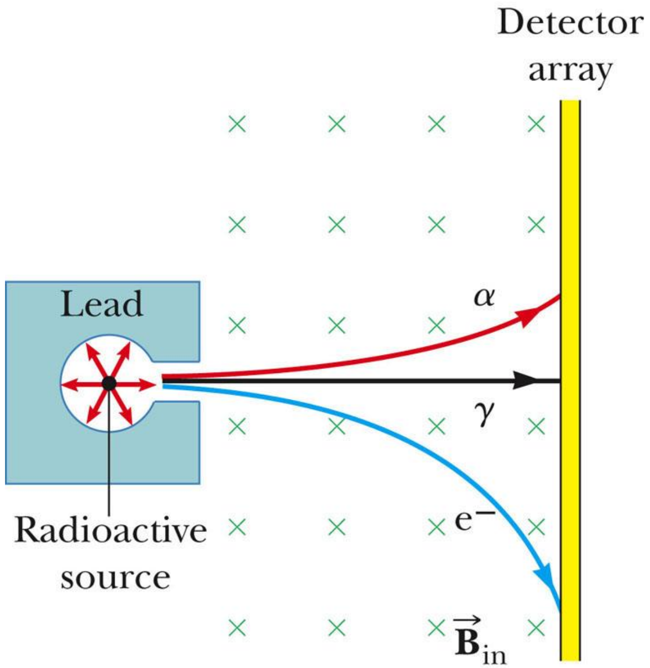
Thomson



cathode ray

Radioactivity

- Unstable nuclei decay to more stable nuclei
- Can emit 3 types of radiation in the process



