Modern Physics Lab.

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A study on the Basic Property of Superconductivity

Main goals:

- $R(T, H = 0), \chi(T, H = 0)$
- The differences between 1st-order PT and 2nd-order PT by comparing $\chi(T, H \neq 0)$ for a large H.

Additional staffs to be explored if time allows:

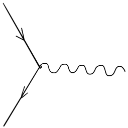
- History dependence of sc.
- Other $R(T; H)$ and $\chi(T; H)$ relations near the critical point.

Why?

Let's have a brief review on the theory of superconductivity and phase transition.

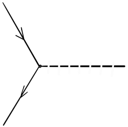
Electron-phonon Interaction

- QED vertex



$$= -ie\gamma^{\mu}$$

- Electron-phonon interactive vertex



$$= -ig$$

- Similar picture, except...

Inner Lines in Feynman Diagrams

- In QED, photon is a spin-1 particle (gauge field)

$$\text{photon inner line} = iD^{\mu\nu}(k) = \frac{-ig^{\mu\nu}}{k^2 + i\epsilon}$$

  Repulsive force $D^{00} < 0$.

- Phonon: a spin-0 particle (abelian real scalar field)

$$\text{phonon inner line} = iD(k) = \frac{i}{k^2 - m^*(k)^2 + i\epsilon}$$

  $D > 0$, attractive force, results in **electron bound state**, i.e. Cooper pair.

- See, for example, Zee, *QFT nut*, Altland, *cond-mat field theory*.

Broken Symmetry and Phase Transition

- Electron field = Spinor, with $U(1)$ symmetry in its Lagrangian without an external field..
- Superconductivity: $U(1)$ symmetry breaking, selecting a particular ground state.
- Symmetry breaking is often related to phase transition $\Rightarrow$ Sc is a new TD phase.

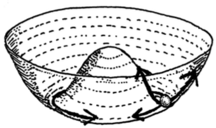


*figure taken from Lancaster/Blundell

Criterions for S.C. State

- Zero resistance:

$$R \sim 0$$

  In lab: a sudden decreasing in R, measured by 4-line method, given by $R = \frac{V_+ - V_-}{2I}$.

- Meissner effect: susceptibility

$$1 + \chi \sim 0$$

  In lab: a sudden increasing in emf output from lock-in amplifier, given by $\varepsilon \sim \chi$.

The Behavior near T_c

- We'd ask the exact form of $R(T, H)$ and $\chi(T, H)$. 1st-order PT: the transition is not **continue**.
- 2nd-order PT: the transition is not **smooth**, we'd like to say, e.g.

$$R(T, H = 0) \sim (T - T_c)^{\beta}$$

$$1 + \chi(T_c, H) \sim H^{\frac{1}{\delta}}, \quad 1 + \chi(T, H = 0) \sim (T - T_c)^{-\gamma}$$

- **Critical exponents** β, γ and δ are NOT integer, i.e. non-analytical behavior.

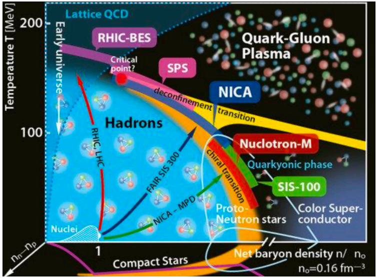
Why Study Critical Exponents

A very fundamental property of PT, but we're largely innocent of.

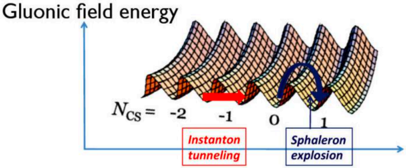
- Everyday phenomenons: vapor.
- In AMO and CMP: BEC.
- In hep-th: Higgs mechanism.
- In hep-ph/nucl-th: studying the critical exponents reflects our knowledge of symmetry and vacuum (e.g. QCD topological nontrivial vacuum leads to a lot of interesting phenomenons such as CME/CSE, and QCD phase diagram is still largely unknown).

QCD Phase Diagram and QCD Vacuum

- QCD Phase Diagram: unknown boundary, even don't know whether PT or not.



- Non-trivial QCD vacuum, topological transition.



Theoretic Calculation

- Mean Field method, of equivalently, Landau 2nd-order PT theory

$$F(m, T) = F_0 + a(T)m^2 + b(T)m^4$$

  giving $\beta = \frac{1}{2}$, dimensional independence. However, we know that PT doesn't occur in $d = 1$ and in $d = 2$, one can give exactly $\beta = \frac{1}{8}$ from the first principle.
- Why MF fails? Fluctuations.
- In $d = 3$, unable to solve critical exponents exactly. Must find other solution.

Renormalization Group (RG)

- Taking fluctuations into our concern by generalizing Landau theory by considering a field $\phi(\vec{x})$ in d dimension space

$$F[\phi] = \int d^d\vec{x}\, \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4.$$

  No term higher than $O(\vec{\nabla}^2)$ ensures locality.

- Partition function as a path integral

$$Z = \int \mathcal{D}\phi e^{-\beta F[\phi]}.$$

- In $d = 4$, equivalent to ϕ^4 theory in quantum field theory after Wick rotation, which is renormalizable. β functions for m^2 and λ.

RG flows

The behavior of RG flows near W_2 gives critical exponents as a function of ϵ.

