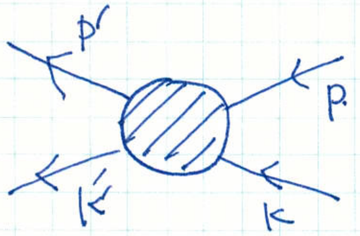
⓪ The Reduction formula

Given $\mathcal{L}(\varphi, \partial\varphi)$ we calculate S-matrix by suming over all possible Feynman Graphs

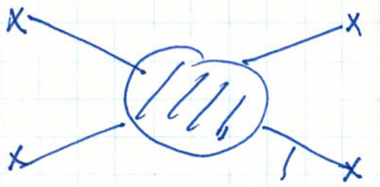


— Fig (1)

1) off shell ~~ma~~ S-matrix

$$\mathcal{L} \Rightarrow \mathcal{L} + \rho(x)\,\phi(x)$$

and calculate $\langle 0|S|0\rangle$ to fourth order in ρ.



internal line

— Fig (2)

Fig (2) is same as Fig (1) but now external legs became internal $\rightarrow \dfrac{i}{p^2 - m^2}$ with p^2 not necessarily m^2.

ii) Interaction Picture of getting i)

$$\langle 0|S|0\rangle = T\langle 0|\, \exp -i\int \rho(x)\,\phi(x)\, d^4x\, |0\rangle$$

$$\Big\downarrow \ \frac{\delta}{\delta\rho_1\,\delta\rho\,\delta\rho\,\delta\rho}\Big|_{\rho=0} \Rightarrow \text{Fig (2)}$$

$$= T\langle 0|\, \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\,|0\rangle$$

Fourier transforming the last equation

$$\langle k', p' | S | k, p \rangle = (K\text{-}F.) \int dx_1 \cdots d^4x_4 \, (i)^4 (\Box_1+m^2) \cdots (\Box_4+m^2)$$

$$e^{ik'\cdot x_1} e^{+ip'\cdot x_2} e^{-ik\cdot x_3} e^{-ip\cdot x_4}$$

$$\times \, T\langle 0 | \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) | 0 \rangle \quad \cdots (1)$$

when on mass shell → gives S-matrix

we can put only part of the particle off-mass shell.

$$\langle k', p' | S | k, p \rangle = (K\text{-}F.) \int d^4x_1 x_2^4 \, (i)^2 (\Box_1+m^2)(\Box_2+m^2)$$

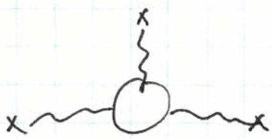
$$e^{ik'\cdot x} e^{-ik\cdot x_2} \, T\langle p | \phi(x_1) \phi(x_2) | p \rangle.$$

Chiral Perturbation & Strong Interaction Physics

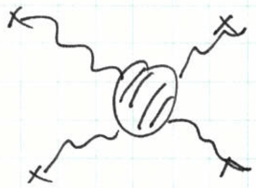
◎ Introduction

목적 : n-point correlation function of currents 을 구하는게 목적.

mass , width.



Form Factor



Scattering length etc.
decay, process.

접근 방법.

① Small momenta. ($q^2 < \Lambda_{Had}^2$)

Symmetry consideration + Perturbation in m_q, q^2

\+ Technical Assumption.

$\Rightarrow$ Effective Lagrangian ($\mathcal{L}_{eff}$) for QCD at Low momenta.

② Intermediate momenta $(q^2 \sim \Lambda_{Had}^2)$

Symmetry + Operator Product Expansion

+ Technical Assumption

$\Rightarrow$ QCD sum rules. Hadron Parameters.

$m_\rho, f_\rho, f_\pi, \cdots$

③ High momenta $(q^2 > \Lambda_{HAD}^2)$

Perturbative QCD.

⓪ Symmetries – Currents – Commutators.

⓪ Why currents.

Old days used to understand Weak decay by Currents.

A1) Weak decay

leptonic

$$l \to l' + \bar{\nu}_e + \nu_e$$

ex) $\mu \to e + \nu_\mu + \bar{\nu}_e$ (set basic strength)

Semi leptonic.

$h \to l + \nu_e$ $\qquad \pi \to \mu + \bar{\nu}_\mu$

$h \to h + l + \bar{\nu}_e$ $\qquad K^- \to K^0 + e + \bar{\nu}_e$

non-leptonic

$h \to h + h'$ $\qquad K^+ \to \pi^+, \pi^0.$

Dynamics

All these processes are well described by Fermi-Theory

$$\mathcal{L} = \frac{G}{\sqrt{2}} \left(J_\mu^+ J_\mu + h.c \right)$$

where

$$J^\mu = J_l^\mu + J_{ha.}^\mu$$

$G = 1.0 \times 10^{-5}\, m_p^{-2}$ from leptonic decay.

