ÖZYEĞİN UNIVERSITY

EE393 Python for Engineers

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online

23.11.2020

2020-2021 Fall Semester

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Agenda

- Short review
- NumPy
- SciPy
- An introduction to matplotlib
- Many examples selected from engineering fundamentals
- Homework

(Download to your local)

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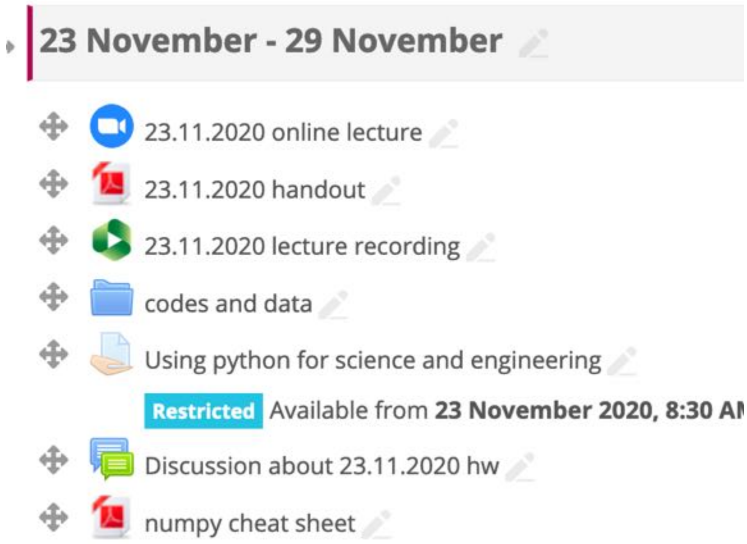
23.11.2020 | LMS resources



Learning objectives for 23.11.2020

- Understands how to use Python in science and engineering
- Applies numpy methods to Python programs
- Applies scipy and numpy methods to engineering problem solving
- Plots (x,y) discrete points using several strategies in Python-matplotlib

MIDTERM EXAM

DECEMBER 21, 2020; 08:40

Online
(Details will be announced later)

FINAL EXAM

JANUARY 15, 2021; 09:00

Online
(Details will be announced later)

Last week

- openpyxl –working with excel from Python
- Objects in Python
- OOP basics

Scientific/Engineering Python Uses

- Extra features required:
  - fast, multidimensional arrays
    - With homogenous elements inside!!!! e.q. all numbers!
  - libraries of reliable, tested **scientific functions**
  - plotting tools
- **NumPy** is at the core of nearly every scientific Python application or module since it provides a fast **N-d array** datatype that can be manipulated in a vectorized form.

Speedtest
Python vs Numpy

100M numbers are multiplied in a) Python lists, b) Numpy Array; and execution times are measured for comparison

Numpy is approx. 20 (+) times faster

```python
#SPEED comparison
# importing required packages
import time

# size of arrays and lists
size = 100000000

# declaring lists
list1 = range(size)
list2 = range(size)
# declaring arrays
array1 = np.arange(size)
array2 = np.arange(size)

# list
initialTime = time.time()
resultantList = [(a * b) for a, b in zip(list1, list2)]

# calculating execution time
print("Time taken by Lists :",
      (time.time() - initialTime),
      "seconds")

# NumPy array
initialTime = time.time()
resultantArray = array1 * array2
# calculating execution time
print("Time taken by NumPy Arrays :",
      (time.time() - initialTime),
      "seconds")
```

```
Time taken by Lists : 8.10220313072204 6 seconds
Time taken by NumPy Arrays : 0.38413882255542 seconds
```

```
Time taken by Lists : 8.33761191368103 seconds
Time taken by NumPy Arrays : 0.4112420082092285 seconds
```

Python 2.7

```
(base) Orhans-MacBook-Pro:week8 orhang$ python2.7 speedtest.py
('Time taken by Lists :', 65.85366821289062, 'seconds')
('Time taken by NumPy Arrays :', 3.162869930267334, 'seconds')
```

Arrays – Numerical Python (Numpy)

- Lists ok for storing small amounts of one-dimensional data

```
>>> a = [1,3,5,7,9]
>>> print(a[2:4])
[5, 7]
>>> b = [[1, 3, 5, 7, 9], [2, 4, 6, 8, 10]]
>>> print(b[0])
[1, 3, 5, 7, 9]
>>> print(b[1][2:4])
[6, 8]
```

```
>>> a = [1,3,5,7,9]
>>> b = [3,5,6,7,9]
>>> c = a + b
>>> print (c)
[1, 3, 5, 7, 9, 3, 5, 6, 7, 9]
```

- But, can't use directly with arithmetical operators (+, -, *, /, …)
- Need efficient arrays with arithmetic and better multidimensional tools
- **Numpy**

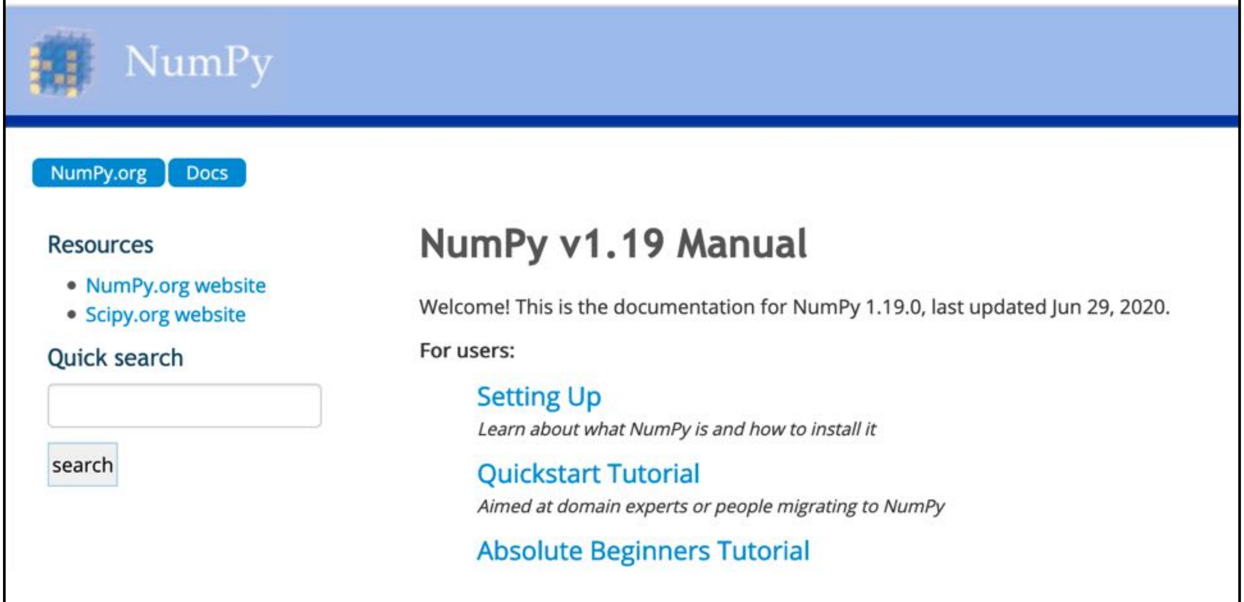
```
>>> import numpy
```

- Similar to lists, but much more capable, except it is fixed in size

What is NumPy?

- NumPy is the fundamental package needed for scientific computing with Python. It contains:
- a powerful N-dimensional array object
- **basic linear algebra functions**
- **basic Fourier transforms**
- **sophisticated random number capabilities**
- tools for integrating Fortran code
- tools for integrating C/C++ code

numpy.org/docs/stable



Numpy methods

- We generally import numpy as follows (as many Python coders do):

import numpy as np

- Main object type for numpy is **nd** array (or, numpy array)
  - Some of the methods are public methods and called as **np.methodname.** For ex. np.array, np.arrange, np.linspace, np.sort etc.
  - Some others are called via an instance such as
    - X = np.array(….)
      X.sum, X.mean etc.

