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WHEN INTUITION MISFIRES

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**HYPER SUDOKUS ARE HARDER  
THAN STANDARD SUDOKUS**

## WHAT IS A HYPER SUDOKU?

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- 4 additional blocks
- in each **hyperblock**, each number from 1 to 9 appears exactly once

							1	
		2					3	4
				5	1			
					6	5		
	7		3				8	
		3						
				8				
5	8					9		
6	9							

# HYPER SUDOKUS ARE EASIER\* THAN STANDARD SUDOKU

							1	
		2					3	4
				5	1			
					6	5		
	7		3				8	
		3						
				8				
5	8					9		
6	9							

/	/						1	
		2					3	4
/	/		/	5	1			
	/		/		6	5		
	7		3				8	
		3						
				8				
5	8					9		
6	9							

\* *hardness* with zChaff: number of added **conflict literals** and **conflict clauses**  
*hardness* with WalkSAT: number of **restarts** and **variable flips**

# EXPERIMENTAL SETUP

## SAT solvers

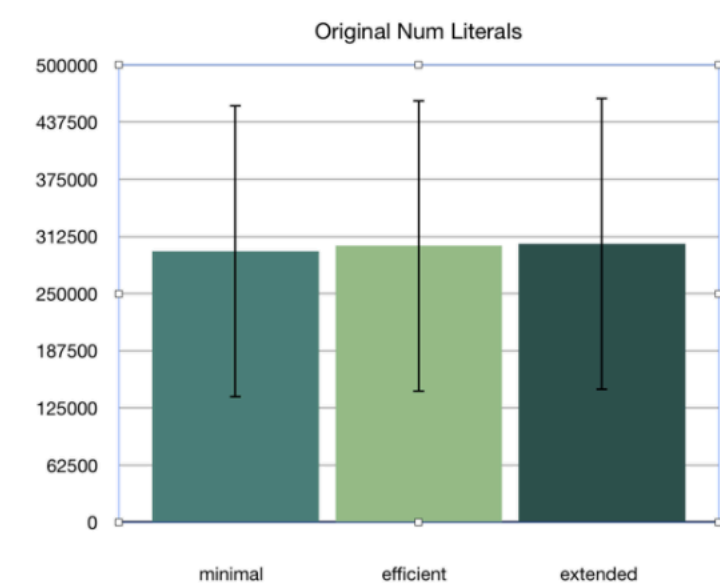
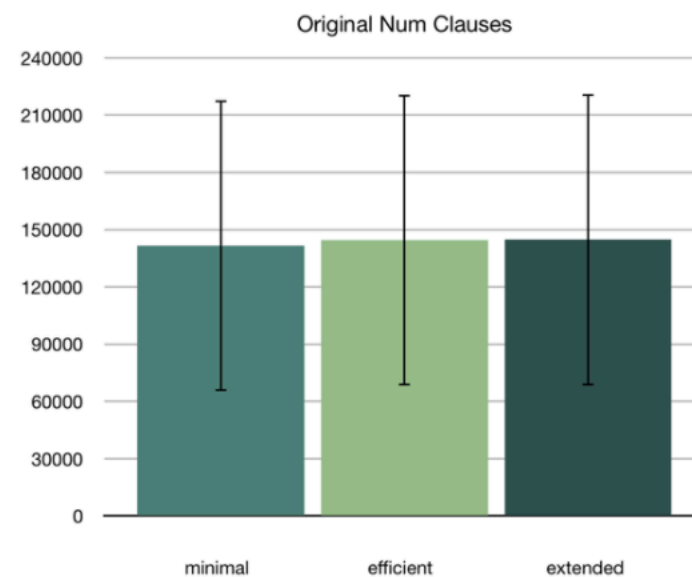
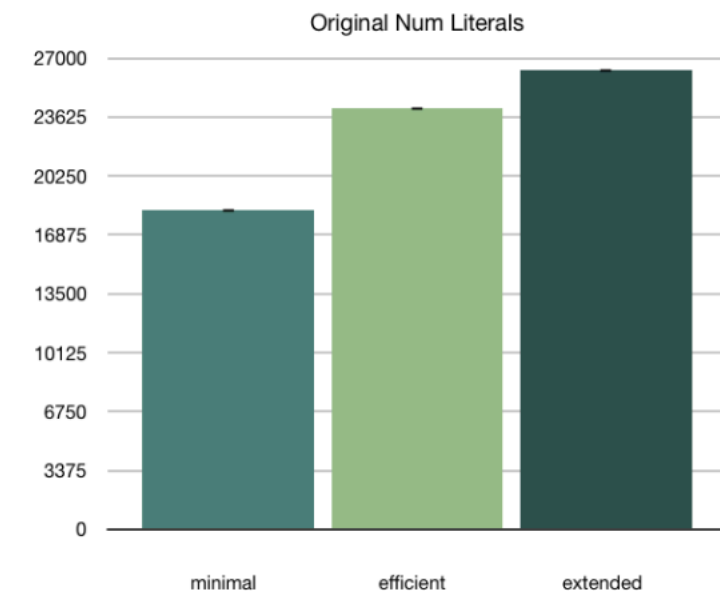
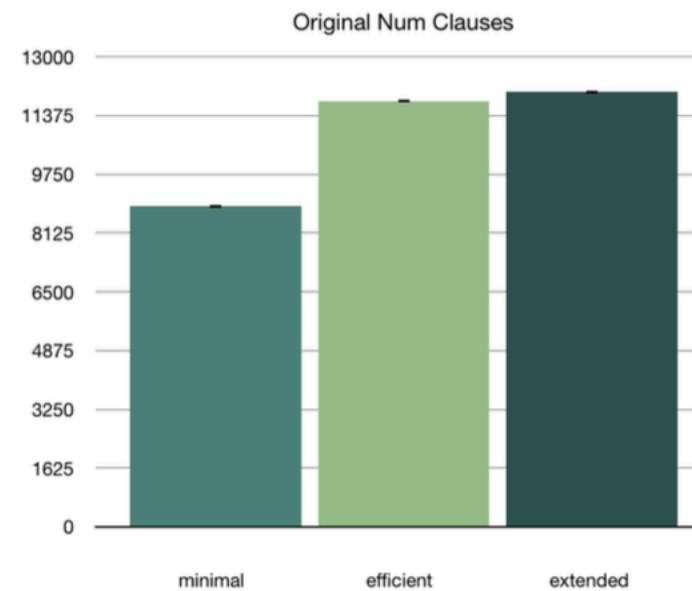
- > PicoSAT
- > zChaff
- > WalkSAT

