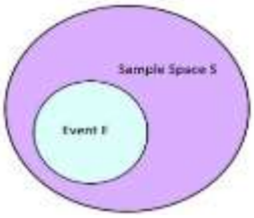
Unit VI

6.	**Probability and Statistics**
6.1	Elementary Probability: Joint and Conditional probability and Bayes theorem
6.2	Discrete Probability Distributions: Binomial and Poisson
6.3	Continuous Probability Distributions: Exponential and Normal

6.1 Elementary Probability: Joint and Conditional probability and Bayes theorem

Random Experiment: An action or an operation which can produce any result or outcome is called a Random Experiment or trail and outcomes are known an events.
Certain Event or Sure Event: An event whose occurrence is inevitable when an experiment is performed is called Certain Events.
Impossible Event: An event which can never occur when an experiment is performed is called an impossible event.
Mutually Exclusive Events: Two events are said to be mutually exclusive if both cannot occur simultaneously (disjoint events).
Independent Events: Two events are said to be independent of each other if occurrence or non-occurrence of one does not affect the occurrence of other.
Exhaustive Events: A group of events is said to be exhaustive if it includes all possible outcomes of experiment.
Sample Space: A set of all possible outcomes of a random experiment is called a sample space. The sample space of a random experiment is denoted by S.



Probability: Let S be a sample space of an experiment and A be any event of this sample space. The probability P(A) of the event A is defined as the real valued set function which associates a real value corresponding to a subset A of the sample space .The probability P(A) satisfies the following three axioms:

- **a.** $P(A) \geq 0$.
- **b.** $P(S) = 1$.
- **c.** If A_1, A_2,……….A_n are finite mutually exclusive events then $P(A_1 \cup A_2 \cup ....A_n) = P(A_1) + P(A_2) + P(A_3) + ... + P(A_n)$.

Algebra of Events: If A and B are two events on sample space S, then

- **a.** $A \cup B \Rightarrow$ **Occurrence of A or B.**
- **b.** $A \cap B \Rightarrow$ **Occurrence of A and B.**

Addition Theorem on Probability
Probability that either of events A or B will occur is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For mutually exclusive events, $P(A \cap B) = 0$

$$\therefore P(A \cup B) = P(A) + P(B).$$

Joint Probability, Conditional Probability and Bayes Theorem

6.1.1 Joint Probability: It is simply the likelihood that two events will happen at the same time. Joint probability is calculated by multiplying the probability of event A, expressed as P(A), by the probability of event B, expressed as P(B).It is denoted by $P(A \cap B)$.

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$.

OR

$P(A \cap B) = P(A) \times P(B)$ where A and B are independent events.

Example: **A company has estimated that the probabilities of success for three products introduced in the market are 1/3, 2/5, and 2/3 respectively. Assuming independence, find (i) the probability that the three products are successful (ii) the probability that none of the product is successful.**

Solution: Consider the following events:

A: First product is successful, and B= Second Product is successful and C=Third product is successful.

$$P(A) = \frac{1}{3}, P(B) = \frac{2}{5}, P(C) = \frac{2}{3}.$$

(i) Required probability $= P(A \cap B \cap C) = P(A).P(B).P(C) = \frac{4}{45}$.

Required probability $= P(A^C \cap B^C \cap C^C) = P(A^C).P(B^C).P(C^C) = \frac{2}{15}$.

Example: **In two successive throws of a pair of dice, determine the probability of getting a total of 8 each time.**

Solution: Let A denote the event of getting a total of 8 in first throw and B be the event of getting a total of 8 in second throw. Then $P(A) = \frac{5}{36}, P(B) = \frac{5}{36}$.

Required probability $= P(A \cap B) = P(A).P(B) = \frac{25}{1296}$.

