

Parallel and Distributed Computing

First Project

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1 Introduction

This report presents the algorithms, performance results and respective analysis of the first project in the Parallel and Distributed Computing course. The objective of this project was to evaluate and compare the performance of different memory access patterns and parallelization techniques.

Matrix multiplication was chosen as the central problem due to its relative simplicity and its suitability for testing various performance-related aspects. In particular, we examine how operating systems and hardware handle memory access. Specifically, most CPUs employ techniques such as caching, where data is temporarily stored closer to the CPU for faster access, and prefetching, where data is retrieved in contiguous blocks (cache lines). Consequently, when one element is accessed, the processor often fetches adjacent elements as well and keeps them cached. To observe this effect, we implemented three single-core algorithms. Additionally, to explore how different parallelization strategies can improve performance, we also implemented two multi-core algorithms (see Algorithms Explanation section).

By developing the code for these tasks, we became more familiar with modern and high performance C++ and Lua, since we chose it as our alternative implementation language. Moreover, we also had to research about matrix multiplication algorithms and, in order to implement multi-core solutions, we explored and experimented with the OpenMP standard, allowing us to learn more about CPU multithreading.

2 Algorithms Explanation

To explore how cache locality and parallel computing affect matrix multiplication, we implemented five different algorithms. Although cache-related behavior is largely independent of the programming language, we additionally developed Lua implementations to demonstrate that performance differences arise from algorithmic choices rather than language specifics. In spite of these algorithms performing the same operations and producing the same result, the order of operations can significantly affect memory access patterns and thus performance.

2.1 Naive Matrix Multiplication

The *naive* matrix multiplication algorithm is closely related to its hand calculation. This method for multiplication of two matrices, $A \times B = C$, where A is a matrix $m \times n$, B is $n \times p$ and C is $m \times p$, obtains each cell (i, j) in the solution by doing the dot product of row i of A with column j of B .

Implementing this algorithm is fairly straightforward, as we only need to iterate through every row and column of the solution matrix, and, for each cell, perform the linear combination of the values as shown in Figure 1.

As we can expect from the 3 nested loops, the overall time complexity for the matrix multiplication algorithm is $\Theta(n^3)$. This time complexity is maintained among

```
for (i = 0; i < m; i++)
  for (j = 0; j < p; j++)
    for (k = 0; k < n; k++)
      C[i][j] += A[i][k] * B[k][j];
```

Figure 1: Overview of Naive Matrix Multiplication Algorithm

all the implemented algorithms, therefore performance impacts are mainly caused by cache accesses and parallelism.

2.2 Line Matrix Multiplication

The line matrix multiplication algorithm is very similar to the previous algorithm. The difference is that, instead of calculating the result matrix cell by cell, we instead focus on calculating and storing some terms of the dot product line by line. Since matrices are stored in a row-major format, accessing data consecutively along a row can improve cache locality, and, consequently reduce execution time.

The only difference is that the second and third loops switch places, as shown in figure 2.

```
for (i = 0; i < m; i++)
  for (k = 0; k < n; k++)
    for (j = 0; j < p; j++)
      C[i][j] += A[i][k] * B[k][j];
```

Figure 2: Overview of Line Matrix Multiplication Algorithm

2.3 Block Matrix Multiplication

The last single-core algorithm implemented is the block matrix multiplication algorithm. This approach expands on top of the previous one and the same logic applied to calculate sub-results line by line is now used to calculate the solution block-by-block. The main idea is that, now, we have divided the matrices into smaller blocks and the operations are performed block by block. This approach can further enhance cache locality by keeping calculations within a more localized area for both matrices, potentially improving overall efficiency.

For this, we need 6 nested loops as shown in Figure 3.

```
for (I = 0; I < m; I += block_size)
  for (K = 0; K < n; K += block_size)
    for (J = 0; J < p; J += block_size)
      for (i = I; i < min(I + block_size, m); i++)
        for (k = K; k < min(K + block_size, n); k++)
          for (j = J; j < min(J + block_size, p); j++)
            C[i][j] += A[i][k] * B[k][j];
```

Figure 3: Overview of Block Matrix Multiplication Algorithm

2.4 Parallel Matrix Multiplication

Besides the previous three single-core algorithms, we also implemented two variations of the line algorithm using the OpenMP API for parallel programming in C++. The next sections show an explanation and overview of the implementations.

2.4.1 First Version

The first approach aims to parallelize the outermost loop i of the line algorithm. There will be a division of i values between the different threads so that each one works on separate rows at the same time (they also get their own k and j variables). For this approach, the threads all run independently and then synchronize once, when the loop finishes.

