# nlcor: Nonlinear Correlation

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## Purpose

Estimate nonlinear correlations using `nlcor`. Yields a correlation estimate between 0 and 1, and the adjusted p value. The p value indicates if the estimated correlation is statistically significant.

## Description

Correlations are commonly used in various data mining applications. Typically linear correlations are estimated. However, the data may have a nonlinear correlation but little to no linear correlation. If, for example, we are performing data exploration using automated techniques on many variables, such nonlinearly correlated variables can easily be overlooked.

Nonlinear correlations are quite common in real data. Due to this, nonlinear models, such as SVM, are employed for regression, classification, etc. However, there are not many approaches to estimate nonlinear correlations. If developed, it will find application in data exploration, variable selection, and other areas.

In this package, we provide an implementation of a nonlinear correlation estimation method using an adaptive local linear correlation computation in `nlcor`. The function `nlcor` returns the nonlinear correlation estimate, the corresponding adjusted p value, and an optional plot visualizing the nonlinear relationships.

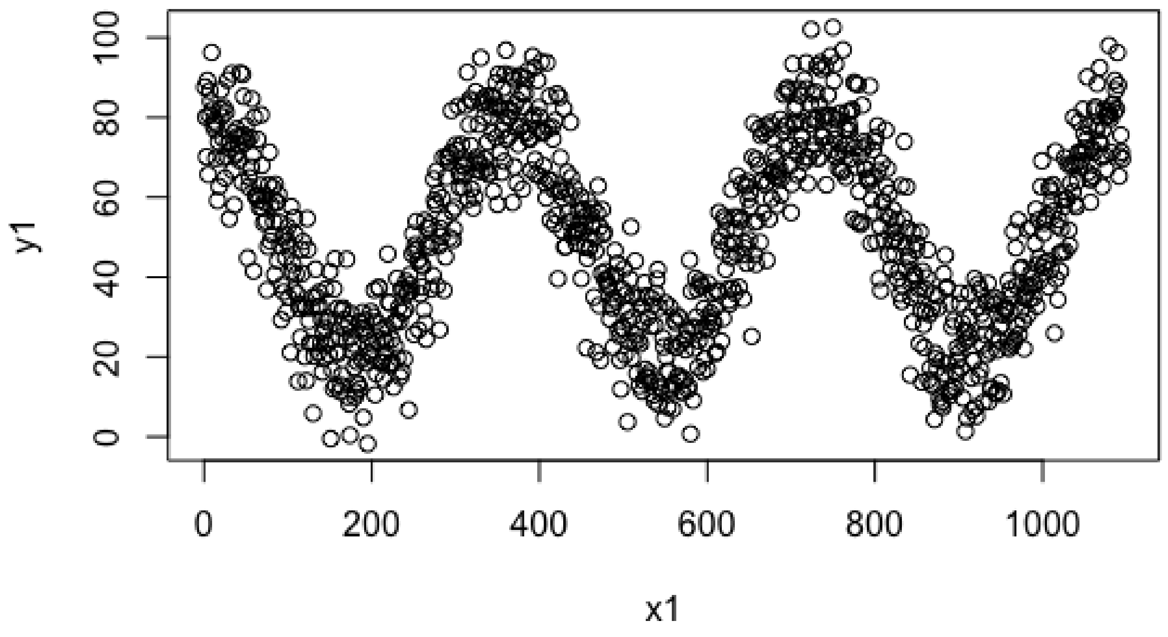
The correlation estimate will be between 0 and 1. The higher the value the more is the nonlinear correlation. Unlike linear correlations, a negative value is not valid here. Due to multiple local correlation computations, the net p value of the correlation estimate is adjusted (to avoid false positives). The plot visualizes the local linear correlations.

In the following, we will show its usage with a few examples. In the given examples, the linear correlations between `x` and `y` is small, however, there is a visible nonlinear correlation between them. This package contains the data for these examples and can be used for testing the package.