Tutorial	
1.	Find the probability of getting sum as 9, when two dices are thrown. **(Ans 1/9)**
2.	A bag contains four white and two black balls and second contains three of each color. A bag is selected at random, and a ball is drawn at random from the bag chosen. What is the probability that the ball is drawn white? **(Ans 7/12)**

6.1.2 Conditional Probability

The probability that B will occur with the given condition that A has already occurred is called the conditional probability of B and is denoted by $P(B|A)$ .It can also be interpreted as probability of B with A as sample space.

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)}.$$

Similarly we can define $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

The above definition written in product form give the multiplication theorem for probability

$P(A \cap B) = P(A|B) \times P(B)$.

Remark: For independent events, if probability of A remains constant whether B has occurred or not, then A and B are independent events. i.e. $P(A|B)=P(A)$.

$\therefore P(A\cap B)=P(A)P(B)$.

is known as multiplication theorem for independent events.

Example: **A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?**

Solution: Let E be the event that 'number 4 appears at least once' and F be the event that the' the sum of the numbers appearing is 6'.

Then E={ (4,1),(4,2),(4,3), (4,4),(4,5),(4,6),(1,4),(2,4),(3,4),(5,4), (6,4)}

F= {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}

$P(E)=\frac{11}{36}$ And $P(F)=\frac{5}{36}$.

$E\cap F$ = {(2, 4), (4, 2)}.

$P(E\cap F)=\frac{2}{36}$.

Hence the required conditional probability $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{2}{36}}{\frac{5}{36}}=\frac{2}{5}$.

Tutorial	
1.	A bag contains 3 red and 4 white balls. Two draws are made without replacement. What is the probability that both balls are red? **(ANS 1/7)**
2.	The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday? **(ANS 0.15)**

6.1.3 Bayes Theorem

Bayes Theorem (for two events) :

Let A and B are two mutually exclusive and exhaustive events. If we know the conditional probability P(B/A) and the probability P(A), then the Bayes' theorem tells that we can compute the conditional probability P(A/B) as follows: $P(A/B)=\frac{P(B/A)P(A)}{P(B)}$ with $P(B)\neq 0$.

Bayes Theorem (General) : Let $A_1, A_2, \ldots, A_n$ be n mutually exclusive and exhaustive events with $P(A_i)\neq 0$ for i=1,2,3……n in a sample space S. Let B be an event that can occur in combination with one off the events $A_1, A_2, \ldots, A_n$ with $P(B)\neq 0$.The probability of the events A_i when the event B has actually occurred is given by

$$P(A_i|B)=\frac{P(A_i)P(B|A_i)}{\sum_{i=1}^{n}P(A_i)P(B|A_i)}.$$

Example **A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that the red ball is drawn from bag B.**

Solution Let events B_1, B_2, be the following:

B_1: the event that the ball is drawn from bag A.

B_2: the event that the ball is drawn from bag B.

Clearly B_1, B_2 are mutually exclusive and exhaustive events and hence they represent partition of the sample space.

Let E be the event that the ball drawn is red. Given that

$P(B_1) = \frac{1}{2}$ $P(B_2) = \frac{1}{2}$.

$P(E|B_1)$ =Probability that the ball drawn is red given that it is drawn from bag A= $\frac{3}{5}$.

Similarly $P(E|B_2) = \frac{5}{9}$.

Hence, by Bayes Theorem

$$P(B_2 \mid E) = \frac{P(B_2)P(E \mid B_2)}{P(B_1)P(E \mid B_1) + P(B_2)P(E \mid B_2)} = \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}} = \frac{25}{52}.$$

Tutorial	
(i)	Three urn contains 6 red and 4 black; 4 red and 6 black; 5 red and 5 black balls respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is red. Find the probability that it is drawn from the first urn.**(Ans 2/5)**
(ii)	In a factory that manufactures bolts, machines A, B, and C manufacture 25%, 35%, and 40% of the bolts respectively and out of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolts is drawn at a random from the product and is found to be defective. What is the probability that it is manufactured by machine B?**(ANS 28/69)**

6.2 Discrete Probability Distributions: Binomial and Poisson

6.2.1 Random Variables

Random Variables: A random variable X is a real valued function of the elements of the sample space of a random experiment. In other words, a variable which takes the real values, depending on the outcome of a random experiment is called a random variable.