For this, we used OpenMP's `parallel for` and `private variable` pragmas, as shown in Figure 4.

```
#pragma omp parallel for private(i, k, j)
for (i = 0; i < m; i++)
    for (k = 0; k < n; k++)
        for (j = 0; j < p; j++)
            C[i][j] += A[i][k] * B[k][j];
```

Figure 4: Overview of First Parallel Matrix Multiplication Algorithm

2.4.2 Second Version

The second approach aims to parallelize the innermost loop j of the line algorithm. The thread pool will work on the same row at the same time but on different columns. In this case, the threads need to be synchronized every time the inner loop finishes, which happens $i \times k$ times, leading to some overhead.

For this, we used OpenMP's `parallel region`, `for` and `private variable` pragmas, as it can be seen in Figure 5.

```
#pragma omp parallel private(i, k)
for (i = 0; i < m; i++)
    for (k = 0; k < n; k++)
        #pragma omp for private(j)
        for (j = 0; j < p; j++)
            C[i][j] += A[i][k] * B[k][j];
```

Figure 5: Overview of Second Parallel Matrix Multiplication Algorithm

3 Performance Metrics

The performance of the algorithms described in the previous sections was evaluated using the following metrics:

- **Time:** The time required to execute the algorithm. This is a crucial metric for comparing the speed and efficiency of multi-core implementations.
- **Speedup:** Defined by the ratio of the sequential execution time T_{seq} to the parallel execution time T_{par} :

$$\text{Speedup} = \frac{t_{\text{seq}}}{t_{\text{par}}}.$$

This indicates how much faster the parallel algorithm runs compared to the sequential version. A higher speedup indicates more effective utilization of parallel resources.

- **Efficiency:** Defined by the ratio of the speedup to the number of threads p :

$$\text{Efficiency} = \frac{\text{Speedup}}{p}.$$

This measures how effectively the algorithm scales with an increasing number of threads. An efficiency of 1 (or 100%) indicates perfect scaling.

- **FLOPS:** The number of floating-point operations per second, given by the formula

$$\text{FLOPS} = \frac{2 \times n^3}{t},$$

where $2 \times n^3$ represents the total number of arithmetic operations (one multiplication and one addition per iteration) in the matrix multiplication, and t is the total execution time. Higher FLOPS values indicate more numerical computations for the same amount of time.

- **Cache Misses:** To evaluate memory access patterns and their performance impact, we use the number of cache misses, such as L1_DCM (Level 1 Data Cache Misses), L2_DCM (Level 2 Data Cache Misses), and L3_TCM (Level 3 Total Cache Misses). Each cache miss forces the processor to fetch data from a slower cache level or main memory, incurring extra clock cycles. Reducing cache misses can improve time execution by minimizing these wasted cycles.

4 Results and Analysis

To benchmark the implementations previously discussed, we executed the test set shown below 5 times, storing the results for analysis. All algorithms were conducted on a university computer, with an Intel Core i7-9700 with 64KB of L1 cache, 256KB L2 cache and 12MB L3 cache shared among cores (more info in the Appendix section), operating under Ubuntu and C++ version compiled using g++ with the -O2 flag. The charts produced are available in the Appendix section. The tests performed were the following:

- Naive, Line and Parallel implementations (across both C++ and Lua) with input matrices from 600×600 to 3000×3000 in size increments of 400.

- Line, Block and Parallel implementations (in C++) with input matrices from 4096×4096 to 10240×10240 in size increments of 2048, with block sizes 128, 256 and 512.

4.1 Performance Evaluation of Single Core Implementations

The first comparison in the single core implementations, evaluated both in C++ and Lua, is between the naive and the line approach. The results show that, independently of the language of implementation, the execution times are smaller for the latter. It can also be observed that execution time is faster in C++ when compared to Lua. Evidence for this can be seen in graphs 6 and 7.

The main reason why the line implementation is more performant than the naive approach is due to sequential data processing, which reduces cache misses, as shown in graph 8. Due to the prefetching mechanism, when one memory address is accessed, adjacent data is also loaded into memory. This promotes cache locality, ensuring that subsequent memory accesses hit the cache instead of requiring slower memory fetches.

Having established the advantages of the line implementation, we can try to further optimize the code to reduce cache misses during memory accesses. The best approach for this is to subdivide the matrix calculation and perform it in small blocks, limiting the memory region being processed. We decided to test this with different matrix and block sizes, and the results can be consulted in figures 9, 10, 11, 12 and 13.

Overall, the block algorithms are proportionally better than the line algorithms in relation to the L1 and L2 cache misses, as shown in figures 9, 11 and 12.

However, this is not the case for matrix size of 8192×8192 , where the block algorithms with block sizes of 256 and 512 suffer a spike in execution time (and consequent decrease in Mflops), shown in charts 9 and 10. These results were caused by the number of cache misses that occur on the L3 cache, suggesting that these block sizes don't fit well in this cache.

According to the memory hierarchy, the cost cache misses on L3 is much higher than L1 and L2, because it requires data to be fetched from an even lower level, therefore, the total execution is higher than the expected. To verify this, we ran extra tests to measure the L3_TCM and the results are shown in 13.

