Generators of the SU(3) group - IV

☞ Subalgebra

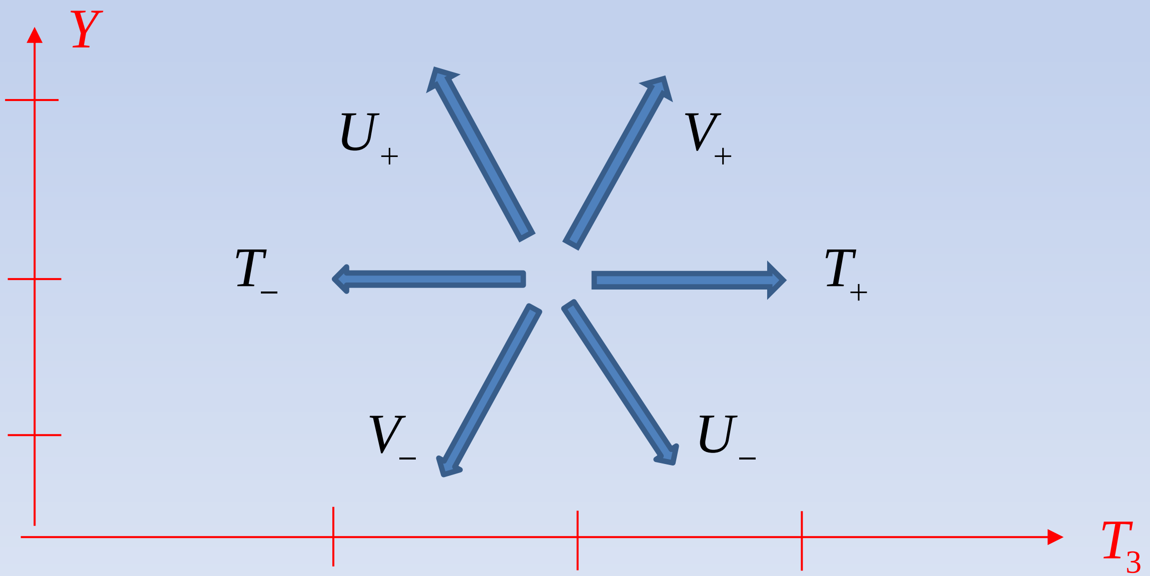
$$[T_3, T_\pm] = \pm T_\pm \qquad [T_+, T_-] = 2T_3$$