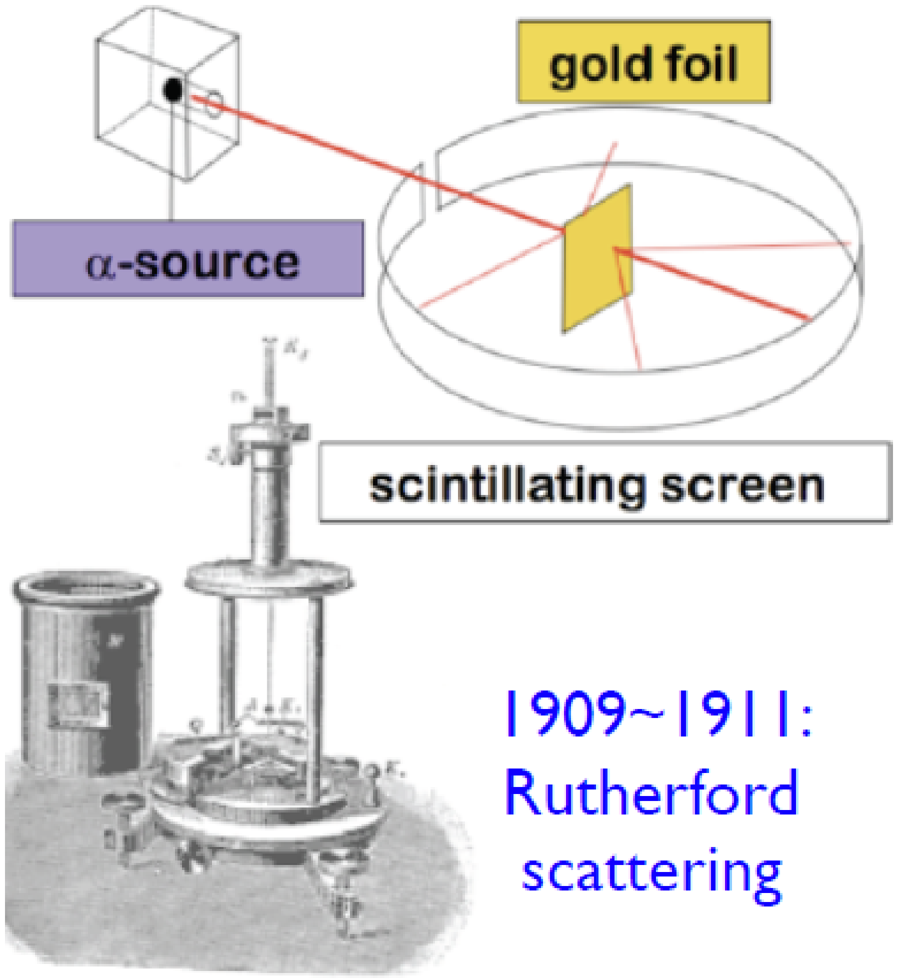
α particles: ^4_2He nuclei

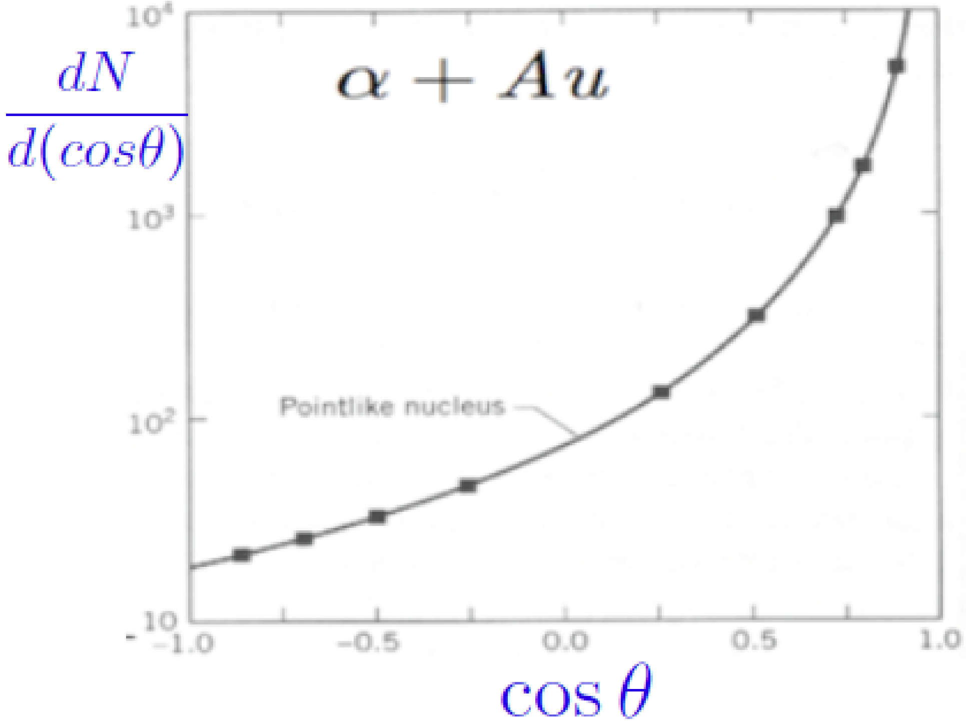
β particles: e^-, e^+

γ rays: high energy photons

A positron (e^+) is the antiparticle of the electron (e^-)

Discovering the Nucleus





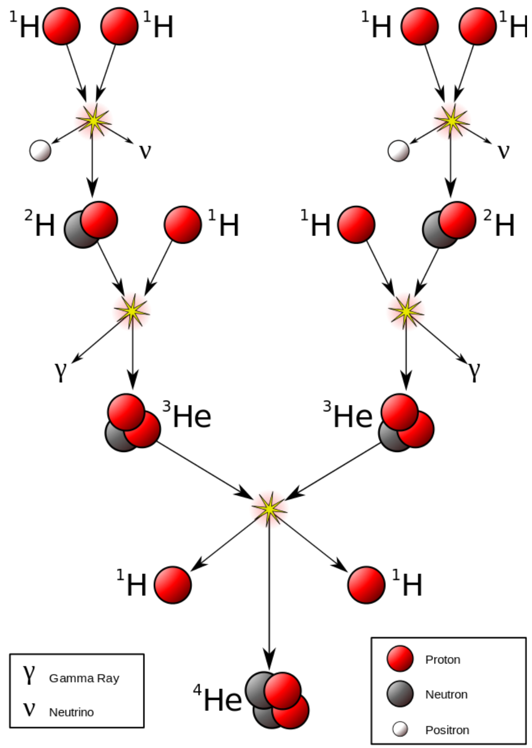
1909~1911: Rutherford scattering

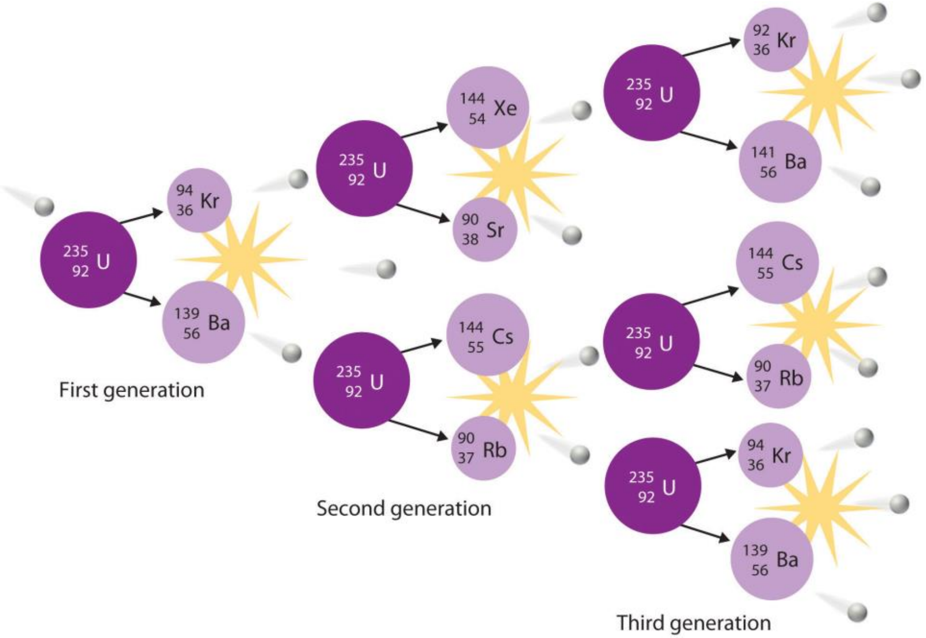
$$\left(\frac{d\sigma}{d\ \cos\theta}\right)_R = \frac{\pi}{2} Z^2 \alpha^2 \left(\frac{\hbar c}{KE}\right)^2 \frac{1}{(1-\cos\theta)^2}$$

