

# 习题

我没有答案

高寒

gaoh26@mail2.sysu.edu.cn

## 1.1.1

Compute the partition function of a quantum harmonic oscillator with frequency  $\omega$  and energy levels

$$E_n = \hbar\omega \left( n + \frac{1}{2} \right) \quad n \in \mathbf{Z}$$

Find the average energy  $U$  and entropy  $S$  as a function of temperature  $T$ .

## 1.1.2

Einstein constructed a simple model of a solid as  $N$  atoms, each of which vibrates with the same frequency  $\omega$ . Treating these vibrations as a harmonic oscillator, show that at high temperatures,  $k_B T \gg \hbar\omega$ , the Einstein model correctly predicts the Dulong-Petit law for the heat capacity of a solid,

$$C_V = 3Nk_B$$

At low temperatures, the heat capacity of many solids is experimentally observed to tend to zero as  $c_V \sim T^3$ . Was Einstein right about this?

## 1.2

A particle moving in one dimension has Hamiltonian

$$H = \frac{p^2}{2m} + \lambda q^4$$

Show that the heat capacity for a gas of  $N$  such particles is  $C_V = 3Nk_B/4$ . Explain why the heat capacity is the same regardless of whether the particles are distinguishable or indistinguishable.

## 1.3

A non-relativistic particle is confined in a spherical container with radii  $R$  with potential

$$V(r) = \begin{cases} -h & , r < a \\ 0 & , a \leq r < R \\ +\infty & , r > R \end{cases}$$

Firstly predict the rough behavior of the pressure for a gas of  $N$  such particles as a function of  $T$ , then determine  $P(T)$  by statistics.

## 2.1

Show that

1.

$$C_P - C_V = T \left( \frac{\partial V}{\partial T} \right)_P \left( \frac{\partial P}{\partial T} \right)_V = -T \left( \frac{\partial V}{\partial T} \right)_P^2 \left( \frac{\partial P}{\partial V} \right)_T$$

2.

$$\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial P}{\partial T} \right)_V - P$$

3.

$$\left( \frac{\partial U}{\partial P} \right)_T = -T \left( \frac{\partial V}{\partial T} \right)_P - P \left( \frac{\partial V}{\partial P} \right)_T$$

## 2.1

4.

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T \left(\frac{\partial^2 P}{\partial T^2}\right)_V$$

5.

$$\left(\frac{\partial C_P}{\partial P}\right)_T = -T \left(\frac{\partial^2 V}{\partial T^2}\right)_P$$

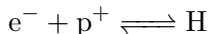
From equations above, show that for any non-ideal gas with constant  $C_V$  and  $C_P$ , the equation of state can be written as

$$(C_P - C_V)T = (P + a)(V + b)$$

where  $a$  and  $b$  are constants.

## 2.2

Consider the neutral gas of electrons, protons and Hydrogen. They undergo a reaction as



You know from Quantum Mechanics that the Hydrogen atom has binding energy  $E = -I$  ( where  $I = 13.6$  eV). Let the number of Hydrogen atoms be  $N_H = (1 - x)N$  and the number of electrons and protons be  $N_e = N_p = xN$  with  $x \in [0, 1]$ .

1. What is the equilibrium condition for the gas?
2. From the condition above, show that

$$\frac{x^2}{1-x} = \frac{V}{N} \left( \frac{m_e m_p}{2\pi \hbar^2 m_H} \right)^{3/2} (k_B T)^{3/2} e^{-I/k_B T}$$