4.2 Performance Evaluation of Multi-Core Implementations

Besides the single core algorithms, we also implemented two multi-core approaches, based on the line algorithm, using OpenMP. As expected, both multi-core algorithms have better overall performance when compared to the single core line approach.

By distributing the operation load across different processing units, these approaches are able to perform a larger number of floating point operations per second and, consequently, have smaller execution times, as we can see in graphs 14 and 15.

Nonetheless, the multi-core implementations follow different ideas, as discussed in the Algorithms section, and have substantially different results. While still being faster than the sequential code, the first implementation is able to execute 3 to 4 times more Mflops than the second one, as seen in 15.

While both take advantage of parallelism, the second approach has an associated synchronization overhead after each iteration of the inner parallelized loop, which occurs $size^2$ times, leading to sizeable overheads increasing quadratically with matrix size. Nonetheless, it will still be faster than the single-core algorithm because the number of operations performed by each thread also grows.

As a result of this, we observe faster execution times for the first parallel approach which uses parallelism on the outermost loop, only requiring synchronization in the end. The comparison between these two algorithms is clear when analyzing their speedup and efficiency.

While the second approach has a speedup of slightly over $1.0\times$ for matrices over around 1000×1000 and efficiency below 0.2, the first implementation is at least $4\times$ faster (usually even around $5\times$) and has an efficiency over 0.5. These conclusions can be seen in the charts 16 and 17.

5 Conclusion

In conclusion, this project allowed us to deepen our understanding of fundamental programming concepts, such as data accesses, with emphasis on the impacts of cache misses, prefetching mechanisms and parallel computing. By implementing multiple valid approaches to the same problem and comparing their performance metrics, we were able to verify the concepts taught in the theoretical classes.

Developing the code across two languages, in our case C++ and Lua, also provided us with better insights in those languages, particularly C++, where we explored the Performance API (PAPI) for benchmarking and OpenMP for the multi-core programming.

Overall, the project successfully met its objectives, giving us hands-on experience with the intricacies of parallel computing and demonstrating the performance capabilities of programs developed with memory layout and multi-core mechanisms in mind.

6 Appendix

6.1 Comparison Charts

NOTE: The following charts are a summary of the data collected from the performance tests executed, which can be found in the project's repository. Each test was performed five times, being also run an extra time for C++ line and block multiplications to gather L3 cache misses data. The median was used in the chart generation to represent the data for each value set.

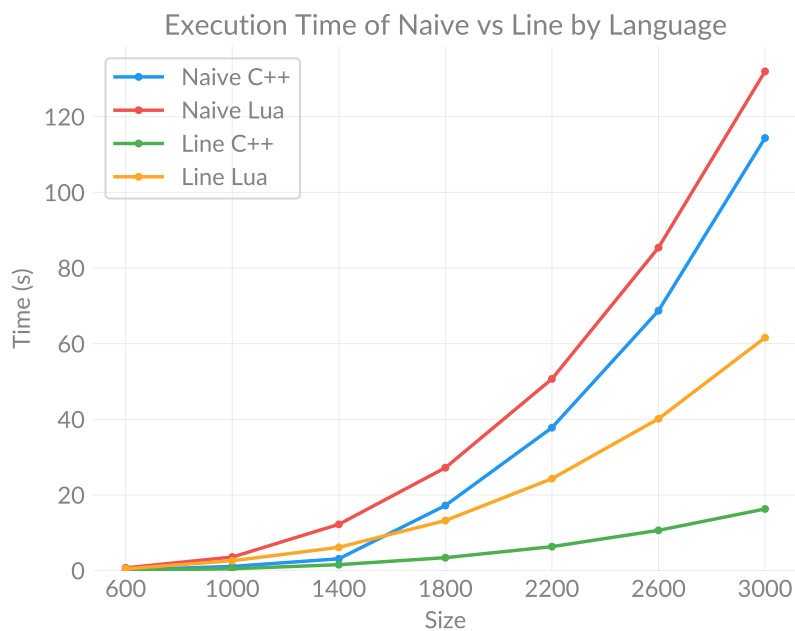


Figure 6: Time comparison between naive and line multiplication, in both C++ and Lua