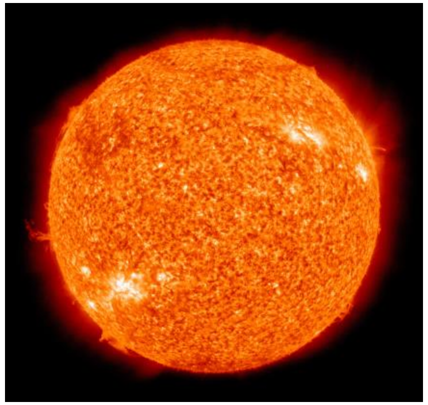
HW1: Derive Rutherford formula

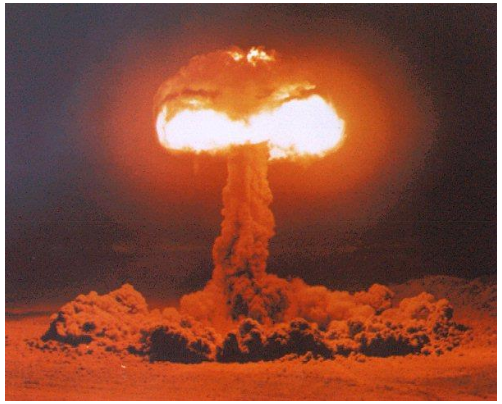
A beautiful experiment with many essential elements of modern high energy collision experiments

Fusion and Fission







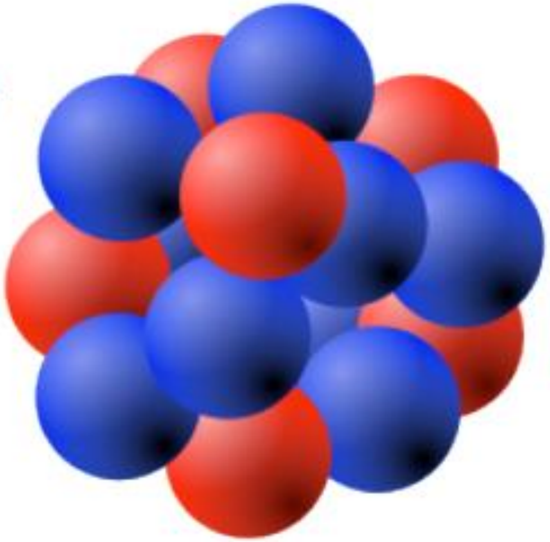


Structure of Nucleus

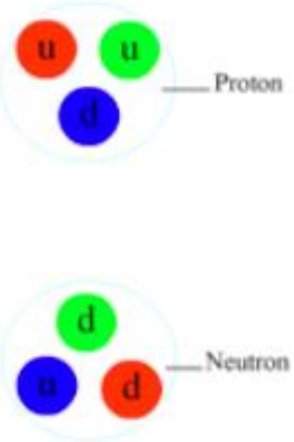
Chadwick

Mismatch between
atomic number & atomic mass
led to the conjecture and
discovery of NEUTRON.
(why was it difficult to find?)

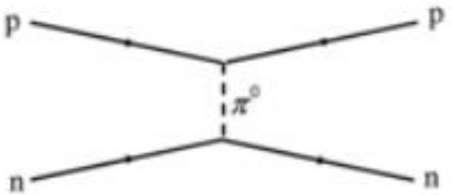
A nucleus is made of
Z protons + (A-Z) neutrons



What holds the NUCLEONS together?



The nuclear force:
short range, very STRONG

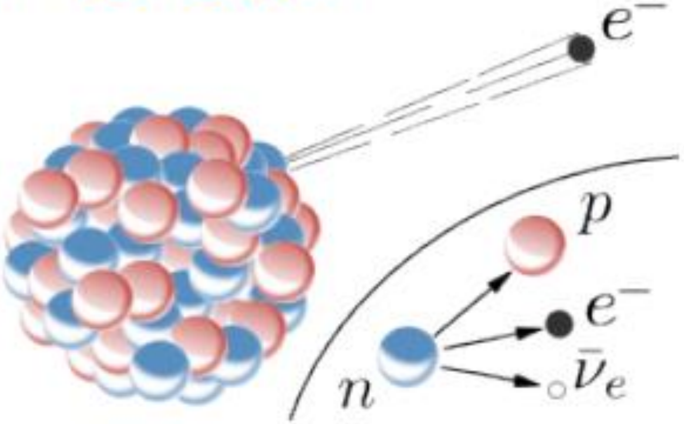


Yukawa

There is yet another force...

Nuclear beta-decay was observed:

(Z,A) --> (Z+1,A) + (e-) + ?

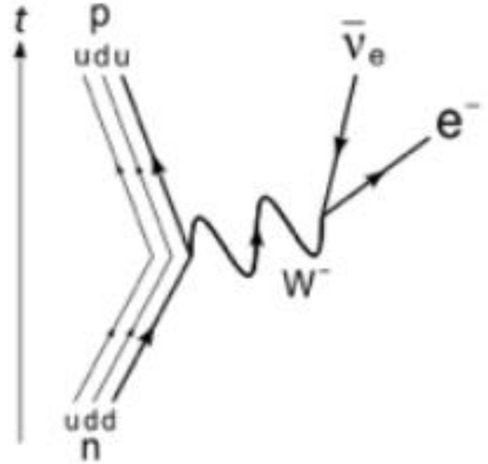


* *prediction of neutrino*

* *a new type of force: WEAK FORCE*

Pauli

Fermi



$$\sigma \approx G_F^2 E^2$$

$$\frac{G_F}{(\hbar c)^3} = \frac{\sqrt{2}}{8}\frac{g^2}{m_W^2} = 1.16637(1) \times 10^{-5}\ \mathrm{GeV}^{-2}\ .$$

Continuing Exploration of Strong Force

1950~1960: A burst of discovering new particles with strong interaction by virtue of **particle accelerators & detectors** (bubble chamber)

