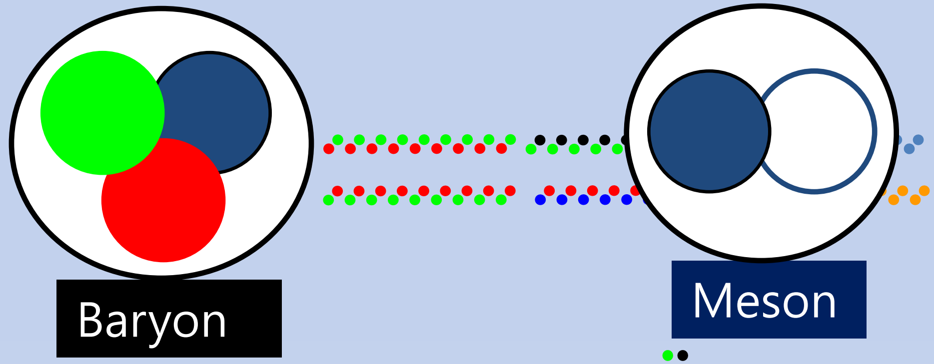
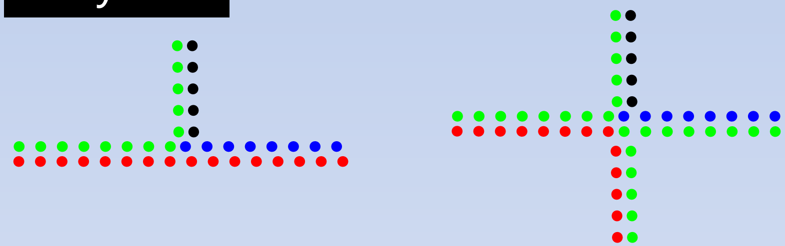
QED *vs* **QCD**

	QED	QCD
Charge		
Boson		
Boson interaction		
β function	$\beta(e)=\dfrac{e^3}{12\pi^2}$	$\beta(g)=-\left(11-\dfrac{2n_f}{3}\right)\dfrac{g^3}{16\pi^2}$

- QCD Lagrangian density

☞ Lagrangian density for QCD

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \qquad A_\mu = \frac{\lambda^a}{2} A_\mu^a = T^a A_\mu^a$$

$$L = -\frac{1}{4}\mathrm{Tr}\left[F_{\mu\nu}F_{\mu\nu}\right] + \bar{\psi}\left(iD_\mu\gamma^\mu - m\right)\psi$$

$$D_\mu\psi = \left(\partial_\mu - igA_\mu\right)\psi$$

$$F_{\mu\nu} = \frac{1}{-ig}\left[D_\mu, D_\nu\right] = T^a\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c\right)$$

• Appendix : Dirac equation

☞ Dirac equation: Field equation for spin ½ particle

$$\left(i\partial_\mu \gamma^\mu - m\right)\psi = 0$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

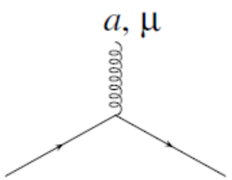
$$E^2 = p^2 + m^2$$

$$\psi = \begin{pmatrix} \psi_{+\uparrow} \\ \psi_{+\downarrow} \\ \psi_{-\uparrow} \\ \psi_{-\downarrow} \end{pmatrix}$$

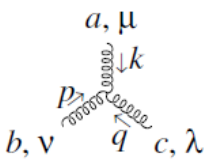
- QCD Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \overline{\psi}^j (i \not{D}^k_j - m\delta^k_j)\psi_k = -\frac{1}{2}\mathrm{Tr}F^2 + \overline{\psi}(i\not{D} - m)\psi \ .$$

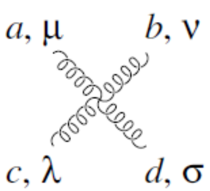
$$\mathcal{L} = \mathcal{L}_0 + gA^a_\mu \bar{\psi}\gamma^\mu T^a \psi - g f^{abc}(\partial_\mu A^a_\nu)A^{b\mu}A^{c\nu} - g^2 f^{eab} f^{ecd} A^a_\mu A^b_\nu A^{c\mu} A^{d\nu} \ .$$



$$= \quad ig\gamma^\mu T^a$$



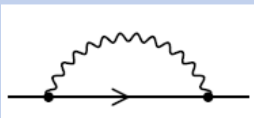
$$= \quad g f^{abc}[g^{\mu\nu}(k-p)^\lambda + g^{\nu\lambda}(p-q)^\mu + g^{\lambda\mu}(q-k)^\nu]$$

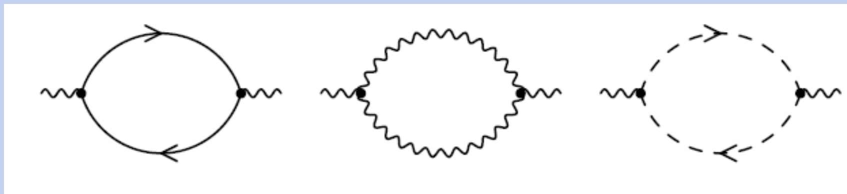


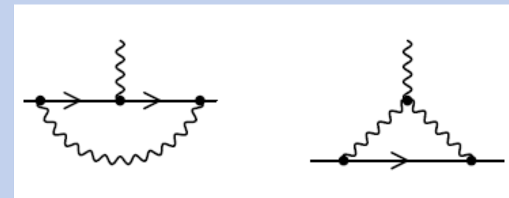
$$= \quad \begin{aligned} -ig^2[&f^{abe}f^{cde}(g^{\mu\lambda}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\lambda}) + \\ &f^{ace}f^{bde}(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\sigma}g^{\nu\lambda}) + \\ &f^{ade}f^{bce}(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\sigma\nu})] \end{aligned}$$

- QCD Running coupling constant

$$\mu\frac{d}{d\mu}F = \left(\mu\frac{\partial}{\partial\mu} + \beta(g_s)\frac{\partial}{\partial g_s}\right) F(E,\mu,g_s) = 0\,.$$



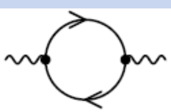




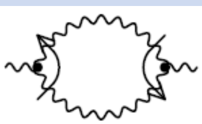
☞ Asymptotic freedom

$$\frac{d\log\alpha_s(\mu)}{d\log\mu} = -2\beta(\alpha_s(\mu))$$

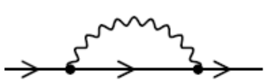
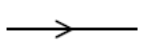
$$\beta(\alpha_s) = \beta_0\frac{\alpha_s}{4\pi} + \cdots \qquad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$$

 $= T_F$ 

$$\mathrm{Tr}\, t^a t^b = T_F\delta^{ab}$$

 $= C_A$  or $if^{acd}if^{bdc} = C_A\delta^{ab}$

$$C_A = 2T_F N_c$$

 $= C_F$  or $t^a t^a = C_F$

$$C_F = T_F\left(N_c - \frac{1}{N_c}\right)$$