### Example 1.

A data with cyclic nonlinear correlation.

```r
plot(x1, y1)
```

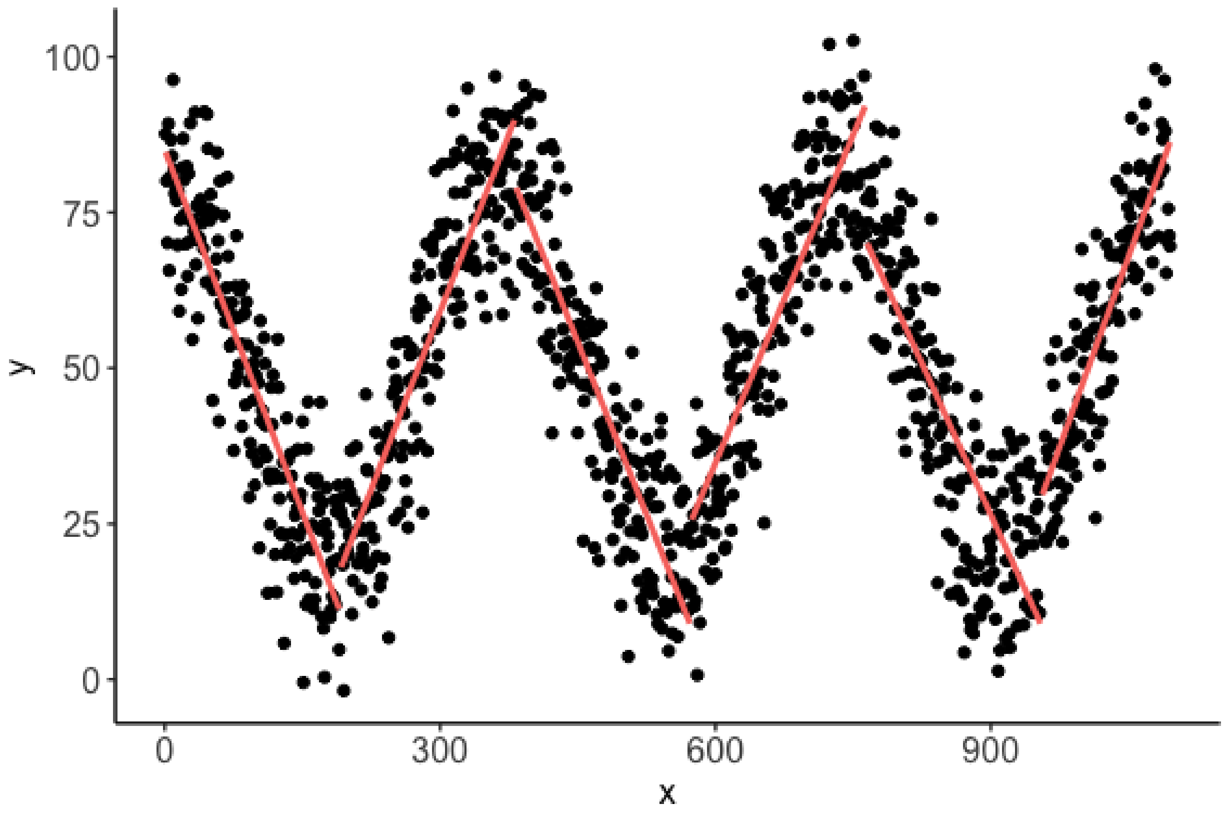


The linear correlation of the data is,

```r
cor(x1, y1)
#> [1] 0.008001837
```

As expected, the correlation is close to zero. We estimate the nonlinear correlation using `nlcor`.

```r
c <- nlcor(x1, y1, plt = T)
c$cor.estimate
#> [1] 0.8688784
c$adjusted.p.value
#> [1] 0
print(c$cor.plot)
```

