

Bonus Lab

Deadline: 3–7 June 2024

Equations and Systems of equations in \mathbb{R}

1. Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{pmatrix}$$

and the vector $b = (3, 13, 4)$. Find the LU decomposition of A and solve the system $Ax = b$.

2. Solve the following system using the Jacobi, Gauss-Seidel, respectively the Relaxation method:

$$\begin{cases} 5x_1 - 2x_2 + 3x_3 = -1 \\ -3x_1 + 9x_2 + x_3 = 2 \\ 2x_1 - x_2 + 7x_3 = 3 \end{cases}$$

3. Compute the first three Newton iterates for the nonlinear problem $f(x) = 4x^3 - 2x^2 + 3$ with $x_0 = -1$.
4. Compute x_2, x_3, x_4 for the Secant method applied to $f(x) = x^3 - \sin x + 4x^2 + 6x + 9$ with $x_0 = 8$ and $x_1 = 7$.
5. What is x_2 if $x_0 = 0$ and $x_1 = 1$ in the application of the Secant method to $f(x) = x^2 - 2$?
6. Apply the Bisection algorithm to find a positive root of $x^2 - 4x \sin x + (2 \sin x)^2 = 0$.

Remark: 1-6 (0.5p)