$$[U_3, U_\pm] = \pm U_\pm \qquad [U_+, U_-] = 2U_3 \equiv {}^3\!/_2\, Y - T_3$$

$$[V_3, V_\pm] = \pm V_\pm \qquad [V_+, V_-] = 2V_3 \equiv {}^3\!/_2\, Y + T_3$$

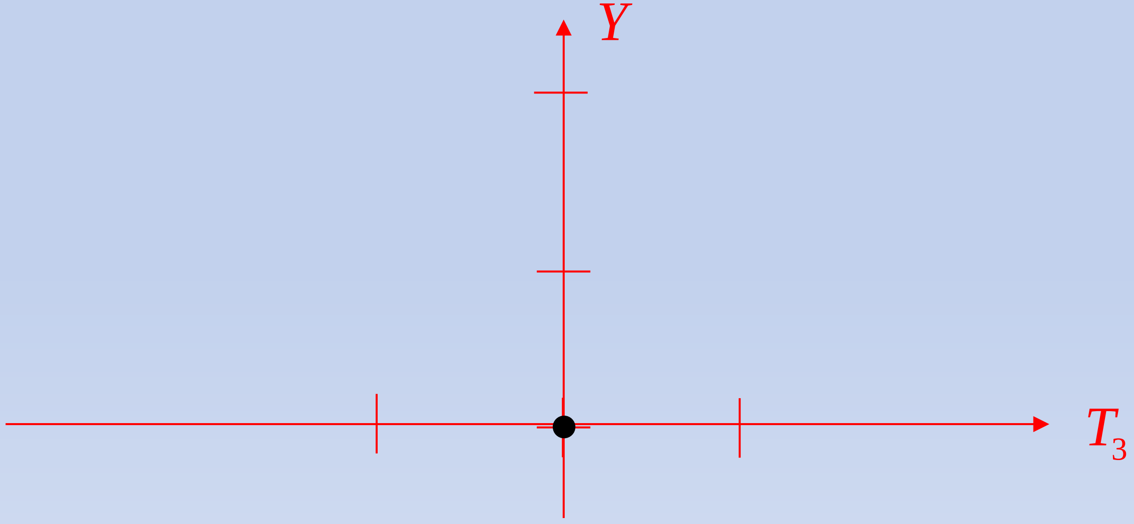
☞ Raising operators in T3 and Y space

$$[T_3, U_\pm] = \mp \frac{1}{2} U_\pm \qquad [T_3, V_\pm] = \pm \frac{1}{2} V_\pm \qquad [Y, U_\pm] = \pm U_\pm \qquad [Y, V_\pm] = \pm V_\pm$$



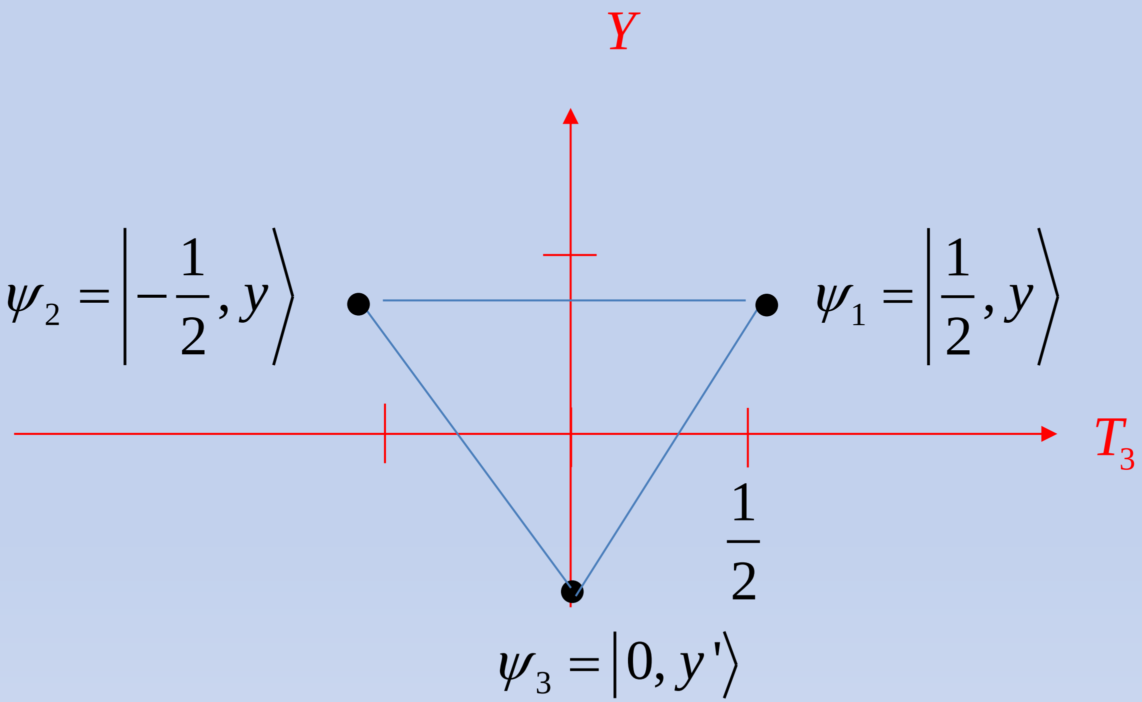
- Trivial Representation

☞ Singlet $\left|t_3, y\right\rangle = \left|0, 0\right\rangle$



- Smallest non-trivial Representation $[3]$ or (1,0)