## Encodings

- > minimal
- > efficient
- > extended
- + hyper-constraints

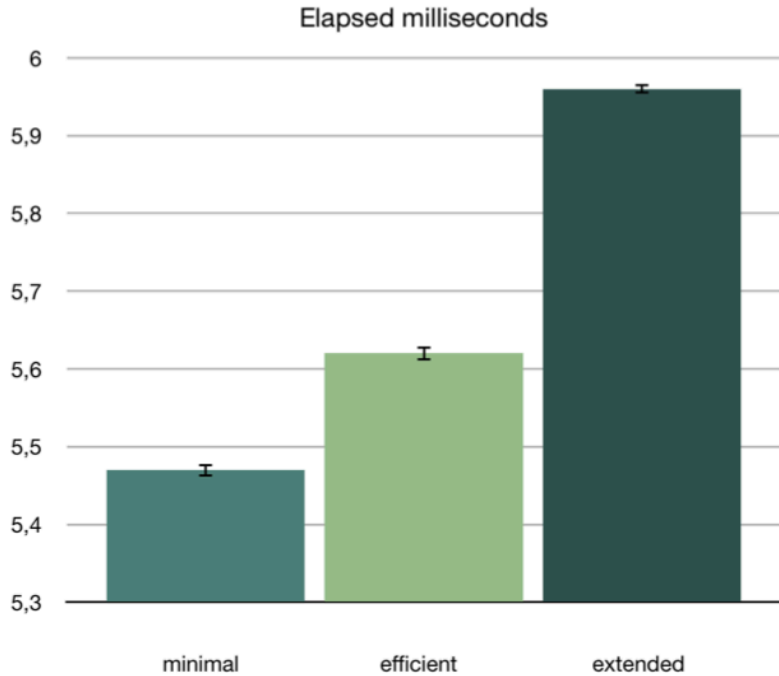
