COMPLEMENTS OF GEOMETRY - SEMINAR 5

1. In an acute triangle ABC, AD, BE and CF are heights. Show that the line AD is the angle bisector of $\angle EDF$

2. Let $ABCD$ be a cyclic quadrilateral. A line parallel to BC intersects AB, CD at E, F respectively. Show that A, D, F, E are concyclic.

3. Let the incircle ω of triangle ABC touch BC, CA, and AB at D, E, and F respectively. Let Y_1, Y_2, Z_1, Z_2, and M be the midpoints of BF, BD, CE, CD, and BC, respectively. Let $Y_1Y_2 \cap Z_1Z_2 = X$. Prove that $MX \perp BC$.

Problems for presentation

Presentation 1. Let A, B, C, D be four distinct points on a line, in that order. The circles with diameters AC and BD intersect at X and Y. Let P be a point on the line XY such that $P \notin BC$. The line CP intersects the circle with diameter AC at C and M, and the line BP intersects the circle with diameter BD at B and N. Prove that the lines AM, DN and XY are concurrent.

Presentation 2. Let ABC be a triangle, H its orthocenter and consider A'', B'', C'' the midpoints of $[AH]$, $[BH]$ and $[CH]$. Show that the diameters of the Euler circle of $\triangle ABC$ that pass through A'', B'' and C'' are the perpendicular bisectors of the ortic triangle corresponding to $\triangle ABC$. (Recall the the ortic triangle of $\triangle ABC$ is $\triangle A_1B_1C_1$, where A_1, B_1, C_1 are the feet of the perpendiculars from the vertices of $\triangle ABC$).

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