Seminar 7 (2024)

1. It was found that the probability to log on to a computer from a remote terminal is 0.7. Let X denote the number of attempts, which are independent, that must be made to gain access to the computer:
a) Write the probability distribution of X.
b) Write the cumulative distribution function of X.
c) Compute the probability that at most 4 attempts must be made to gain access to the computer.
d) Compute the probability that at least 3 attempts must be made to gain access to the computer.

2. The time, in minutes, it takes to reboot a certain system is a continuous variable with the density function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} c(4-x)^2, & \text{if } 0 < x < 4 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Compute the constant c.
(b) Compute the probability that the system takes between 1 and 2 minutes to reboot.
(c) Compute the probability that the system takes at least 1 minute to reboot.

3. Find the density function of the volume V of a cube, whose edge X is a random variable uniformly distributed on $[0, 2]$.

$$X \sim Unif[0,2] \iff f(x) = \begin{cases} \frac{1}{2}, & x \in [0,2] \\ 0, & x \notin [0,2] \end{cases} \text{ is the density function of the } Unif[0,2] \text{ distribution}$$

4. The time to failure T, in hours of operating time, of a television set subject to random voltage surges has exponential $Exp(\frac{1}{500})$ distribution.
(a) Compute the cumulative distribution function of T.
(b) Compute the probability that the unit operates successfully more than 400 hours.
(c) Suppose the unit has operated successfully for 400 hours. What is the (conditional) probability it will operate for another 500 hours?

$$T \sim Exp\left(\frac{1}{500}\right) \iff f_T(t) = \begin{cases} 0, & \text{if } t \leq 0 \\ \frac{1}{500} e^{-\frac{t}{500}}, & \text{if } t > 0\,. \end{cases}$$

5. A random number generator produces independently a sequence of numbers between 2 and 5. Each of these can be considered an observed value of a random variable uniformly distributed on the interval $[2, 5]$. Ten numbers are generated. What is the probability that seven or more numbers are less than or equal to 4.7?

6. Six identical electronic devices are installed at one time. The units fail independently, and the time to failure, in days, of each is a random variable with exponential distribution $Exp(\frac{1}{30})$. A maintenance check is made at fifteen days. What is the probability that at least four are still operating at the maintenance check?

7. Let $F : \mathbb{R} \to \mathbb{R}$ be defined by

$$F(x) \quad = \quad \begin{cases} 0, & \text{if } \ x < -4 \\ \dfrac{a(x+4)}{|x|+b}, & \text{if } \ x \geq -4\,, \end{cases}$$

where $a, b \in \mathbb{R}$ are parameters. For what values of $a, b \in \mathbb{R}$ the function F is the cumulative distribution function of a continuous random variable X? Find the density function of X when $P(-1 < X < 1) = 0.4$.