Numpy array

- The most important object defined in NumPy is an N-dimensional array type called **ndarray**. It describes the collection of items of the same type. Items in the collection can be accessed using a zero-based index as we did for Python lists.
- Every item in an ndarray takes the same size of block in the memory. Each element in ndarray is an object of data-type object (called **dtype**).
- Basic numpy array is created using **numpy.array**

An array is a central data structure of the NumPy library. An array is a grid of values and it contains information about the raw data, how to locate an element, and how to interpret an element. It has a grid of elements that can be indexed in various ways. The elements are all of the same type, referred to as the array **dtype.**

Numpy – Creating vectors (or 1D arrays)

- From lists
  - numpy.array

```
#creating 1D arrays (vectors)
# as vectors from lists
a = np.array([1,3,5,7,9]) #a is a 1-d array
b = np.array([3,5,6,7,9])
c = a + b
print (c)
print (type(c))
print (c.shape)
print (c.dtype)
#Accessing array elements
print ("First: ", a[0])
print ("Last minus 1: ", a[-2])
for x in a:   #ndarrays are iterable
    print (x, end=" ")
```

```
[ 4  8 11 14 18]
<class 'numpy.ndarray'>
(5,)
int64
First:  1
Last minus 1:  7
1 3 5 7 9
```

numpy.ipynb

Data Types

- **bool_** :Boolean (True or False) stored as a byte
- **int_** :Default integer type (same as C long; normally either int64 or int32)
- **intc** : Identical to C int (normally int32 or int64)
- **intp** : Integer used for indexing (same as C ssize_t; normally either int32 or int64)
- **int8** : Byte (-128 to 127)
- **int16** : Integer (-32768 to 32767)
- **int32** : Integer (-2147483648 to 2147483647)
- **int64** : Integer (-9223372036854775808 to 9223372036854775807)
- **uint8** : Unsigned integer (0 to 255)
- **uint16** : Unsigned integer (0 to 65535)

int8, int16, int32, int64 can be replaced by equivalent string i1, i2, i4 etc.

Data Types

- **uint32** : Unsigned integer (0 to 4294967295)
- **uint64** : Unsigned integer (0 to 18446744073709551615)
- **float_** : Shorthand for float64
- **float16** : Half precision float: sign bit, 5 bits exponent, 10 bits mantissa
- **float32** : Single precision float: sign bit, 8 bits exponent, 23 bits mantissa
- **float64** : Double precision float: sign bit, 11 bits exponent, 52 bits mantissa
- **double** : Shorthand for float64
- **longdouble** : Shorthand for float96 or float128 (-1.7E4932 .. -1.7E4932)
- **complex_** : Shorthand for complex128
- **complex64** : Complex number, represented by two 32-bit floats (real and imaginary components)
- **complex128** : Complex number, represented by two 64-bit floats (real and imaginary components)

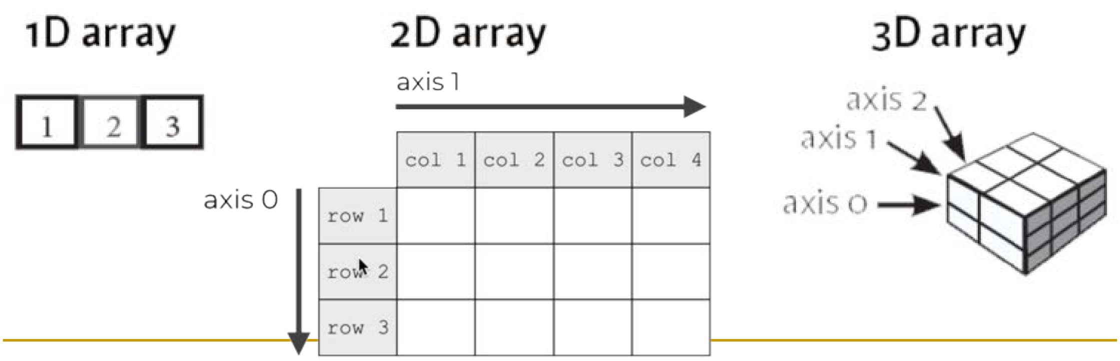
Creating n-dim arrays & shape

```
#CREATING A MATRIX
#more than one dimension!
l = [[1, 2, 3], [3, 6, 9], [2, 4, 6]]  # create a list
a = np.array(l)  # convert a list to an array
print(a)
print (a.shape)
print(a.dtype)  # get type of an array
M = np.array([[1, 2], [3, 4]])
print (M.shape)
print (M.dtype)
```

```
[[1 2 3]
 [3 6 9]
 [2 4 6]]
(3, 3)
int64
(2, 2)
int64
```

Numpy – Creating arrays

- There are a number of ways to initialize new numpy arrays, for example from
  - a Python list or tuples
  - using functions that are dedicated to generating numpy arrays, such as **arange**, **linspace**, etc.
  - reading data from files (csv, excel, txt etc.)



```python
#checking number of dimensions, shape and size
import numpy as np
a = np.array(1+2j)
b = np.array([0,2,4,6,8])
c = np.array([[1,2,3], [5,8,13]])
c2 = np.array([[1,2,3], [5,8,13],[1,1,1]])
d = np.array([[[1, 2, 3,0], [5,8,13,0]],
              [[21,34,55,0], [89,144,233,0]],
              [[21,34,55,0], [89,144,233,0]]])
print("dimension of a: {}, shape: {}".format(a.ndim, a.shape))
print("dimension of b: {}, shape: {}".format(b.ndim, b.shape))
print("dimension of c: {}, shape: {}".format(c.ndim, c.shape))
print("dimension of c2: {}, shape: {}".format(c2.ndim, c2.shape))
print("dimension of d: {}, shape: {}, size: {}".format(d.ndim, d.shape, d.size))
print (d)
```

```
dimension of a: 0, shape: ()
dimension of b: 1, shape: (5,)
dimension of c: 2, shape: (2, 3)
dimension of c2: 2, shape: (3, 3)
dimension of d: 3, shape: (3, 2, 4), size: 24
[[[  1   2   3   0]
  [  5   8  13   0]]

 [[ 21  34  55   0]
  [ 89 144 233   0]]

 [[ 21  34  55   0]
  [ 89 144 233   0]]]
```

ndim, shape, size

Special arrays : empty, zeros, ones

```python
import numpy as np
x = np.empty([3,2], dtype = int)
print (x)
#values are randomly assigned values in the memory
```

```
[[4607182418800017408                   0]
 [4611686018427387904                   0]
 [4613937818241073152                   0]]
```

```python
# array of five zeros. Default dtype is float
import numpy as np
x = np.zeros(5)
print (x)
```

```
[0. 0. 0. 0. 0.]
```

```python
# array of five ones. Default d
import numpy as np
x = np.ones(5)
print (x)
y = np.ones([5,3], dtype=int)
print (y)
```

```
[1. 1. 1. 1. 1.]
[[1 1 1]
 [1 1 1]
 [1 1 1]
 [1 1 1]
 [1 1 1]]
```

```python
#identity matrix
I = np.eye(5)
print (I)
```

```
[[1. 0. 0. 0. 0.]
 [0. 1. 0. 0. 0.]
 [0. 0. 1. 0. 0.]
 [0. 0. 0. 1. 0.]
 [0. 0. 0. 0. 1.]]
```

asarray

```python
#asarray
#This function is similar to numpy.
#It is useful for converting Python sequence into ndarray
import numpy as np
x = [1,2,3]
a = np.asarray(x, dtype = float)
print (a)
```

```
[1. 2. 3.]
```

LOOK!

numpy.ipynb

Numpy – Creating matrices

```python
import numpy as np
# create a list
l = [[1, 2, 3], [3, 6, 9], [2, 4, 6]]
a = np.array(l) # convert a list to an array
print(a)
print (a.shape)
print(a.dtype) # get type of an array
print ("row:", a[0])
print ("column:", a[:,1])

M = np.array([[1, 2], [3, 4]])
print (M.shape)
print (M.dtype)

M = np.array([[1, 2], [3, 4]], dtype=complex)
print (M)
```

```
[[1 2 3]
 [3 6 9]
 [2 4 6]]
(3, 3)
int64
row: [1 2 3]
column: [2 6 4]
(2, 2)
int64
[[1.+0.j 2.+0.j]
 [3.+0.j 4.+0.j]]
```

ONLY ONE TYPE!!!!!!!!! FOR ALL ARRAY ELEMENTS!!!!!