Types of Random Variables:

(a) Discrete Random Variable.

(b) Continuous Random Variables.

Discrete Random Variable: Let X be a random variable. If the number of values of X is countable then X is called discrete random variable. E.g. Number of heads in toss of 3 coins, Number of floor of a building.

Continuous Random Variable: A random variable X which assume all real value within a given interval is called as a continuous random variable. Thus the possible values of a continuous random variables are unaccountably infinite. E.g. Age of the person, Height of a person in inches, Temperature.

6.2.2 Discrete Probability Distribution

Probability distribution of a random variable is the set of all its possible values together with their respective probabilities. Let X be a discrete random variable which takes the values $x_1, x_2, x_3, \ldots\ldots, x_n$. The probability of each possible outcome x_i is $p_i = p(x_i) = P(X = x_i)$ for *i=1,2,3,…,n*. The number $p(x_i), i = 1,2, \ldots.$ must satisfy the following conditions:

(i) $p(x_i) \geq 0$, for all values of *i*

(ii) $\sum_{i=1}^{\infty} p(x_i) = 1$

The function $p(x_i)$ is called the probability or probability mass function or probability density function of the random variable X. The probability distribution of the random variable in the form table can be represent as follows:

$X = x_i$	x_1	x_2	x_3	……	x_n
$p(x_i) = P(X = x_i)$	$p(x_1)$	$p(x_2)$	$p(x_3)$	……	$p(x_n)$

Let X be a discrete random variable which takes the values $x_1, x_2, \ldots..$ such that $x_1 < x_2 < \ldots..$ with probabilities $p(x_1), p(x_1), \ldots..$ such that $p(x_i) \geq 0$ for all values of *i* and $\sum_{i=1}^{\infty} p(x_i) = 1$.
The distribution function $F(x)$ of the discrete random variable X is defined by $F(x) = P(X \leq x) = \sum_{i=1}^{x} p(x_i)$; where x is any integer. The function $F(x)$ is also called cumulative distribution function.

Example: A fair die is tossed once. If the random variable is getting an even number, find the probability distribution of *X*.
Sol: when a fair die is tossed, sample space $S = \{1,2,3,4,5,6\}$.
Let *X* be the random variable of getting an even number.
X can take the values 0 and 1.

$P(X = 0) = P(1, 3, 5) = \frac{3}{6} = \frac{1}{2}$

$P(X = 1) = P(2, 4, 6) = \frac{3}{6} = \frac{1}{2}$.

Hence the probability distribution of X is

$X = x_i$	0	1
$p(x_i) = P(X = x_i)$	$\frac{1}{2}$	$\frac{1}{2}$

Tutorial:

1. State with reasons whether the following represent the probability mass function of a random variable:

(i)

$X = x_i$	0	1	2	3
$p(x_i) = P(X = x_i)$	0.4	0.3	0.2	0.1

(ii)

$X = x_i$	0	1	2	3
$p(x_i) = P(X = x_i)$	1/2	1/3	1/6	1/4

(iii)

	$X = x_i$	0	1	2	3
	$p(x_i) = P(X = x_i)$	-1/2	1/2	1/4	3/4
2.	If the random variable X takes the value 1, 2, 3 and 4 such that 2P(X=1) =3P(X=2) =P(X=3) =5P(X=4). Find the probability distribution.				

