Faculty of Mathematics and Computer Science
Babeş-Bolyai University, Cluj-Napoca
Numerical Analysis
MIE - semester IV

Laboratory 9

Deadline: 13 – 17 May 2024

Conditioning number of a matrix

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1. Let us consider the system $Ax = b$, where

$$A = \begin{pmatrix} 10 & 8 & -2 & 7 \\ 2 & 3 & 0 & 1 \\ -3 & 2 & 6 & 9 \\ 7 & 5 & 9 & 10 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 5 \\ -8 \\ 3 \\ 12 \end{pmatrix}$$

   - Solve the system and find its conditioning number (use the MatLab function *cond*).
   - Solve the system $Ax_2 = b_2$, where

$$b_2 = \begin{pmatrix} 5.6 \\ -8.2 \\ 3.9 \\ 12.4 \end{pmatrix}$$

   - Compute the input relative error $\frac{\|b-b_2\|}{\|b\|}$ and the output relative error $\frac{\|x-x_2\|}{\|x\|}$.
   - Solve the system $A_3x_3 = b$, where

$$A_3 = \begin{pmatrix} 10 & 8.54 & -2.13 & 7.2 \\ 2.12 & 3.1 & 0.6 & 0.99 \\ -3 & 1.83 & 5.7 & 9.1 \\ 6.77 & 5.23 & 8.87 & 10 \end{pmatrix}$$

   - Compute the input relative error $\frac{\|A-A_3\|}{\|A\|}$ and the output relative error $\frac{\|x-x_3\|}{\|x\|}$.

2. Find the conditioning numbers of the Vandermonde matrices $V_1(t_k)$, where $t_k = \frac{1}{k}$ for $k = \overline{1,n}$ and $n = \overline{10,15}$.

3. Find the conditioning numbers of the Vandermonde matrices $V_2(t_k)$, where $t_k = -1 + \frac{2k}{n}$ for $k = \overline{1,n}$ and $n = \overline{10,15}$.

4. Find the conditioning numbers of the Hilbert matrices $H_n(h_{ij})$, where $h_{ij} = \frac{1}{i+j-1}$ for $1 \leq i,j \leq n$ and $n = \overline{10,15}$.

Remark: 1 (1p), 2-4 (0.5p)