

Topology on \mathbb{R}

Exercise 1

Use \checkmark when the set is closed, and a \times when it is not :

$(-1, 2]$	$(-1, 1)$	$[-1, 1]$	$\mathbb{R} \setminus \{1\}$	$\{1, 2, 3\}$	$\mathbb{R} \setminus (0, 1)$	\mathbb{Z}	\mathbb{Q}	$\mathbb{R} \setminus \mathbb{Q}$	\mathbb{R}

Exercise 2

Specify LB, UB, inf, sup, min max for the following sets:

$$\begin{aligned}
 A &= \bigcup_{n \in \mathbb{N} \setminus \{1\}} \left(-1 + \frac{1}{n}, 1 - \frac{1}{n}\right), & B &= \bigcup_{n \in \mathbb{N}} \left[-1 + \frac{1}{n}, 1 - \frac{1}{n}\right] \\
 C &= \bigcap_{n \in \mathbb{N} \setminus \{1\}} \left(-1 + \frac{1}{n}, 1 - \frac{1}{n}\right) & D &= \bigcap_{n \in \mathbb{N}} \left[-1 + \frac{1}{n}, 1 - \frac{1}{n}\right] \\
 E &= \bigcup_{n \in \mathbb{N}} \left[-1 - \frac{1}{n}, 1 + \frac{1}{n}\right] & F &= \bigcap_{n \in \mathbb{N}} \left(-1 - \frac{1}{n}, 1 + \frac{1}{n}\right)
 \end{aligned}$$

Exercise 3

Check weather the following sets are open and closed:

$$\begin{aligned}
 A &= \bigcup_{n \in \mathbb{N} \setminus \{1\}} \left(-1 + \frac{1}{n}, 1 - \frac{1}{n}\right), & B &= \bigcup_{n \in \mathbb{N}} \left[-1 + \frac{1}{n}, 1 - \frac{1}{n}\right] \\
 C &= \bigcap_{n \in \mathbb{N} \setminus \{1\}} \left(-1 + \frac{1}{n}, 1 - \frac{1}{n}\right) & D &= \bigcap_{n \in \mathbb{N}} \left[-1 + \frac{1}{n}, 1 - \frac{1}{n}\right] \\
 E &= \bigcup_{n \in \mathbb{N}} \left[-1 - \frac{1}{n}, 1 + \frac{1}{n}\right] & F &= \bigcap_{n \in \mathbb{N}} \left(-1 - \frac{1}{n}, 1 + \frac{1}{n}\right)
 \end{aligned}$$

Excercise 4 Fill in the following table and give a complete proof for the examples at **1,3,5,6,9,11** :

Nr.	A	LB(A)	inf A	max A	UB(A)	sup A	max
1	$(-\infty, -1] \cup (2, +\infty)$						
2	$(-1, 9] \cup [10, 20)$						
3	$\left((-1, 9] \cup [10, 20)\right) \cap \mathbb{N}$						
4	$\{1, 2, 3\}$						
5	\mathbb{N}						
6	$\mathbb{R} \setminus \{1, 2, 3\}$						
7	$\mathbb{R} \setminus \mathbb{N}$						
8	\mathbb{Z}						
9	$\mathbb{R} \setminus \mathbb{Z}$						
10	\mathbb{Q}						
11	$\mathbb{R} \setminus \mathbb{Q}$						
12	\mathbb{R}						

Excercise 5 Fill in the following table, and try to prove **1,3,5,6,9,11** :

Nr.	A	int A	bd A	cl A	ext A	Izo A	
1	$(-\infty, -1] \cup (2, +\infty)$						
2	$(-1, 9] \cup [10, 20)$						
3	$\left((-1, 9] \cup [10, 20)\right) \cap \mathbb{N}$						
4	$\{1, 2, 3\}$						
5	\mathbb{N}						
6	$\mathbb{R} \setminus \{1, 2, 3\}$						
7	$\mathbb{R} \setminus \mathbb{N}$						
8	\mathbb{Z}						
9	$\mathbb{R} \setminus \mathbb{Z}$						
10	\mathbb{Q}						
11	$\mathbb{R} \setminus \mathbb{Q}$						
12	\mathbb{R}						

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