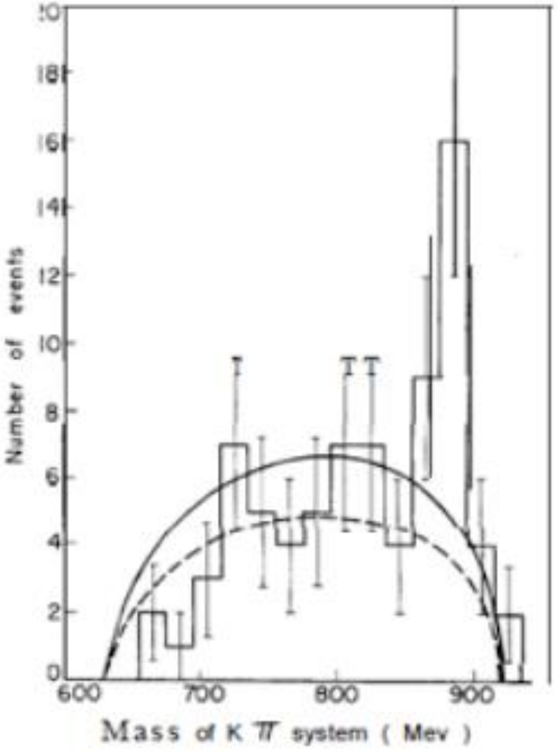
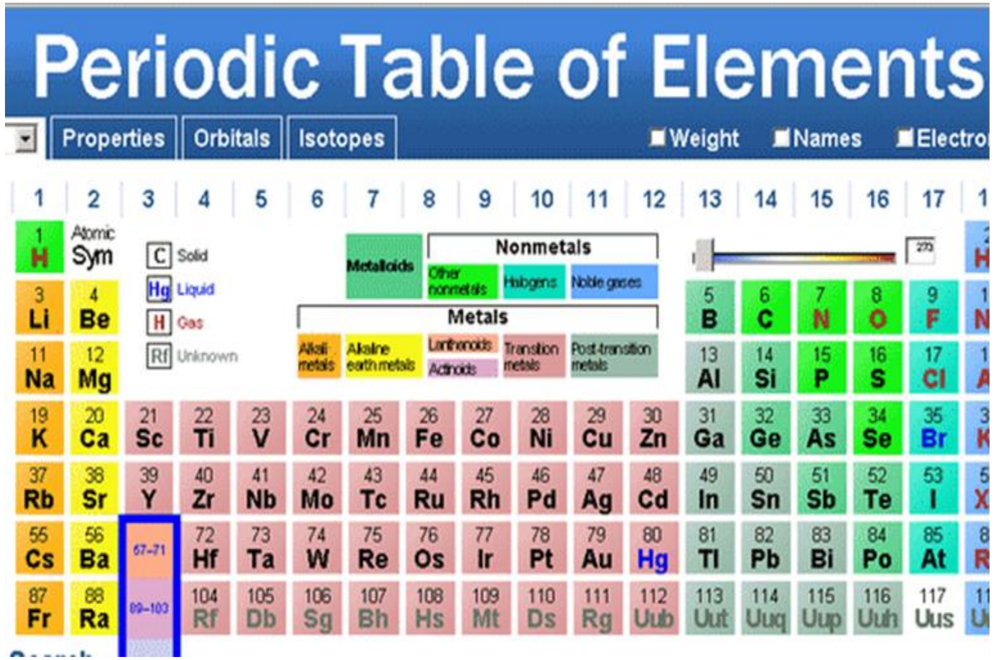
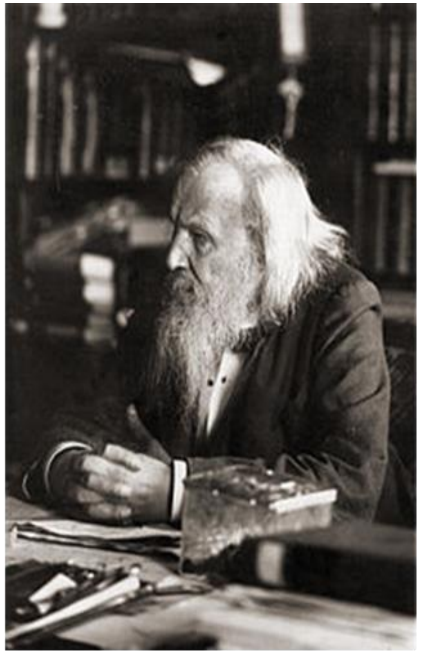


Fig. 15. Discovery of the K^* (890).

- After the discovery of proton and neutron, physicists continued to look for new “elementary particles”. In the 1950-60s, more than a hundred of “elementary particles” are found, question: are they all elementary?
- In chemistry, elements fill into the periodic table which was used to predict new elements and inspired the atomic model.