Try this to get an error !!!
M[0,0]=12
M[1,0]='hello'

Numpy – Matrices

```python
#USE OF MATRICES
print("Matrix a:\n", a)
print("First row:", a[0]) # this is just like a list of lists
print("element at (1,2) :", a[1, 2])   # arrays can be given comma separated indices
print(a[1, 1:3]) # and slices

print(a[:,1])
a[1, 2] = -3
print("a changed as\n",a)

a[:, 0] = [2, -1, 5]
print ("see how first row is changed")
print(a)
```

```
Matrix a:
 [[ 0  2  3]
 [ 9  6 -3]
 [ 8  4  6]]
First row: [0 2 3]
element at (1,2) : -3
[ 6 -3]
[2 6 4]
a changed as
 [[ 0  2  3]
 [ 9  6 -3]
 [ 8  4  6]]
see how first row is changed
[[ 2  2  3]
 [-1  6 -3]
 [ 5  4  6]]
```

Numpy – array creation and use

```
#array creation and use
x, y = numpy.mgrid[0:5, 0:5] # similar to meshgrid in MATLAB
print (x)
# random data
z=numpy.random.rand(5,5)
print (z)
```

```
[[0 0 0 0 0]
 [1 1 1 1 1]
 [2 2 2 2 2]
 [3 3 3 3 3]
 [4 4 4 4 4]]
[[0.17706244 0.17677506 0.76115697 0.43130308 0.95218228]
 [0.61385839 0.91252151 0.30113525 0.47876756 0.48179825]
 [0.96204896 0.64342828 0.6630705  0.19714807 0.32601357]
 [0.65244553 0.14365803 0.97918597 0.6870939  0.95028749]
 [0.51279853 0.26362755 0.90909429 0.32577498 0.52959444]]
```


numpy.arrange and numpy.linspace

- **arrange** returns an **ndarray** object containing evenly spaced values within a given range.
- In **linspace**, instead of step size, the number of evenly spaced values between the interval is specified.

```
#Creating numpy arrays -using creation functions
x = np.arange(0, 10, 1) # arguments: start, stop, step
print (x)
y=np.linspace(0, 10, 25) #arguments: start,stop,no.of points
print (y)
y = y.astype('complex')
print (y)
```

```
[0 1 2 3 4 5 6 7 8 9]
[ 0.          0.41666667  0.83333333  1.25        1.66666667  2.08333333
  2.5         2.91666667  3.33333333  3.75        4.16666667  4.58333333
  5.          5.41666667  5.83333333  6.25        6.66666667  7.08333333
  7.5         7.91666667  8.33333333  8.75        9.16666667  9.58333333
 10.        ]
[ 0.        +0.j  0.41666667+0.j  0.83333333+0.j  1.25      +0.j
  1.66666667+0.j  2.08333333+0.j  2.5       +0.j  2.91666667+0.j
  3.33333333+0.j  3.75      +0.j  4.16666667+0.j  4.58333333+0.j
  5.        +0.j  5.41666667+0.j  5.83333333+0.j  6.25      +0.j
  6.66666667+0.j  7.08333333+0.j  7.5       +0.j  7.91666667+0.j
  8.33333333+0.j  8.75      +0.j  9.16666667+0.j  9.58333333+0.j
 10.        +0.j]
```

dtype could be used in both functions

np.arrange

```
#np.arrange different uses
x = np.arange(10)
print ("x =",x)
y = np.arange(5,10)
z = np.arange(5,10,0.5)
print (z)

#reshape to a different dimension
zz = np.arange(1,10).reshape(3,3)
print (zz)
```

```
x = [0 1 2 3 4 5 6 7 8 9]
[5.  5.5 6.  6.5 7.  7.5 8.  8.5 9.  9.5]
[[1 2 3]
 [4 5 6]
 [7 8 9]]
```

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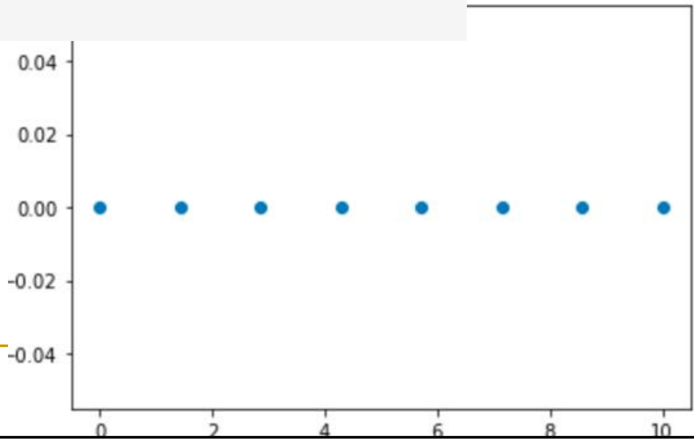
```
#linspace
import numpy as np
import matplotlib.pyplot as plt
fig = plt.figure()
ax = fig.add_subplot(111)
N = 8
y = np.zeros(N)
x1 = np.linspace(0, 10, N, endpoint=True)
p1 = plt.plot(x1, y, 'o')
ax.set_xlim([-0.5,10.5])
plt.show()
```

np.linspace



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Miscellaneous array creations : mgrid and random

```
#array creation and use
#mgrid
#random values in ndarrays
x,y = np.mgrid[0:5, 0:5] # similar to meshgrid in MATLAB
print (x)
print (y)
# random data
z=np.random.rand(5,5)
print (z)
```

```
[[0 0 0 0 0]
 [1 1 1 1 1]
 [2 2 2 2 2]
 [3 3 3 3 3]
 [4 4 4 4 4]]
[[0 1 2 3 4]
 [0 1 2 3 4]
 [0 1 2 3 4]
 [0 1 2 3 4]
 [0 1 2 3 4]]
[[0.12385852 0.30939883 0.94205462 0.19855433 0.20146846]
 [0.66198873 0.9794651  0.33698499 0.11377853 0.45960751]
 [0.00726439 0.2761222  0.18953797 0.66304801 0.94782969]
 [0.15115426 0.90193745 0.13085806 0.8968881  0.2364141 ]
 [0.49409317 0.66690493 0.52205047 0.88732595 0.60540875]]
```


Diagonal matrices

```
#Creating arrays
# a diagonal matrix
a=np.diag([1,2,3])
print (a)
b = np.zeros(5)
print(b)
print (b.dtype)
n = 1000
my_int_array = np.zeros(n, dtype=np.int)
print (my_int_array.dtype)
c = np.ones((3,3))
print (c)
```

```
[[1 0 0]
 [0 2 0]
 [0 0 3]]
[0. 0. 0. 0. 0.]
float64
int64
[[1. 1. 1.]
 [1. 1. 1.]
 [1. 1. 1.]]
```

```
#useful ndarray functions : sum, max, min, mean
x = np.array([[1,2,3],[4,5,6],[7,8,9]])
print ("x array is\n",x)
print ("ndim:",x.ndim)
print ("Axis-0 sum is {}".format(x.sum(axis=0)))
print ("Axis-1 sum is {}".format(x.sum(axis=1)))
print ("Axis-0 max is {}".format(x.max(axis=0)))
print ("Axis-1 min is {}".format(x.min(axis=1)))
print ("Axis-1 mean is {}".format(x.mean(axis=1)))
print ("Global sum is {}".format(x.sum()))
```

```
x array is
 [[1 2 3]
 [4 5 6]
 [7 8 9]]
ndim: 2
Axis-0 sum is [12 15 18]
Axis-1 sum is [ 6 15 24]
Axis-0 max is [7 8 9]
Axis-1 min is [1 4 7]
Axis-1 mean is [2. 5. 8.]
Global sum is 45
```

Useful ndarray functions working on axis

Numpy – other array methods

```
#other functions
#note that you can specify axis value in each of the functions
arr = np.array([4.5, 2.3, 6.7, 1.2, 1.8, 5.5])
print (arr.sum())
print (arr.mean())
print (arr.std())
print (arr.max())
print (arr.min())
print (div_by_3.all())
print (div_by_3.any())
print (div_by_3.sum())
print (div_by_3.nonzero())
```

```
22.0
3.6666666666666665
2.028683207293725
6.7
1.2
False
True
3
(array([2, 5, 8]),)
```

Numpy – Creating arrays from a file

testdata.txt

```
"Stn", "Data", "Tg", "qTg", "Tn", "qTn", "Tx", "qTx"
001, 19010101,   -49, 00,   -68, 00,   -22, 40
001, 19010102,   -21, 00,   -36, 30,   -13, 30
001, 19010103,   -28, 00,   -79, 30,    -5, 20
001, 19010104,   -64, 00,   -91, 20,   -10, 00
001, 19010105,   -59, 00,   -84, 30,   -18, 00
001, 19010106,   -99, 00,  -115, 30,   -78, 30
001, 19010107,   -91, 00,  -122, 00,   -66, 00
001, 19010108,   -49, 00,   -94, 00,    -6, 00
001, 19010109,    11, 00,   -27, 40,    42, 00
```

```
#file I/O
import os
print(os.system('head testdata.txt'))
data = np.genfromtxt('testdata.txt', delimiter=',', skip_header=1)
print (data.shape)
print (data)
np.savetxt('datasaved.txt', data)
```

```
0
(9, 8)
[[ 1.0000000e+00  1.9010101e+07 -4.9000000e+01  0.0000000e+00
  -6.8000000e+01  0.0000000e+00 -2.2000000e+01  4.0000000e+01]
 [ 1.0000000e+00  1.9010102e+07 -2.1000000e+01  0.0000000e+00
  -3.6000000e+01  3.0000000e+01 -1.3000000e+01  3.0000000e+01]
 [ 1.0000000e+00  1.9010103e+07 -2.8000000e+01  0.0000000e+00
  -7.9000000e+01  3.0000000e+01 -5.0000000e+00  2.0000000e+01]
```