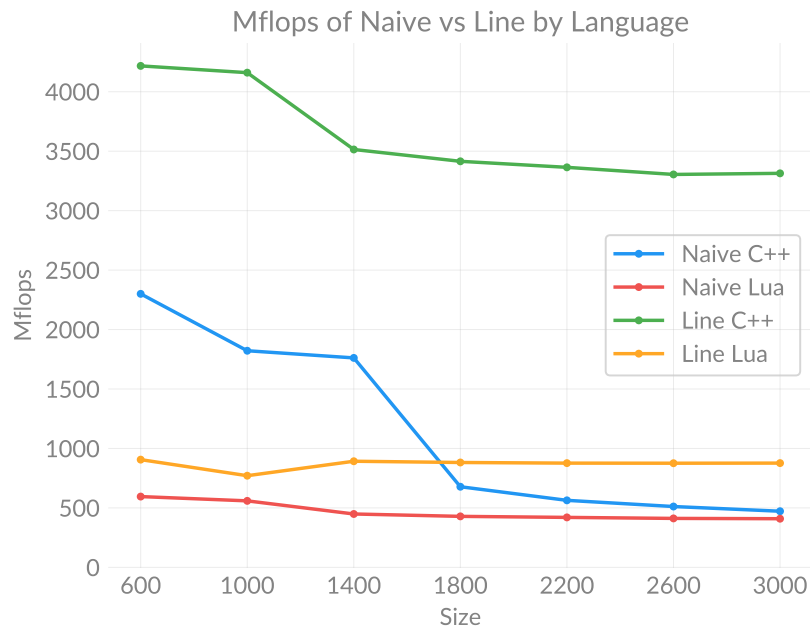


Figure 7: Mflops comparison between naive and line multiplication, in both C++ and Lua

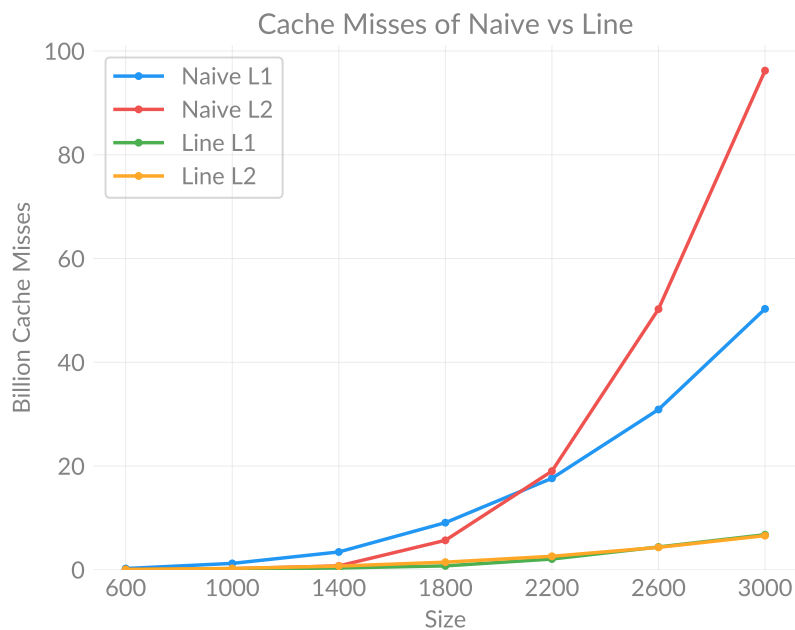


Figure 8: Cache misses comparison between naive and line multiplication, in C++

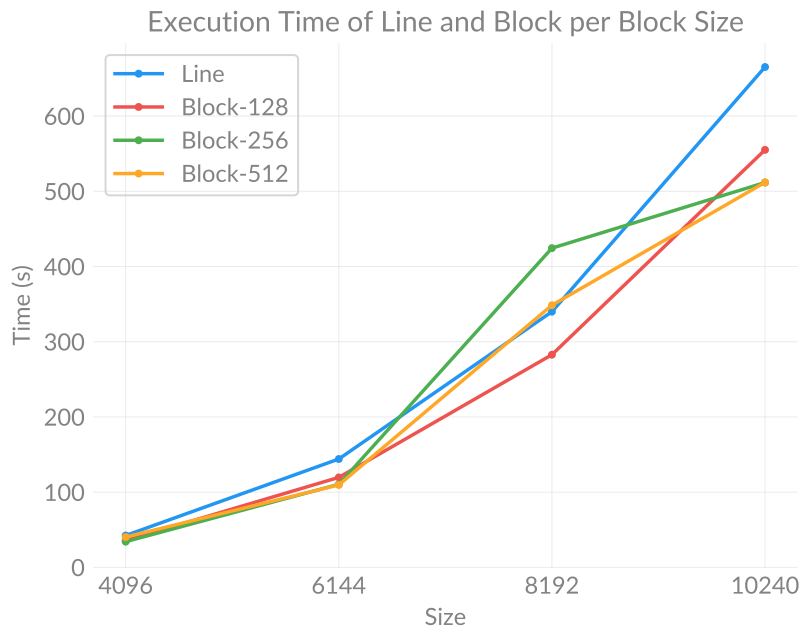


Figure 9: Time comparison between line and block multiplication, with different block sizes