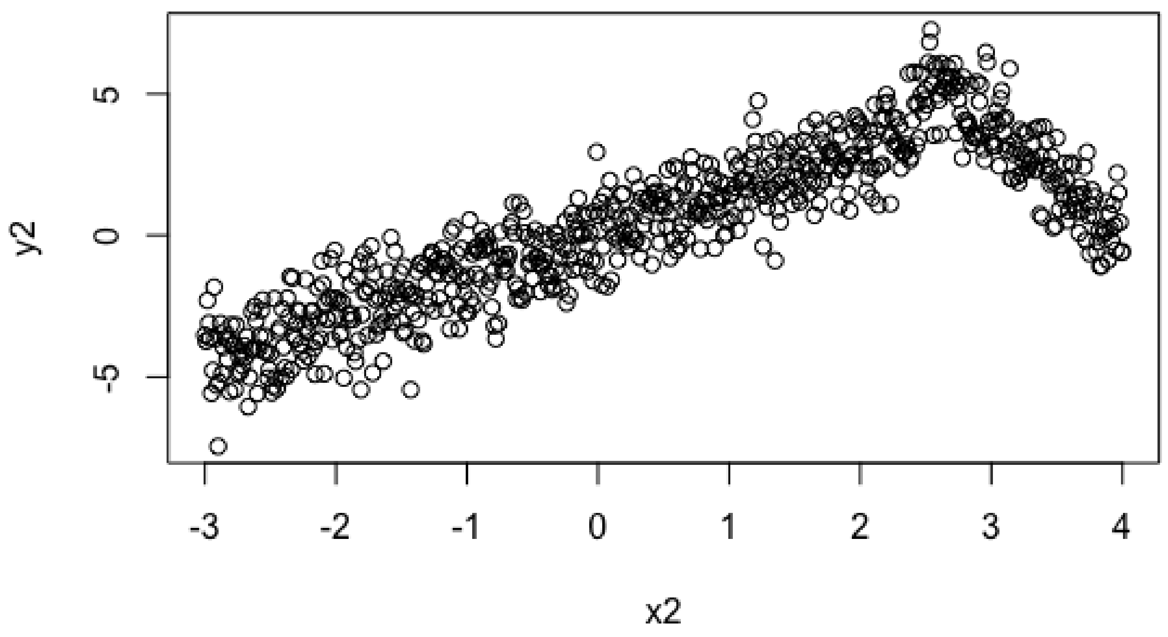


The plot shows the piecewise linear correlations present in the data.

**Example 2.**

A data with non-uniform piecewise linear correlations.

```
plot(x2, y2)
```



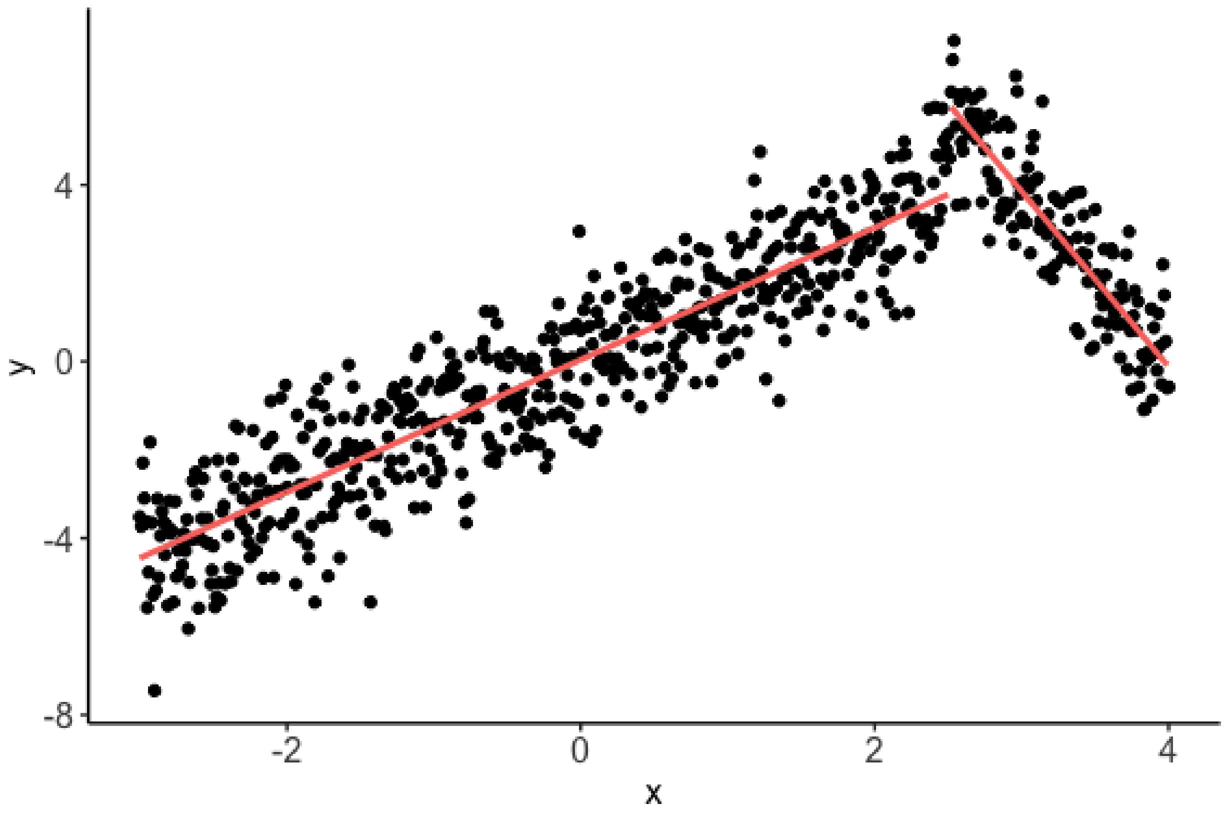
The linear correlation of the data is,

```
cor(x2, y2)
#> [1] 0.828596
```

