CHAROTAR UNIVERSITY OF SCIENCE AND TECHNOLOGY

CE245-Data Structure and Algorithms

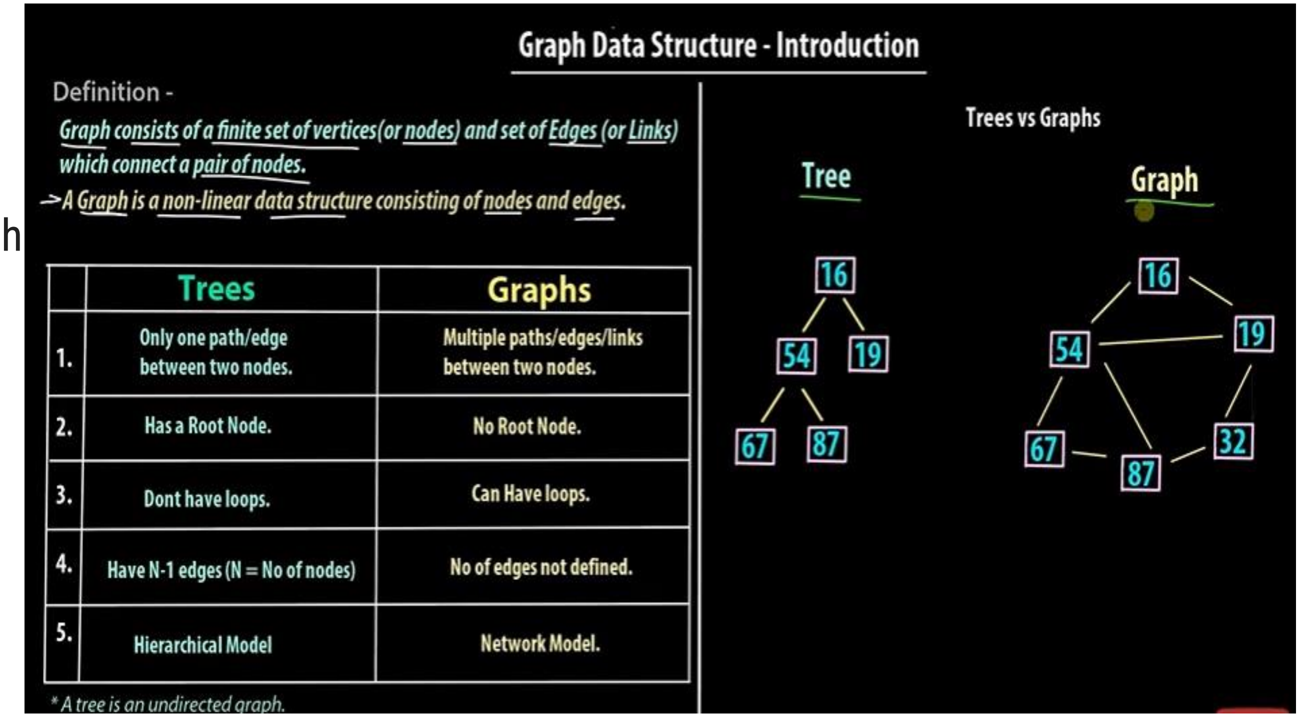
Unit- 3

Non-Linear Data Structure
Graph

Devang Patel Institute of Advance Technology and Research

Graphs

- What is Graph?
- Representation of Graph
  - Matrix representation of Graph
  - Linked List representation of Graph
- Elementary Graph Operations
  - Breadth First Search (BFS)
  - Depth First Search (DFS)
  - Spanning Trees
  - Minimal Spanning Trees
  - Shortest Path



