Differentiable Functions

Exercise 1: Determine the n-th derivative of the following functions:

a) $f:(-1,\infty)\to\mathbb{R}$ defined by $f(x)=(1+x)^r$, where $r\in\mathbb{R}$;

b) $f:(-1,\infty)\to\mathbb{R}$ defined by $f(x)=x\cdot\ln(1+x)$;

c) $f:(-\infty,-1)\to\mathbb{R}$ defined by $f(x)=x\cdot\ln(1-x)$;

d) $f:(-1,1)\to\mathbb{R}$ defined by $f(x)=\sqrt{3x+4}$;

e) $f:(-\frac{1}{2},\infty)\to\mathbb{R}$ defined by $f(x)=\frac{1}{\sqrt{2x+1}}$.

Exercise 2: Determine the n-th derivative of the following functions:

a) $f:\mathbb{R}\setminus\{-\frac{b}{a}\}\to\mathbb{R}$ defined by $f(x)=\frac{1}{ax+b}$;

b) $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\sin(ax+b)$;

c) $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\cos(ax+b)$;

d) $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=e^{ax+b}$.

Exercise 3: Compute the derivatives of the following functions

a) $f:(0,\infty)\to\mathbb{R}$ defined by $f(x)=x^x$;

b) $f:(0,\infty)\to\mathbb{R}$ defined by $f(x)=x^{\frac{1}{x}}$;

c) $f:(0,\pi)\to\mathbb{R}$ defined by $f(x)=\sin x^x$;

d) $f:(0,\infty)\to\mathbb{R}$ defined by $f(x)=x^{\sin x}$;

Exercise 4: Prove that $\frac{1}{x+1}<\ln(x+1)-\ln x<\frac{1}{x}$ for all $x>0$.

Exercise 5:

a) Prove that for all $n\in\mathbb{N}$ if holds

$$na^{n-1}<\frac{b^n-a^n}{b-a}<nb^{n-1}$$

for all $a,b\in(0,+\infty)$ with $a<b$.

Excercise 6:

Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=x+|x-1|$$

for all $x\in\mathbb{R}$.

a) Prove that f has side derivatives at $x_0 = 1$;

b) Compute the side derivates of f at $x_0 = 1$;

c) Is f differentiable on the left at $x_0 = 1$? What about on the right?

d) Does f have a derivative at $x_0 = 1$?

e) Is f differentiable at $x_0 = 1$?