The linear correlation is quite high in this data. However, there is significant and higher nonlinear correlation present in the data. This data emulates the scenario where the correlation changes its direction after a point. Sometimes that change point is in the middle causing the linear correlation to be close to zero. Here we show an example when the change point is off center to show that the implementation works in non-uniform cases.

We estimate the nonlinear correlation using `nlcor`.

```
c <- nlcor(x2, y2, plt = T)
c$cor.estimate
#> [1] 0.897205
c$adjusted.p.value
#> [1] 0
print(c$cor.plot)
```

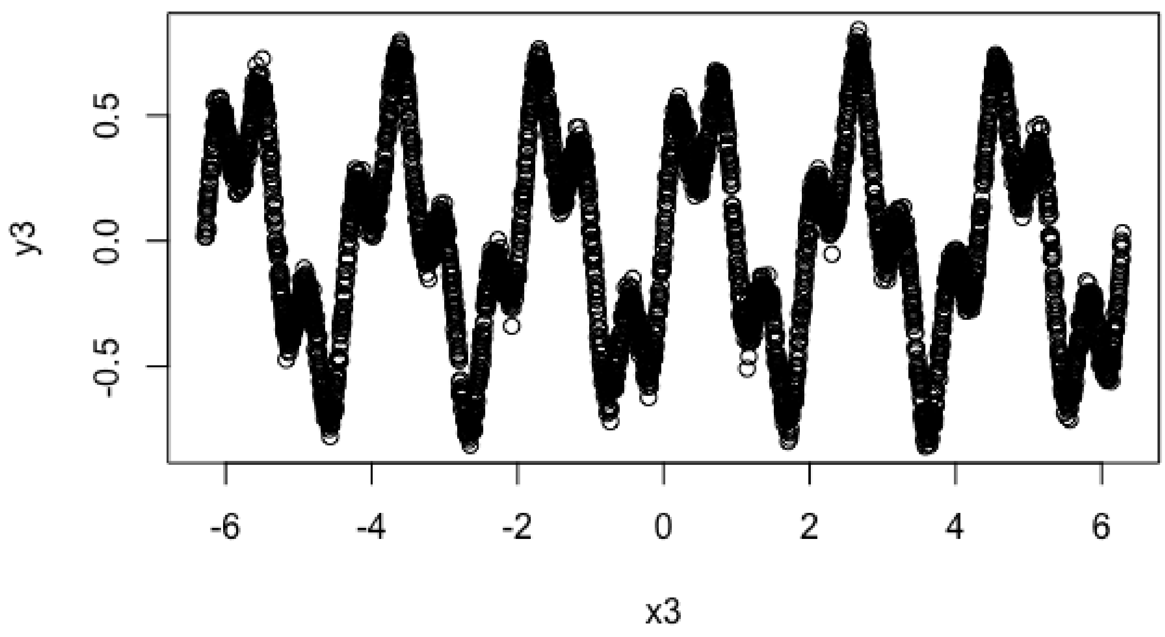


It is visible from the plot that `nlcor` could estimate the piecewise correlations in a non-uniform scenario. Also, the nonlinear correlation comes out to be higher than the linear correlation.