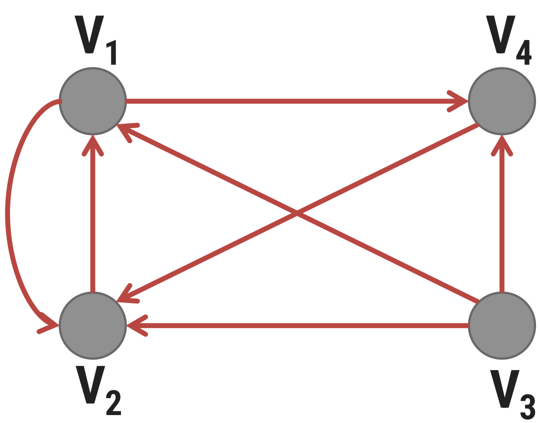
Adjacency matrix

- A **diagrammatic representation** of a **graph** may have limited usefulness. However such a representation **is not feasible** when number of **nodes** an **edges** in a graph **is large**
- It is easy to store and manipulate matrices and hence the graphs represented by them in the computer
- Let **G = (V, E)** be a simple **diagraph** in which **$V = \{v_1, v_2, \ldots, v_n\}$** and the **nodes** are assumed to be **ordered** from **v_1** to **v_n**
- An n x n matrix **A** is called **Adjacency matrix** of the graph G whose **elements** are **a_{ij}** are given by

$$a_{ij} = \begin{cases} 1 & if (V_i, Vj) \in E \\ 0 & otherwise \end{cases}$$

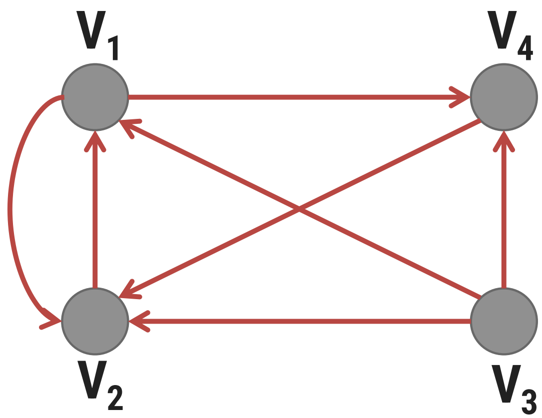
Adjacency matrix

- An **element** of the adjacency matrix is either **0** or **1**
- Any **matrix** whose **elements are either 0 or 1** is called **bit matrix** or **Boolean matrix**
- For a given graph G =m (V, E), an **adjacency matrix** depends upon the ordering of the elements of V
- For different ordering of the elements of V we get different adjacency matrices.



$$A = \begin{array}{c c} & \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} & \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Adjacency matrix



$$A = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline V_1 & 0 & 1 & 0 & 1 \\ V_2 & 1 & 0 & 0 & 0 \\ V_3 & 1 & 1 & 0 & 1 \\ V_4 & 0 & 1 & 0 & 0 \end{array}$$

- The **number of elements** in the **i^{th} row** whose **value is 1** is equal to the **out-degree** of node V_i
- The **number of elements** in the **j^{th} column** whose **value is 1** is equal to the **in-degree** of node V_j
- For a **NULL graph** which consist of only n nodes but no edges, the **adjacency matrix** has **all its elements 0**. i.e. the adjacency matrix is the NULL matrix

Power of Adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A^2 = A \times A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 3 & 0 & 2 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