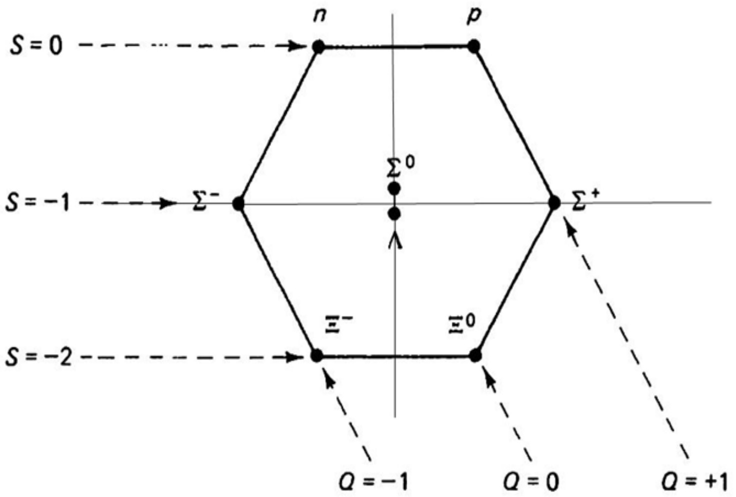


Mendeleev mid-19th century

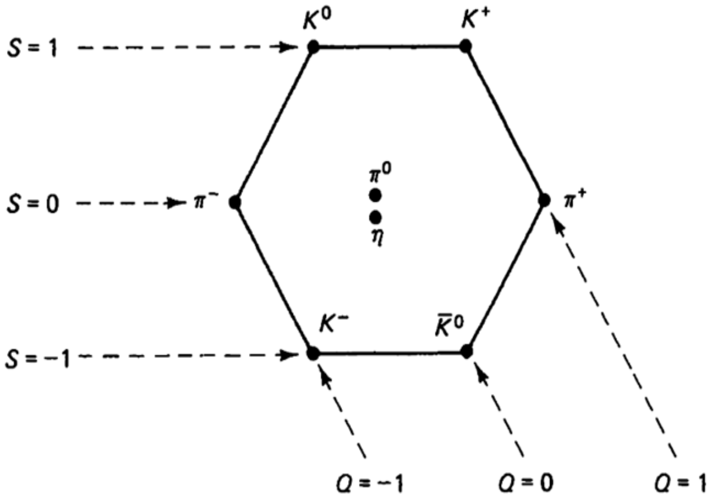
W. Lamb:” …the finder of a new elementary particle used to be rewarded by a Nobel Prize, but such a discovery now ought to be punished by a $10,000 fine”

- So how about to make a “periodic table of elementary particles” ?
- The eightfold way of Gell-Mann and Neemann (1961):
  1) spin=0, mesons; spin=1/2, baryons;
  2) put the particles on the plane of S-Q;
  3) they observe very regular patterns
  for the first eight lightets baryons and mesons

S: strangeness introduced by Gell-Mann to explain the “strange” behavior of some particles: they are produced always in pair and decay very slowly