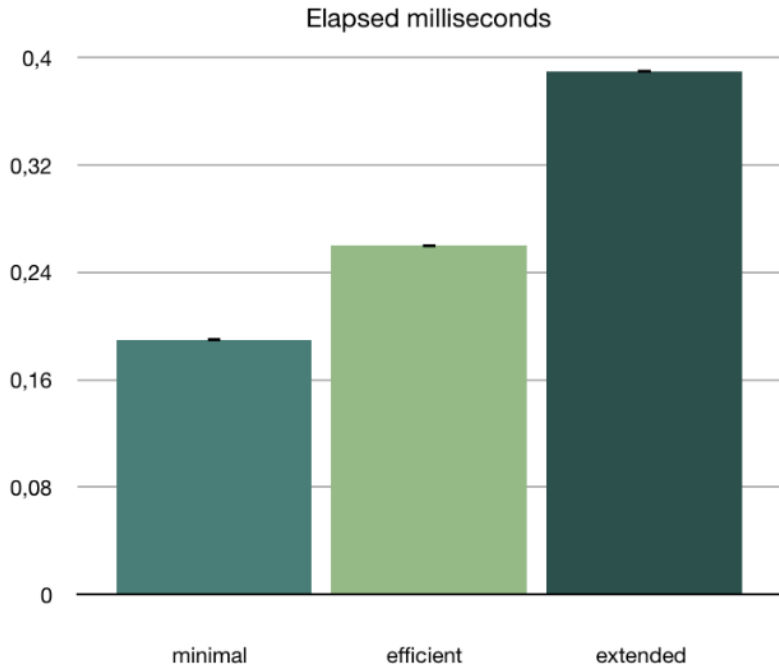
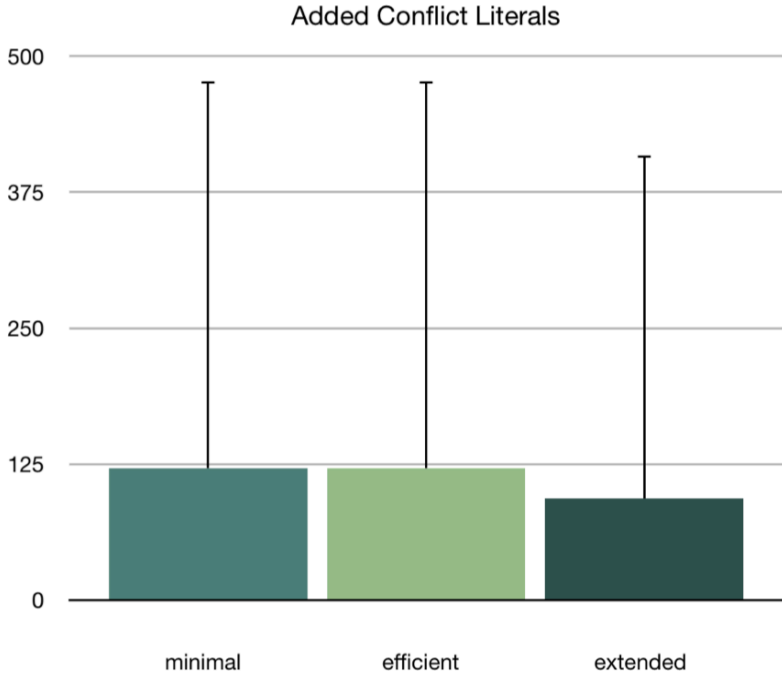
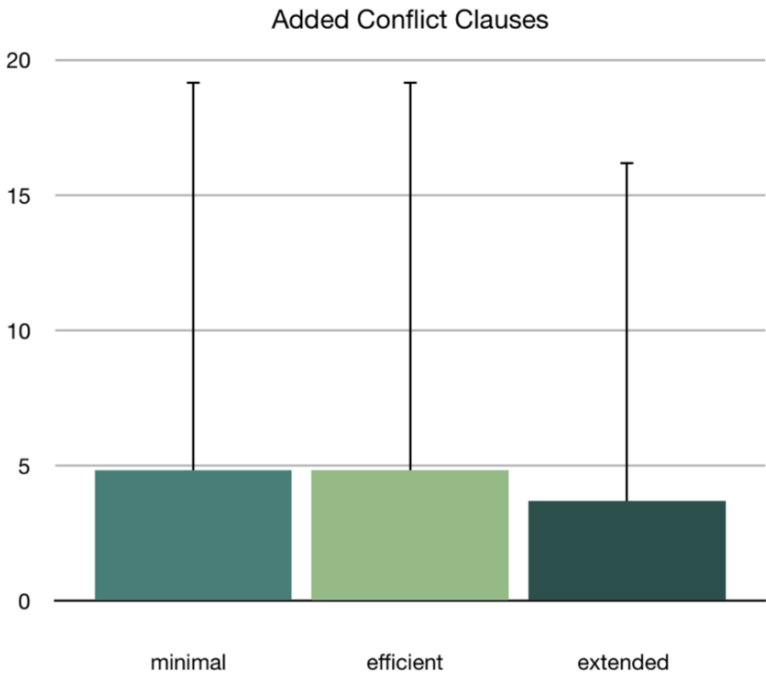
## Data sets