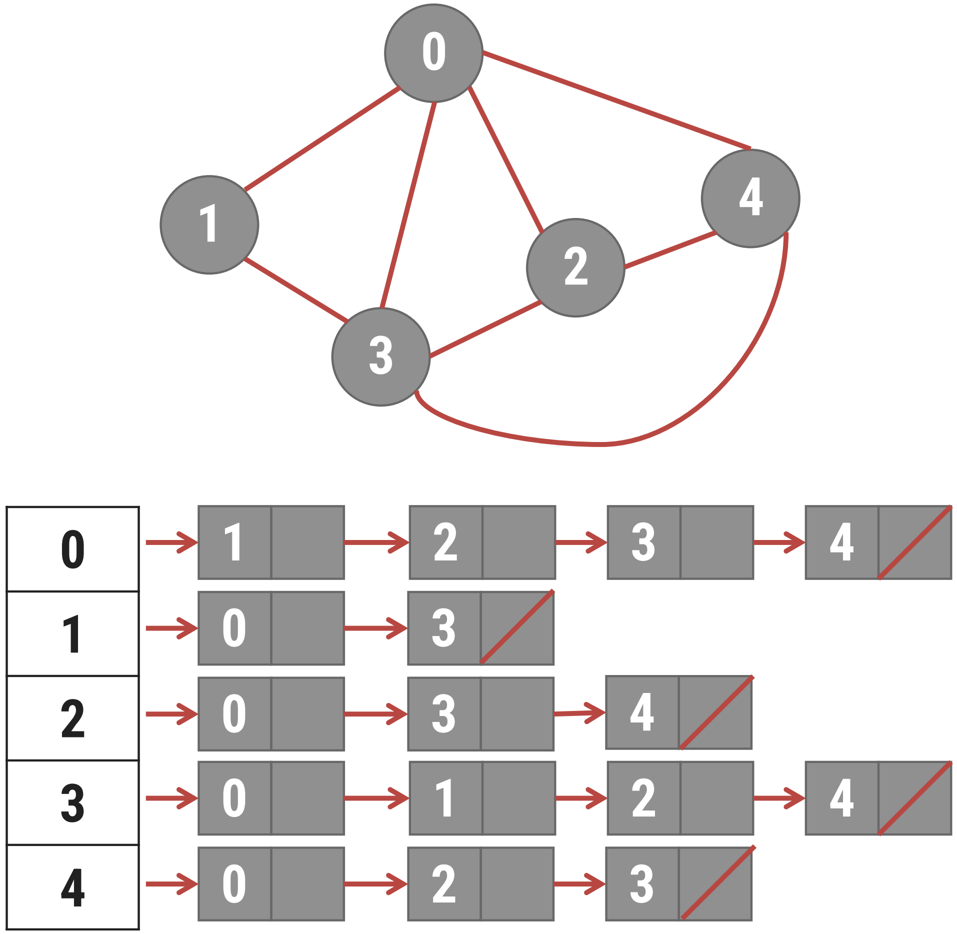
- Entry of **1** in **i^{th}** row and **j^{th}** column of **A** shows existence of an **edge (V_i, V_j)**, that is a **path of length 1**
- Entry in **A^2** shows **no of different paths** of **exactly length 2** from node **V_i** to **V_j**
- Entry in **A^3** shows **no of different paths** of **exactly length 3** from node **V_i** to **V_j**

Path matrix or reachability matrix

- Let **G = (V,E)** be a simple diagraph which contains **n nodes** that are assumed to be ordered.
- A **n x n** matrix **P** is called **path matrix** whose elements are given by

$$P_{ij} = \begin{cases} 1, if\ there\ exists\ path\ from\ node\ V_i\ to\ V_j \\ 0, otherwise \end{cases}$$

Adjacency List Representation



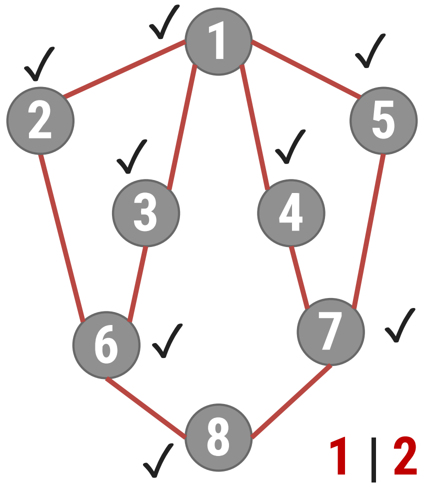
Graph Traversal

- Two Commonly used Traversal Techniques are
  - Breadth First Search (BFS)
  - Depth First Search (DFS)