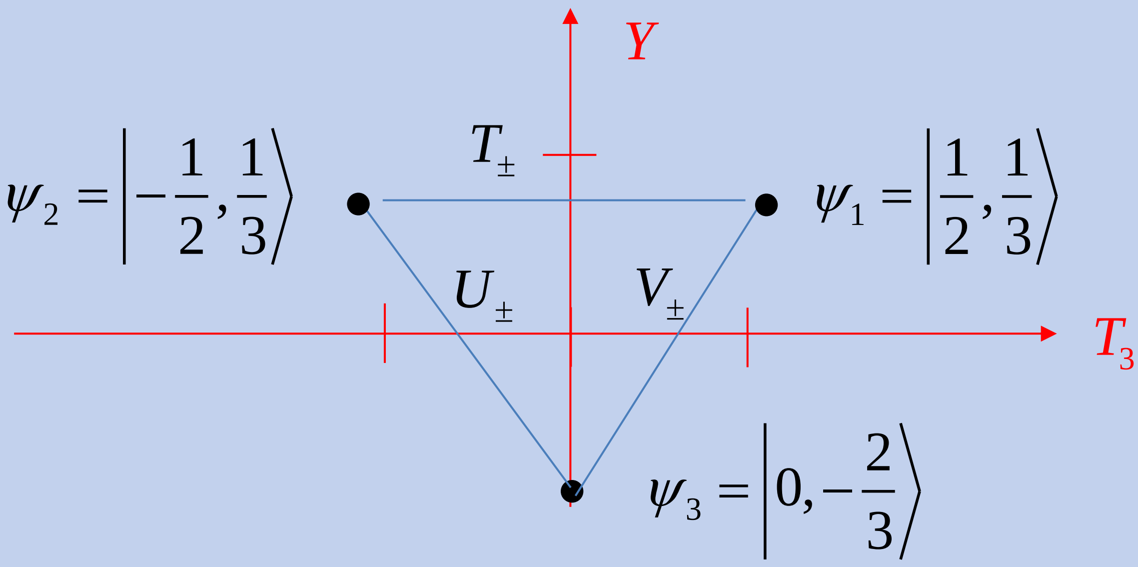
☞ Triplet: has smallest SU(2) subgroup T=1/2



☞ From U spin $$U_3\psi_1 = \left(\frac{3}{4}Y - \frac{1}{2}T_3\right)\left|\frac{1}{2}, y\right\rangle = 0 \;\rightarrow\; y = \frac{1}{3}$$

☞ From U spin $$U_3\psi_3 = \left(\frac{3}{4}Y - \frac{1}{2}T_3\right)\left|0, y'\right\rangle = -\frac{1}{2}\psi_3 \;\rightarrow\; y' = -\frac{2}{3}$$

