SEMINAR 9

1) Show that the Abelian group $(\mathbb{R}_+^*, \cdot)$ is an $\mathbb{R}$-vector space with the external operation $*$ defined by

$$\alpha * x = x^\alpha,\ \alpha \in \mathbb{R},\ x \in \mathbb{R}_+^*\,.$$

2) Let V be a K-vector space an let M be a set. Show that V^M is a K-vector space with the pointwise operations on V^M, i.e.

$$(f+g)(x) = f(x) + g(x),\ (\alpha f)(x) = \alpha f(x),\ \forall f, g \in V^M,\ \forall \alpha \in K.$$

3) Can one organize a finite set M as a vector space over an infinite field K?

4) Let $p \in \mathbb{N}$ be a prime. Can one organize the Abelian group $(\mathbb{Z}, +)$ as a vector space over the field $(\mathbb{Z}_p, +, \cdot)$?