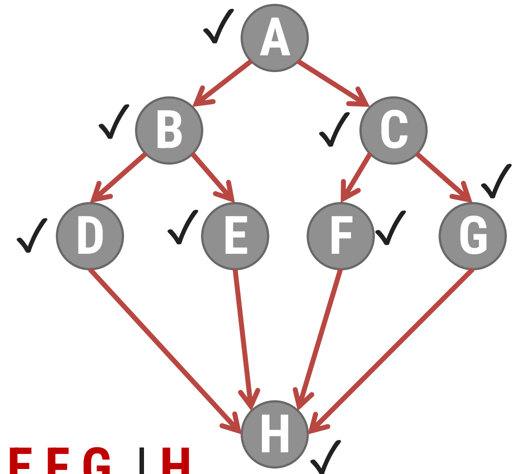
Breadth First Search (BFS)

- This methods **starts** from vertex $\mathbf{V_0}$
- $\mathbf{V_0}$ is marked as **visited**. All **vertices adjacent to $\mathbf{V_0}$** are **visited next**
- Let vertices adjacent to V_0 are V_1, V_2, V_2, V_4
- V_1, V_2, V_3 and V_4 are marked visited
- All unvisited vertices adjacent to V_1, V_2, V_3, V_4 are visited next
- The method **continuous until all vertices** are **visited**
- The algorithm for BFS has to maintain a list of vertices which have been visited but not explored for adjacent vertices
- The vertices which have been visited but not explored for adjacent vertices can be stored in **queue**

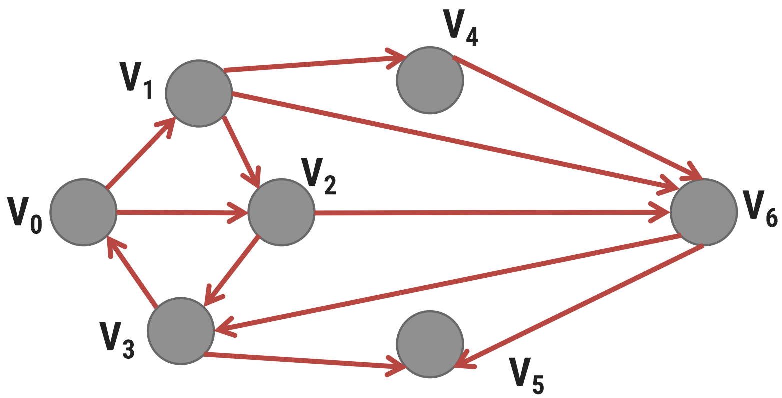
Breadth First Search (BFS)



1 | 2 3 4 5 | 6 7 | 8



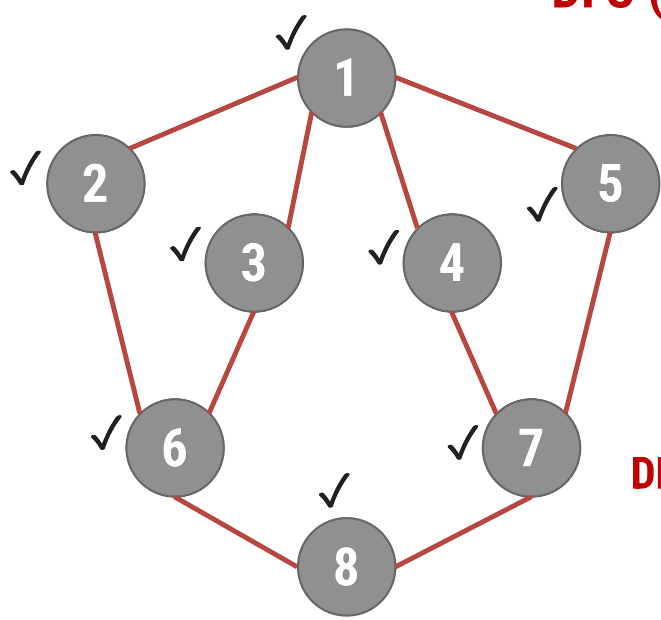
A | B C | D E F G | H



V_0 | V_1 V_2 | V_4 V_6 V_3 | V_5

Depth First Search (DFS)

