SEMINAR 8

1) Are these matrices invertible? If yes, find their inverses:

a) $\begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 1 \\ 4 & 1 & 4 \end{pmatrix}$; b) $\begin{pmatrix} 3 & 4 & 2 \\ 6 & 8 & 5 \\ 9 & 12 & 10 \end{pmatrix}$; c) $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$.

Definition. A square matrix resulted from the identilastty matrix after performing only one elementary operation is called **elementary matrix**.

2) Show that any elementary matrix has an inverse and that the inverse of any elementary matrix is also an elementary matrix.

3) Let $m, n \in \mathbb{N}^*$. Show that any elementary operation on a matrix $A = (a_{ij}) \in M_{m,n}(K)$ is the result of the multiplication of A with an elementary matrix. More precisely, any elementary operation on the rows (columns) of A results by multiplying A on the left (right) side with the elementary matrix resulted by performing the same elementary operation on I_m (I_n, respectively).

4) (HOMEWORK) Let $n \in \mathbb{N}^*$. For any elementary matrix $E \in M_n(K)$ and any matrix $A \in M_n(K)$ we have

$$\det(EA) = \det E \cdot \det A = \det(AE).$$

5) Show that any invertible matrix is a product of elementary matrices.

6) Let $n \in \mathbb{N}^*$. For any matrices $A, B \in M_n(K)$ we have $\det(AB) = \det A \cdot \det B$.