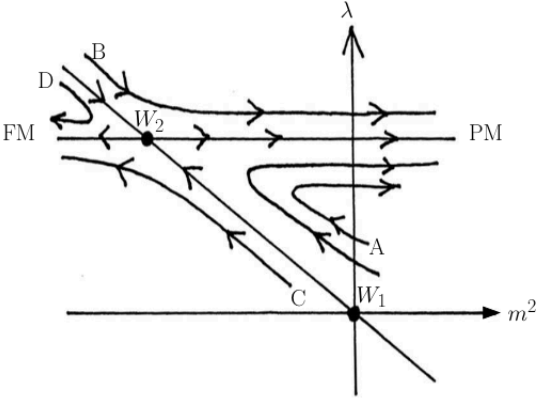


*figure taken from Lancaster/Blundell

Results from RG as a ϵ expansion

- However, we live in $d = 3$ space.
- Consider $d = 4 - \epsilon$ Euclidean space, RG flows (β functions) give, e.g.

$$\beta = \frac{1}{2} - \frac{\epsilon}{6} + O(\epsilon^2)$$

- Then take $\epsilon = 1$ for $d = 3$, we have $\beta = \frac{1}{3}$.
  Experimentally/Numerically on a lattice, $\beta \approx 0.3264$.

We Can Never Know the Exact Answer

- The result is only from ϵ expansion and in order to get high order corrections, we have to calculate a HUGE number of Feynman diagrams, which is too arduous (and probably not converge).
- RG is also known for its mathematically bad reputation.
- So measuring the critical exponents from experiment is of great importance from both practical and theoretic aspects. That's why...

Our Plan

1. A phenomenological study: measuring critical temp. T_c by two methods (resistance and susceptibility) discussed in sect.1.
2. A study on the dependence of transition temp. $T(H)$ between external magnetic field H.
3. A detailed study on S.C. from the prospect of critical exponents (Our question above).

Let's Keep Focus and Don't Be too Ambitious

- We don't plot $R(T, H)$ our for every (T, H) because it's meaningless from theoretic aspect.
- What we are interested in are 2nd-order PTs, i.e., $R(T, H = 0), \chi(T, H = 0)$ and $\chi(T_c, H)$.
- Heavily relies on the precision of measuring T_c. Therefore we focus on a particular sample and repeat as many time as possible.
- Since the critical exponents of PT are known for universality, focusing on a particular sample would't make us lose any generality.

Procedures

$\boxed{\text{Measuring } R \text{ and } \chi \text{ both from HT} \rightarrow \text{LT and from LT} \rightarrow \text{HT}}$

$\Downarrow$

$\boxed{\text{Fit the value of } T_c.}$

$\Downarrow$

$\boxed{\text{Precisely control the temp. at } T_c \text{ and measure } \chi(T_c, H).}$

$\Downarrow$

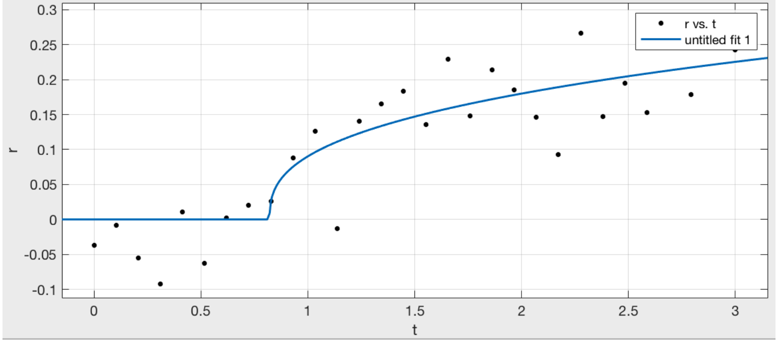
$\boxed{\text{Measuring } \chi(T, H \neq 0) \text{ to study the difference btw 1st \& 2nd-order PT.}}$

$\Downarrow$

$\boxed{\text{Fit our datas for } (T_c, \alpha, \beta, \delta)}$

A Simulative Data Processing

1. Set $\beta = 0.5$, $T_c = 1$, adding a gaussian noise to every point.
2. Naively fitting data points by LS gives $T_c = 0.82 \pm 0.08$, $\beta = 0.4 \pm 0.3$



3. Too large uncertainty in β, a lot of data points needed.

Tables for Data Recording

表 14: 电阻变化的临界指数测量，第一次升温测量

温度 T/K	正向电压 $V_+/\mu\mathrm{V}$	反向电压 $V_-/\mu\mathrm{V}$

Figure: Table for measuring $R(T, H = 0)$.

表 18: 磁化率变化的临界指数测量，第一次升温测量

温度 T/K	输出差分电压 R/V	输出差分电压信号幅角 θ

Figure: Table for measuring $\chi(T, H = 0)$.

More Tables

表 24: 临界温度下磁化率磁场变化

磁场 B_0/T	输出差分电压 R/V	输出差分电压信号幅角 θ
0.0		

Figure: Table for measuring $\chi(T_c, H)$.

表 25: 一级相变时磁化率测量，第一次升温测量

温度 T/K	输出差分电压 R/V	输出差分电压信号幅角 θ

Figure: Table for measuring $\chi(T, H \neq 0)$ for 1st-order PT.

Thank You for Your Attentions

We hope you are safe and healthy at home.