Binomial Probability Distribution: Consider $\boldsymbol{n}$ independent trial of a random experiment which results in either success or failure. Let $\boldsymbol{p}$ be the probability of success remaining constant every time and $q = 1 - p$ be the probability of failure. The probability of x successes and n-x failure is given by

$$\boldsymbol{P(X = x) = nCx\, p^x q^{n-x}, x = 0, 1, 2, 3, \dots\dots n, where\ p + q = 1}$$

Conditions for Binomial Distribution

(i) The number of trials n is finite.
(ii) There are only two possible outcomes, success or failure.
(iii) The trials are independent of each other.
(iv) The probability of success p is constant for each trial.

Mean of the Binomial Distribution: $\boldsymbol{np}$.
Variance of the Binomial Distribution: $\boldsymbol{npq}$.
Standard Deviation of Binomial Distribution: $\sqrt{\boldsymbol{npq}}$.

Example **It has been claimed that in 60% of all solar heat installations the utility bill is reduced by at least one-third. Accordingly, what are the probabilities that the utility bill will be reduced by at least one third in (i) four of five installations? (ii) at least four of five installations?**

Solution: Let p be the probability that the utility bill is reduced by one-third in the solar heat installations.

$\boldsymbol{p}$=60%=0.6, $q = 1 - p = 1 - 0.6 = 0.4\ , n = 5$

Probability the the utility bill is reduced by one-third in x installation out of 5 installations

$$P(X = x) = nCxp^x q^{n-x}.$$

(i) Probability that the utility bill is reduced by one-third in 4 of 5 installation**s**

$$P(X = 4) = 5C4(0.6)^4(0.4)^1 = 162/625.$$

(ii) Probability that the utility bill is reduced by one third in at least 4 of 5 installation

$$P(X \geq 4) = P(X = 4) + P(X = 5) = \frac{1053}{3125} = 0.337.$$

Tutorial	
1.	If 10% of the screws produced by a machine are defective, find the probability that out of 5 screws chosen at random (i) none is defective, (ii) one is defective and (iii) at most two are defective. **Ans (i) $\mathbf{0.5905}$ (ii) $\mathbf{0.32805}$ (iii) $\mathbf{0.9914}$**
2.	Two dice are thrown five times. Find the probability of getting the sum as 7 (i) at least once (ii) two times. **Ans (i) $\boldsymbol{\frac{4651}{7776}}$ (ii) $\boldsymbol{\frac{625}{3888}}$**
3.	If the probability of a defective bolt is $\frac{1}{8}$, find the mean and variance for the distribution of 640 defective bolts. **Ans 70**

Poisson Probability Distribution: It is a limiting case of binomial distribution under the following conditions:

(i) The number of trials should be infinitely large.

(ii) The probability of successes p for each trial should be very small.

(iii) $np = \lambda$ should be finite, mean of poisson distribution where λ is a constant.

Formula for Poisson distribution if the probability of x is given by

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} x = 01,2,3, \dots \dots$$

Remark: $\boldsymbol{nCxp^x q^{n-x} \simeq \frac{e^{-np}(np)^x}{x!}}$.

Example: A manufacturer of cotterpins knows that 5% of his products are defective. If he sells cotterpins in boxes of 100 and guarantees that not more that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality?

Solution: Let p be the probability of a pin being defective.

p=5%=0.05, n=100.

Since p is very small and n is large, Poisson distribution is used.

$\lambda = np = 100 \times 0.05 = 5$.

Let X be the random variable which denotes the number of defective pins in a box of 100.

Probability of x defective pins in a box of 100

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-5}5^x}{x!}, x = 0,1,2,3, \dots \dots$$

Probability that a box will fail to meet the guarantee quality

$$P(X > 10) = 1 - P(X \leq 10)$$

$P(X > 10) = 1 - \sum_{x=0}^{10} \frac{e^{-5}5^x}{x!} = 0.0137.$

Tutorial	
1.	If 2% of light bulbs are defective, find the probability that (i) at least one is defective and (ii) exactly 7 are defective in a sample of 100. **Ans (i) $\boldsymbol{0.8647}$ (ii) $\boldsymbol{0.0034}$**
2.	Six coins are tossed 6400 times. Using the Poisson distribution, what is the approximate probability of getting six heads 10 times? **Ans (i) $\boldsymbol{\frac{e^{-100}100^{10}}{10!}}$**
3.	A manufacturer, who produces medicine bottle, finds that 0.1% of the bottle are defective .The bottle are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottle. Using Poisson distribution, find the probability of getting no defective bottles. **Ans 0.6065**

