Homework 8 08.12.2022

~~Derivable~~ Differentiable Function

Ex 1: a) $f:(-1,\infty)\to\mathbb{R}$ $f(x)=(1+x)^r,\ r\in\mathbb{R}$

$f'(x)=\cancel{(r-1)(1+x)}\ r(1+x)^{r-1}\cdot 1$

$f''(x)=r(r-1)(1+x)^{r-2}$

$f^{(3)}(x)=r(r-1)(r-2)(1+x)$

$f^{(n)}(x)=\begin{cases}0, & n>r\\ \frac{r!}{(r-n)!}(1+x), & n\le r\end{cases}$

b) $f:(-1,\infty)\to\mathbb{R},\ f(x)=x\cdot\ln(1+x)$

$f'(x)=\ln(1+x)+\frac{x}{1+x}$

$f''(x)=\frac{1}{1+x}+\frac{1(1+x)-x}{(1+x)^2}=\frac{1}{1+x}+\frac{1}{x+1^2}=\frac{x+2}{(1+x)^2}$

~~$f^{(3)}(x)=\frac{-1}{(1+x)^2}+\frac{-2(1+x)}{(1+x)^{32}}$~~ $f^{(3)}(x)=\frac{(1+x)^2-2(x+2)\cancel{(x+1)}}{(1+x)^{43}}=\frac{-3-x}{(1+x)^3}$

~~$f^{(4)}(x)=\frac{-2(1+x)}{(1+x)^{43}}+\frac{4(1+x)^2}{(1+x)^{42}}$~~ $f^{(4)}(x)=\frac{-(1+x)^{3}+3(x+3)(1+x)}{(1+x)^{65}}$

~~$f^{(5)}(x)=\frac{6(1+x)^3}{(1+x)^{63}}-\frac{8(1+x)}{(1+x)^{43}}$~~ $=\frac{-1-2x-x^2+3x+9}{(1+x)^5}=$

~~$f^{(6)}(x)=\frac{18(1+x)^3}{(1+x)^6}-24(1+x)$~~ $=\frac{8-x-x^2}{(1+x)^5}$

c) $f:(-\infty,-1)\to\mathbb{R},\ f(x)=x\cdot\ln(1-x)$

$f'(x)=\ln(1-x)-\frac{x}{1-x}$

$f''(x)=\frac{1}{1-x}-\frac{\cancel{1-x+x}}{(1-x)^2}=\frac{-x}{(1-x)^2}$

$f^{(3)}(x)=\frac{-(1-x)^2+2x(1-x)}{(1-x)^{43}}=\frac{-1-x}{(1-x)^3}$

~~$f^{(4)}(x)=$~~

d) $f(x)=\sqrt{3x+4}\Rightarrow f'(x)=\frac{1}{2\sqrt{3x+4}}\cdot 3=\frac{3}{2\sqrt{3x+4}}$

$f^{(n)}(x)=\frac{3^n}{2^n\sqrt{3x+4}}$

e) $f(x)=\frac{1}{\sqrt{2x+4}}\Rightarrow f'(x)=\frac{-\frac{1}{2\sqrt{2x+4}}}{2x+4}=\frac{-\frac{1}{\sqrt{2x+4}}}{2x+4}=$

$=-\frac{1}{\sqrt{2x+4}}\cdot\frac{1}{2x+4}=\frac{-1}{(2x+4)\sqrt{2x+4}}=$

$f^{(n)}(x)=\frac{(-1)^n}{\sqrt{(2x+4)}^{2n+1}}$

Ex 2: a) $f(x) = \frac{1}{ax+b} \Rightarrow f'(x) = \frac{-a}{(ax+b)^2}$ $\quad f''(x) = \frac{-2a(ax+b)}{(ax+b)^4} =$

$= \frac{-2a}{(ax+b)^3}$ $\quad f^{(3)}(x) = \frac{+6a\cancel{(ax+b)^2}}{(ax+b)^{\cancel{6}4}} = \frac{6a}{(ax+b)^4}$

$f^{(n)}(x) = (-1)^n \cdot \frac{n! \cdot a}{(ax+b)^{n+1}}$

b) $f(x) = \sin(ax+b) \Rightarrow f'(x) = +\cos(ax+b) \cdot a \Rightarrow f''(x) = -a^2 \sin(ax+b)$

$\cancel{f^{(n)}(x) = \begin{cases} a^n \sin(ax+b), & n \text{ is even} \\ a^n \cos(ax+b), & n \text{ is odd} \end{cases}}$

c) $\cancel{f^{(n)}(x) = \begin{cases} -a^n \sin(ax+b), & n \text{ is odd} \\ a^n \cos(ax+b), & n \text{ is even} \end{cases}}$

~~d) $f(x) = e^{ax+b}$~~

$f^{(n)}(x) = \begin{cases} a^n \cdot \cos(ax+b), & n \% 4 = 1 \\ -a^n \cdot \sin(ax+b), & n \% 4 = 2 \\ -a^n \cos(ax+b), & n \% 4 = 3 \\ a^n \sin(ax+b), & n \% 4 = 0 \end{cases}$

c) $f(x) = \cos(ax+b)$ $\quad f'(x) = -a \sin(ax+b)$ $\quad f''(x) = -a^2 \sin(ax+b)$

$f^{(n)}(x) = \begin{cases} -a^n \sin(ax+b), & n \% 4 = 1 \\ -a^n \cos(ax+b), & n \% 4 = 2 \\ a^n \sin(ax+b), & n \% 4 = 3 \\ a^n \cos(ax+b), & n \% 4 = 0 \end{cases}$

d) $f(x) = e^{ax+b} \Rightarrow f'(x) = e^{(ax+b)} \cdot a$ $\quad f^{(n)}(x) = a^n e^{ax+b}$

Ex 3:

Ex 4: $\frac{1}{x+1} < \ln(x+1) - \ln x < \frac{1}{x} \Leftrightarrow \frac{1}{x+1} < \ln \frac{x+1}{x} < \frac{1}{x} \mid ()'$

$\frac{\cancel{x}\,-1}{(x+1)^2} < \frac{\cancel{x}}{x+1} \cdot \frac{x-x-1}{x^2} < \frac{-1}{x^2} \mid \cancel{(x+1)^2}\, x^2$

$\cancel{-1}\ \frac{1}{x^2} < \frac{1}{x(x+1)} < \frac{1}{(x+1)^2} \mid \cdot x^2 \cdot (x+1)^2$ $\cancel{(x+1)^2}$

$(x+1)^2 < x(x+1) < x^2$ $\quad x^2+2x+1 < x^2+x < x^2 \mid -x^2$

$2x+1 < x < 0$ $\quad x+1 < 0$ $\quad x < -1$

Exo: 5) $f(x) = x + |x-1| = \begin{cases} x - x + 1 = 1, & x < 1 \\ x + x - 1 = 2x - 1, & x \geq 1 \end{cases}$

$\lim_{x \nearrow 1} f(x) = 1$

$\lim_{x \searrow 1} f(x) = \lim_{x \searrow 1} 2x - 1 = 1$