**Example 3.**

A data with higher and multiple frequency variations.

```
plot(x3, y3)
```



The linear correlation of the data is,

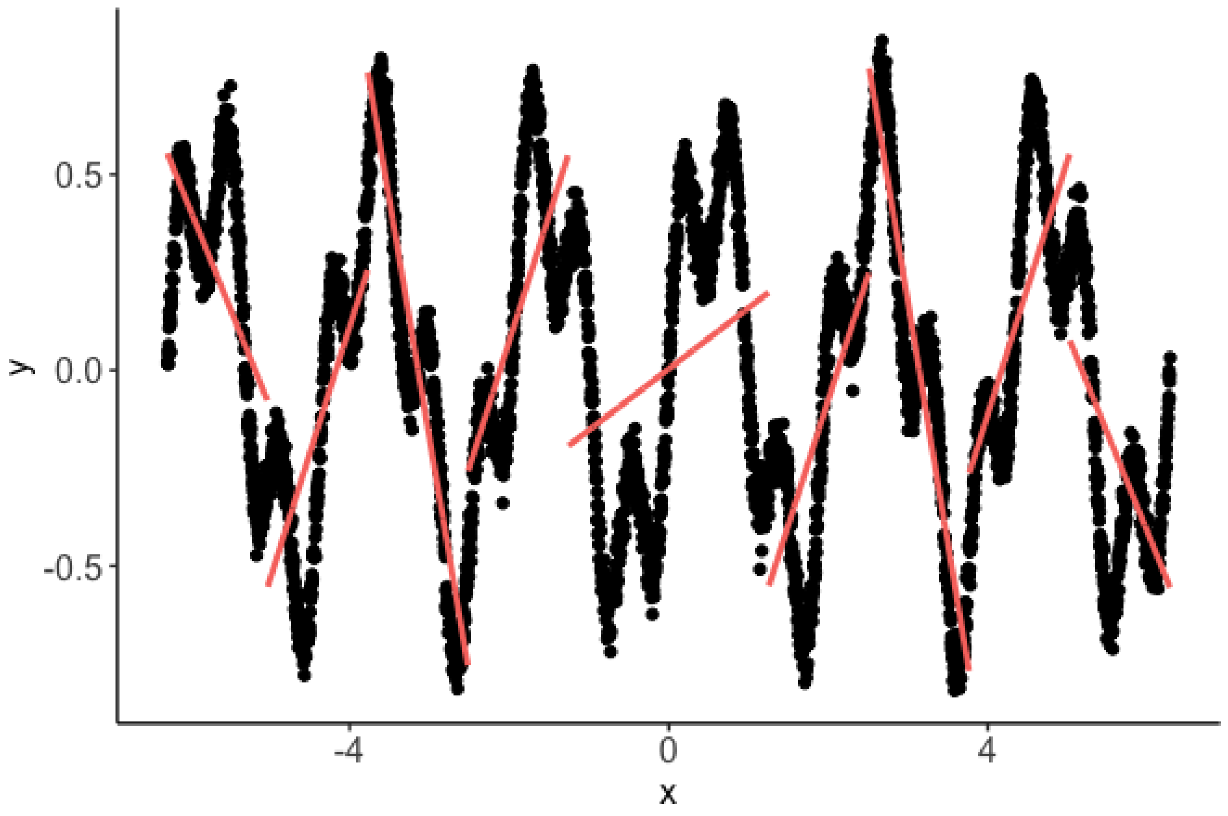
```
cor(x3, y3)
#> [1] -0.1337304
```

The linear correlation is expectedly small, albeit not close to zero due to some linearity.

Here we show we can refine the granularity of the correlation computation.