Numpy – file i/o

```
#file I/O
M = np.random.rand(3,3)
print (M)
np.save('saved-matrix.npy', M)     #binary format!
N=np.load('saved-matrix.npy')
print (N)
print (N.ndim, N.shape)
```

```
[[0.51996506 0.80997384 0.73765913]
 [0.26754792 0.30598719 0.44882472]
 [0.26466518 0.87206017 0.99212161]]
[[0.51996506 0.80997384 0.73765913]
 [0.26754792 0.30598719 0.44882472]
 [0.26466518 0.87206017 0.99212161]]
2 (3, 3)
```

Numpy – object copying

Two ndarrays are mutable and may be views to the same memory:

```
#numpy arrays are mutable
#be careful when you create
#another copy of an nd array
x = np.array([1,2,3,4])
y = x
print (x is y)
print (id(x), id(y))
x[0] = 9
print (y)
x[0] = 1
z = x[:]
print (x is z)
print (id(x), id(z))
x[0] = 8
print (z)
```

```
True
140216049723232 140216049723232
[9 2 3 4]
False
140216049723232 140216049762352
[8 2 3 4]
```

```
x = np.array([1,2,3,4])
y = x.copy()
print (x is y)
print (id(x), id(y))
x[0] = 9
print (x)
print (y)
```

```
False
140216049724512 140216049765632
[9 2 3 4]
[1 2 3 4]
```

(Faster)

numpy basic operations

- Operations on ndarrays are elemet-wise

```
#numpy ndarray basic operations
a = np.array([3,4,5])
b = np.ones(3)
print ("a-b:", a - b)

a = np.array([[1,2],[3,4]])
b = np.array([[1,2],[3,4]])
print ("a:\n",a)
print ("b:\n",b)
print ("a*b:\n", a * b)
print ("dot product of a and b:\n",np.dot(a,b))
```

```
a-b: [2. 3. 4.]
a:
 [[1 2]
 [3 4]]
b:
 [[1 2]
 [3 4]]
a*b:
 [[ 1  4]
 [ 9 16]]
dot product of a and b:
 [[ 7 10]
 [15 22]]
```

Special operators: +=, -=, *=, /=, **= , //=, %=

```
#special addition and
#multiplication operators
a = np.zeros((2,2),dtype='float')
a += 5
print (a)
a *= 5
print (a)
a = a + 2*a
print (a)
```

```
[[5. 5.]
 [5. 5.]]
[[25. 25.]
 [25. 25.]]
[[75. 75.]
 [75. 75.]]
```


Concatenation of arrays

```
#Concatenation of arrays
a = np.array([1,2,3])
b = np.array([4,5,6])
c = np.array([7,8,9])
a_h = np.hstack([a,b,c]) #horizontal stack
a_v = np.vstack([a,b,c])
print (a_h)
print (a_v)
```

```
[1 2 3 4 5 6 7 8 9]
[[1 2 3]
 [4 5 6]
 [7 8 9]]
```

Array sort

```
#array sort
arr = np.array([4.5, 2.3, 6.7, 1.2, 1.8, 5.5])
arr.sort()  # acts on array itself
print(arr)
x = np.array([4.5, 2.3, 6.7, 1.2, 1.8, 5.5])
print ("x is\n",x)
xx=np.sort(x)
print (xx)
print(x)
s = x.argsort() #sorts indice no.s wrt corresponding values
print (s)
print (x[s])
```

```
[1.2 1.8 2.3 4.5 5.5 6.7]
x is
 [4.5 2.3 6.7 1.2 1.8 5.5]
[1.2 1.8 2.3 4.5 5.5 6.7]
[4.5 2.3 6.7 1.2 1.8 5.5]
[3 4 1 0 5 2]
[1.2 1.8 2.3 4.5 5.5 6.7]
```


Miscellaneous uses

```
a = numpy.arange(4.0)
b = a * 23.4
c = b/(a+1)
c += 10
print (c)
arr = numpy.arange(100, 200)
select = [5, 25, 50, 75, -5]
print(arr[select])  # can use integer lists as indices
arr = numpy.arange(10, 20 )
div_by_3 = arr%3 == 0  # comparison produces boolean array
print(div_by_3)
print(arr[div_by_3])  # can use boolean lists as indices
arr = numpy.arange(10, 20) . reshape((2,5))
print (arr)
```

```
[10.   21.7  25.6  27.55]
[105 125 150 175 195]
[False False  True False False  True False False  True False]
[12 15 18]
[[10 11 12 13 14]
 [15 16 17 18 19]]
```

Common numpy functions useful for engineers

```
#common mathematical functions
x = np.arange(1,5)
result = np.sqrt(x) * np.pi
print (result)
print (np.power(2,4)) #much faster than python equivalent
print (x.max() - x.min())

#exponential & log
arr = np.array([10,8,4])
print(np.exp(arr)) #e^x
print (np.log(arr)) #ln(x), base is e
print ("e :", np.e, "pi:", np.pi)
print (np.log10(arr)) #log(x), base is 10
print (np.log2(arr)) #log(x), base is 2

#rounding
print ("\nROUNDING")
arr = np.array([20.8999,67.89899,54.23409])
print(np.around(arr,2)) #round off with 2 decimals
print(np.floor(arr)) #largest integer less than input number
print(np.ceil(arr))  #smallest integer greater than input num

#trigonometric
print ("\nTRIGONOMETRIC FUNCTIONS")
arr = np.array([0, 30, 60, 90, 120, 150, 180])
print(np.sin(arr * np.pi / 180)) #sine function
print(np.cos(arr * np.pi / 180)) #cosine
```

Algebraic
Rounding
Logarithm
Trigonometry
Complex numbers

Numpy – statistics

More functions are available in **scipy**

```
#statistics
a = np.array([1, 4, 3, 8, 9, 2, 3], float)
print ("median:",np.median(a))
b = np.array([[1, 2, 1, 3], [5, 3, 1, 8]], float)
c = np.corrcoef(b)
print ("Correlation:", c)
d = np.corrcoef(a,a)
print ("Correlation:", d)
a = np.array([1,2,3,4,6,7,8,9])
b = np.array([2,4,6,8,10,12,13,15])
c = np.array([-1,-2,-2,-3,-4,-6,-7,-8])
print (np.corrcoef([a,b,c]))
print ("Covariance: ", np.cov(a)) #covariance
print ("Variance: ", np.var(a)) #covariance
print ("Standard deviation: ", np.std(a)) #covariance
```

```
median: 3.0
Correlation: [[1.         0.72870505]
 [0.72870505 1.        ]]
Correlation: [[1. 1.]
 [1. 1.]]
[[ 1.          0.99535001 -0.9805214 ]
 [ 0.99535001  1.         -0.97172394]
 [-0.9805214  -0.97172394  1.        ]]
Covariance:  8.571428571428571
Variance:  7.5
Standard deviation:  2.7386127875258306
```

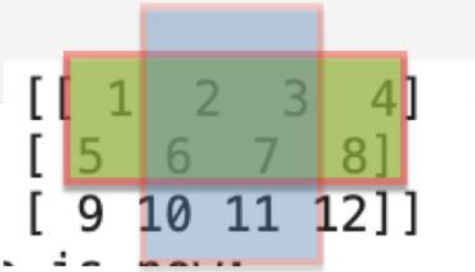
Linear Algebra

Matrix operations

```python
#numpy linear algebra package : linalg
A = np.array([[6, 1, 1],
              [4, -2, 5],
              [2, 8, 7]])
# Rank of a matrix
print("Rank of A:", np.linalg.matrix_rank(A))

# Trace of matrix A
print("\nTrace of A:", np.trace(A))

# Determinant of a matrix
print("\nDeterminant of A:", np.linalg.det(A))

# Inverse of matrix A
print("\nInverse of A:\n", np.linalg.inv(A))

print("\nMatrix A raised to power 3:\n",
           np.linalg.matrix_power(A, 3))

#solving linear eqn systems
b = [1,2,3]
x = np.linalg.solve(A,b)
print ("Solution vector:", x)

#Eigenvalues and eigen vectors
#Creating an array using diag function
a = np.diag((1, 2, 3))
print("Array is :",a)

# calculating an eigen value using eigvals() function
c = np.linalg.eigvals(a)
print("Eigen value is :",c)

# calculating an eigen value using eig() function
c, d = np.linalg.eig(a)
print("Eigen value is :",c)
print("Eigen vector is :",d)
```