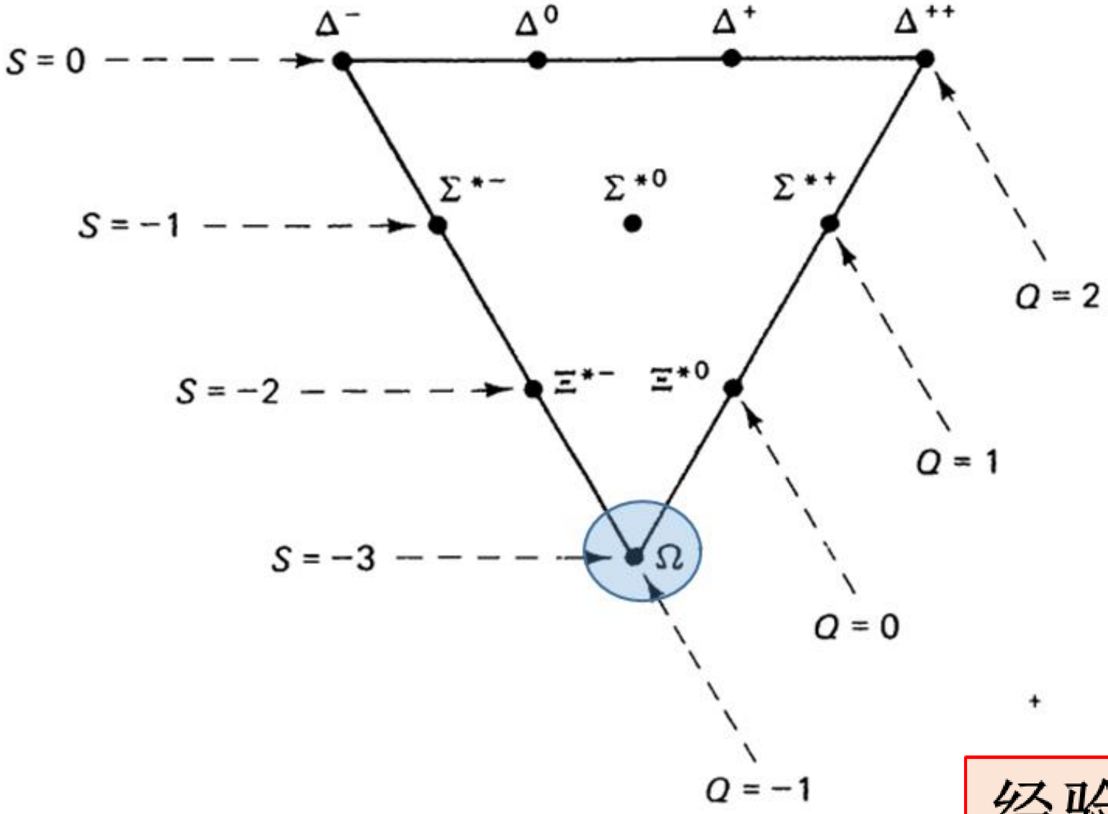


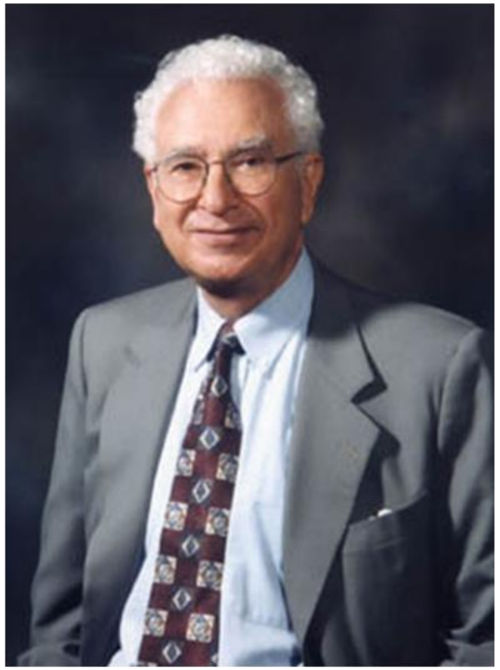
The baryon octet



The meson octet

- Heavier baryons fit into a incomplete decuplet. Gell-Mann predicted: there must be a baryon of Q=-1 and S=-3!
- Found in 1964, the Omega-minus baryon.

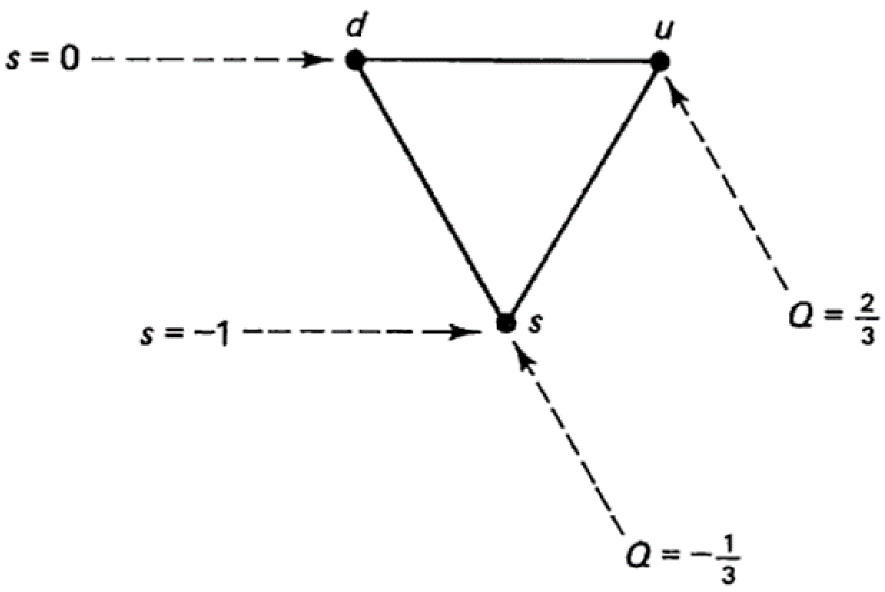




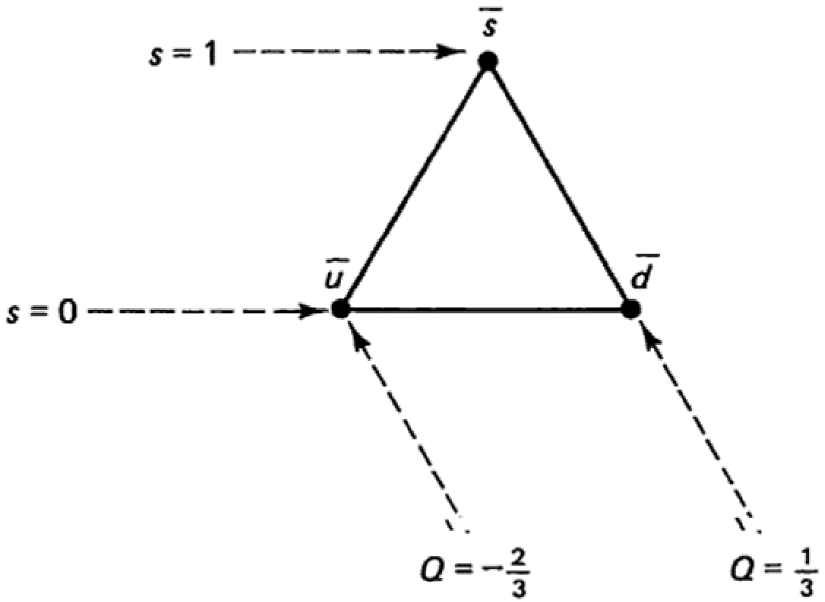
Gell-Mann

经验： Classification is the first stage in the development of any science——Griffiths

- Natural question: the periodic table of chemical elements are explained by more elementary constituents, electrons and nuclei; what can explain the eightfold way? What are the more elementary constituents?
- Quark model of Gell-Mann and Zweig (1964): 3 types (flavour) of quarks, spin-1/2.