- Rederiving commutation relations related to raising and lowering using $[3]$



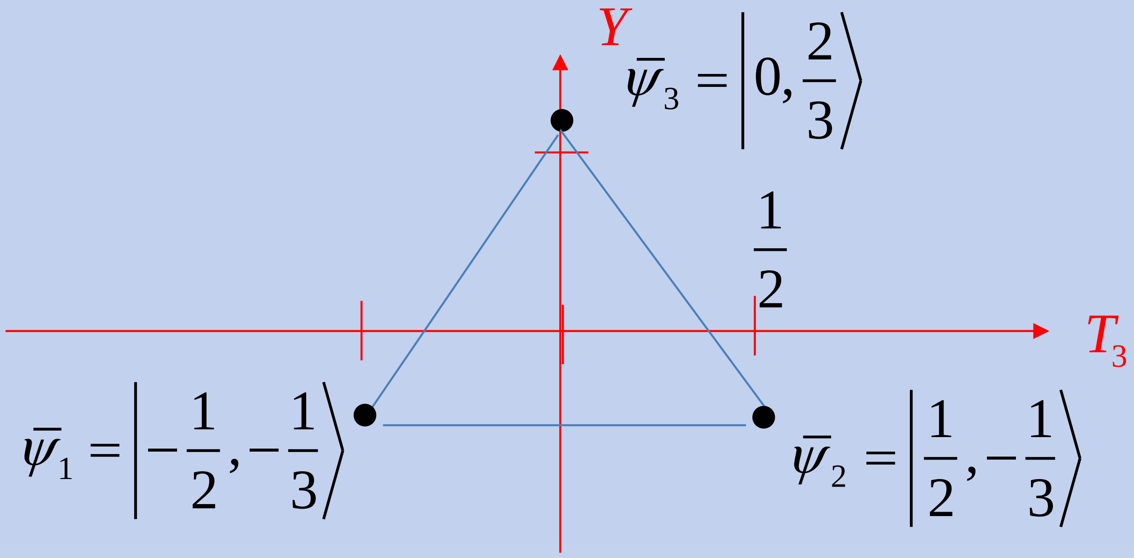
☞ $\left[T_3, T_\pm\right] = T_3 T_\pm - T_\pm T_3 = \pm T_\pm$ Apply to either $\psi_{1,2}$

☞ $\left[T_+, V_+\right] = \left[T_+, U_-\right] = \left[U_+, V_+\right] = 0$

$$\left[T_+, V_-\right] = -U_- \qquad \left[T_+, U_+\right] = V_+ \qquad \left[U_+, V_-\right] = T_-$$

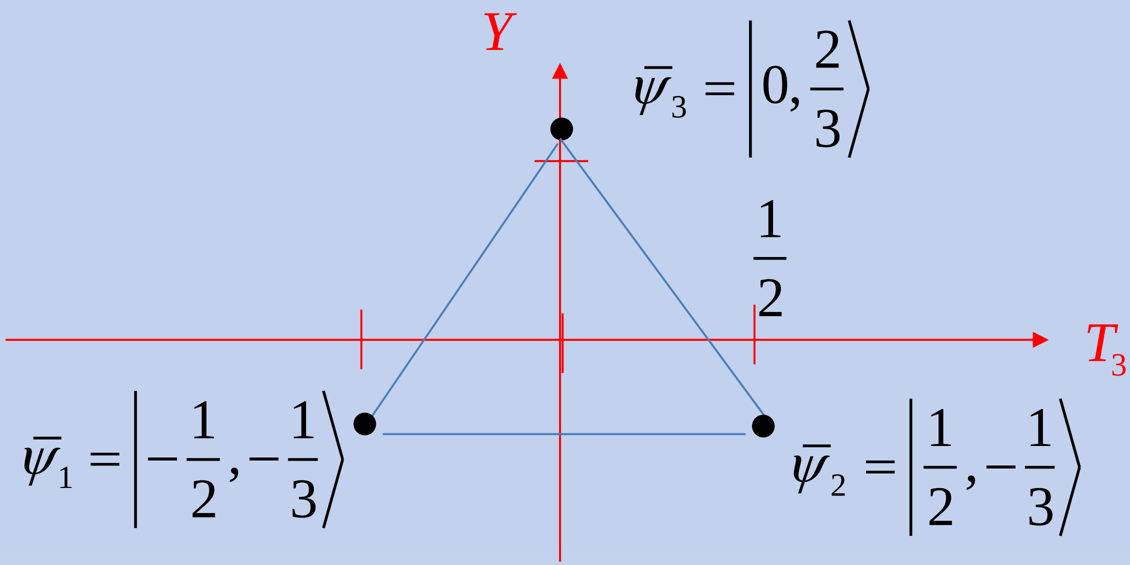
- Another Smallest non-trivial Representation $\left\lfloor \bar{3} \right\rfloor$ or (0,1)

