Optics, hw 4

Han Gao, 17353019

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9.42:

Solution: Look at the source via each optical element. If the brightness is dimed about a half, what you're holding is then a linear polarizer; if the brightness reminds almostly unchanged, then it is a quarter-wave plate.

9.43:

Solution: A $\mathscr{L}$-state has the light vector as

$$\vec{E}(z) = E_0 \cos(kz - \omega t)\vec{e}_x - E_0 \cos(kz - \omega t - \frac{\pi}{2})\vec{e}_y. \tag{1}$$

After traversing, E_y is retarded for $\frac{pi}{4}$ in phase. The light vector is then

$$\begin{aligned}\vec{E}(z) &= E_0 \cos(kz - \omega t)\vec{e}_x - E_0 \cos(kz - \omega t - \frac{\pi}{2} - \frac{\pi}{4})\vec{e}_y \\ &= E_0 \cos(kz - \omega t)\vec{e}_x - \frac{\sqrt{2}}{2} E_0[\cos(kz - \omega t) - \sin(kz - \omega t)]\vec{e}_y,\end{aligned} \tag{2}$$

which represents a left-handed $\mathscr{E}$-state showed in fig. 1.

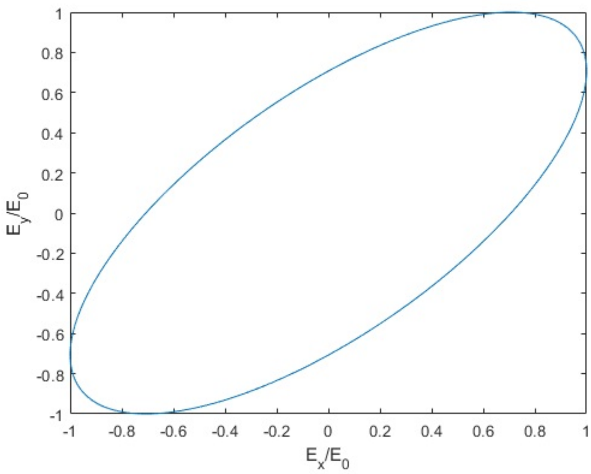

Figure 1: Polarization of the emerging light

9.48:

Solution: Rotatory for given solution is

$$\beta = \frac{66.45^\circ}{10 \text{ cm}} \times \frac{10 \text{ g}/1000 \text{ cm}^3}{1 \text{ g/cm}^3} = 0.006645^\circ/\text{cm} \tag{3}$$

Orientation is then

$$\alpha = \beta l = 0.006645^\circ/\text{cm} \times 100 \text{ cm} = +0.6645^\circ \tag{4}$$

9.54:

Solution: (a) $(1, 1, 0, 0)$ represents a horizontal $\mathscr{P}$-state (linear polarized in x direction). $(3, 0, 0, 3)$ representes a $\mathscr{R}$-state (right-circularly polarization).

(b) Stokes vector of combined beam is $(1, 1, 0, 0)+(3, 0, 0, 3) = (4, 1, 0, 3)$, which represents an $\mathscr{E}$-state of flux density 4.

(c) Degree of polatization is given by

$$V = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} = \frac{\sqrt{1^2 + 3^2}}{4} = 0.79 \tag{5}$$

(d) A natural light with flux density 2. For any natural light can be decomposed into vertial and horizontal $\mathscr{P}$-state with equal amplitudes, combining two vertial and horizontal $\mathscr{P}$-state, reversely, results in a natural light.

9.69:

Solution: (a)

$$\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} \cos\theta\cos\alpha + \sin\theta\sin\alpha \\ \sin\theta\cos\alpha - \sin\alpha\cos\theta \end{pmatrix} = \begin{pmatrix} \cos(\theta-\alpha) \\ \sin(\theta-\alpha) \end{pmatrix} \tag{6}$$

(b)

$$\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha + i\sin\alpha \\ -\sin\alpha + i\cos\alpha \end{pmatrix} = \frac{e^{i\alpha}}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \tag{7}$$

(c)

$$\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha - i\sin\alpha \\ -\sin\alpha - i\cos\alpha \end{pmatrix} = \frac{e^{-i\alpha}}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \tag{8}$$

(d) The filter is an optical rotator, constructed by, for example, sugar solution with specific rotation β and length $l = \alpha/\beta$.

9.71:

Solution: (a) Applying a horizontal $\mathscr{P}$-state yields

$$\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} (1//0) = \begin{pmatrix} 1 \\ i \end{pmatrix}, \tag{9}$$

which stands for a $\mathscr{R}$-state.

Similarly, applying a vertical $\mathscr{P}$-state yiels a $\mathscr{L}$–state.

$$\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} (0//1) = \begin{pmatrix} i \\ 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -i \end{pmatrix}. \tag{10}$$

Therefore, the given matrix represents a quarter-wave plate with its fast axis on the line $y = -x$.

(b) Using the same approach as (a), it can be easily identified that the given matrix represents a quarter-wave plate with its fast axis on the line $y = x$.

Ex.:

Solution: For a Jones vector $\begin{pmatrix} \hat{E}_x \\ \hat{E}_y \end{pmatrix}$, it can be decomposed to fast axis and slow axis as

$$\hat{E}_o = \hat{E}_x \cos\theta + \hat{E}_y \sin\theta; \tag{11}$$

$$\hat{E}_e = -\hat{E}_x \sin\theta + \hat{E}_y \cos\theta, \tag{12}$$

or

$$\begin{pmatrix} \hat{E}_o \\ \hat{E}_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{E}_x \\ \hat{E}_y \end{pmatrix} \tag{13}$$

When emerging, the two components now become

$$\begin{pmatrix} \hat{E}'_o \\ \hat{E}'_e \end{pmatrix} = \begin{pmatrix} \hat{E}_o e^{-i\delta} \\ \hat{E}_e \end{pmatrix} = \begin{pmatrix} e^{-i\delta} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{E}_o \\ \hat{E}_e \end{pmatrix}. \tag{14}$$

Notice that the relationship between $(\hat{E}_o, \hat{E}_e)$ and $(\hat{E}_x, \hat{E}_y)$ is

$$\begin{pmatrix} \hat{E}'_x \\ \hat{E}'_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{E}'_o \\ \hat{E}'_e \end{pmatrix}. \tag{15}$$

The overall relationship between $\vec{E}$ and $\vec{E}'$ is then

$$\begin{pmatrix} \hat{E}'_x \\ \hat{E}'_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\delta} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{E}_x \\ \hat{E}_y \end{pmatrix}. \tag{16}$$

Corresponding Jones matrix is

$$\begin{aligned} J &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\delta} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \\ &= \begin{pmatrix} \sin^2\theta + \cos^2\theta e^{-i\delta} & \frac{1}{2}\sin 2\theta (e^{-i\delta} - 1) \\ \frac{1}{2}\sin 2\theta (e^{-i\delta} - 1) & \cos^2\theta + \sin^2\theta e^{-i\delta} \end{pmatrix}. \end{aligned} \tag{17}$$