Proof by Mathematical Induction

PROVE :- $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

Basis : $n = 1$

$$1 = \frac{1(1+1)}{2}$$

$$1 = \frac{1(2)}{2}$$

$$1 = \frac{2}{2}$$

$$1 = 1$$

L.H.S = R.H.S

INDUCTION :- ASSUME TRUE $n = k$

$$1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}$$

show true $n = k+1$

$$\underbrace{1 + 2 + 3 + \cdots k} + k+1 = \frac{k+1\,(k+1+1)}{2}$$

$$\frac{k(k+1)}{2} + k+1 = \frac{(k+1)(k+2)}{2}$$

$$\frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

$$\frac{(k+2)\ (k+1)}{2} = \frac{(k+1)\ (k+2)}{2}$$

L.H.S = R.H.S

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$$1^2 + 2^2 + 3^2 + \dots n^2 = \frac{n(n+1)(2n+1)}{6}$$

Basis: for $n = 1$

$1^2 = 1$ $= \frac{1 * (1+1)(2(1)+1)}{6}$

L.H.S

$= \frac{1 * 2 * 3}{6}$

$= \frac{6}{6} = 1$ R.H.S

Induction: Assume true $n = k$

$$1^2 + 2^2 + 3^2 + \dots k^2 = \frac{k * (k+1)(2k+1)}{6}$$

Show true $\underline{n = k+1}$

$$\underbrace{1^2 + 2^2 + 3^2 + \dots + k^2} + (k+1)^2 =$$

$$\frac{(k+1)\ ((k+1)+1)(2(k+1)+1)}{6}$$

R.H.S

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= (k+1)\left[\frac{k(2k+1)}{6} + (\cancel{k}+1)\right]$$

$$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$$

$$= (k+1)\left[\frac{2k^2+7k+6}{6}\right]$$

$$= (k+1)\left[\frac{2k^2+4\cancel{k}+3\cancel{k}+6}{6}\right]$$

$$= (k+1)\left[\frac{2k(k+2)+3(k+2)}{6}\right]$$

$$= (k+1)\left[\frac{(2k+3)(k+2)}{6}\right]$$

$$= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

L.H.S