$J_l^\mu = \frac{1}{2} \sum \bar{\nu}_e \gamma^\mu (1-\gamma_5) l \qquad l = e, \mu, \tau.$

& $J_H^\mu = V_\mu + A_\mu.$

Semi-leptonic decay

~~h → h + leptons~~ $\quad i \to f + \text{lepton.}$

$$\langle f, l \,|\, J_\mu^H J_\mu^L \,|\, i \rangle = \langle f | J_\mu^H | i \rangle \underbrace{\langle l | J_\mu^l | 0 \rangle}_{\text{well}}$$

$\therefore$ important to know various properties of

$\underbrace{\langle f | J_\mu^H | i \rangle}$

Example.

a) g_V, g_A. : from Neutron β decay.

$$\langle p | J_\mu^{+} | n \rangle = (\text{K.F.})\, e^{-ik\cdot x}\, \bar{u}_p [\gamma_\mu g_V(k^2) + i \gamma_\mu \gamma_5 g_A(k^2) + O(k^2)]\, u_n.$$

experimentaly $g_V \simeq 1$ $g_A/g_V \simeq 1.25$ $k^2 \simeq 0$.

b) f_π

$$\langle 0 | A_\mu(x) | \pi^- \rangle = (\text{K.F.})\, e^{-ip\cdot x} (i p_\mu F_\pi / \sqrt{2})$$

$F_\pi \to 93$ MeV.

$$\to \langle 0 | \partial^\mu A_\mu(x) | \pi^- \rangle = (\text{K.F.})\, m_\pi^2 F_\pi\, e^{-ip\cdot x}/\sqrt{2} \neq 0$$

$(k^2 - m_\pi^2)^2 \phi$.

$\therefore$ $\phi_{\pi^-} = \sqrt{2}\, \partial_\mu A^\mu / F_\pi m_\pi^2$ + terms that vanish on mass-shell

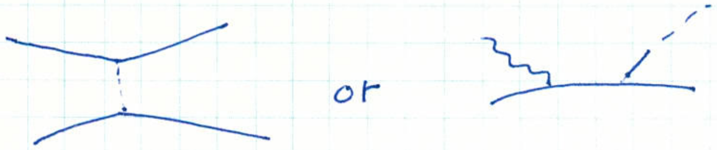
perfectly good pion field usable in reduction formula.

(20) The Goldberger - Treiman Relation & PCAC

define

$$\langle p|\ \phi_{\pi^-}(x)\ |n\rangle = (\text{K.F.})\ e^{-ik\cdot x}\,(k^2-m_\pi^2)^{-1}\,\bar{u}_p\gamma_5 u_n\, g(k^2)\sqrt{2}$$

we only know $g(k^2=m_\pi^2)$ by extrapolating



or

which gives $g(m_\pi^2) = g = 13.5$

From weak decay,

$$\langle p|\ A_\mu(x)\ |n\rangle = i(\text{K.F.})\ e^{-ik\cdot x}\ \bar{u}_p$$

$$\times\left[\gamma_\mu\gamma_5 g_A(k^2) + k_\mu\gamma_5 g_P(k^2) + \sigma_{\mu\nu}\gamma_5 k^\nu g_M(k^2)\right] u_n$$

taking the div.

$$\langle p|\ \partial^\mu A_\mu(x)\ |n\rangle = (\text{K.F.})\ e^{-ik\cdot x}\ \bar{u}_p\gamma_5 u_n\left[-2M\, g_A(k^2) + k^2 g_P(k^2)\right]$$

$(k^2-m_\pi^2)\,\phi$

Now use PCAC Assumption. then

$$\sqrt{2}\, g(k^2)\,/\,(k^2-m_\pi^2) = \left[-2M\, g_A(k^2) + k^2 g_P(k^2)\right]\sqrt{2}/F_\pi m_\pi^2$$

Now assume $g(k^2)$ is slowly varying

then $\underline{g(0) = g(m_\pi^2) = 2M\, g_A(k^2=0)}/F_\pi$.

↳ Experimentally very good

⑥ The gradient-coupling model.

o Obtaining currents from Lagrangian. $\mathcal{L}(\phi^a, \partial_\mu \phi^a)$

dynamics

$\delta \int d^4 \mathcal{L} = 0$ . . . (1)

$$\rightarrow \frac{\partial \mathcal{L}}{\partial \phi_\alpha} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_\alpha} = 0$$

● Currents

let $\delta \phi^\alpha = F_i^\alpha \delta \lambda$. 이라면

assume, that this is a sym of the Lag.

$$\delta \mathcal{L} = \left(\frac{\partial \mathcal{L}}{\partial \phi^a} F_i^\alpha + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^\alpha)} \partial_\mu F_i^\alpha \right) \delta \lambda = 0$$

If $\delta \lambda$ dep on x → then (1) 를 적용할 수 없음.

$$\delta \mathcal{L} = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^\alpha)} F_i^a \partial_\mu \delta x \neq 0.$$

but integral must vanish.

because

$$\int d^4x \left[\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^a)} F_i^a \right] \partial_\mu \delta \lambda = 0$$

$\downarrow$

$$\partial_\mu \left[\underbrace{\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^a)} F_i^a}_{J^\mu} \right] = 0$$

$\therefore$ J^μ : conserved current.