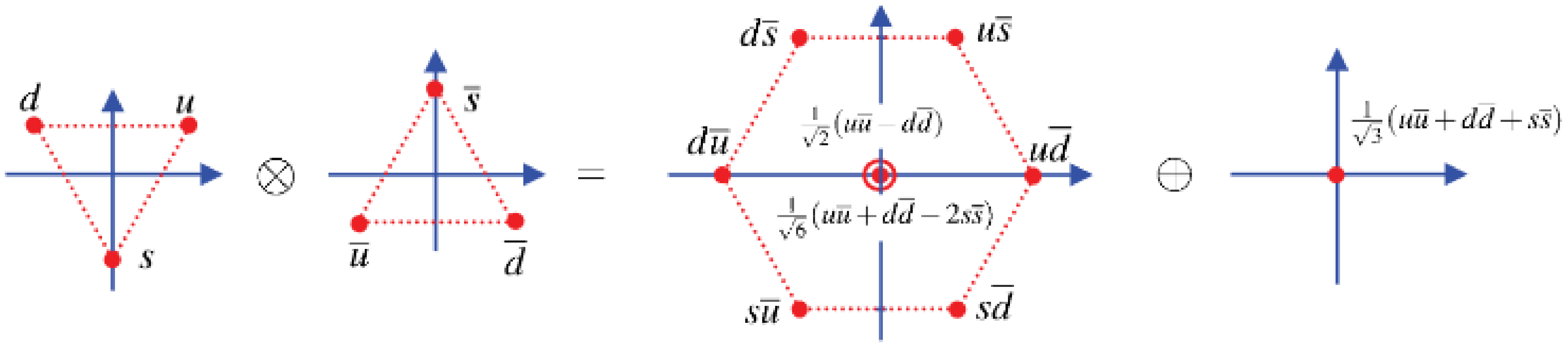
The quarks



The antiquarks

- 3 quarks form a baryon; 1 quark + 1 anti-quark form a meson*.
- Excise: Put the anti-triangle on to the triangle, one obtains the meson octet plus a singlet: $\bar{3}$x3=8+1. Put three triangles together, one obtains two baryon octets, a decuplet and a singlet: 3x3x3=10+8+8+1.

For example:



- **In the language of group theory:** $3\otimes\bar{3}=8\oplus 1$

Let us make an introduction to Symmetry and Group theory

Symmetry and Group theory

Symmetries play a central role in particle physics; one aim of particle physics is to discover the fundamental symmetries of our universe

Symmetries are usually divided to:

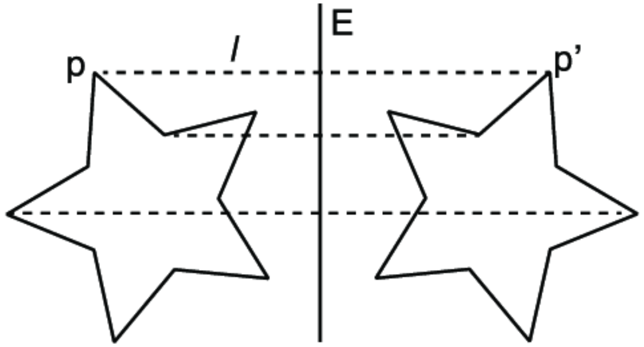
* Space time: include rotations, boosts, translations $x \to x+a$, parity $x \to -x$, time reversal, scale invariance …

* Continous or discrete; Abelian or non-Abelian;

* Internal, global and local (gauged).

Symmetry and Group theory

For example: Mirror symmetry



Operation e: nothing changed
Operation I: reflection w.r.t mirror E

e x e=e; I x I=e; e x I=I; I x e=I

Group Z_2={e,I} with product x

Symmetries can be composed:

* if T_1, T_2 are symmetries, then $T_2 \circ T_1$ is also a symmetry.

* If T is a symmetry, then T^{-1} is also a symmetry.

* The identity $\phi \to \phi$ is a symmetry.

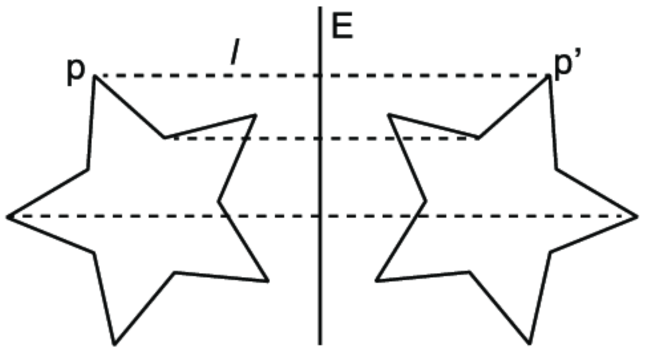
A group is a pair $(G, \cdot)$, where G is a set, $\cdot$ is a product $G \times G \to G$:

- The product is associative, $\forall g1,g2,g3 \in G \; (g1 \cdot g2) \cdot g3 = g1 \cdot (g2 \cdot g3)$;
- There is an identity $e \in G$, such that $\forall g \in G \; e \cdot g = g \cdot e = g$;
- Every element $g \in G$ has inverse $g^{-1} \in G$ such that $g \cdot g^{-1} = g^{-1} \cdot g = e$.

