Dynamical Approach to Current Algebra. (PRL. ~~30~~ 18 (1967) 188.

Recall.

1. Refinition of Current Algebra.

$$[\cancel{A}V\ V] = g_V V \qquad [A\ A] = g_V V \ , [A\ V] = g_V A$$

Redefine all currents by dividing by g_V.

$$V' = \frac{V}{g_V} \qquad A' = \frac{A}{g_V}$$

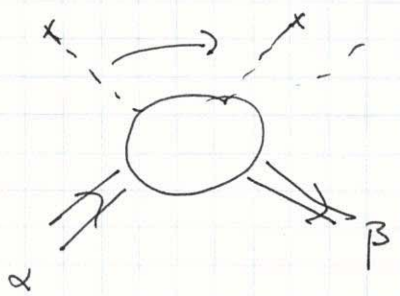
2. Use PCAC $\quad \partial_\mu A_\mu = f_\pi m_\pi^2 \phi \quad \Rightarrow \quad \partial_\mu A'_\mu = \frac{f_\pi}{g_V} m_\pi^2 \phi$

3. ~~Gold~~ berger - Treiman

$$f_\pi g_{\pi NN} = 2 M g_A \qquad \text{because} \quad A' -$$

$\cancel{4}$ Structure of formula by S-matrix.

A) $\cancel{\partial_\mu A^\mu} \langle \cancel{\partial^\lambda A_\lambda}$ ... for



Each soft pion

$$\langle \beta |\ \partial_\mu A'_\mu \qquad | \beta \rangle \times \left(\frac{m_\pi^2}{q^2 - m_\pi^2} \right) \cdot \frac{1}{f_\pi} g_V$$

$$= \langle \beta |\ \partial_\mu A'_\mu \qquad | \beta \rangle (-1)^n \left(\frac{g_{\pi NN}}{2M} \right) \left(\frac{g_V}{g_A} \right)$$

↳ for each soft pion.

즉: Time ordered product of a number of vector and axial-vector current times $(g_{\pi NN}/2M_N) \times (g_V/g_A)$ for each soft pion. -- ①

B) Adler's Theorem.

hook up to external leg, then $\underbrace{\langle N|\partial^\mu A_\mu|N\rangle}_{\downarrow} \times$ without-soft pion $M_{\alpha\beta}$

$\frac{g_A}{g_V}$ or $(g_V/g_A)^{-1}$. -- ②

$\therefore$ ① × ② $= O(g_{\pi NN}/2M_N)$

lowest order in $g_{\pi NN}$ + $\underbrace{O(g_V/g_A)}_{\downarrow}$

because if not hooked up on external leg.

A. A, B) + current commutator.

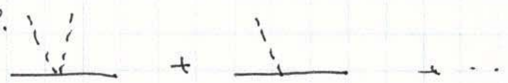
⇒

C) New Prescription Chiral-Dynamics method → immediately gives C.A. resu

1) Choose any Lagrangian which satisfy PCAC & current commutation relation

2) Evaluate to lowest order in G.

3) Write the result in the form dictated by current algebra.

i.e. 

⇒ "trees" of soft pions attached to vector and axial-vector vertices on the external line of a "core" process $\alpha \to \beta$.

⇒ (Non-linear version directly satisfy this so not neccessary if ~~used~~ NL version is used)

4) Supply higher order corrections by multiplying with factors $\underbrace{g_V/g_A}_{\text{from } A)}$ for each soft pion and $\underbrace{g_A/g_V}_{\text{from } B)}$ for each axial-vector vertex on the nth line..

⊛ Linear σ-model.

A model satisfying Current Algebra.

Let us chose a linear representation.

$$U = U_0 + i\tau\cdot U = \sigma + i\tau\cdot\pi.$$

s.t.

$$U' = V_R U V_L^{\dagger} = (1+i\alpha+i\beta)(U_0 + i\tau\cdot U)(1-i\alpha+i\beta)$$

$$= U + i(\alpha+\beta)$$

↓

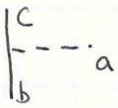
Section 4 참조

$$\mathcal{L} = -\bar{N}\left(\not{\partial} + m_{N0} - G_0(\sigma + i\tau\cdot\pi\gamma_5)\right]N$$

$$-\frac{1}{2}\left[\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} + m_{\pi 0}^2\vec{\pi}^2\right] - \frac{1}{2}\left[\partial_\mu\sigma\,\partial^\mu\sigma + m_{\sigma 0}^2\sigma^2\right]$$