[kaggle.com/bryanpark/sudoku](https://kaggle.com/bryanpark/sudoku)  
[sudokucentral.co.uk/hypersudoku.php](https://sudokucentral.co.uk/hypersudoku.php)



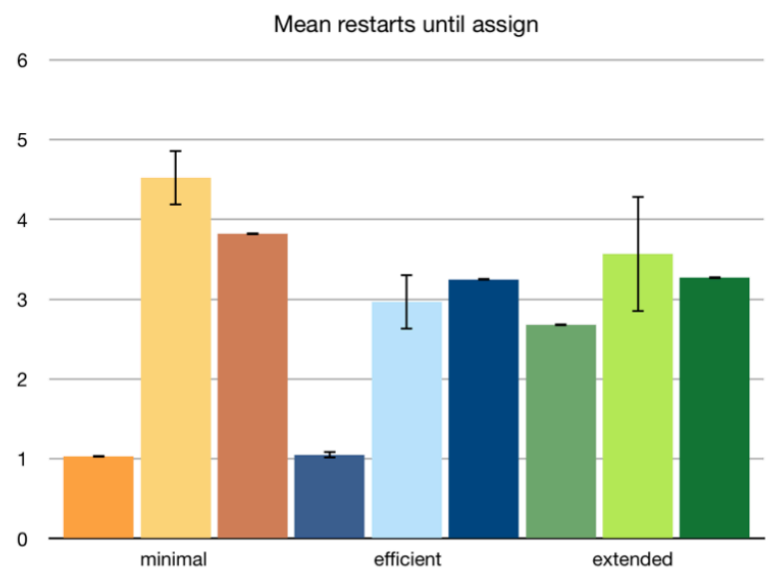
# HYPER SUDOKUS REQUIRE CONFLICT CLAUSES AND LITERALS

## THE EXTENDED ENCODING YIELDS LESS CONFLICT