$f(X) \in \mathbb{R}^d$

Laplace $\left(0, \frac{\Delta}{\epsilon}\right)$ in all components independenty of each other.

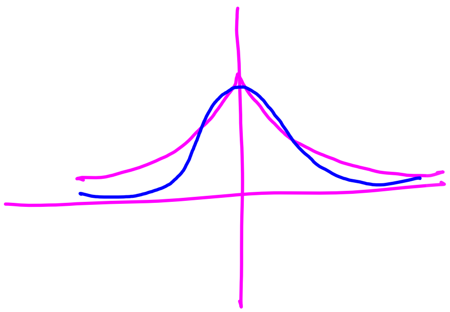
$$\Delta = \max_{\substack{x, x' \\ \text{neighbouring.}}} \| f(x) - f(x') \|_1 = \sum_{i=1}^{d} \left| (f(x))_i - (f(x'))_i \right|$$

Approximate DP

$M : \mathcal{X}^n \rightarrow \mathcal{Y}$ is (ϵ, δ) approximately DP if $\forall$ neighbouring $x, x' \in \mathcal{X}^n$, and all $S \subseteq \mathcal{Y}$

$$P(M(x) \in S) \le e^{\epsilon} P(M(x') \in S) + \delta$$

To achieve this, use the Gaussian Mechanism



Add Gaussian noise $N\left(0, \ln\left(\frac{1}{\delta}\right)\frac{\Delta^2}{\epsilon^2}\right)$

For high dimensional outputs,

$$N\left(0, \ln\left(\frac{1}{\delta}\right)\frac{\Delta_2^2}{\epsilon^2}\right)$$

L2- sensitivity.

$$\Delta_2 = \max_{\substack{x, x' \\ \text{neighbouring}}} \|f(x) - f(x')\|_2 = \sqrt{\sum_{i=1}^{d} (f(x)_i - f(x')_i)^2}$$

We gain in terms of utility

Laplacian noise $\quad Lap\left(0, \frac{\Delta}{\epsilon}\right) \quad Lap\left(0, \frac{d}{n\epsilon}\right)$

$\sum_{i=1}^{d} \frac{1}{n} = \frac{d}{n}$

Gaussian noise $\quad \mathcal{N}\left(0, \frac{\sqrt{d \ln(1/\delta)}}{n\epsilon}\right)$

Properties of ADP

- POST PROCESSING

Say M is (ϵ, δ) DP

$F \circ M$ is (ϵ, δ) DP

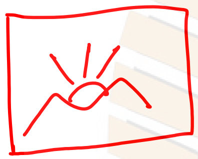
COMPOSITION

$\{M_1, M_2, \ldots, M_k\}$ are all (ϵ, δ) DP

Basic: $(k\epsilon, k\delta)$ DP

Advanced: $\left(\epsilon\sqrt{k \log\left(\frac{1}{\delta'}\right)} + \epsilon\left(e^{\epsilon}-1\right),\ k\delta + \delta'\right)$ - DP

Digital good



Buyers n

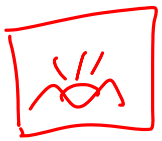
$\boxed{\text{Valuations} \quad v_1, \ldots, v_n}$

How to set price?

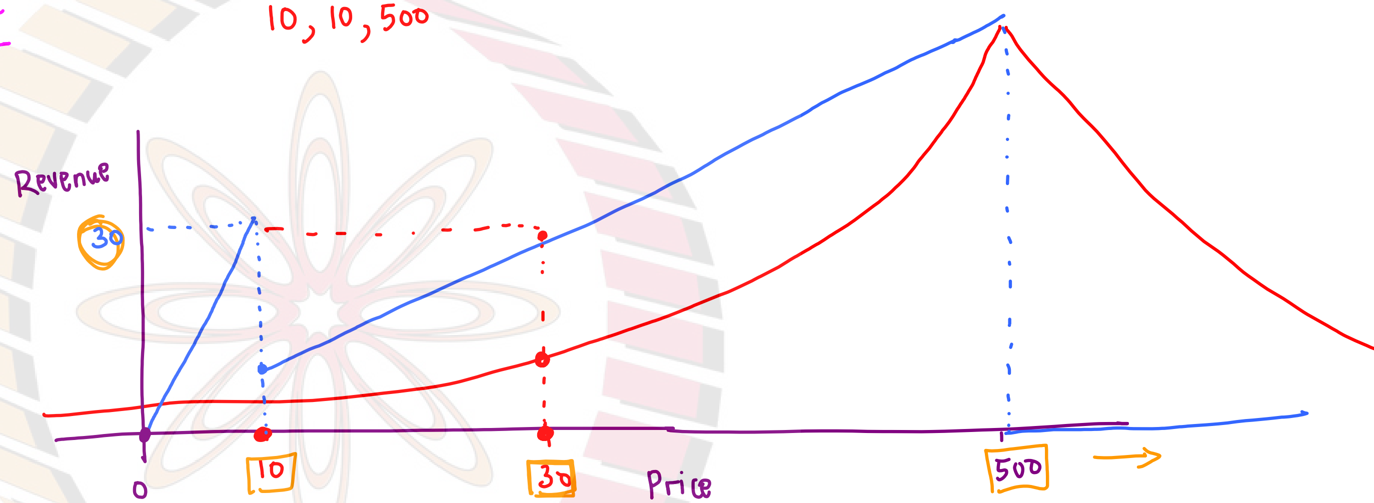
p

$$p\left(\underbrace{\sum_{i=1}^{n} \overset{\text{Indicator}}{\mathbb{1}}(v_i \geq p)}_{\hookrightarrow \ \#\text{Buyers whose valuation} \geq p.}\right)$$

Example



10, 10, 500



EXPONENTIAL MECHANISM (2012, Talwar et.al)

$(10, 10, 500)$

$x \in \mathcal{X}^n$ (valuations)

$\mathcal{H}$ (set of prices)

$S : (\mathcal{X}^n, \mathcal{H}) \to \mathbb{R}$

Revenue function. $\boxed{p \sum_{i=1}^{n} \mathbb{1}(v_i \geq p)}$

$$\Delta = \max_{h \in \mathcal{H}} \max_{x, x' \in \mathcal{X}^n} \left| s(x, h) - s(x', h) \right|$$

Select $h \in \mathcal{H}$ with probability proportional to $\boxed{e^{\frac{\epsilon}{2\Delta} s(x, h)}}$

EM is ϵ-DP

Utility

$$P\left(\underbrace{S\big(x, \underbrace{EM(x)}_{\text{Price}}\big)}_{\text{Revenue}} \le \underbrace{OPT(x)}_{\text{Maximum revenue that could be achieved.}} - \left(\underbrace{\frac{2\Delta}{\epsilon}\left(\ln(|R|) + t\right)}\right)\right) \le e^{-t}$$