

Laboratory 8

Deadline: 29 – 30 April 2024

Quadrature formulas II

1. Let us consider the following integral

$$I = \int_1^{1.5} f(x)dx, \quad \text{where } f(x) = e^{-x^2}.$$

- Evaluate the integral I using the rectangle formula (see Slide 5, Lecture 8).
- Plot the graph of the function and the graph of the rectangle which area approximates the integral using rectangle (midpoint) formula.
- Use the repeated rectangle formula for $n \in \{150, 500\}$ to evaluate the integral (see Slide 6, Lecture 8).

2. Let us consider the following integral

$$I = \int_0^1 f(x)dx, \quad \text{where } f(x) = \frac{2}{1+x^2}.$$

- Approximate the integral using the Romberg algorithm for trapezium formula (precision is $\varepsilon = 10^{-4}$). *Hint: see Slide 9, Lecture 7*
- Approximate the integral using the Romberg algorithm in Aitken's form (precision is given by $\varepsilon = 10^{-4}$). *Hint: see Slide 12, Lecture 7*

3. Let us consider the function $f : [1, 3] \rightarrow \mathbb{R}$ given by $f(x) = \frac{100}{x^2} \sin \frac{10}{x}$ for all $x \in [1, 3]$.

- Plot the graph of the function f .
- Use an adaptive quadrature algorithm for Simpson's formula to approximate the integral $I = \int_1^3 f(x)dx$ with precision $\varepsilon = 10^{-4}$ (see Slide 17, Lecture 7).
- Compare the obtained result with the one obtained applying repeated Simpson's formula for $n = 50$ and $n = 100$ (value: -1.4260247818).

Remark: 1-2 (1p), 3 (0.5p)