Calculus on $\mathbb{R}$ Lect. dr. Anca Grad
Homework 3 **2022**

Sequences of real numbers - 2nd part

Exercise 1: Study the nature (covergence or divergence) of the following sequence of real numbers.

$$x_n = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\dots\left(1 - \frac{1}{n^2}\right),\ n \geq 2.$$

In case it is convergent, compute its limit.

Exercise 2: Determine the limits of the following sequences of real numbers, having as general term:

$$a)\ \frac{5^n}{7^n},\qquad b)\ \frac{4^n + (-4)^n}{6^n},\qquad c)\ \frac{7 - 3n^3}{5n^2 + n},\qquad d)\ \left(2 + \frac{9^n + (-8)^n}{10^n + 1}\right)^{25n^3 - n^2},$$

$$e)\ \frac{1 + 2 + \ldots + n}{n^2},\qquad f)\ \left(\frac{n^3 + 4n + 1}{2n^3 + 5}\right)^{\frac{-2n^4+1}{n^4+3n+1}},\qquad g)\ (\cos(-2013))^n,$$

$$h)\ \left(\frac{2n^5 + 3n + 1}{7n^5 - n^4 + 3}\right)^{\frac{9n - n^4}{8n^3+1}}.$$

Exercise 3: Determine the limits of the following sequences of real numbers, having as general term:

$$a)\ \left(1 + \frac{1}{-5n^3 + 3n}\right)^{n^2 - 5n^3},\quad b)\ (9n^2 + 5)ln\left(1 + \frac{1}{4n^2}\right),$$

$$c)\frac{n^n}{1^1 + 2^2 + ... + n^n}$$

$$d)\frac{x_1 + 2x_2 + ... + nx_n}{n^2},$$

when $(x_n)_{n\in\mathbb{N}}$ is a convergent sequence, with the limit $x \in \mathbb{R}$.

Exercise 4: Determine the limits of the following sequences of real numbers, having as general term:

$$a)x_n = \frac{a^n - a^{-n}}{a^n + a^{-n}},\quad a \neq 0$$

$$b) y_n = \frac{a^n + b^n}{a^{n+1} + b^{n+1}}, \quad a \neq -b$$

$$c) z_n = \frac{1 + a + ... + a^n}{1 + b + ... + b^n}, \quad a, b > 0.$$