Symmetry and Group theory

For example: Mirror symmetry

Operation e: nothing changed
Operation I: reflection w.r.t mirror E



e x e=e; I x I=e; e x I=I; I x e=I

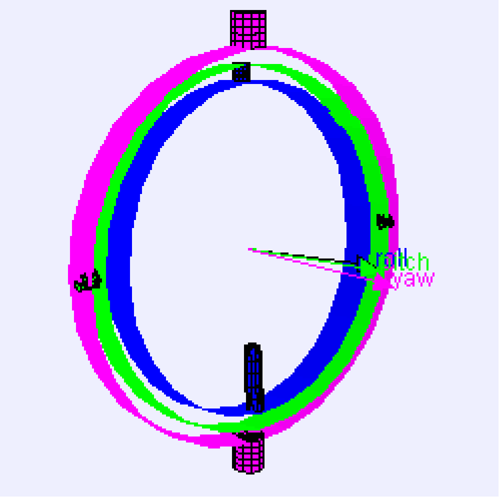
Group Z_2={e,I} with product x

Operation e: =1
Operation I: =-1
Group product x := $\star$ (the normal product)

{1,-1; $\star$} is a representation of Z_2

Symmetry and Group theory

For example: Rotation



Denote a 3D rotation around $\vec{n}$ by an angle ψ as $\mathscr{R}_{\vec{n}}(\psi)$

$$|\mathscr{R}_{\vec{n}}(\psi)\vec{x}|=|\vec{x}|$$

In the case of an infinitesimally small angle $|\delta\psi| \ll 1$, a Taylor expansion of the Rodrigues formula to first order gives

$$\mathscr{R}_{\vec{n}}(\delta\psi)\vec{x} = \vec{x} + \delta\psi\,\vec{n} \times \vec{x} + \mathcal{O}(\delta\psi^2). = \left[\mathbb{1}_3 - \mathrm{i}\delta\psi\, J_{\vec{n}} + \mathcal{O}(\delta\psi^2)\right]\vec{x}$$

The reader is invited to check the validity of this relation by drawing $\vec{x}$ and $\mathscr{R}_{\vec{n}}(\delta\psi)\vec{x}$ in the plane orthogonal to the direction of $\vec{n}$.

Considering the infinitesimal rotations about the three axes of the coordinate system, i.e. about the unit vectors $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$, one finds a basis of $\mathfrak{so}(3)$, namely

$$J_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix} \quad , \quad J_2 = \begin{pmatrix} 0 & 0 & \mathrm{i} \\ 0 & 0 & 0 \\ -\mathrm{i} & 0 & 0 \end{pmatrix} \quad , \quad J_3 = \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

i.e. the ij entry of the matrix J_k reads

$$\boxed{(J_k)_{ij} = -\mathrm{i}\epsilon_{ijk}}$$

where ϵ_{ijk} is the usual totally antisymmetric Levi-Civita[(x)] symbol with $\epsilon_{123} = 1$

Symmetry and Group theory

Such J's form a Lie algebra called so(3):

$$[J_i, J_j] = \mathrm{i}\sum_{k=1}^{3} \epsilon_{ijk} J_k \quad \text{for all } i, j \in \{1, 2, 3\}.$$

A rotation around $\vec{n}$ by an angle ψ

$$\mathscr{R}_{\vec{n}}(\psi) = \mathrm{e}^{-\mathrm{i}\psi(n_1 J_1 + n_2 J_2 + n_3 J_3)}.$$

All such rotations form a Lie group called SO(3)

However, the Lie algebra so(3) generates a Lie group SU(2) which doubles SO(3), i.e., SO(3)=SU(2)/Z_2

Consider a representation of so(3) using the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \quad , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

A SU(2) element is given by

$$\mathcal{U}_{\vec{n}}(\psi) = \mathrm{e}^{-\mathrm{i}\psi\, \vec{n}\cdot\vec{\sigma}/2} = \mathrm{e}^{-\mathrm{i}\psi(n_1\sigma_1 + n_2\sigma_2 + n_3\sigma_3)/2}.$$

Symmetry and Group theory

Using the following to show that ψ is the same rotating angle is SO(3)

$$\left(\cos\frac{\psi}{2}\right)\mathbb{1}_2 - \mathrm{i}\left(\sin\frac{\psi}{2}\right)\vec{n}\cdot\vec{\sigma} = \mathrm{e}^{-\mathrm{i}\psi\,\vec{n}\cdot\vec{\sigma}/2}$$

Thus, $U_{\vec{n}}(2\pi) = -1, U_{\vec{n}}(4\pi) = 1 = U_{\vec{n}}(0)$. One complete rotation in SU(2) gives two complete rotations in SO(3)

Quantum world often described by SU(2) rotation, e.g. the spin

Symmetry and Group theory

Using the following to show that ψ is the same rotating angle is SO(3)

$$\left(\cos\frac{\psi}{2}\right)\mathbb{1}_2 - \mathrm{i}\left(\sin\frac{\psi}{2}\right)\vec{n}\cdot\vec{\sigma} = \mathrm{e}^{-\mathrm{i}\psi\,\vec{n}\cdot\vec{\sigma}/2}$$

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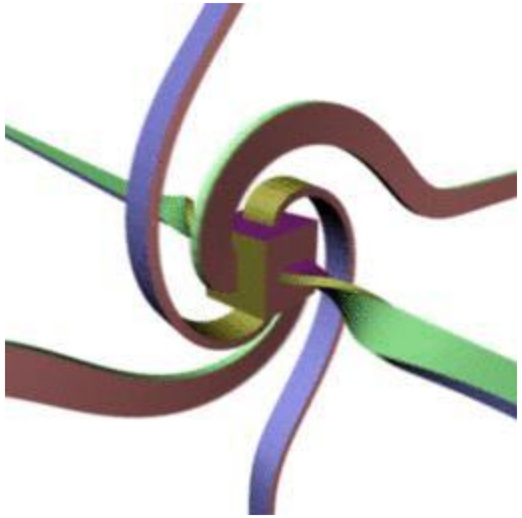

A visualization of SU(2) rotation: the Dirac strings