☞ Anti-Triplet: Quantum numbers are reversed



- Another view for $\left[\bar{3}\right]$ Representation

☞ Anti-Triplet: Quantum numbers are reversed



☞ Commutation relation for SU(3)

$$\left[F_i, F_j\right] = i f_{ijk} F_k$$

$$\left[\bar{F}_i, \bar{F}_j\right] = i f_{ijk} \bar{F}_k \qquad \text{where} \quad \bar{F}_i = -F_i^*$$

- Generators for $\left[\bar{3}\right]$ in terms of previous representation

☞ Note $\bar{F}_i = -F_i^*$ $\qquad F_i = \frac{1}{2}\lambda_i$

$$\bar{\lambda}_1 = -\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \bar{\lambda}_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Longrightarrow \bar{T}_\pm = -F_1 \pm iF_2 = -T_\mp$$

$$\bar{\lambda}_4 = -\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \bar{\lambda}_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \Longrightarrow \bar{V}_\pm = -F_4 \pm iF_5 = -V_\mp$$

$$\bar{\lambda}_6 = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \bar{\lambda}_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \Longrightarrow \bar{U}_\pm = -F_6 \pm iF_7 = -U_\mp$$

$$\bar{\lambda}_3 = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \bar{\lambda}_8 = -\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \Longrightarrow \begin{array}{l} \bar{T}_3 = \bar{F}_3 = -T_3 \\ \bar{Y} = \frac{2}{\sqrt{3}}\bar{F}_8 = -Y \end{array}$$

- Charges of (1,0) or (1,0)

Charge Operator $Q = \frac{1}{2}Y + T_3$

☞ Triplet $[3]$ $Q\left|\frac{1}{2},\frac{1}{3}\right\rangle = \left(\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\right)\left|\frac{1}{2},\frac{1}{3}\right\rangle = \frac{2}{3}\left|\frac{1}{2},\frac{1}{3}\right\rangle$

$$Q\psi_1 = \frac{2}{3}\psi_1 \qquad Q\psi_2 = -\frac{1}{3}\psi_2 \qquad Q\psi_3 = -\frac{1}{3}\psi_3$$

☞ Anti- Triplet $\left[\overline{3}\right]$

$$Q\overline{\psi}_1 = -\frac{2}{3}\overline{\psi}_1 \qquad Q\overline{\psi}_2 = \frac{1}{3}\overline{\psi}_2 \qquad Q\overline{\psi}_3 = \frac{1}{3}\overline{\psi}_3$$

☞ These are the quarks and antiquarks

- Construction of all SU(3) multiplets from $[3]$ $[\bar{3}]$: Outer product

☞ As in SU(2) $\frac{1}{2}\times\frac{1}{2}=1+0$

☞ Construction of the D(p,q) representation from p-quarks and q-antiquarks

$$\left|\frac{1}{2},\frac{1}{3}\right\rangle,\ \left|-\frac{1}{2},\frac{1}{3}\right\rangle,\ \left|0,-\frac{2}{3}\right\rangle \qquad \left|0,\frac{2}{3}\right\rangle,\ \left|\frac{1}{2},-\frac{1}{3}\right\rangle,\ \left|-\frac{1}{2},-\frac{1}{3}\right\rangle$$

- Product state

$$\underbrace{|T_3(1)Y(1)\rangle \cdots |T_3(p)Y(p)\rangle}_{p\text{-quarks}} \times \underbrace{|\bar{T}_3(1)\bar{Y}(1)\rangle \cdots |\bar{T}_3(q)\bar{Y}(q)\rangle}_{q\text{-antiquarks.}}$$

The Irreducible Representation should be

Eigenstates of

$$T_3 = \sum_{i=1}^{p+q} T_3(i) , \quad Y = \sum_{i=1}^{p+q} Y(i).$$

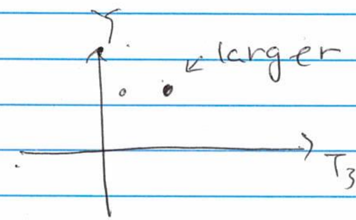
$$\therefore |T_3\ Y\rangle = \left| \sum_{i=1}^{p} T_3(i) + \sum_{i=1}^{q} \bar{T}_3(i) ,\ \sum_{i=1}^{p} Y(i) + \sum_{i=1}^{q} \bar{Y}(i) \right\rangle$$

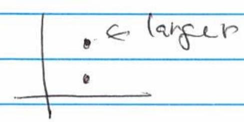
plotting these state in $T_3 - Y$ diagram is called

the "weight" diagram.

