Seminar 2

1. Consider the English alphabet with 21 consonants and 5 vowels (lowercase letters). In how many ways can we choose 6 letters such that we get 4 distinct consonants and 2 distinct vowels, if: a) the letters are not ordered; b) the letters are ordered? Examples: a) {s,e,a,r,c,h}; b) (a,r,c,h,e,s), (c,a,s,h,e,r), (c,h,a,s,e,r).

2. Two numbers are obtained by rolling two dice. Compute the probabilities of the following events:
a) A: "the numbers are equal".
b) B: "the sum of the numbers is even".
c) C: "the sum of the numbers is at most equal to 10."

3. For this problem, we assume that the birthday of every person falls equally likely in any month of the year (i.e., the probability that a person was born in a certain month is $\frac{1}{12}$). Compute the probability that
a) in a group of 5 persons there are at least 2 persons that celebrate their birthdays in the same month?
b) in a group of 5 persons all the birthdays fall in at most two months?

4. For this problem, we assume that the birthday of every person falls equally likely on any day of the year 2003 (i.e., the probability that a person was born in a certain day is $\frac{1}{365}$). Which is the minimum number n such that the probability of the event "at least two persons share a birthday in an arbitrary group of n persons born in 2003" is at least 50%?

5. a) How many solutions $(x_1, \ldots, x_n) \in \mathbb{N}^* \times \cdots \times \mathbb{N}^*$ does the equation $x_1 + \ldots + x_n = k$ ($k, n \in \mathbb{N}^*$, $k \geq n$) have?

6. How many *multisubsets* with $k \in \mathbb{N}$ elements does a set S with $n \in \mathbb{N}^*$ distinct elements have?
A *multisubset*, called also a *combination with repetitions*, is a collection of elements *not necessarily distinct* and *not ordered*. Example: $\{1, 1, 3, 4, 4\}$ is a *multisubset* of the set $\{1, 2, 3, 4\}$.

7. A person sends 10 memes by choosing for each meme a recipient from a list of 20 friends. Compute the probability that the first friend in the list receives exactly 5 memes?

8. 5 balls numbered from 1 to 5 are randomly placed on a line. Compute the probability that:
a) the first and the last balls have even numbers;
b) the first two balls have odd numbers;
c) the balls with even numbers are next to each other;
d) at least two balls that are placed next to each other have the same parity.

9. In how many ways can we split the following marbles among 3 persons:
a) 1 red marble, 1 blue marble, 1 green marble, 1 yellow marble and 1 orange marble?
b) 5 red marbles;
c) 5 red marbles and 3 blue marbles;
d) 5 red marbles, 3 blue marbles and 4 green marbles.