CLAUSES AND CONFLICT LITERALS

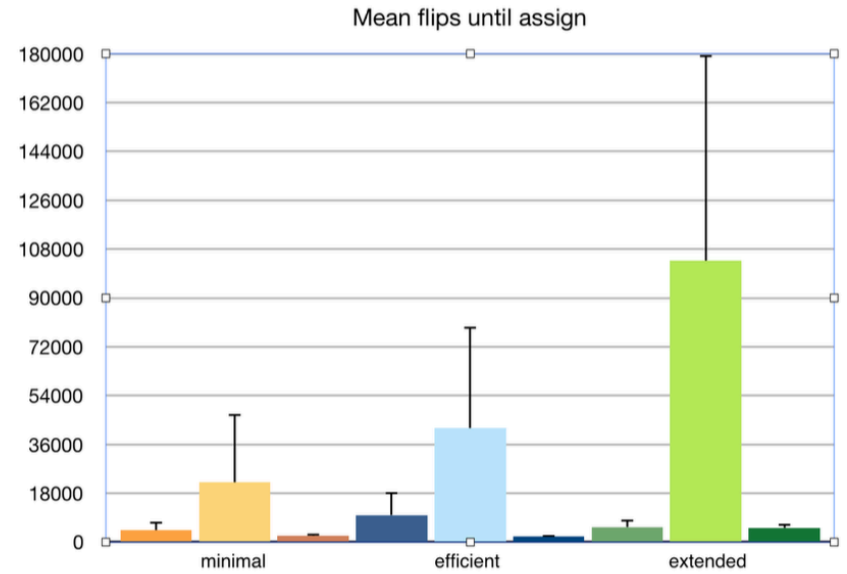


HYPOTHESIS FALSIFIED

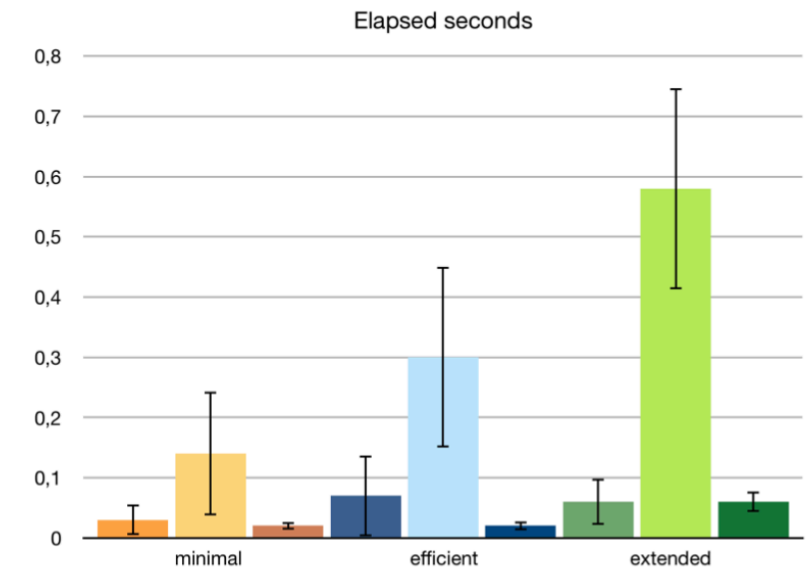
HYPER SUDOKUS REQUIRE MORE VARIABLE FLIPS THAN A STANDARD SUDOKU  
AND APPROXIMATELY THE SAME NUMBER OF RESTARTS



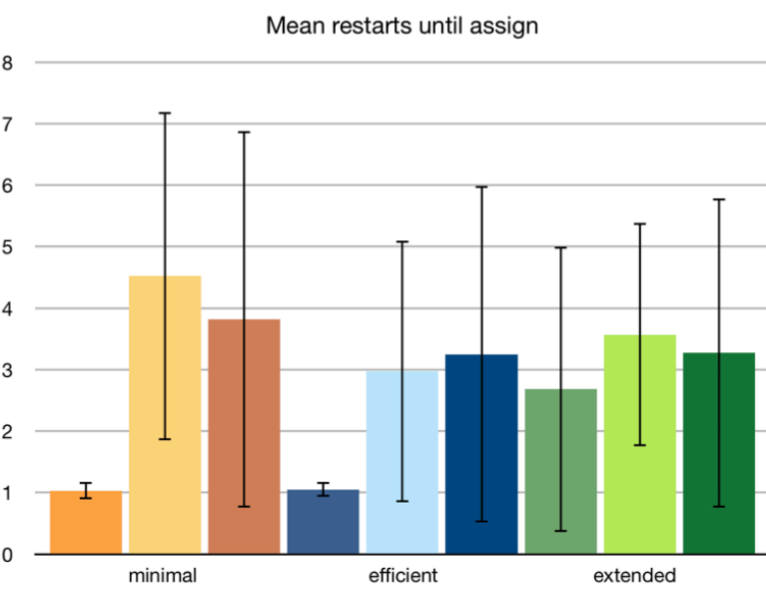
(a)



