<u>Unit III</u>

3.	**Partial Differential Equations and Applications:**
3.1	Formation of Partial Differential Equation
3.2	Lagrange's Linear Differential Equations
3.3	Special types of Nonlinear First Order Partial Differential Equation

Practice work examples

Derive a partial differential equation (by eliminating arbitrary constants) from the following equations:

1. $z = ax + by + ab$
2. $z = ax + (1 - a)y + b$
3. $z = axe^y + \left(\frac{1}{2}\right)a^2 e^{2y} + b$
4. $(x - a)^2 + (y - b)^2 + z^2 = 1$
5. $z = ax + by + a^2 + b^2$

Derive a partial differential equation (by eliminating arbitrary functions) from the following relations:

1. $z = f(x^2 - y) + g(x^2 + y)$
2. $z = f(x^2 - y^2)$
3. $z = xy + f(x^2 + y^2)$
4. $f(x + y + z, x^2 + y^2 - z^2) = 0$

<u>Lagrange's Equation:</u>

Solve the following Lagrange's equation:

1. $xyp + y^2 q = zxy - 2x^2$
2. $z(x + y)p + z(x - y)q = x^2 + y^2$
3. $(\tan x)p + (\tan y)q = \tan z$
4. $(x^2 - y^2 - z^2)p + 2xy\, q = 2xz$

Case-1: Equations involving only p and q, that is, $f(p, q) = 0$

Solve the following Partial differential equations:

1. $p = e^q$
2. $p^2 + 6p + 2q + 4 = 0$
3. $p = q^2$

Case 2: Equations not involving the Independent variables, that is, $f(z, p, q) = 0$

Solve the following Partial differential equations:

1. $p^3 + q^3 - 3pqz = 0$
2. $9(p^2z + q^2) = 4$
3. $p^2 = qz$

Case 3: Separable Equations, that is, $f(x, p) = g(y, q)$

Solve the following Partial differential equations:

1. $p - 3x^2 = q^2 - y$
2. $q(p - \cos x) = \cos y$
3. $yp + xq = pq$

Case-4: Clairaut's Equation, that is, $z = px + qy + f(p, q)$

Solve the following Partial differential equations:

1. $(pq - p - q)(z - px - qy) = pq$
2. $(px + qy - z)^3 = 27pq$
3. $z = px + qy + \frac{q}{p} - p$