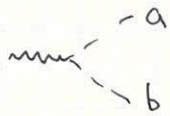
$$-\underbrace{(m_{\sigma 0}^2 - m_{\pi 0}^2)}_{\Delta m_\pi^2}\left[G_0^2/8m_{N0}^2\,(\vec{\pi}^2+\sigma^2)^2 - G_0/2m_{N0}(\vec{\pi}^2+\sigma^2)\sigma\right]$$

Note, Feynman Rules



$-G_0\tau^a_{cb}\gamma^5$,

$i\delta_{bc}G_0$



$i\Delta m_\pi^2\,\frac{G_0}{2m_{N0}}\,\delta_{ab}$

4) π-N scattering

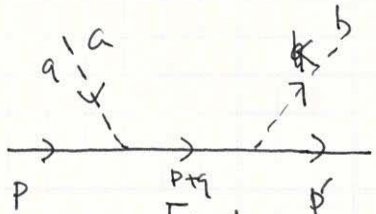


Fig 1 $= p+q-k$

+

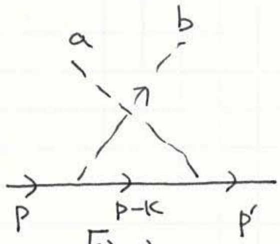


Fig 2

+

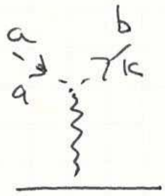


Fig 3.

1.

$$G_0^2\,\bar{u}(p')\gamma^5\tau^b\,\frac{i}{\not{p}+\not{q}-m_N}\,\tau^a\gamma^5 u(p) \quad + \quad G_0^2\,\bar{u}(p')\,\tau^a\gamma^5\,\frac{i}{\not{p}-\not{k}-m_{N0}}\,\gamma^5\tau^b\,u(p)$$

① ②

$$+\,G_0^2\,\bar{u}(p')\,u(p)\,\frac{i}{(q-k)^2-m_\sigma^2}\;i\Delta m_\pi^2\,\frac{i}{2m_{N0}}\,\delta_{ab} \quad \times 2$$

③

Weinberg Tomozawa term from linear sigma model

I. CORRELATION FUNCTION

$$M = \bar{u}(p')\gamma^5\tau^b \frac{i}{\not{p}+\not{q}-m}\tau^a\gamma^5 u(p) + \bar{u}(p')\gamma^5\tau^b \frac{i}{\not{p}-\not{k}-m}\tau^a\gamma^5 u(p) \tag{1}$$

There will be an extra sigma exchange term in the $N(p)+\pi(q)\to N(p')+\pi(k)$ scattering. Note $p+q=p'+k$.