- It is like preorder traversal of tree
- Traversal can start from any vertex V_i
- V_i is visited and then all vertices adjacent to V_i are traversed recursively using DFS



DFS (G, 1) is given by

Step 1: Visit (1)

Step 2: DFS (G, 2) → **DFS (G, 2):**
DFS (G, 3)
DFS (G, 4)
DFS (G, 5)

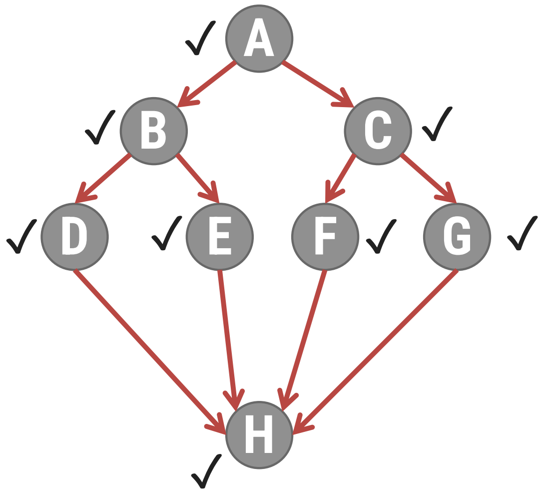
DFS (G, 2):
Step1: Visit(2)
Step 2: DFS (G, 6) → **DFS (G, 6):**

DFS (G, 6):
Step1: Visit(6)
Step 2: DFS (G, 3)
DFS (G, 8)

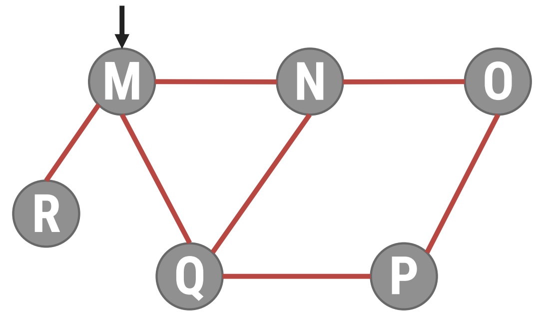
DFS of given graph starting **from** node **1** is given by

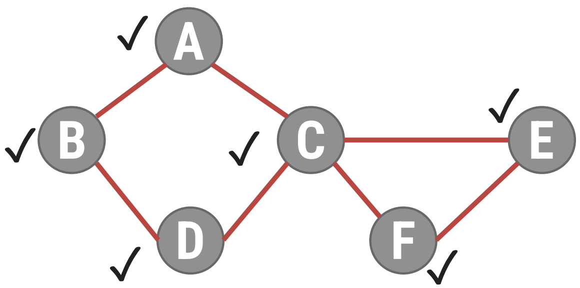
1 2 6 3 8 7 4 5

Depth First Search (DFS)



