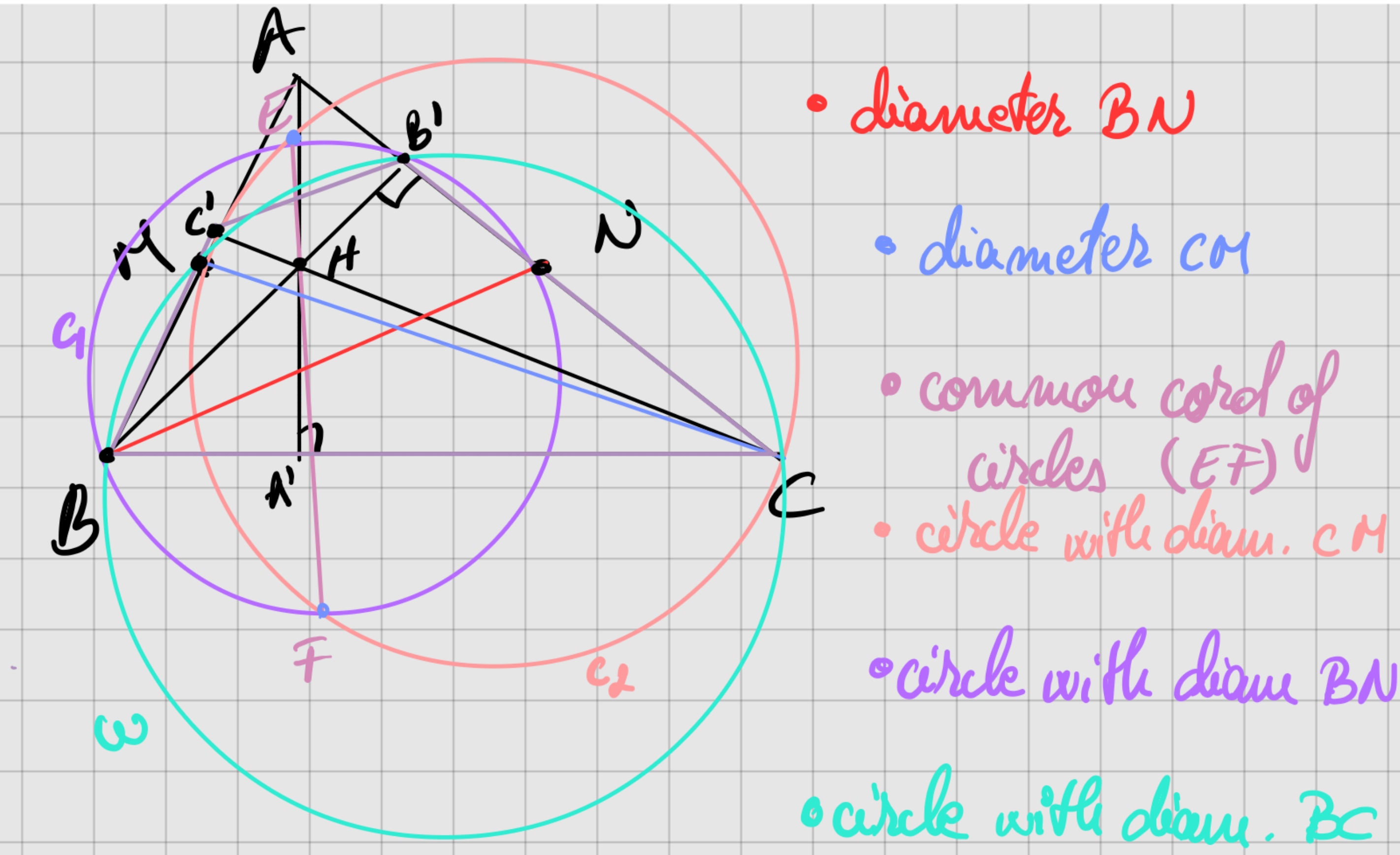
Seminar 4.

Presentation 2. Let ABC be a triangle which is not right. Denote by H the orthocenter of this triangle and let M and N be points on its sides AB and AC. Prove that the common chord of circles with diameters CM and BN passes through H.



Let AA', BB', CC' be the altitudes in ΔABC

C_1 - circle of diameter BN ; C_2 - circle of diameter CM

C_1 passes through B' (foot of altitude B) ; C_2 passes through C'

$\sphericalangle BC'C = \sphericalangle CB'C = 90^\circ \Rightarrow$ quadrilateral $BCB'C'$ cyclic

Let BC be the diameter of circle ω containing B, C, B', C'

$$p(H, \omega) = -HC \cdot HC' = -HB \cdot HB' \text{ (power of } H \text{ w.r.t } \omega) \quad (1)$$

$$\left.\begin{array}{l} p(H, C_1) = -HB \cdot HB' \\ p(H, C_2) = -HC \cdot HC' \end{array}\right\} \overset{(1)}{\Longrightarrow} \underline{p(H, C_1) = p(H, C_2)} \quad (2)$$

Let E and F be intersection points of C_1, C_2.

$$\left.\begin{array}{l} p(E, C_1) = p(E, C_2) = 0 \\ p(F, C_1) = p(F, C_2) = 0 \end{array}\right\} \Rightarrow \underline{EF = \text{radical axis of } C_1, C_2} \quad (3)$$

(2), (3) $\Rightarrow$ H lies on EF