6.3 Continuous Probability Distributions: Exponential and Normal

Probability density function :

Let $f(x)$be a continuous function, then Mean=$\int_{-\infty}^{\infty} x.f(x)\,dx$

and Variance=$\int_{-\infty}^{\infty} (x - \bar{x})^2.f(x)\,dx, \bar{x} = mean$

$f(x)$ of continuous random variable x is called probability density function if

(i) $f(x) \geq 0$ for every values of x.

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(iii) $\int_a^b f(x)\,dx = P(a < x < b)$

Tutorial	
1.	A function $f(x)$ is defined as follows $f(x) = \begin{cases} 0\,, & x < 2 \\ \frac{1}{18}(2x+3)\,, & 2 \le x \le 4 \\ 0\,, & x > 4 \end{cases}$. Show that it is a probability density function.
2.	A continuous random variable has p.d.f. $f(x) = \begin{cases} ke^{-\frac{x}{5}}, x \ge 0 \\ 0\,, \ elsewhere \end{cases}$ then find the value of k. **Ans** $\frac{1}{5}$

Example The probability density function $f(x)$ of a continuous random variable x is defined by $f(x) = \begin{cases} \frac{A}{x^3}, & 5 \le x \le 10 \\ 0, & otherwise \end{cases}$. Find the value of A.

Solution: Here $f(x) = \frac{A}{x^3},\ 5 \le x \le 10$

Since $f(x)$ is probability density function, so

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\therefore \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{b} f(x)\,dx + \int_{b}^{\infty} f(x)\,dx = 1$

$\int_{5}^{10} \frac{A}{x^3}\,dx = 1 \Rightarrow \left[-\frac{A}{2x^2}\right]_5^{10} = 1$

A=200/3

Exponential Distribution: The continuous random variable X has exponential distribution if its density function is given by

$$f_X(x) = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}} & , x > 0, \beta > 0 \\ 0 & , elsewhere \end{cases}$$

Example: **The probability density function of the length of the time that a person speaks over phone is**

$$\boldsymbol{f_X(x) = \begin{cases} \frac{1}{6} e^{-\frac{x}{6}} & , x > 0 \\ 0 & , \ otherwise \end{cases}}.$$

What is the probability that the person will talk for (a) more than 8 minutes (b) less than 4 minutes (c) between 4 and 8 minutes?

Solution: $\int_0^{\infty} f_X(x)dx = \int_0^{\infty} \frac{1}{6} e^{\frac{-x}{6}}\,dx,$

(a) $P(X > 8) = \int_8^{\infty} \frac{1}{6} e^{-\frac{x}{6}} dx = 0.2635.$

(b) $P(X < 4) = \int_0^{4} \frac{1}{6} e^{-\frac{x}{6}} dx = 0.4865.$

(c) $P(4 \le X \le 8) = \int_4^{8} \frac{1}{6} e^{-\frac{x}{6}} dx = 0.5134 - 0.2635 = 0.2499.$

Tutorial	
1.	In a construction site, 3 lorries unload materials per hour, on an average. The probability density function is given by $f_X(x) = \begin{cases} 3e^{-3x} & ,x > 0 \\ 0 & ,\ otherwise \end{cases}$. What is the probability that the time between arrivals of successive lorries will be (i) at least 30 minutes (ii) less than than 10 minutes? **Ans (i) 0.223 (ii) 0.393**

Normal Distribution:

The most important continuous probability distribution in the entire field of statistics is the normal distribution. Its graph, called the normal curve, is the bell-shaped curve of Figure, which approximately describes many phenomena that occur in nature, industry, and research.
For example, physical measurements in areas such as meteorological experiments, rainfall studies, and measurements of manufactured parts are often more than adequately explained with a normal distribution. In addition, errors in scientific measurements are extremely well approximated by a normal distribution.
A continuous random variable X having the bell-shaped distribution (refer figure (a)) is called a **normal random variable. (See the table of Normal distribution for values) on last page)**

The mathematical equation for the probability distribution of the normal variable depends on the two parameters mean (μ) and standard deviation (σ). Hence, we denote the values of density of X by $n(x; \mu, \sigma)$.

Normal Distribution: The density of the normal random variable X, with mean μ and variance σ^2, is given by

$$n(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < \mu < \infty, \sigma > 0,$$

Where $\pi = 3.14159 \ldots$ and $e = 2.71828 \ldots ..$

Note: once μ and σ are specified, the normal curve is completely determined.

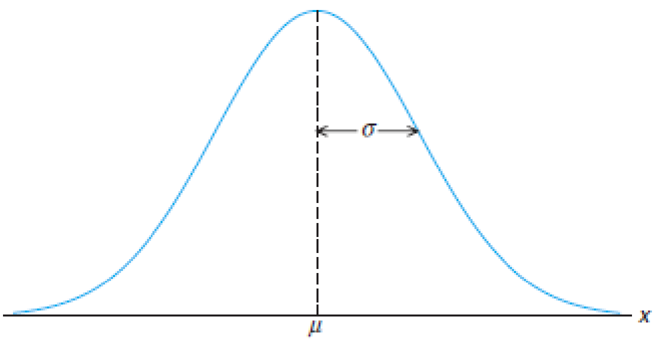


Figure (a) The normal curve

Properties of the Normal distribution (normal curve):

(i)Normal curve is a bell-shaped symmetrical curve about the ordinate $X = \mu$. The ordinate is maximum at $X = \mu$.
(ii)The curve is symmetrical about the y-axis. The mean, median and mode coincide at origin.
(iii)The normal curve approaches the horizontal axis asymptotically as we proceed in either direction away from the mean.
(iv)The total area under the curve and above the horizontal axis is equal to 1 (one). It gives the total probability of the random variable X taking values between $-\infty$ to ∞.

Mathematically, $P(-\infty < X < \infty) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 1$

(v)The ordinate at $X = \mu$ divides the area under the normal curve into two equal parts, i.e., $\int_{-\infty}^{u} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \int_{u}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{2}$.

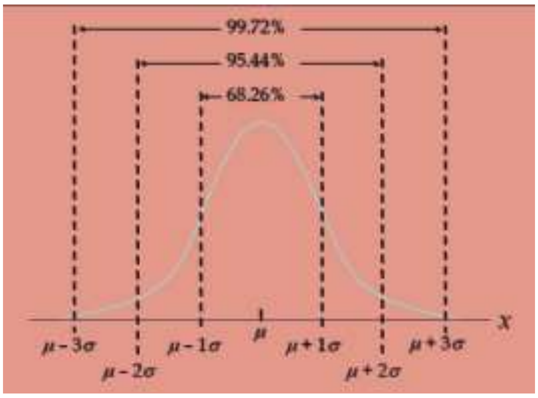
(vi)The curve has its points of inflection (the point at which curvature changes) at $x = \mu \pm \sigma$.