Array slicing –sub arrays

```python
# ARRAY INDEXING
# Create the following  2 array with shape (3, 4)
# [[ 1  2  3  4]
#  [ 5  6  7  8]
#  [ 9 10 11 12]]
a = np.array([[1,2,3,4], [5,6,7,8], [9,10,11,12]])
print ("a is :\n", a)

# Use slicing to pull out the subarray consisting of the first 2 rows
# and columns 1 and 2; b is the following array of shape (2, 2):
# [[2 3]
#  [6 7]]
b = a[:2, 1:3]
print ("b is now:\n",b)
```




```
a is :
 [[ 1  2  3  4]
 [ 5  6  7  8]
 [ 9 10 11 12]]
b is now:
 [[2 3]
 [6 7]]
```

```
#misc ndarray operations
print(a[0])  # this is just like a list of lists
print(a[1][2])  #print a single value ==> 7 will be written
print(a[1, 2])  # arrays can be given comma separated indices
print(a[1, 1:3])  # and slices
print(a[:,1])
a[1, 2] = 7
print(a)
a[:, 0] = [0, 9, 8]
print(a)
```

```
[1 2 3 4]
7
7
[6 7]
[ 2  6 10]
[[ 1  2  3  4]
 [ 5  6  7  8]
 [ 9 10 11 12]]
[[ 0  2  3  4]
 [ 9  6  7  8]
 [ 8 10 11 12]]
```


Boolean array indexing

```
#BOOLEAN ARRAY INDEXING
a = np.array([[1,2], [3, 4], [5, 6]])
bool_idx = (a > 2)  # Find the elements which are bigger than 2;
                    # this returns a numpy array of Booleans of the same
                    # shape as a, where each slot of bool_idx tells
                    # whether that element of a is > 2.

print (bool_idx)
print (a[a > 2])
```

```
[[False False]
 [ True  True]
 [ True  True]]
[3 4 5 6]
```

Using arrays wisely

- Array operations are implemented in C or Fortran
- Optimised algorithms - i.e. fast!
- Python loops (i.e. for i in a:…) are much slower
- **Prefer array operations over loops, especially when speed important**
- Also produces shorter code, often more readable

Numpy – arrays, matrices

For **two dimensional** arrays NumPy defined a special matrix class in module matrix. Objects are created either with matrix() or mat() or converted from an array with method **asmatrix().**

Note that the statement **m = mat(a)** creates a copy of array 'a'.

Changing values in 'a' will not affect 'm'.

On the other hand, method **m = asmatrix(a)** returns a new reference to the same data. Changing values in 'a' will affect matrix 'm'.

```
#matrices -different definitions
m = numpy.mat([[1,2],[3,4]])
print (m)
a = numpy.array([[1,2],[3,4]])
print (a)
m = numpy.mat(a)
print (m)
#or
a = numpy.array([[1,2],[3,4]])
print (a)
m = numpy.asmatrix(a)
print (m)
```

```
[[1 2]
 [3 4]]
[[1 2]
 [3 4]]
```

Numpy – matrices

Array and matrix operations may be quite different!

```
#array vs matrix
a = numpy.array([[1,2],[3,4]])
m = numpy.mat(a) # convert 2-d array to matrix
m = numpy.matrix([[1, 2], [3, 4]])
print (a[0]) # result is 1-dimensional
print (m[0])    # result is 2-dimensional
print (a*a)    # element-by-element multiplication
print (m*m)    # (algebraic) matrix multiplication
print (a**3)    # element-wise power
print (m**3)    # matrix multiplication m*m*m
print (m.T) # transpose of the matrix
print (m.H)  # conjugate transpose (differs from .T for complex matrices)
print (m.I) # inverse matrix
```

```
[1 2]
[[1 2]]
[[ 1  4]
 [ 9 16]]
[[ 7 10]
 [15 22]]
[[ 1  8]
 [27 64]]
[[ 37  54]
```


Numpy – matrices

- Operator *, dot(), and multiply():
  - For array, '*' **means element-wise multiplication**, and the dot() function is used for matrix multiplication.
  - For matrix, '*'**means matrix multiplication**, and the multiply() function is used for element-wise multiplication.
- Handling of vectors (rank-1 arrays)
  - For array, the vector shapes 1xN, Nx1, and N are all different things. Operations like A[:,1] return a rank-1 array of shape N, not a rank-2 of shape Nx1. Transpose on a rank-1 array does nothing.
  - For matrix, rank-1 arrays are always upgraded to 1xN or Nx1 matrices (row or column vectors). A[:,1] returns a rank-2 matrix of shape Nx1.
- Handling of higher-rank arrays (rank > 2)
  - array objects can have rank > 2.
  - matrix objects always have exactly rank 2.
- Convenience attributes
  - array has a .T attribute, which returns the transpose of the data.
  - matrix also has .H, .I, and .A attributes, which return the conjugate transpose, inverse, and asarray() of the matrix, respectively.
- Convenience constructor
  - The array constructor takes (nested) Python sequences as initializers. As in array([[1,2,3],[4,5,6]]).
  - The matrix constructor additionally takes a convenient string initializer. As in matrix("[1 2 3; 4 5 6]")

Numpy – matrix mathematics

```
#array math
A = numpy.array([[n+m*10 for n in range(5)] for m in range(5)])
v1 = numpy.arange(0, 5)
print (A)
print (v1)
print (numpy.dot(A,A))
print (numpy.dot(A,v1))
print (numpy.dot(v1,v1))
```

```
[[ 0  1  2  3  4]
 [10 11 12 13 14]
 [20 21 22 23 24]
 [30 31 32 33 34]
 [40 41 42 43 44]]
[0 1 2 3 4]
[[ 300  310  320  330  340]
 [1300 1360 1420 1480 1540]
 [2300 2410 2520 2630 2740]
 [3300 3460 3620 3780 3940]
 [4300 4510 4720 4930 5140]]
[ 30 130 230 330 430]
30
```

```
#Alternatively, we can cast the array objects to the type matrix.
#This changes the behavior of the standard arithmetic
#operators +, -, * to use matrix algebra.
M = numpy.matrix(A)
v = numpy.matrix(v1).T
print (v)
print (M*v)
print (v.T * v)   # inner product
# standard matrix algebra applies
print (v + M*v)
```

```
[[0]
 [1]
 [2]
 [3]
 [4]]
[[ 30]
 [130]
 [230]
 [330]
 [430]]
[[30]]
[[ 30]
 [131]
 [232]
 [333]
 [434]]
```


scipy

- SciPy is a scientific python open source to perform Mathematical, Scientific and Engineering Computations.
- The SciPy library depends on **NumPy**, which provides convenient and fast N-dimensional array manipulation.
- The SciPy library is built to work with NumPy arrays and provides many user-friendly and efficient numerical practices such as routines for numerical integration and optimization.
- Together, they run on all popular operating systems, are quick to install and are free of charge.
- SciPy is easy to use, but powerful enough to depend on by some of the world's leading scientists and engineers.

scipy core sub-packages

scipy.cluster	Vector quantization / Kmeans
scipy.constants	Physical and mathematical constants
scipy.fftpack	Fourier transform
scipy.integrate	Integration routines
scipy.interpolate	Interpolation
scipy.io	Data input and output
scipy.linalg	Linear algebra routines
scipy.ndimage	n-dimensional image package
scipy.odr	Orthogonal distance regression
scipy.optimize	Optimization
scipy.signal	Signal processing
scipy.sparse	Sparse matrices
scipy.spatial	Spatial data structures and algorithms
scipy.special	Any special mathematical functions
scipy.stats	Statistics

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```
import scipy.constants as sc
print ("Pi: {} , elementary charge: {}, Planck: {}".format(sc.pi, sc.e, sc.h))
print ("Speed of light: {} , gravitational constant: {}, Avogadro: {}".format(sc.c, sc.G, sc.Avogadro))
print ("Standard aptodphere in pascal {} , Boltzman constant: {}, km/h to m/s: {}".format(sc.atm, sc.k, sc.kmh))
print ("And many more!!!!!")
```

```
Pi: 3.141592653589793 , elementary charge: 1.602176634e-19, Planck: 6.62607015e-34
Speed of light: 299792458.0 , gravitational constant: 6.6743e-11, Avogadro: 6.02214076e+23
Standard aptodphere in pascal 101325.0 , Boltzman constant: 1.380649e-23, km/h to m/s: 0.2777777777777778
And many more!!!!!
```

$$\int_0^1 e^{-x^2}\, dx = \frac{1}{2}\sqrt{\pi}\ \mathrm{erf}(1) \approx 0.746824$$

