2. The following two curves are known to lie on a surface $S$:

(a) $\mathbf{r}_1(t) = (2\sqrt{2}\cos t, \sqrt{2}\sin t, -3)$, $t \in \mathbb{R}$;

(b) $\mathbf{r}_2(t) = (2, \sqrt{2}\sin t, -3\sqrt{2}\cos(t)), t \in \mathbb{R}$.

Find the equation for the tangent plane to $S$ at the point given by $\mathbf{r}_1(\frac{\pi}{4}) = \mathbf{r}_2(\frac{\pi}{4})$.

$$r_1\left(\frac{\pi}{4}\right) = \left(2\sqrt{2}\cos\left(\frac{\pi}{4}\right), \sqrt{2}\sin\left(\frac{\pi}{4}\right), -3\right).$$

So $\cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

Then we have:

$$r_1(t) = r_1\left(\frac{\pi}{4}\right) = \left(2\sqrt{2}\cdot\frac{\sqrt{2}}{2}, \sqrt{2}\cdot\frac{\sqrt{2}}{2}, -3\right) = (2, 1, -3).$$

$$\boxed{\text{So } r_1\left(\frac{\pi}{4}\right) = (2, 1, -3).}$$

For $r_2(t)$, we have:

$$r_2\left(\frac{\pi}{4}\right) = \left(2, 2\sqrt{2}\sin\left(\frac{\pi}{4}\right), -3\sqrt{2}\cos\left(\frac{\pi}{4}\right)\right).$$

$$= \left(2, 2\sqrt{2}\cdot\frac{\sqrt{2}}{2}, -3\sqrt{2}\cdot\frac{\sqrt{2}}{2}\right) = (2, 1, -3).$$

$$\boxed{r_2\left(\frac{\pi}{4}\right) = (2, 1, -3)}$$

The common point is $P(2,1,-3)$.

$$r_1'(t) = \left( \frac{d}{dt}(2\sqrt{2}\cos t), \frac{d}{dt}(\sqrt{2}\sin t), \frac{d}{dt}(-3) \right)$$

$$= (-2\sqrt{2}\sin t, \sqrt{2}\cos t, 0).$$

For $t = \frac{\pi}{4}$, we have:

$$r_1'\left(\frac{\pi}{4}\right) = \left(-2\sqrt{2} \cdot \frac{\sqrt{2}}{2}, \sqrt{2} \cdot \frac{\sqrt{2}}{2}, 0\right).$$

$$\boxed{r_1'\left(\frac{\pi}{4}\right) = (-2, 1, 0).}$$

$$r_2'(t) = \left(0, \frac{d}{dt}(\sqrt{2}\sin t), \frac{d}{dt}(-3\sqrt{2}\cos t)\right)$$

$$= (0, \sqrt{2}\cos t, \sqrt{2}\sin t).$$

$$r_2'\left(\frac{\pi}{4}\right) = \left(0, \sqrt{2} \cdot \frac{\sqrt{2}}{2}, 3\sqrt{2} \cdot \frac{\sqrt{2}}{2}\right).$$

$$\boxed{r_2'\left(\frac{\pi}{4}\right) = (0, 1, 3).}$$

The normal vector N:

$$N = r_1'\left(\frac{\pi}{4}\right) \times r_2'\left(\frac{\pi}{4}\right)$$

$$N = \begin{vmatrix} i & j & k \\ -2 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix} = 3i + 6j - 2k.$$

$$\Rightarrow N = (3, 6, -2).$$

<u>Eq. of the tangent plane:</u>

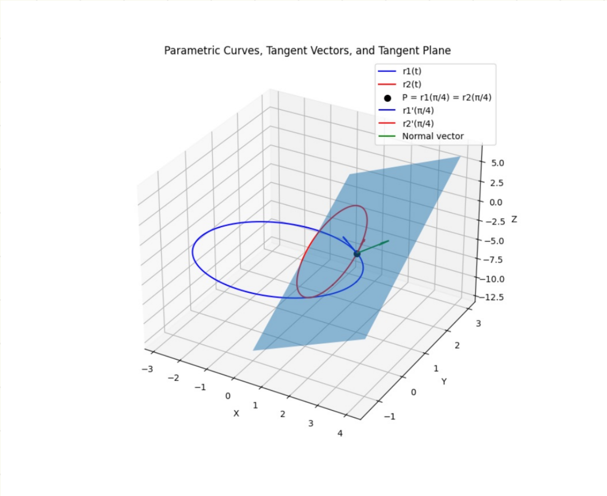
$(2, 1, -3)$.

$$3(x-2) + 6(y-1) - 2(z+3) = 0.$$

$$3x - 6 + 6y - 6 - 2z - 6 = 0.$$

$$3x + 6y - 2z - 18 = 0.$$

$\Rightarrow$ $\boxed{\text{eq. of the plane: } 3x + 6y - 2z = 18.}$




Here is the picture of the problem, generated by python code.