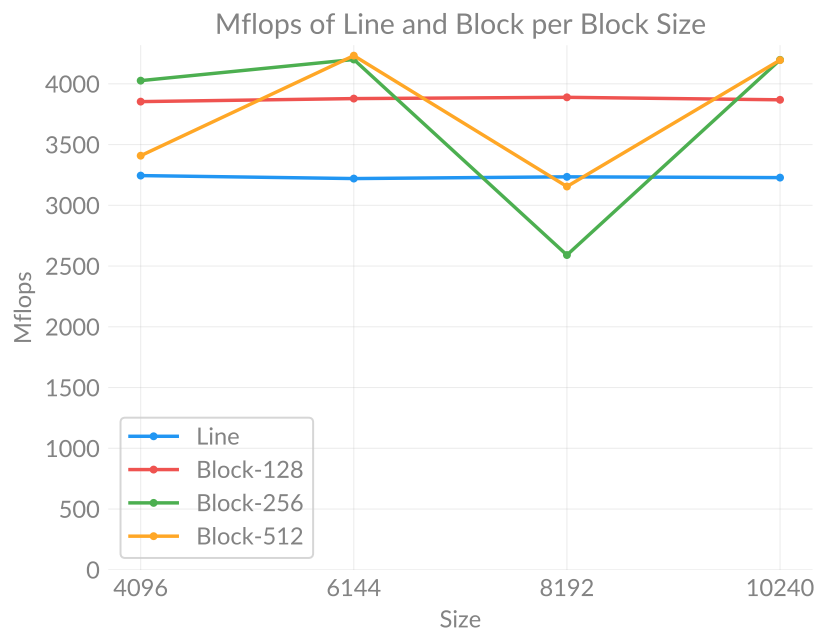


Figure 10: Mflops comparison between line and block multiplication, with different block sizes

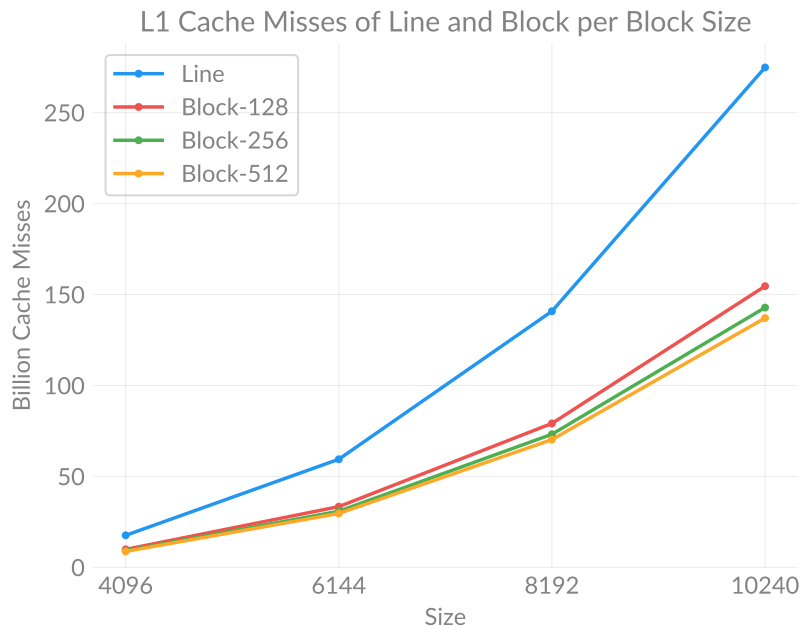


Figure 11: L1 cache miss comparison between line and block multiplication, with different block sizes

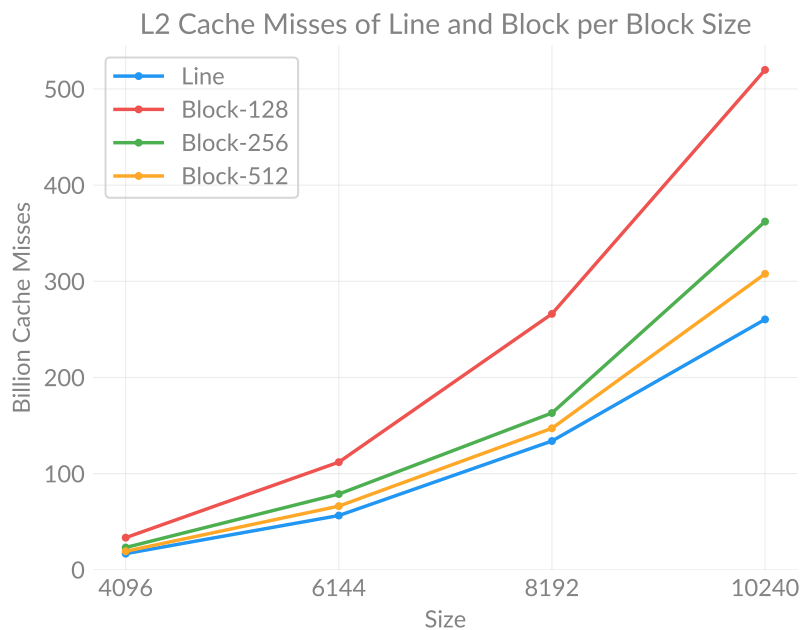


Figure 12: L2 cache miss comparison between line and block multiplication, with different block sizes