$|T_3 , Y\rangle$ is called "larger" than $|T_3' , Y'\rangle$

if $T_3 > T_3'$ or $T_3 = T_3'$ and $Y > Y'$





since $T_3^{max}(i) = \frac{1}{2}$ $\quad Y_3^{max}(i) = \frac{1}{3}$

for largest States $\quad \bar{T}_3^{max}(i) = +\frac{1}{2}$ $\quad \bar{Y}_3^{max}(i) = -\frac{1}{3}$

$$\left(T_3^{total}\right)_{max} = \frac{p+q}{2} \qquad \left(Y^{total}\right)_{max} = \frac{p-q}{3}.$$

- Casimir formula using highest weight state

☞ $$C_1 = F_1^2 + F_2^2 + F_3^2 + F_4^2 + F_5^2 + F_6^2 + F_7^2 + F_8^2$$

$$= T_- T_+ + T_3 + F_3^2 + V_- V_+ + V_3 + U_+ U_- - U_3 + F_8^2$$

☞ Apply this to highest weight state

$$C_1 \left| \frac{p+q}{2}, \frac{p-q}{3} \right\rangle = \left(T_- T_+ + T_3 + F_3^2 + V_- V_+ + V_3 + U_+ U_- - U_3 + F_8^2 \right) \left| \frac{p+q}{2}, \frac{p-q}{3} \right\rangle$$

using $$U_3 \equiv \frac{3}{4} Y - \frac{1}{2} T_3, \quad V_3 \equiv \frac{3}{4} Y + \frac{1}{2} T_3, \quad F_8 = \frac{\sqrt{3}}{2} Y$$

$$= \left(0 + T_3 + F_3^2 + 0 + \frac{3}{4} Y + \frac{1}{2} T_3 + 0 - \frac{3}{4} Y + \frac{1}{2} T_3 + \frac{3}{4} Y^2 \right) \left| \frac{p+q}{2}, \frac{p-q}{3} \right\rangle$$

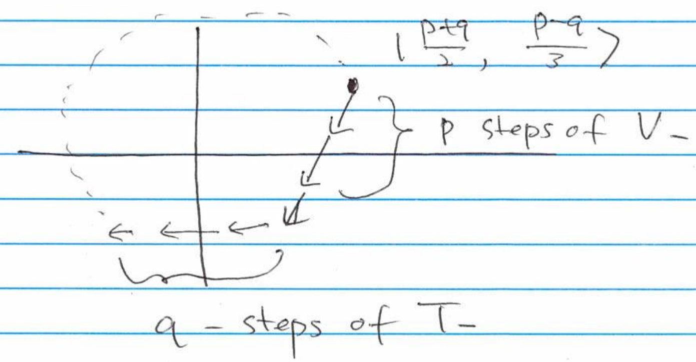
$$= \left((p+q) + \frac{1}{4} (p+q)^2 + \frac{3}{4} \left(\frac{p-q}{3} \right)^2 \right) \left| \frac{p+q}{2}, \frac{p-q}{3} \right\rangle$$

$$\rightarrow C_1 = \frac{1}{3} \left(p^2 + q^2 + pq \right) + p + q$$

Start from

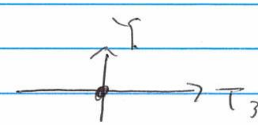
$|\frac{p+q}{2}, \frac{p-q}{3}\rangle$ and obtain all states

using $T_\pm$, $U_\pm$, $V_\pm$

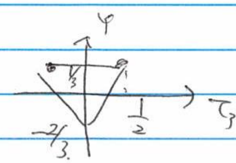


then boundaries are fixed.