° Gradient coupling

$$\mathcal{L} = \mathcal{L}_0 - i\frac{g}{2M}\bar{\psi}\gamma_\mu\gamma_5\tau\psi\cdot\partial^\mu\phi.$$

$$\delta\phi = \delta\lambda \qquad \delta\psi = 0$$

$$A_\mu = \partial_\mu\phi - i\frac{g}{2M}\bar{\psi}\gamma_\mu\gamma_5\tau\psi$$

also if pion mass term exists

$$\partial^\mu A_\mu = -m_\pi^2\phi$$

$$A_\mu = \frac{2Mg_A}{g} A_\mu$$

PCAC, Goldberger Trieman $g(k^2) = \text{const.}$

⓪ Adler's Rule for emission of one soft pion

참고: M. Bando . T. Kugo . K. Yamawaki Phys. Rept. 164 #4&5 (1988) 217

Consider.

$i \to f + \pi$ $\xrightarrow{\text{can be related to}}$ $i \to f$

Consider.

$\langle f | A_\mu^a | i \rangle = M_\mu^a(q) e^{iqx}$ $q = -P_i + P_f$

By PCAC

$\phi = \partial_\mu A^\mu / f_\pi m_\pi^2$.

∴ taking derivative

$\langle f | \partial_\mu A_\mu^a | i \rangle = i q^\mu M_\mu^a(q) = f_\pi^a m_\pi^2 \langle f | \phi | i \rangle$

$\equiv f_\pi^a m_\pi^2 \cdot \frac{i}{q^2 - m_\pi^2} \tilde{G}_{ij}^\pi(i \to f + \pi^a)$ ... ①

where

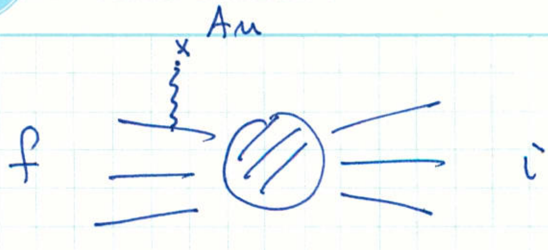
$\tilde{G}_{ij}^\pi(q^2 = m_\pi^2) = G_{ij}^\pi(i \to f + \pi^a)$ physical Amplitude

from ①

$\frac{f_\pi^a m_\pi^2}{q^2 - m_\pi^2} i \tilde{G}_{ij}^\pi(q^2) = i q^\mu M_\mu(q)$. ... ②

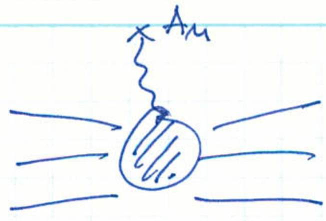
soft limit

$i \tilde{G}_{ij}^\pi(q^2 = 0) \Rightarrow 0$ unless $M_\mu(q)$ has a pole.



(a)

or

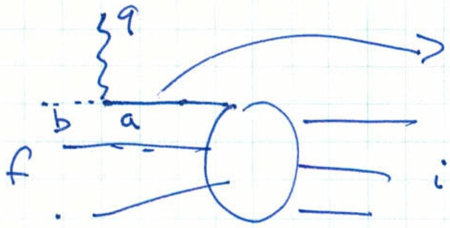


(b)

(b) Type of Diagram certainly ~~than~~ is not divergent.
at $q \to 0$

(a) What about this type

suppose i, f are nucleons. →



$$\frac{1}{(p+q)^2 - m_N^2} = \frac{1}{2p\cdot q + q^2}$$ ∵ on-mass shell nucleon

$$= \langle f - a + b \mid i \rangle \times \frac{i}{2p\cdot q + q^2} \times \langle b \mid A_\mu \mid a \rangle$$

↑ ↑ nucleon state

$$= \langle f \mid i \rangle \frac{i}{2p\cdot q + q^2} \langle b \mid A_\mu \mid a \rangle \quad \cdots \quad (3)$$

Substituting (3) into (2) and taking soft pion limit.

$$-f_\pi^a \, i \, \tilde{G}_{ij}^\pi = \langle f \mid i \rangle \frac{i \, i q^\mu \langle b \mid A_\mu \mid a \rangle}{2p\cdot q} = \langle f \mid i \rangle (+ g_A(0))$$

$$\therefore \; f_\pi^a G_{ij}^\pi = i g_A \langle f \mid i \rangle + \text{other attachment}.$$

if i, j are single particle states

corresponds to internal interaction

so vanishes

$\therefore \quad G^{\pi}_{HH'}(q^2) = 0$ Adler zero.

derivative coupling.