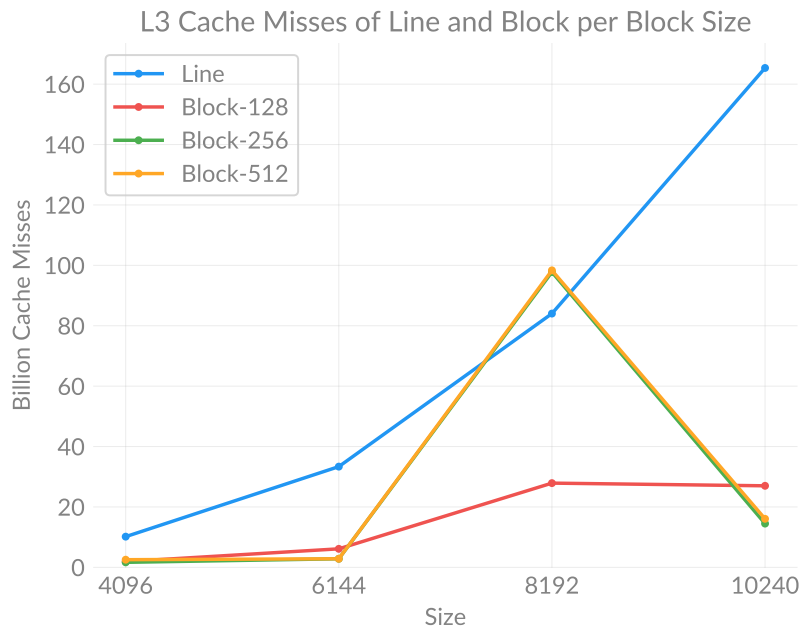


Figure 13: L3 cache miss comparison between line and block multiplication, with different block sizes

