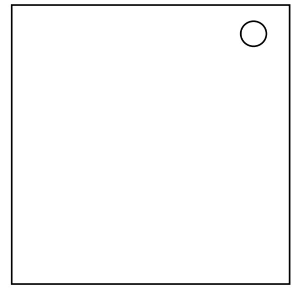
On the Subject of Partial Derivatives



You won't be getting partial credit...

This module displays a trinomial function in terms of three variables: **x, y,** and **z.** In each term of the trinomial, the variables are taken to integer powers between 0 and 5 (inclusive), and they may be multiplied by each other. Coefficients will appear on each term, and the terms will be added to each other. The module also contains a colored LED that may change color on each of the three stages.

The goal of this module is to evaluate a first, second, and third partial derivative of the function displayed based on the current color of the LED, and evaluate the partial derivative at points determined by the edgework of the bomb. Each of these partial derivatives is its own stage, and the current stage is indicated by the number of small white LEDs that are lit on the bottom of the module.

How to take a partial derivative

The **derivative** of a function measures a function's output's sensitivity with respect to a change in its input. The derivative of a single-variable function evaluted at a point measures the function's instantaneous rate of change (slope) at that particular point.

While learning calculating derivatives takes up a sizeable portion of an introductory calculus class, only a single type of function will appear in this module: polynomials. For a monomial $\mathbf{f = Cx^n}$, the derivative $\mathbf{D_x}$ is equal to $\mathbf{Cnx^{n-1}}$. The derivative of the sum of monomials is equal to the sum of the derivatives of the monomials. For example, if $\mathbf{f = 6x^4-x^2+9x-1}$, then $\mathbf{D_x = 24x^3-2x+9}$. The value of the derivative of a function at a point can be found by plugging in the coordinates of the point into the derivative. For example, with the above function, $\mathbf{D_x(1) = 24(1^3)-2(1)+9 = 31}$.

The **partial derivative** of a multivariable function is the derivative of the function taken with respect to exactly one variable, treating all other variables as constants. For a partial derivative of **f** taken with respect to **x,** this is denoted $\mathbf{D_xf}$. A second partial derivative can be taken with respect to the same or a different variable; for example, two second partial derivatives of **f** are $\mathbf{D_{xx}f}$ and $\mathbf{D_{xy}f}$. For example, for the function $\mathbf{g = 2x^3y^2-7xy^4}$, $\mathbf{g_x = 6x^2y^2-7y^4}$ and $\mathbf{g_{xy} = 12x^2y-28y^3}$. Evaluated at a point, $\mathbf{g_x(1,2) = 6(1)^2(2)^2-7(2)^4 = -88}$ and $\mathbf{g_{xy}(1,2) = 12(1)^2(2)-28(2)^3 = -200}$. By Clairaut's Theorem, $\mathbf{D_{xy}f = D_{yx}f}$.

Determining which variable to derive with respect to

On each stage, there is a lit LED on the right side of the module. Its color will be primary (red, yellow, or blue) or secondary (orange, green, or purple), and may change between stages. The following table represents the colors representing each variable in each stage of the module.

	x	**y**	**z**
Stage 1	Red	Yellow	Blue
Stage 2	Blue	Red	Yellow
Stage 3	Yellow	Blue	Red

For each stage, if the LED is a primary color, then derive with respect to the variable associated with that variable in the table. Otherwise, derive with respect to the variable associated with the color that is **not** a component of the secondary color.

Determining the values of x, y, and z

The partial derivative of the displayed function is to be calculated for certain values of **x**, **y**, and **z** at each stage, as desrcibed in the below table.

Let SN[i] represent the value of the **i**th character in the serial number. The values of all letters in the serial number are equal to their one-indexed position in the alphabet; A=1, B=2, ..., Y=25, Z=26. The following abbreviations are used in the table.

- B: Batteries
- BH: Battery holders
- LI: Lit indicators
- UI: Unlit indicators
- P: Ports
- PP: Port Plates

All values in the following table must be taken modulo 10. If this results in a variable equalling 0, then the value of that variable at that stage is 1.

	x	**y**	**z**
Stage 1	SN[1] + UI	SN[5] − LI	BH + PP
Stage 2	SN[3] − PP	B + BH	SN[6] − LI
Stage 3	UI + B	SN[4] − P	SN[2] + P

Solving the module

Evaluate the obtained partial derivative of the displayed function at the point representing the correct values of **x**, **y**, and **z** and input it on the keypad*. A correct answer on Stage 1 or Stage 2 will light a white LED on the bottom of the module, and the larger colored LED may change colors. A correct answer on Stage 3 will solve the module. If the value entered on the keypad does not equal to the value of the correct partial derivative evaluated at the correct point, then a strike will be incurred.

*If after evaluating a partial derivative at a certain point an answer less than 0 or greater than 10,000 is obtained, input the last four digits of the absolute value of the number. Do not add leading zeroes to any answer.