```
#Numerical Integration
import scipy.integrate
from numpy import exp

def f(x):
    return exp(-x**2)
i = scipy.integrate.quad(f, 0, 1)
print (i)
```

```
(0.7468241328124271, 8.291413475940725e-15)
```

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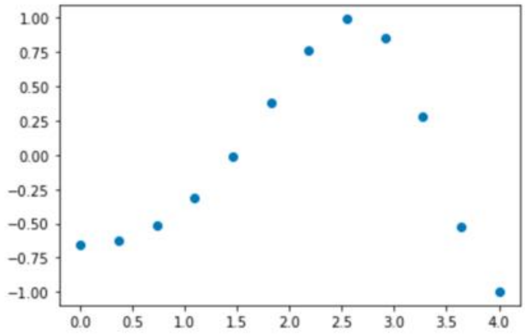
scipy interpolation

```python
#Interpolation
import numpy as np
from scipy import interpolate
import matplotlib.pyplot as plt

x = np.linspace(0, 4, 12)
y = np.cos(x**2/3+4)
print (x,y)
```

```
[0.         0.36363636 0.72727273 1.09090909 1.454545
 2.18181818 2.54545455 2.90909091 3.27272727 3.636363
5364362 -0.61966189 -0.51077021 -0.31047698 -0.007154
  0.76715099  0.99239518  0.85886263  0.27994201 -0.5
```

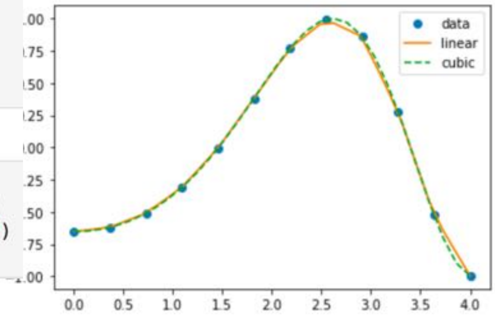
```python
#plot points
plt.plot(x, y,'o')
plt.show()
```



```python
#linear and cubic interpolations
f1 = interpolate.interp1d(x, y, kind = 'linear')
f2 = interpolate.interp1d(x, y, kind = 'cubic')
print (f2(3.7))
```

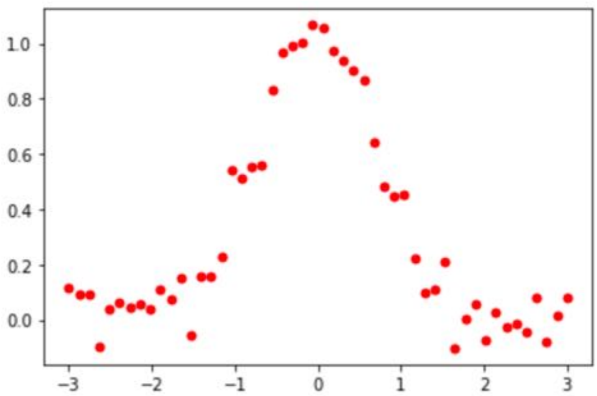
```
-0.6503130110905226
```

```python
xnew = np.linspace(0, 4,30)
plt.plot(x, y, 'o', xnew, f1(xnew), '-', xnew, f2(xnew), '--')
plt.legend(['data', 'linear', 'cubic','nearest'], loc = 'best')
plt.show()
```

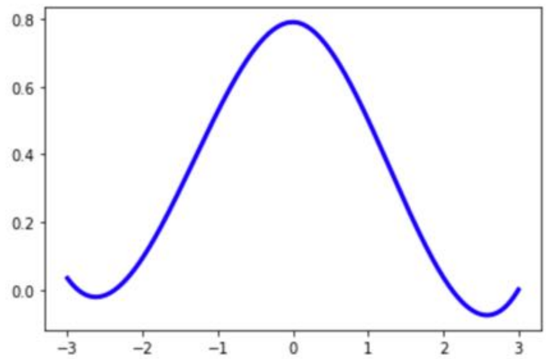


Cubic spline

```python
#scipy cubic spline
import matplotlib.pyplot as plt
from scipy.interpolate import UnivariateSpline
x = np.linspace(-3, 3, 50)
y = np.exp(-x**2) + 0.1 * np.random.randn(50)
plt.plot(x, y, 'ro', ms = 5)
plt.show()
```



```python
spl = UnivariateSpline(x, y)
xs = np.linspace(-3, 3, 1000)
#fine smoothing
spl.set_smoothing_factor(1.1)
plt.plot(xs, spl(xs), 'b', lw = 3)
plt.show()
```



linalg

```
#scipy linalg
#importing the scipy and numpy packages
from scipy import linalg
import numpy as np

#Declaring the numpy arrays
a = np.array([[1, 3, 5], [2, 5, 1], [2, 3, 8]])
b = np.array([10, 8, 3])

#Passing the values to the solve function
x = linalg.solve(a, b)

#printing the result array
print (x)
```

```
[-9.28  5.16  0.76]
```

$$x + 3y + 5z = 10$$

$$2x + 5y + z = 8$$

$$2x + 3y + 8z = 3$$

Root finding

```
import numpy as np
from scipy.optimize import root
def func(x):
    return x*2 + 2 * np.cos(x)
sol = root(func, 0.3, method="lm")
print ("Solution response :\n", sol)
print ("Solution:", sol["x"])
print ("Error:", sol["fun"])
```

```
Solution response :
     cov_x: array([[0.08925456]])
      fjac: array([[-3.34722404]])
       fun: array([0.])
      ipvt: array([1], dtype=int32)
   message: 'The relative error between twc
0.000000'
      nfev: 12
       qtf: array([2.49797583e-09])
    status: 2
   success: True
         x: array([-0.73908513])
Solution: [-0.73908513]
Error: [0.]
```

- **lm**: Use least squares with Levenberg-Marquardt
- **hybr**: Find the roots of a multivariate function using MINPACK's hybrd and hybrj routines (modified Powell method)
- broyden1
- broyden2
- anderson
- linearmixing
- diagbroyden
- excitingmixing
- kyrlov
- df-sane

$$x^2 + 2\cos(x) = 0$$

Optimization

```python
from scipy.optimize import minimize

def eqn(x):
  return x**2 + x + 2

#use Broyden-Fletcher-Goldfarb-Shanno algorithm
mymin = minimize(eqn, 0, method='BFGS')
print(mymin)
```

```
      fun: 1.75
 hess_inv: array([[0.50000001]])
      jac: array([0.])
  message: 'Optimization terminated successfully.'
     nfev: 8
      nit: 2
     njev: 4
   status: 0
  success: True
        x: array([-0.50000001])
```


(backup slides)

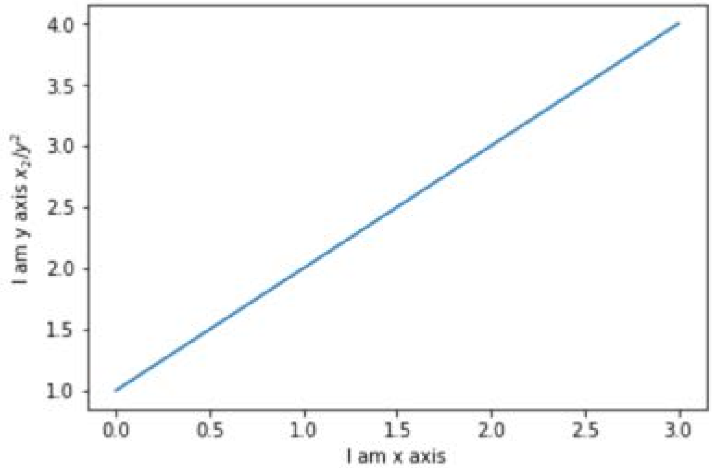
- We'll first investigate an important python library: **matplotlib**

matplotlib

Simple Plot. The most basic plot(), with text labels

```
import matplotlib.pyplot as plt
plt.plot([1,2,3,4])
plt.ylabel('I am y axis $x_2/{y^2}$')
plt.xlabel('I am x axis')
plt.show()
```

Only y data is provided. x automatically becomes [0,1,2,3]



LaTeX is possible in texts

See that data is given as a list

matplotlib

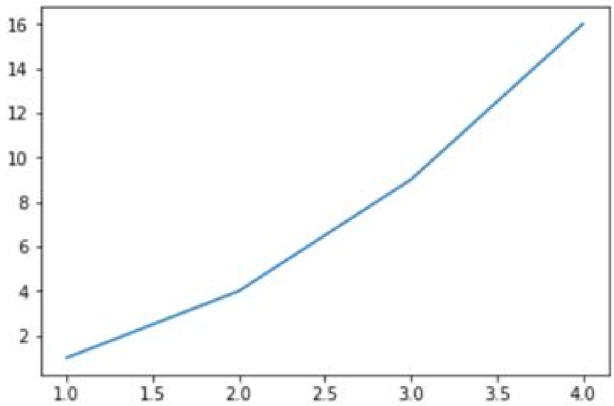
plot() is a versatile command, and will take an arbitrary number of arguments. For example, to plot x versus y, you can issue the command:

plt.plot([1,2,3,4], [1,4,9,16])

there is an optional third argument which is the format string that indicates the color and line type of the plot.