Ex∘ $D(0,0)$

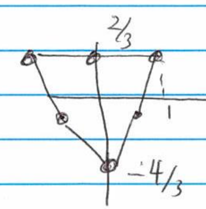


∘ $D(1,0) = [3]$

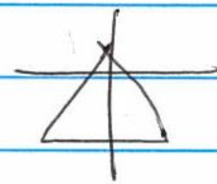


∘ $D(0,1) = [\bar{3}]$

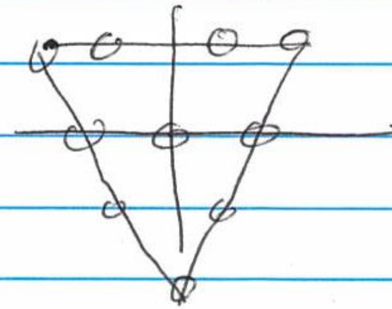
∘ $D(2,0) = [6]$



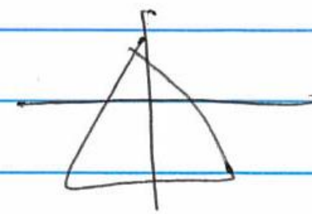
- $D(0,2)$



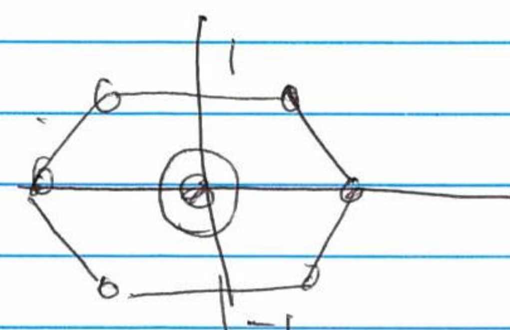
- $D(3,0) = [10]$



- $D(0,3) = [\overline{10}]$



- $D(1,1) = [8]$



Ex) 8-6 The Pseudoscalar Meson.

Spin = 0 parity − : pesudoscalar Meson.

As we saw from Home work., they form

[8] = D(1, 1) Representation.

Ex) 8-8 Scalar Meson.

Spin = 0, parity = +.

Not so easy to identify because of large width.

Still D(1, 1) Rep.

Ex) 8-9 Vector meson

~~Spin = ~~ parity = −

Spin = 1

K^{*0}, K^{*+}, $\rho^{\pm,0}$, ϕ, ω, K^{*-}, K^{*0}

⇒ Nine Particles because D(1, 1) + D(0, 0)

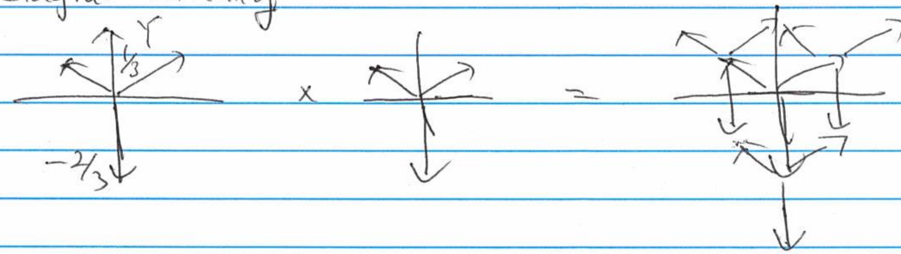
↑ Singlet.