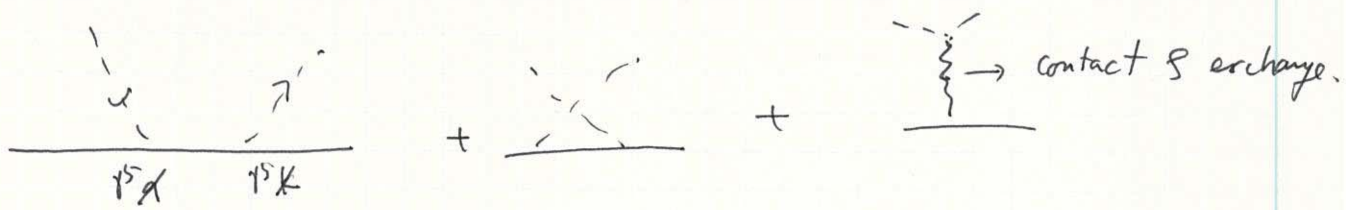
$$\begin{aligned}
M &= \bar{u}(p')\left(\frac{\not{p}'+m}{2m}\right)\gamma^5\tau^b \frac{i}{\not{p}+\not{q}-m}\tau^a\gamma^5\left(\frac{\not{p}+m}{2m}\right)u(p) + \bar{u}(p')\left(\frac{\not{p}'+m}{2m}\right)\gamma^5\tau^a \frac{i}{\not{p}-\not{k}-m}\tau^b\gamma^5\left(\frac{\not{p}+m}{2m}\right)u(p)\\
1 &= \bar{u}(p')\left(\frac{\not{p}+\not{q}+m}{2m}-\frac{\not{k}}{2m}\right)\gamma^5\tau^b \frac{i}{\not{p}+\not{q}-m}\tau^a\gamma^5\left(\frac{\not{p}+\not{q}+m}{2m}-\frac{\not{q}}{2m}\right)u(p)\\
&\quad +\bar{u}(p')\left(\frac{\not{p}-\not{k}+m}{2m}+\frac{\not{q}}{2m}\right)\gamma^5\tau^a \frac{i}{\not{p}-\not{k}-m}\tau^b\gamma^5\left(\frac{\not{p}-\not{k}+m}{2m}+\frac{\not{k}}{2m}\right)u(p)\\
2 &= \bar{u}(p')\tau^b\gamma^5\left(\frac{-\not{p}-\not{q}+m}{2m}+\frac{\not{k}}{2m}\right)\frac{i}{\not{p}+\not{q}-m}\gamma^5\tau^a\left(\frac{\not{p}+\not{q}+m}{2m}-\frac{\not{q}}{2m}\right)u(p)\\
&\quad +\bar{u}(p')\tau^a\gamma^5\left(\frac{-\not{p}+\not{k}+m}{2m}-\frac{\not{q}}{2m}\right)\frac{i}{\not{p}-\not{k}-m}\gamma^5\tau^b\left(\frac{\not{p}-\not{k}+m}{2m}+\frac{\not{k}}{2m}\right)u(p)\\
3 &= i\bar{u}(p')\tau^b\gamma^5\left(-\frac{1}{2m}+\frac{\not{k}}{2m}\frac{1}{\not{p}+\not{q}-m}\right)\gamma^5\tau^a\left(\frac{\not{p}+\not{q}+m}{2m}-\frac{\not{q}}{2m}\right)u(p)\\
&\quad +i\bar{u}(p')\tau^a\gamma^5\left(-\frac{1}{2m}-\frac{\not{q}}{2m}\frac{i}{\not{p}-\not{k}-m}\right)\gamma^5\tau^b\left(\frac{\not{p}-\not{k}+m}{2m}+\frac{\not{k}}{2m}\right)u(p)\\
4 &= i\bar{u}(p')\tau^b\gamma^5\left(-\frac{1}{2m}\right)\gamma^5\tau^a u(p) + i\bar{u}(p')\tau^b\gamma^5\left(\frac{\not{k}}{2m}\frac{1}{\not{p}+\not{q}-m}\right)\gamma^5\tau^a\left(\frac{\not{p}+\not{q}+m}{2m}-\frac{\not{q}}{2m}\right)u(p)\\
&\quad +i\bar{u}(p')\tau^a\gamma^5\left(-\frac{1}{2m}\right)\gamma^5\tau^b u(p) + i\bar{u}(p')\tau^a\gamma^5\left(-\frac{\not{q}}{2m}\frac{i}{\not{p}-\not{k}-m}\right)\gamma^5\tau^b\left(\frac{\not{p}-\not{k}+m}{2m}+\frac{\not{k}}{2m}\right)u(p)\\
5 &= i\bar{u}(p')\tau^b\gamma^5\left(-\frac{1}{2m}\right)\gamma^5\tau^a u(p) + i\bar{u}(p')\tau^b\gamma^5\left(\frac{\not{k}}{2m}\right)\gamma^5\tau^a\left(-\frac{1}{2m}\right)u(p) + i\bar{u}(p')\tau^b\gamma^5\left(\frac{\not{k}}{2m}\frac{1}{\not{p}+\not{q}-m}\right)\gamma^5\tau^a\left(-\frac{\not{q}}{2m}\right)u(p)\\
&\quad +i\bar{u}(p')\tau^a\gamma^5\left(-\frac{1}{2m}\right)\gamma^5\tau^b u(p) + i\bar{u}(p')\tau^a\gamma^5\left(\frac{\not{q}}{2m}\right)\gamma^5\tau^b\left(\frac{1}{2m}\right)u(p) + i\bar{u}(p')\tau^a\gamma^5\left(-\frac{\not{q}}{2m}\frac{i}{\not{p}-\not{k}-m}\right)\gamma^5\tau^b\left(\frac{\not{k}}{2m}\right)u(p)
\end{aligned} \tag{2}$$

use $\tau^a\tau^b = \delta^{ab} + i\epsilon^{abc}\tau^c$.