(vii) Probabilities lies in normal curve as follows:

a) $P\,(\mu - \sigma < x < \mu + \sigma) = 68\ \%$
(i.e. about 68% of the values will lie between $(\mu - \sigma)$ and $(\mu + \sigma)$)

b) $P\,(\mu - 2\sigma < x < \mu + 2\sigma) = 95.5\%$
(i.e. about 95% of the values will lie between $(\mu - 2\sigma)$ and $(\mu + 2\sigma)$)

c) $P\,(\mu - 3\sigma < x < \mu + 3\sigma) = 99.7\ \%$
(i.e. about 99.7% of the values will lie between $(\mu - \sigma)$ and $(\mu + \sigma)$)



Probability of a Normal random variable in an interval:

Let X be a normal random variable with mean (μ) and standard deviation (σ). The probability of X lying in the interval (x_1, x_2) is given by

$P(x_1 < X < x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$

Hence, the probability is equal to the area under the normal curve between the ordinates $X = x_1$ and $X = x_2$ respectively.

$P(x_1 < X < x_2)$ can be evaluated easily by converting a normal random variable into another random variable.

Let $z = \frac{x-\mu}{\sigma}$ be a new random variable. The distribution of Z is also normal.

Thus if X be a normal random variable with mean (μ) and standard (σ) then $z = \frac{x-\mu}{\sigma}$ is also normal random variable with mean 0 and standard deviation is 1.

Since the parameters of the distribution of Z are fixed, it is known as standard normal distribution.

$$P(x_1 \le X \le x_2) = P\left[\left(\frac{x_1 - \mu}{\sigma}\right) \le X \le \left(\frac{x_2 - \mu}{\sigma}\right)\right] = P(z_1 \le Z \le z_2)$$

Where $z_1 = \frac{x_1-\mu}{\sigma}$ and $z_2 = \frac{x_2-\mu}{\sigma}$.

This probability is equal to the area under the standard normal curve between the ordinates Z= z_1 and Z= z_2 respectively.

Example: The manufacturer knows from his experience that the resistance of the resistors he produces is normal with $\mu = 100\, ohms$ and $SD = \sigma = 2\ ohms$ What percentage of resistors will have resistance between 98 ohms and 102 ohms?

Solution: Let X be the random variable which denotes the resistance of the resistors.

$\mu = 100\, ohms$ and $SD = \sigma = 2\ ohms$.

$$Z = \frac{X - \mu}{\sigma}.$$

When X=98 , $Z = \frac{98-100}{2} = -1$.

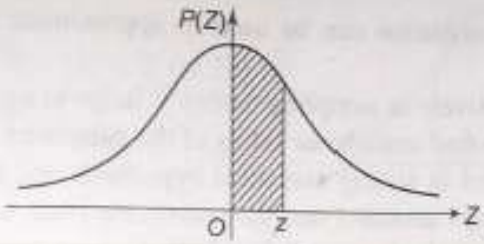
When X=102, $Z = \frac{102-100}{2} = 1$.

$$\begin{aligned} P(98 \le X \le 102) &= P(-1 \le Z \le 1) \\ &= P(-1 \le Z \le 0) + P(0 \le Z \le 1) \\ &= P(0 \le Z \le 1) + P(0 \le Z \le 1)(By Symmetry) \\ &= 2P(0 \le Z \le 1) = 2(0.3413) = 0.6826 \end{aligned}$$

Hence the percentage of resistors have resistance between 98 ohms and 102 ohms=68.26%.

Tutorial	
1.	X is normally distributed and the mean of X is 30 and SD is 5.Find the probability of the following : (i) $X \geq 45$ (ii) $26 \leq X \leq 40$. **Ans (i) 0.7653 (ii) 0.0013**
2.	The average seasonal rainfall in a place is 16 inches with an SD of 4 inches. What is the probability that the rainfall in that place will be between 20 and 24 inches in a year? $\boldsymbol{Ans : 0.1359}$
3.	The daily sales of a firm are normally distributed with mean of Rs 8000 and variance of Rs 10000.What is the probability that on a certain day the sales will be less than Rs 8210? **Ans (i) 0.482**

Standard Normal (Z) Table, Area between 0 and z



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3990	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4115	0.4131	0.4147	0.4162
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990