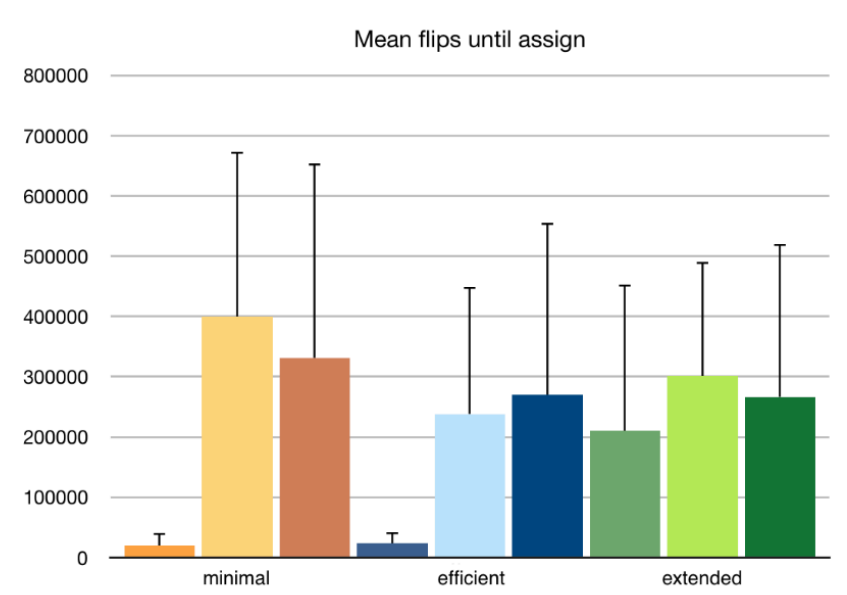
(b)



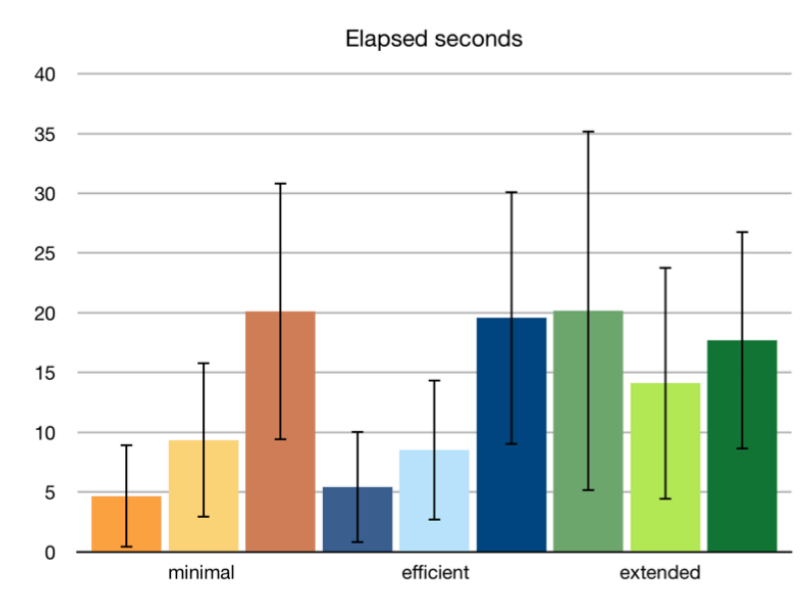
(c)



(d)



(e)



(f)

- walksat -novelty
- walksat -rnovelty
- walksat -best

# FOR A SAT SOLVER, IT APPEARS TO BE EASIER TO SOLVE A STANDARD SUDOKU THAN A HYPER SUDOKU.

- ▶ Intuitively, Hyper Sudokus allow for a more constrained logical reasoning, but the very form of these constraints (additional clauses and literals) negatively affects the performance of the tested SAT solvers.
- ▶ For an accessible SAT problem, performance of solvers is inversely proportional to the number of clauses and literals.
- ▶ Unless a SAT problem requires a more complex or explicit encoding, it is best to opt for the encoding that minimises the number of clauses and literals.