$$\begin{aligned}
M &= -\frac{i}{m}\bar{u}(p')\delta^{ab}u(p) + \frac{i}{4m^2}\bar{u}(p')\delta^{ab}\gamma^5\left(-\not{k}+\not{q}\right)\gamma^5\tau^a u(p) + \frac{i}{4m^2}\bar{u}(p')\imath\epsilon^{abc}\gamma^5\left(\not{k}+\not{q}\right)\gamma^5\tau^a u(p)\\
&\quad +i\bar{u}(p')\tau^b\gamma^5\left(\frac{\not{k}}{2m}\frac{1}{\not{p}+\not{q}-m}\right)\gamma^5\tau^a\left(-\frac{\not{q}}{2m}\right)u(p) + i\bar{u}(p')\tau^a\gamma^5\left(-\frac{\not{q}}{2m}\frac{i}{\not{p}-\not{k}-m}\right)\gamma^5\tau^b\left(\frac{\not{k}}{2m}\right)u(p)\\
&= -\frac{i}{m}\bar{u}(p')\delta^{ab}u(p) + \frac{i}{4m^2}\bar{u}(p')\imath\epsilon^{abc}\gamma^5\left(\not{k}+\not{q}\right)\gamma^5\tau^a u(p)\\
&\quad -\frac{i}{4m^2}\bar{u}(p')\tau^b\gamma^5\not{k}\frac{1}{\not{p}+\not{q}-m}\gamma^5\tau^a\not{q}u(p) - \frac{i}{4m^2}\bar{u}(p')\tau^a\gamma^5\not{q}\frac{1}{\not{p}-\not{k}-m}\gamma^5\tau^b\not{k}u(p)
\end{aligned} \tag{3}$$

The first term cancels the sigma exchange term, the second term is the Weinberg-Tomozawa term, the third is the Adler term (derivative coupling).

즉 we can rewrite it as a sum of



This is more close to current algebra relation.

so what is wrong with the linear result.
it has contact term such as $N\gamma^5\pi N$ $\sigma\pi^2$ which is not quite in accord with PCAC.

So use chiral symmetry to transform N & π field where there is only pion-derivative coupling to Nucleon.

* Non-linear σ-model.

So transform to a New field.

$$N = (1+\vec{\xi}^2)^{-1/2}(1+i\gamma_5\vec{\tau}\cdot\vec{\xi})N'$$

s.t.

$$\bar{N}[m_{N0} - G_0(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)]N = \bar{N}'[\underbrace{m_{N0} - G_0\sigma'}]N'$$

$$\Downarrow$$

$$\equiv [(m_{N0}-G_0\sigma)^2 + G_0^2\vec{\pi}^2]^{1/2}$$

the required $\vec{\xi}$ is

$$\vec{\xi} = G_0\vec{\pi}[m_{N0} - G_0\sigma + \{(m_{N0}-G_0\sigma)^2 + G_0^2\vec{\pi}^2\}^{1/2}]^{-1} \equiv \frac{G_0}{2m_{N0}}\vec{\pi}'$$

then

$$\mathcal{L} = -\bar{N}'[\not{\partial} + m_{N0} - G_0\sigma' + i\gamma^\mu\left(1+\frac{G_0\vec{\pi}'^2}{4m_{N0}^2}\right)^{-1}\left\{\left(\frac{G_0}{2m_{N0}}\right)\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\pi}' + \left(\frac{G_0^2}{4m_{N0}}\right)^2\vec{\tau}\cdot\vec{\pi}'\times\partial_\mu\vec{\pi}'\right\}]N$$

$$-\frac{1}{2}[\partial_\mu\sigma'\partial^\mu\sigma' + m_{\sigma 0}^2\sigma'^2] - (m_{\sigma 0}^2 - m_{\pi 0}^2)\left[\left(\frac{G_0^2}{8m_{N0}^2}\right)\sigma'^4 - \left(\frac{G_0}{2m_{N0}}\right)\sigma'^3\right]$$

$$-\frac{1}{2}\left(1-\frac{G_0\sigma'}{m_{N0}}\right)\left[\left(1+\frac{G_0^2\vec{\pi}'^2}{4m_{N0}^2}\right)^{-2}\partial_\mu\vec{\pi}'\partial^\mu\vec{\pi}' + \left(1+\frac{G_0^2\vec{\pi}'^2}{4m_{N0}^2}\right)^{-1}m_{\pi 0}^2\vec{\pi}'^2\right].$$

$\Rightarrow$ this satisfies current algebra for all values of $m_{\sigma 0}$,

so if $m_{\sigma 0} \to 0$ all internal σ contribution will vanish.

and σ' field can be dropped everywhere.

$\to$ note σ' is chiral invariant and plays no role in maintaining the chiral invariance of $\mathcal{L}$.

including ingredients $\Rightarrow$ eg. CS).

Important Further developements

- S. Weinberg. PRD 166 (1568) (~~1969~~ (. 1968 년)
- Li & Pagels PRL <u>26</u> (71) 1204. ⇒ non-analytic terms
  coeff. can be calculated.
- m_π, f_π to 1-loop Gasser & Zapeda. NP B174 (1980) 445
- On the Rationale of Effective Lagrangian
  (S. Weinberg. Physica, 96A, (1979) 327.