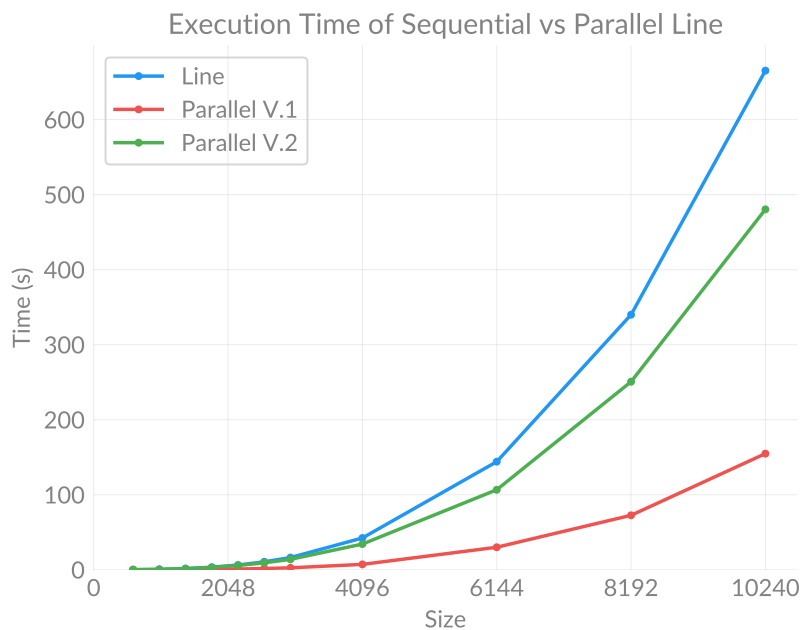


Figure 14: Time comparison between sequential and parallel line multiplication

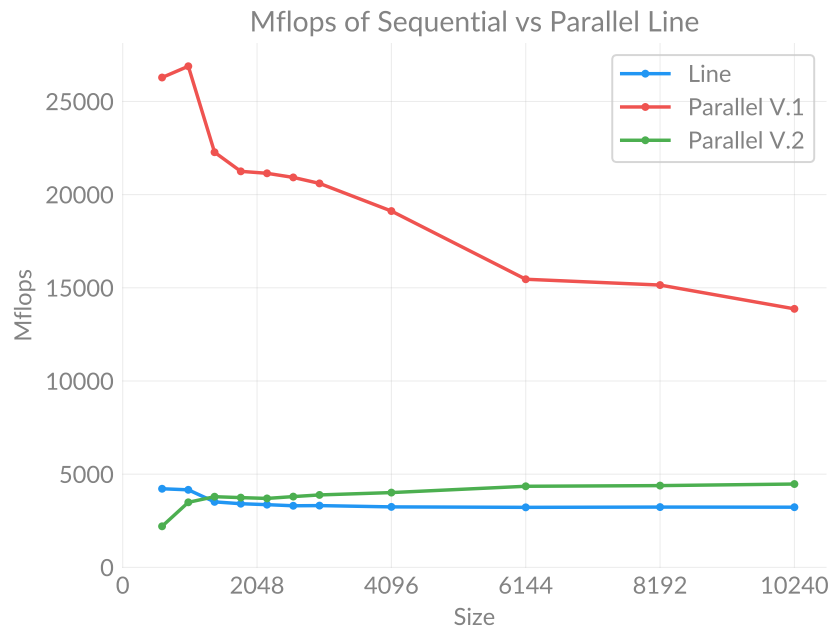


Figure 15: Mflops comparison between sequential and parallel line multiplication

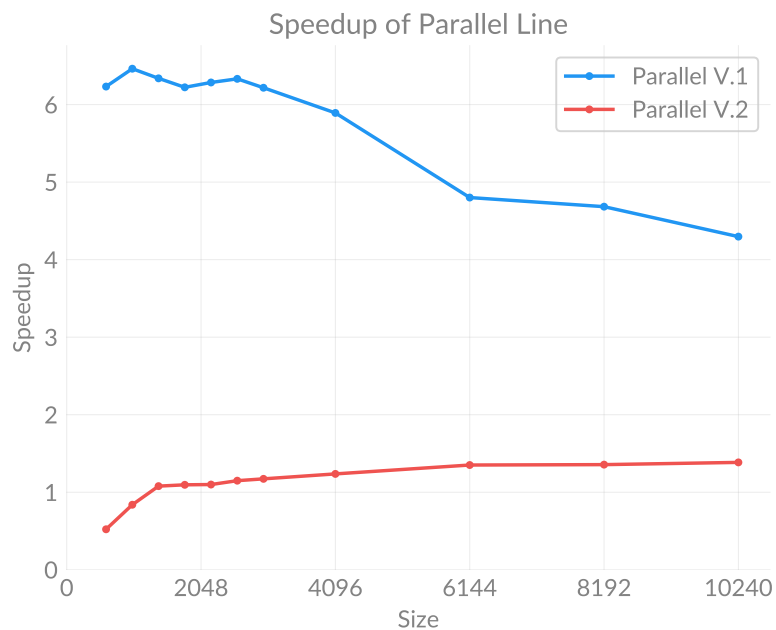


Figure 16: Speedup comparison between versions of parallel line multiplication, relative to the sequential version

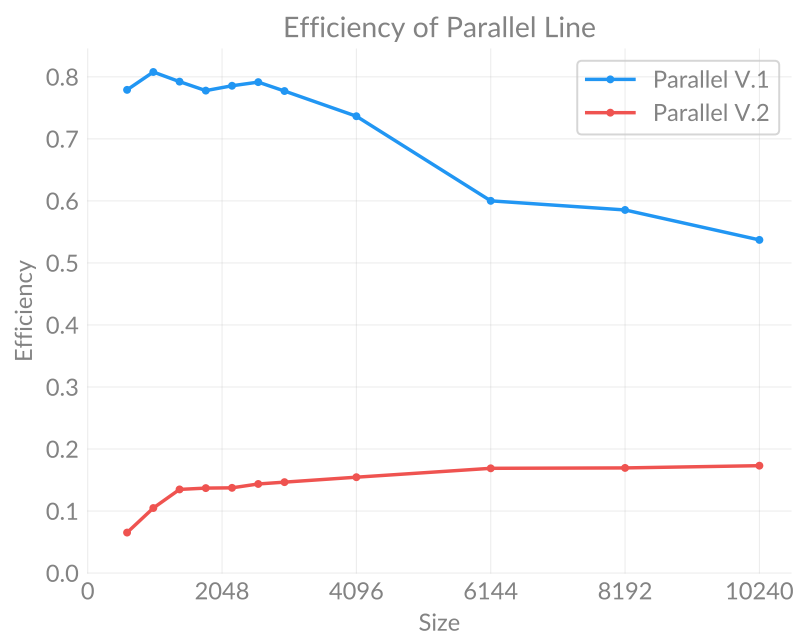


Figure 17: Efficiency comparison between versions of parallel line multiplication, relative to the sequential version

6.2 PC Specifications

```
$ papi_avail -a
Available PAPI preset and user defined events plus hardware information.
```

```
-----
PAPI version          : 7.1.0.0
Operating system      : Linux 6.5.0-15-generic
Vendor string and code : GenuineIntel (1, 0x1)
Model string and code  : Intel(R) Core(TM) i7-9700 CPU @ 3.00GHz (158, 0x9e)
CPU revision          : 13.000000
CPUID                 : Family/Model/Stepping 6/158/13, 0x06/0x9e/0x0d
CPU Max MHz           : 4700
CPU Min MHz           : 800
Total cores            : 8
SMT threads per core  : 1
Cores per socket      : 8
Sockets                : 1
Cores per NUMA region : 8
NUMA regions           : 1
Running in a VM        : no
Number Hardware Counters : 10
Max Multiplex Counters : 384
Fast counter read (rdpmc): yes
-----
```

```
=====
PAPI Preset Events
=====
```

