Faculty of Mathematics and Computer Science
Babeş-Bolyai University, Cluj-Napoca
Numerical Analysis
MIE - semester IV

Laboratory 3

Deadline: 25 – 29 March 2024

Newton interpolation. The Aitken/Neville algorithm

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1. Let us consider

x	1.2	1.6	2	2.4	3.1	4
$\lg x$	0.07918	0.20412	0.30103	0.38021	0.49136	0.60206

   - Using the Newton interpolation polynomial approximate $\lg(2.5)$ and $\lg(3.26)$. Compare with the values $\lg(2.5) = 0.39794$ and $\lg(3.26) = 0.5132176$.
   - Plot the data from the previous table and the Newton interpolation polynomial.
   - Estimate the maximum interpolation error $E = \max \left| f(y_i) - (N_m f)(y_i) \right|$ for $y_1 = \frac{i}{10}$, $i = \overline{10, 30}$, where $f(y) = \lg y$.

2. In order to study the number of points obtained by the students in our Numerical Analysis Seminary, we construct the following table

Points	1	2	3	4	5
Students	30	26	22	32	28

   - According to Newton interpolation polynomial, approximate how many students are expected to obtain 3.5 points during the seminary.
   - Plot the data given in the table and the corresponding Newton interpolation polynomial.

3. Let $f : [0, 5] \to \mathbb{R}$ be given by $f(x) = e^{\cos x}$ and let us consider 15 equidistant interpolation points in $[0, 5]$. Plot the interpolation points, the function f and the Newton interpolation polynomial on $[0, 5]$.

4. Approximate $\sqrt{178}$ with precision $\varepsilon = 10^{-3}$ using the Aitken algorithm.

5. Use Neville's algorithm to approximate $\sqrt{5}$ for
   - $f(x) = 5^x$ and 5 equidistant nodes in $[-2, 2]$;
   - $g(x) = \sqrt{x}$ and 5 equidistant nodes in $[0, 5]$.

Remark: 5 problems $\times$ 0.5p = 2.5p