```
plt.plot([1,2,3,4], [1,4,9,16])
```

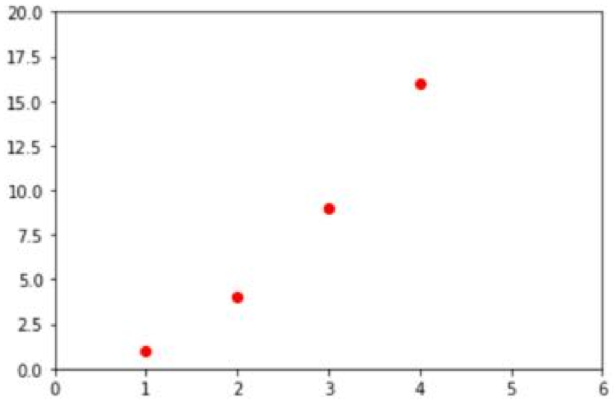
```
[<matplotlib.lines.Line2D at 0x1135c3860>]
```



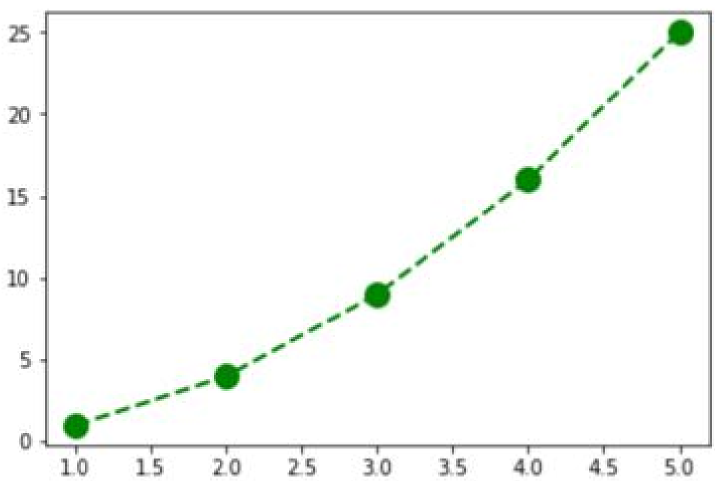
matplotlib –miscellaneous examples

```
import matplotlib.pyplot as plt
plt.plot([1,2,3,4], [1,4,9,16], 'ro')
plt.axis([0, 6, 0, 20])
plt.show()
```

r ➔ red
o ➔ circle

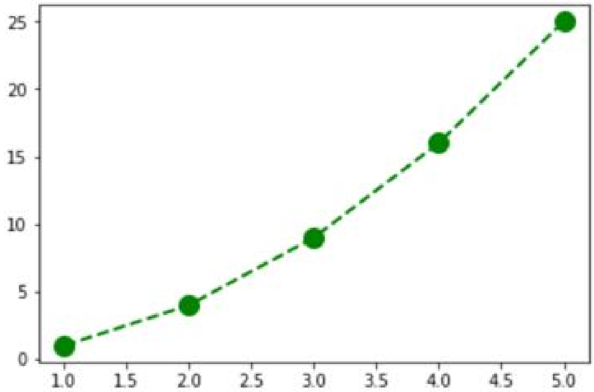



```
x=[1,2,3,4,5]
y=[1,4,9,16,25]
plt.plot(x, y, 'go--', linewidth=2, markersize=12)
plt.show();
```



Try changing values in **plt.plot**

```
x=[1,2,3,4,5]
y=[1,4,9,16,25]
plt.plot(x, y, color='green', marker='o', linestyle='dashed', linewidth=2, markersize=12)
plt.show()
```

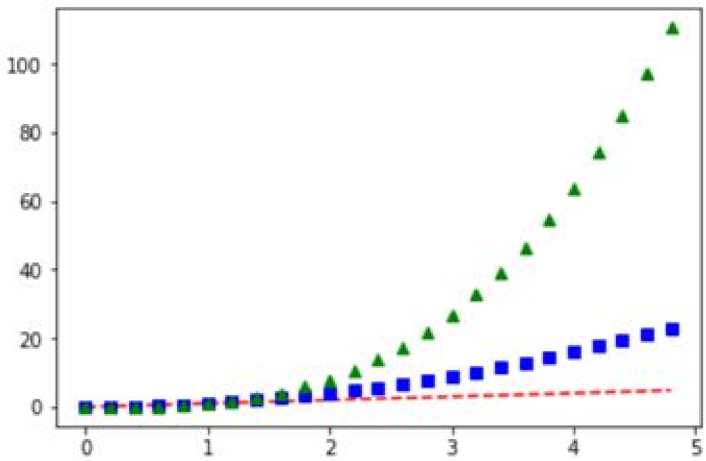


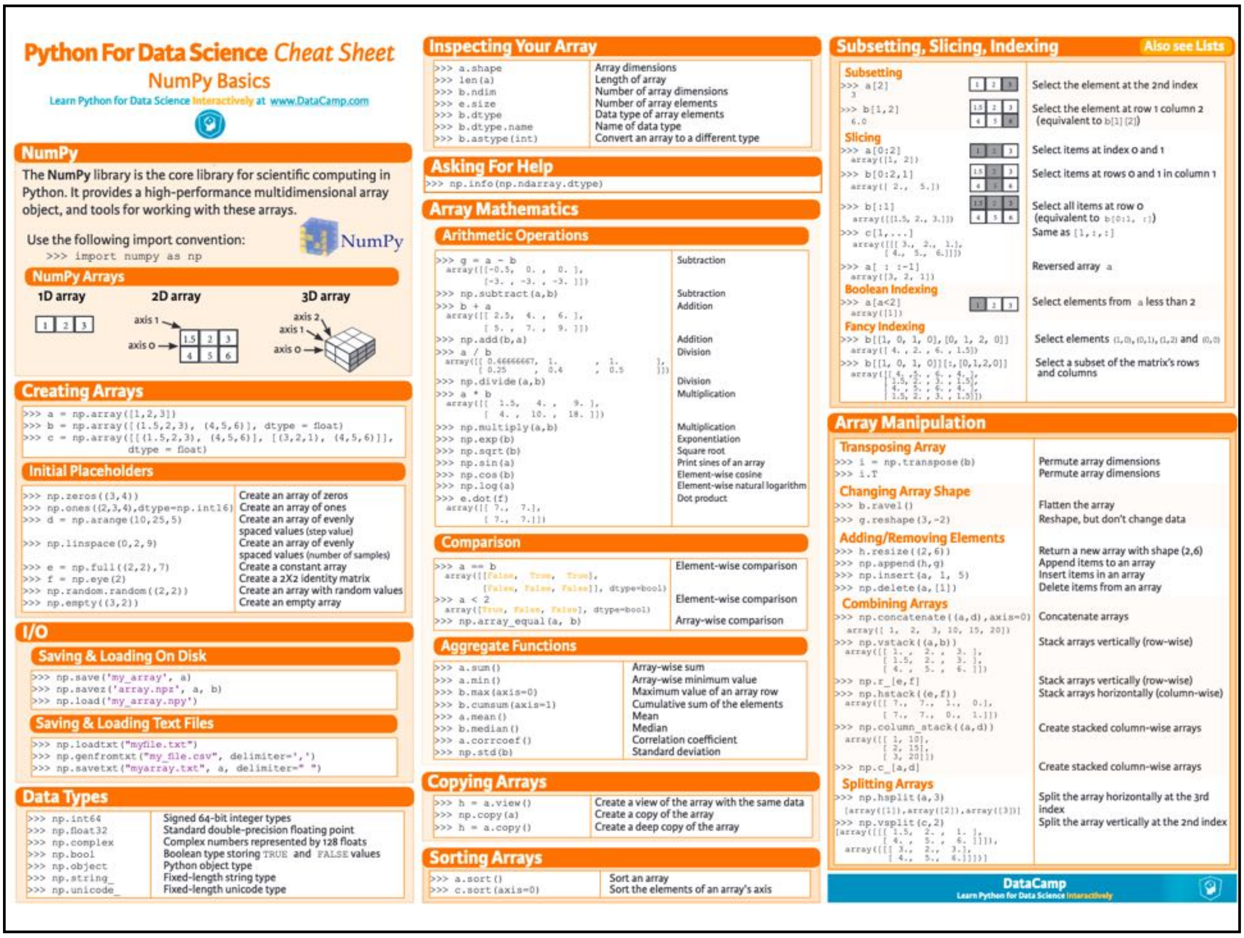
Linestyles: dashed, dotted, solid,- , -- , -. , . , :

Markers: o x X . , v < > ^ 1 2 3 4 8 s p P * h H d D |

Matplotlib –many graphics at once