Name	Code	Deriv	Description (Note)
PAPI_L1_DCM	0x80000000	No	Level 1 data cache misses
PAPI_L1_ICM	0x80000001	No	Level 1 instruction cache misses
PAPI_L2_DCM	0x80000002	Yes	Level 2 data cache misses
PAPI_L2_ICM	0x80000003	No	Level 2 instruction cache misses
PAPI_L1_TCM	0x80000006	Yes	Level 1 cache misses
PAPI_L2_TCM	0x80000007	No	Level 2 cache misses
PAPI_L3_TCM	0x80000008	No	Level 3 cache misses
PAPI_CA_SNP	0x80000009	No	Requests for a snoop
PAPI_CA_SHR	0x8000000a	No	Requests for exclusive access to shared cache line
PAPI_CA_CLN	0x8000000b	No	Requests for exclusive access to clean cache line
PAPI_CA_ITV	0x8000000d	No	Requests for cache line intervention
PAPI_L3_LDM	0x8000000e	No	Level 3 load misses
PAPI_TLB_DM	0x80000014	Yes	Data translation lookaside buffer misses
PAPI_TLB_IM	0x80000015	No	Instruction translation lookaside buffer misses
PAPI_L1_LDM	0x80000017	No	Level 1 load misses
PAPI_L1_STM	0x80000018	No	Level 1 store misses
PAPI_L2_LDM	0x80000019	No	Level 2 load misses
PAPI_L2_STM	0x8000001a	No	Level 2 store misses
PAPI_PRF_DM	0x8000001c	No	Data prefetch cache misses
PAPI_MEM_WCY	0x80000024	No	Cycles Stalled Waiting for memory writes
PAPI_STL_ICY	0x80000025	No	Cycles with no instruction issue
PAPI_FUL_ICY	0x80000026	Yes	Cycles with maximum instruction issue
PAPI_STL_CCY	0x80000027	No	Cycles with no instructions completed
PAPI_FUL_CCY	0x80000028	No	Cycles with maximum instructions completed
PAPI_BR_UCN	0x8000002a	Yes	Unconditional branch instructions
PAPI_BR_CN	0x8000002b	No	Conditional branch instructions
PAPI_BR_TKN	0x8000002c	Yes	Conditional branch instructions taken
PAPI_BR_NTK	0x8000002d	No	Conditional branch instructions not taken
PAPI_BR_MSP	0x8000002e	No	Conditional branch instructions mispredicted
PAPI_BR_PRC	0x8000002f	Yes	Conditional branch instructions correctly predicted
PAPI_TOT_INS	0x80000032	No	Instructions completed
PAPI_LD_INS	0x80000035	No	Load instructions
PAPI_SR_INS	0x80000036	No	Store instructions
PAPI_BR_INS	0x80000037	No	Branch instructions
PAPI_RES_STL	0x80000039	No	Cycles stalled on any resource
PAPI_TOT_CYC	0x8000003b	No	Total cycles
PAPI_LST_INS	0x8000003c	Yes	Load/store instructions completed
PAPI_L2_DCA	0x80000041	No	Level 2 data cache accesses
PAPI_L3_DCA	0x80000042	Yes	Level 3 data cache accesses
PAPI_L2_DCR	0x80000044	No	Level 2 data cache reads
PAPI_L3_DCR	0x80000045	No	Level 3 data cache reads
PAPI_L2_DCW	0x80000047	Yes	Level 2 data cache writes
PAPI_L3_DCW	0x80000048	No	Level 3 data cache writes

PAPI_L2_ICH	0x8000004a	No	Level 2 instruction cache hits
PAPI_L2_ICA	0x8000004d	No	Level 2 instruction cache accesses
PAPI_L3_ICA	0x8000004e	No	Level 3 instruction cache accesses
PAPI_L2_ICR	0x80000050	No	Level 2 instruction cache reads
PAPI_L3_ICR	0x80000051	No	Level 3 instruction cache reads
PAPI_L2_TCA	0x80000059	Yes	Level 2 total cache accesses
PAPI_L3_TCA	0x8000005a	No	Level 3 total cache accesses
PAPI_L2_TCR	0x8000005c	Yes	Level 2 total cache reads
PAPI_L3_TCR	0x8000005d	Yes	Level 3 total cache reads
PAPI_L2_TCW	0x8000005f	Yes	Level 2 total cache writes
PAPI_L3_TCW	0x80000060	No	Level 3 total cache writes
PAPI_SP_OPS	0x80000067	Yes	Floating point operations; optimized to count scaled single ↪ precision vector operations
PAPI_DP_OPS	0x80000068	Yes	Floating point operations; optimized to count scaled double ↪ precision vector operations
PAPI_VEC_SP	0x80000069	Yes	Single precision vector/SIMD instructions
PAPI_VEC_DP	0x8000006a	Yes	Double precision vector/SIMD instructions
PAPI_REF_CYC	0x8000006b	No	Reference clock cycles

Of 59 available events, 18 are derived.