Faculty of Mathematics and Computer Science
Babeş-Bolyai University, Cluj-Napoca
Numerical Analysis
MIE - semester IV

Laboratory 7

Deadline: 22 – 26 April 2024

Quadrature formulas I

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1. Let us consider the following integral

$$I = \int_0^1 f(x)dx, \quad \text{where} \quad f(x) = \frac{3}{1+2x^2}.$$

   - Approximate the integral using trapezium formula.
   - Plot the graph of the function and the graph of the trapezium with vertices $(0,0)$, $(0, f(0))$, $(1, f(1))$ and $(1,0)$.
   - Approximate the integral using Simpson's formula

2. Approximate the double integral

$$I = \int_{1.2}^{3} \int_{1}^{1.6} \ln(2x+y)dxdy$$

   using trapezium formula for double integrals.

   Hint: trapezium formula for double integrals is given by

$$\int_a^b \int_c^d f(x,y)dxdy \approx \frac{(b-a)(d-c)}{16}\Bigg[f(a,c)+f(a,d)+f(b,c)+f(b,d) + 2f\left(\frac{a+b}{2},c\right)+2f\left(\frac{a+b}{2},d\right)+2f\left(a,\frac{c+d}{2}\right) + 2f\left(b,\frac{c+d}{2}\right)+4f\left(\frac{a+b}{2},\frac{c+d}{2}\right)\Bigg]$$

3. Evaluate the following integral that arises in electrical field theory

$$H(p,R) = \frac{60R}{R^2-p^2}\int_0^{2\pi}\left[1-\left(\frac{p}{R}\right)\sin x\right]^{1/2}dx,$$

   for $R = 110$ and $p = 75$ using the repeated trapezium formula for two given values of n.

4. Evaluate the integral

$$I = \int_0^{\pi} \frac{dx}{5 + \sin 25x}$$

using the repeated Simpson's formula for $n = 12$ and $n = 30$.

5. Let us consider the following *error function*:

$$E(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

Use the repeated Simpson's formula to evaluate $E(0.5)$ for $n = 4$ and $n = 10$. Estimate the accuracy of your result and compare with the correct value $E(0.5) = 0.520499876$.

Remark: 1-5 (0.5p)