```
import matplotlib.pyplot as plt
t= [float(x)/10.0 for x in range(0,50,2)]
t2=[x**2 for x in t]
t3=[x**3 for x in t]
plt.plot(t,t,'r--', t,t2,'bs', t,t3,'g^')
plt.show()
```





Python For Data Science Cheat Sheet

NumPy Basics

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NumPy

The NumPy library is the core library for scientific computing in Python. It provides a high-performance multidimensional array object, and tools for working with these arrays.

Use the following import convention:
```
>>> import numpy as np
```

NumPy Arrays

1D array, 2D array, 3D array (axis 0, axis 1, axis 2)

Creating Arrays

```
>>> a = np.array([1,2,3])
>>> b = np.array([(1.5,2,3), (4,5,6)], dtype = float)
>>> c = np.array([[(1.5,2,3), (4,5,6)], [(3,2,1), (4,5,6)]],
                 dtype = float)
```

Initial Placeholders

`>>> np.zeros((3,4))`	Create an array of zeros
`>>> np.ones((2,3,4),dtype=np.int16)`	Create an array of ones
`>>> d = np.arange(10,25,5)`	Create an array of evenly spaced values (step value)
`>>> np.linspace(0,2,9)`	Create an array of evenly spaced values (number of samples)
`>>> e = np.full((2,2),7)`	Create a constant array
`>>> f = np.eye(2)`	Create a 2X2 identity matrix
`>>> np.random.random((2,2))`	Create an array with random values
`>>> np.empty((3,2))`	Create an empty array

I/O

Saving & Loading On Disk

```
>>> np.save('my_array', a)
>>> np.savez('array.npz', a, b)
>>> np.load('my_array.npy')
```

Saving & Loading Text Files

```
>>> np.loadtxt("myfile.txt")
>>> np.genfromtxt("my_file.csv", delimiter=',')
>>> np.savetxt("myarray.txt", a, delimiter=" ")
```

Data Types

`>>> np.int64`	Signed 64-bit integer types
`>>> np.float32`	Standard double-precision floating point
`>>> np.complex`	Complex numbers represented by 128 floats
`>>> np.bool`	Boolean type storing TRUE and FALSE values
`>>> np.object`	Python object type
`>>> np.string_`	Fixed-length string type
`>>> np.unicode_`	Fixed-length unicode type

Inspecting Your Array

`>>> a.shape`	Array dimensions
`>>> len(a)`	Length of array
`>>> b.ndim`	Number of array dimensions
`>>> e.size`	Number of array elements
`>>> b.dtype`	Data type of array elements
`>>> b.dtype.name`	Name of data type
`>>> b.astype(int)`	Convert an array to a different type

Asking For Help

```
>>> np.info(np.ndarray.dtype)
```

Array Mathematics

Arithmetic Operations

`>>> g = a - b`	Subtraction
`>>> np.subtract(a,b)`	Subtraction
`>>> b + a`	Addition
`>>> np.add(b,a)`	Addition
`>>> a / b`	Division
`>>> np.divide(a,b)`	Division
`>>> a * b`	Multiplication
`>>> np.multiply(a,b)`	Multiplication
`>>> np.exp(b)`	Exponentiation
`>>> np.sqrt(b)`	Square root
`>>> np.sin(a)`	Print sines of an array
`>>> np.cos(b)`	Element-wise cosine
`>>> np.log(a)`	Element-wise natural logarithm
`>>> e.dot(f)`	Dot product

Comparison

`>>> a == b`	Element-wise comparison
`>>> a < 2`	Element-wise comparison
`>>> np.array_equal(a, b)`	Array-wise comparison

Aggregate Functions

`>>> a.sum()`	Array-wise sum
`>>> a.min()`	Array-wise minimum value
`>>> b.max(axis=0)`	Maximum value of an array row
`>>> b.cumsum(axis=1)`	Cumulative sum of the elements
`>>> a.mean()`	Mean
`>>> b.median()`	Median
`>>> a.corrcoef()`	Correlation coefficient
`>>> np.std(b)`	Standard deviation

Copying Arrays

`>>> h = a.view()`	Create a view of the array with the same data
`>>> np.copy(a)`	Create a copy of the array
`>>> h = a.copy()`	Create a deep copy of the array

Sorting Arrays

`>>> a.sort()`	Sort an array
`>>> c.sort(axis=0)`	Sort the elements of an array's axis

Subsetting, Slicing, Indexing

Also see Lists

Subsetting	
`>>> a[2]`	Select the element at the 2nd index
`>>> b[1,2]`	Select the element at row 1 column 2 (equivalent to b[1][2])
Slicing	
`>>> a[0:2]`	Select items at index 0 and 1
`>>> b[0:2,1]`	Select items at rows 0 and 1 in column 1
`>>> b[:1]`	Select all items at row 0 (equivalent to b[0:1, :])
`>>> c[1,...]`	Same as [1,:,:]
`>>> a[ : :-1]`	Reversed array a
Boolean Indexing	
`>>> a[a<2]`	Select elements from a less than 2
Fancy Indexing	
`>>> b[[1, 0, 1, 0], [0, 1, 2, 0]]`	Select elements (1,0),(0,1),(1,2) and (0,0)
`>>> b[[1, 0, 1, 0]][:,[0,1,2,0]]`	Select a subset of the matrix's rows and columns

Array Manipulation

Transposing Array	
`>>> i = np.transpose(b)`	Permute array dimensions
`>>> i.T`	Permute array dimensions
Changing Array Shape	
`>>> b.ravel()`	Flatten the array
`>>> g.reshape(3,-2)`	Reshape, but don't change data
Adding/Removing Elements	
`>>> h.resize((2,6))`	Return a new array with shape (2,6)
`>>> np.append(h,g)`	Append items to an array
`>>> np.insert(a, 1, 5)`	Insert items in an array
`>>> np.delete(a,[1])`	Delete items from an array
Combining Arrays	
`>>> np.concatenate((a,d),axis=0)`	Concatenate arrays
`>>> np.vstack((a,b))`	Stack arrays vertically (row-wise)
`>>> np.r_[e,f]`	Stack arrays vertically (row-wise)
`>>> np.hstack((e,f))`	Stack arrays horizontally (column-wise)
`>>> np.column_stack((a,d))`	Create stacked column-wise arrays
`>>> np.c_[a,d]`	Create stacked column-wise arrays
Splitting Arrays	
`>>> np.hsplit(a,3)`	Split the array horizontally at the 3rd index
`>>> np.vsplit(c,2)`	Split the array vertically at the 2nd index

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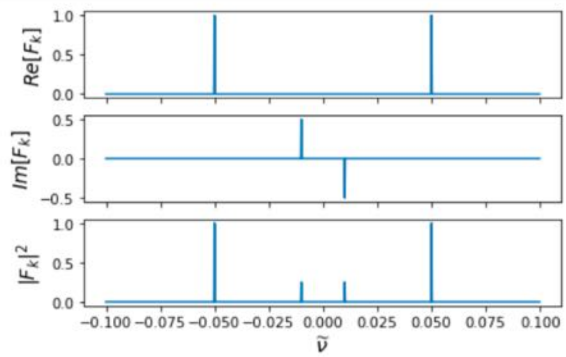
(OPTIONAL) Numpy FFT

```python
#numpy FFT :-)
#optional topic
from numpy import fft
import numpy as np
import matplotlib.pyplot as plt
n = 1000    # Number of data points
dx= 5.0     # Sampling period (in meters)
x = dx*np.arange(0,n)    # x coordinates
w1 = 100.0  # wavelength (meters)
w2 = 20.0   # wavelength (meters)
fx= np.sin(2*np.pi*x/w1) + 2*np.cos(2*np.pi*x/w2) # signal
Fk= fft.fft(fx)/n     # Fourier coefficients (divided by n)
nu= fft.fftfreq(n,dx)  # Natural frequencies
Fk= fft.fftshift(Fk)   # Shift zero freq to center
nu= fft.fftshift(nu)   # Shift zero freq to center
f, ax = plt.subplots(3,1,sharex=True)
ax[0].plot(nu, np.real(Fk))    # Plot Cosine terms
ax[0].set_ylabel(r'$Re[F_k]$', size = 'x-large')
ax[1].plot(nu, np.imag(Fk))          # Plot Sine terms
ax[1].set_ylabel(r'$Im[F_k]$', size = 'x-large')
ax[2].plot(nu, np.absolute(Fk)**2)  # Plot spectral power
ax[2].set_ylabel(r'$\vert F_k\vert ^2$', size = 'x-large')
ax[2].set_xlabel(r'$\widetilde{\nu}$', size = 'x-large')
plt.show()
```



SEE YOU NEXT WEEK!!!

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