8.6 Rules for the Reduction of Direct Products of SU(3) multiplets

— $[3] \times [3]$

$| T_3(1)\, Y(1) \rangle \, | T_3(2)\, Y(2) \rangle$

diagrammatically



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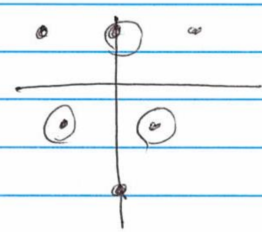

⊙ : two states

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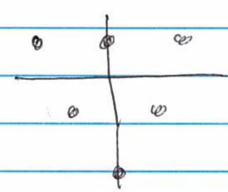

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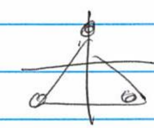
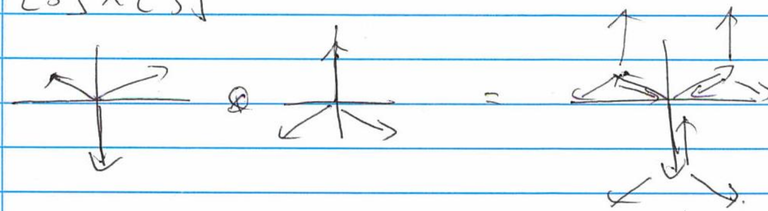
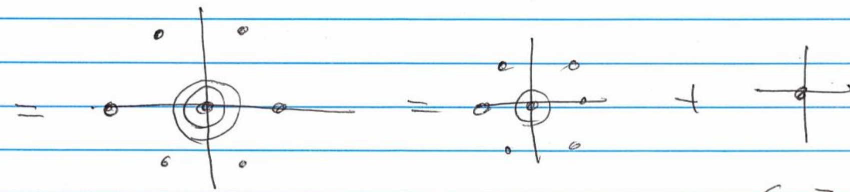


[6] [3].

In a Representation, outer states are single states

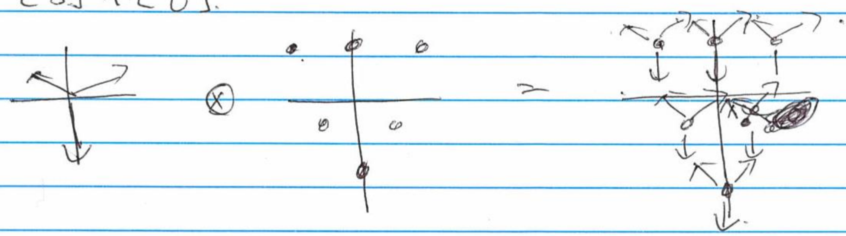
— [3] x [3]

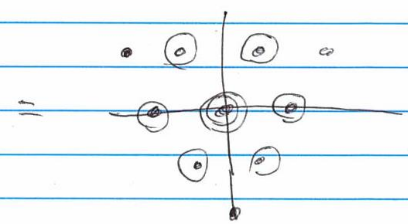


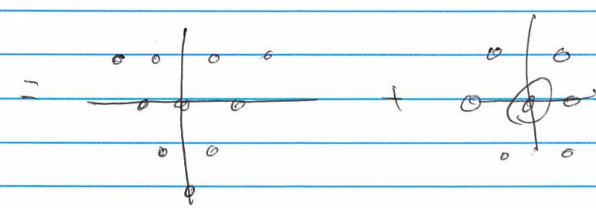


[8] [1]

— [3] x [6]







[10] [8]

☞ Quark flavor : 3 light flavours SU(3) + 3 heavy flavours

Quark	Symbol	Spin	Charge	Baryon Number	S	C	B	T	Mass*
Up	U	1/2	+2/3	1/3	0	0	0	0	1.7-3.3 MeV
Down	D	1/2	-1/3	1/3	0	0	0	0	4.1-5.8 MeV
Charm	C	1/2	+2/3	1/3	0	+1	0	0	1270 MeV
Strange	S	1/2	-1/3	1/3	-1	0	0	0	101 MeV
Top	T	1/2	+2/3	1/3	0	0	0	+1	172 GeV
Bottom	B	1/2	-1/3	1/3	0	0	-1	0	4.19 GeV(MS) 4.67 GeV(1S)

☞ Each quark has additional quantum number: SU(3) color