⓪ Current commutators & symmetry.

For processes involving more than a single pion,
In using PCAC, we need current commutators.

1) Vector – Vector commutator

why is $g_V = 1$.

Suppose J_h^μ has same structure as J_e^μ in weak current.

then $g_V = 1 = g_A$.

Suppose in Addition CVC i.e. conserved vector current, then $g_V = 1$ even after Renormalization ~~of~~ by Strong Interaction.

What is a conserved vector current in strong interaction

$V_\mu^a = \alpha I_\mu^a$ → isospin current.

then $\alpha = 2 g_V$.

Now

$$[I_0^a(x,0), I_0^b(y,0)] = i\epsilon_{abc} I_0^c(x,0)\,\delta^3(x-y) + \text{Schwinger terms.}$$

Schwinger terms
→ terms that dissappear when $\int d^3x$

proof)

from before. Noether theorem.

Suppose Lag has Isospin Symmetry.

$\mathcal{L}[\phi_\alpha, \partial_\mu \phi_\alpha]$ is inv. under

$$\phi_\alpha \Rightarrow \phi_\alpha - i\varepsilon T^a_{\alpha\beta}\phi_\beta. \qquad a \text{ 방향으로}$$

Conserved current associated with this is

$$\delta_a \mathcal{L} = -i\varepsilon \frac{\partial \mathcal{L}}{\partial \phi_\alpha} T^a_{\alpha\beta} - i\varepsilon \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_\alpha} T^a_{\alpha\beta}\partial_\mu \phi_\beta$$

$$= -i\varepsilon \left[\frac{\partial \mathcal{L}}{\partial \phi_\alpha} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_\alpha} \right] T^a_{\alpha\beta}\phi_\beta - i\varepsilon\, \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} T^a_{\alpha\beta}\phi_\beta \right]$$

$$= \varepsilon\, \partial_\mu J^a_\mu = 0$$

$$\therefore \quad J^a_\mu = -i \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_\alpha} T^a_{\alpha\beta}\phi_\beta$$

$$\therefore \quad \dot{Q}^a = 0$$

$$\& \quad J^0_a = -i T^a_{\alpha\beta}\Pi_\alpha \phi_\beta$$

Now Quantize

$$[\phi_\alpha(x), \Pi_\beta(y)]_{x^0=y^0} = i\delta^3(\vec{x}-\vec{y})$$

Then

define (to avoid ordering ambiguity)

$$J^0_a = -i T^a_{\alpha\beta}\, \frac{1}{2}\left[\Pi_\alpha \phi_\beta \underset{\text{Boson or fermion}}{\overset{\pm}{\mp}} \phi_\beta \Pi_\alpha\right]$$

$$[J_a^0(x), J_b^0(y)]_{x^0=y^0} = -i[T^a, T^b]_{\alpha\delta} \frac{1}{2}[\pi_\alpha \phi_\delta + \phi_\delta \pi_\alpha] \delta^3(\vec{x}-\vec{y})$$

$$= \delta^3(\vec{x}-\vec{y}) J^0_{[a,b]}(x).$$

If $T^a_{\alpha\beta}$ form an algebra $[T^a, T^b] = i f^{abc} T^c$

$$\Rightarrow [J_a^0(x), J_b^0(y)]_{x^0=y^0} = i f^{abc} J_c^0(x) \delta^3(x-y)$$

(There hold even when symmetry is broken)

Q.E.D

$$\therefore [V_0^a(x,0), V_0^b(y,0)] = 2i g_V \epsilon_{abc} V_0^c(x,0) \delta^3(x-y).$$

2.) Vector - Axial commutator

If V_μ^a is isospin current, A_μ^a is isotriplet

$$[V_0^a(x,0), A_0^b(y,0)] = 2i g_V \epsilon_{abc} A_0^c(x,0) \delta^3(x-y)$$

3) Axial-Axial commutator

in case of lepton $1+i\gamma_5$ $1-i\gamma_5$ are orthogonal projection.

$$\therefore [(V_0^a + A_0^a), (V_0^b - A_0^b)] = 0$$

$$\Rightarrow [A_0^a(x,0), A_0^b(y,0)] = 2i g_V \epsilon_{abc} V_0^c(x,0) \delta^3(x-y)$$