Under default settings, the output of `nlcor` will be,

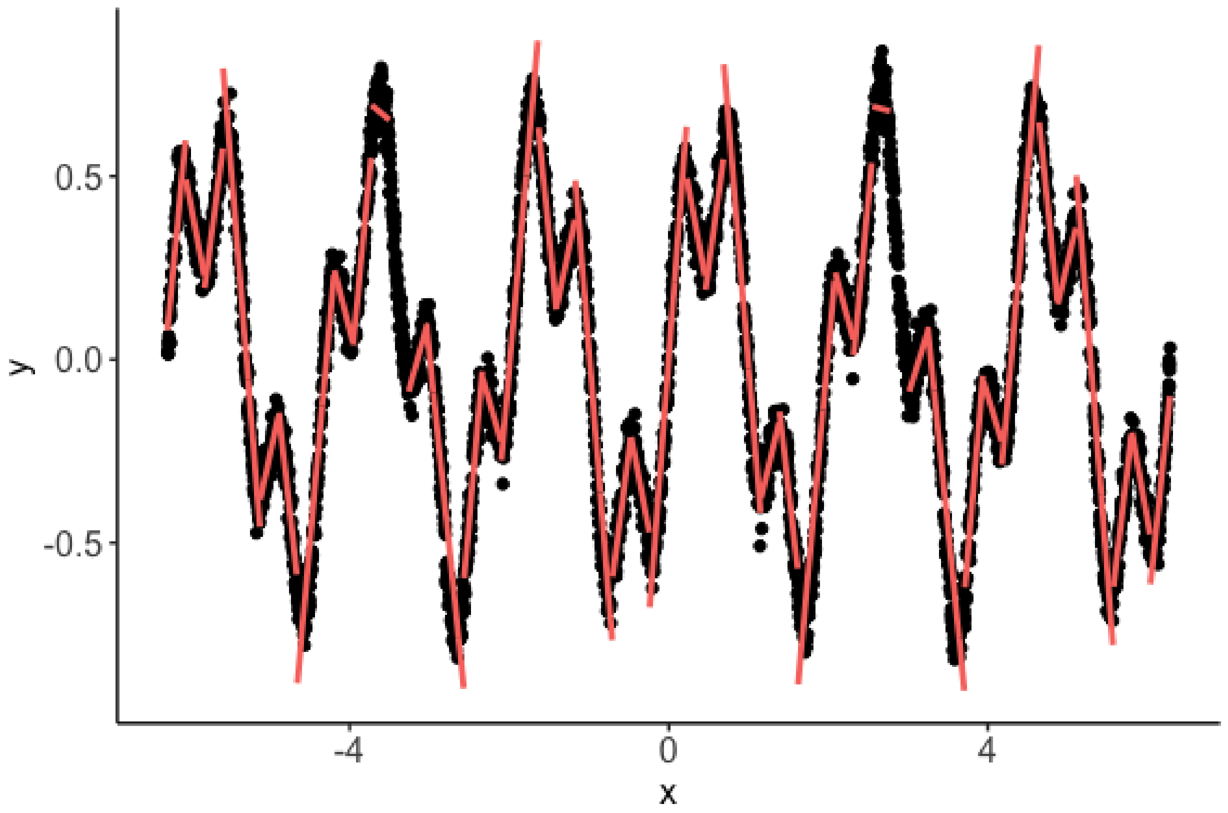
```
c <- nlcor(x3, y3, plt = T)
c$cor.estimate
#> [1] 0.7090148
c$adjusted.p.value
#> [1] 0
print(c$cor.plot)
```



As can be seen in the figure, `nlcor` overlooked some of the local relationships. We can refine the correlation estimation by changing the `refine` parameter. The default value of `refine` is set as 0.5. It can be set as any value between `0` and `1`. A higher value enforces higher refinement. However, higher refinement adversely affects the p value. Meaning, the resultant correlation estimate may be statistically insignificant (similar to overfitting). Therefore, it is recommended to avoid over refinement.

In this data, we rerun the correlation estimation with `refine = 0.9`.

```r
c <- nlcor(x3, y3, refine = 0.9, plt = T)
c$cor.estimate
#> [1] 0.8534956
c$adjusted.p.value
#> [1] 2.531456e-06
print(c$cor.plot)
#> Warning: Removed 148 rows containing missing values (geom_path).
```



As can be seen in the figure, `nlcor` could identify the granular piecewise correlations. In this data, the p value still remains extremely small—the correlation is *statistically significant*.

### Summary

This package provides an implementation of an efficient heuristic to compute the nonlinear correlations between numeric vectors. The heuristic works by adaptively identifying multiple local regions of linear correlations to estimate the overall nonlinear correlation. Its usages are demonstrated here with few examples.

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### Support

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