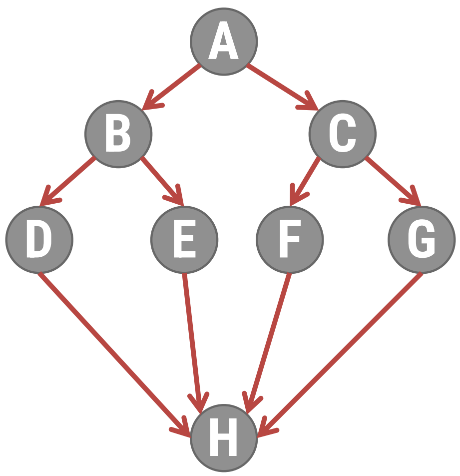
A B D H E C F G

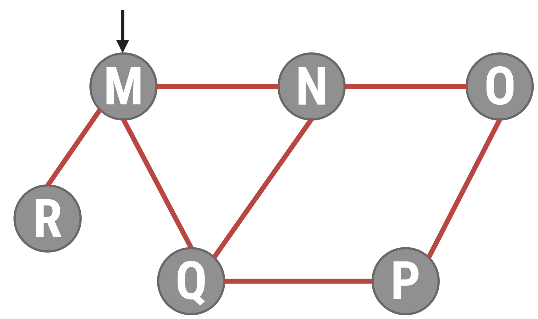


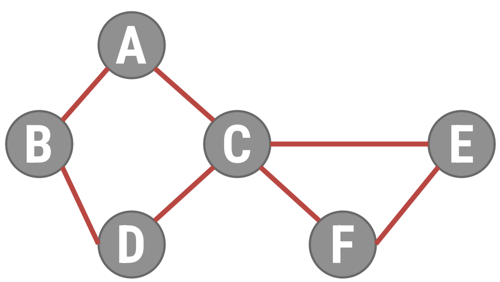


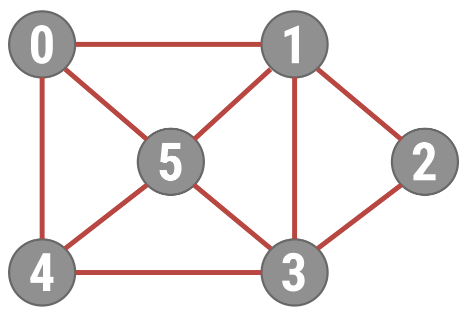
A B D C F E

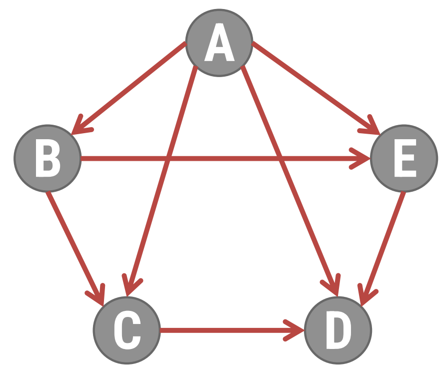
Write DFS & BFS of following Graphs











Procedure : DFS (vertex V)

- This procedure **traverse the graph G in DFS** manner.
- V is a starting vertex to be explored.
- Visited[] is an array which tells you whether particular vertex is visited or not.
- W is a adjacent node of vertex V.
- S is a Stack, PUSH and POP are functions to insert and remove from stack respectively.

Procedure : DFS (vertex V)

```
1. [Initialize TOP and Visited]
   visited[] ← 0
   TOP ← 0
2. [Push vertex into stack]
   PUSH (V)
3. [Repeat  while stack is not Empty]
   Repeat Step 3 while stack is not empty
       v  ← POP()
       if   visited[v] is 0
       then visited [v] ← 1
            for all W adjacent to v
               if   visited [w] is 0
              then PUSH (W)
            end for
       end if
```

Procedure : BFS (vertex V)

- This procedure **traverse the graph G in BFS** manner
- **V** is a **starting vertex** to be explored
- Q is a queue
- visited[] is an array which tells you whether particular vertex is visited or not
- W is a adjacent node f vertex V.

Procedure : BFS (vertex V)

```
1. [Initialize Queue & Visited]
   visited[] ← 0
   F ← R ← 0
2. [Marks visited of V as 1]
   visited[v] ← 1
3. [Add vertex v to Q]
   InsertQueue(V)
4. [Repeat while Q is not Empty]
   Repeat while Q is not empty
     v ← RemoveFromQueue()
     For all vertices W adjacent to v
       If   visited[w] is 0
      Then visited[w] ← 1
           InsertQueue(w)
```