P_2:

$z^2 = x^2 + y^2$

$x^2 + y^2 + z^2 = 1$

~~Hence~~ To show they are orthogonal at every point of intersection, we need to demonstrate that the normals to these surfaces are perpendicular at their points of intersection

First we find the gradient of each surface

For the cone, the gradient is:

$$\nabla F_1 = \left(\frac{\partial F_1}{\partial x}, \frac{\partial F_1}{\partial y}, \frac{\partial F_1}{\partial z}\right) = (-2x, -2y, 2z)$$

For the sphere, the gradient is:

$$\nabla F_2 = \left(\frac{\partial F_2}{\partial x}, \frac{\partial F_2}{\partial y}, \frac{\partial F_2}{\partial z}\right) = (2x, 2y, 2z)$$

~~We~~ The points of intersection need to satisfy both equations.

We substitute $z^2 = x^2 + y^2$ into the sphere's equation:

$$x^2 + y^2 + (x^2 + y^2) = 1 \Rightarrow 2(x^2 + y^2) = 1 \rightarrow x^2 + y^2 = \frac{1}{2} \Rightarrow z^2 = \frac{1}{2}$$

The points of intersection are:

$$(x, y, z) \text{ s.t. } x^2 + y^2 = \frac{1}{2} \text{ and } \frac{1}{z} = \cancel{\frac{1}{\sqrt{2}}} \pm \frac{1}{\sqrt{2}}$$

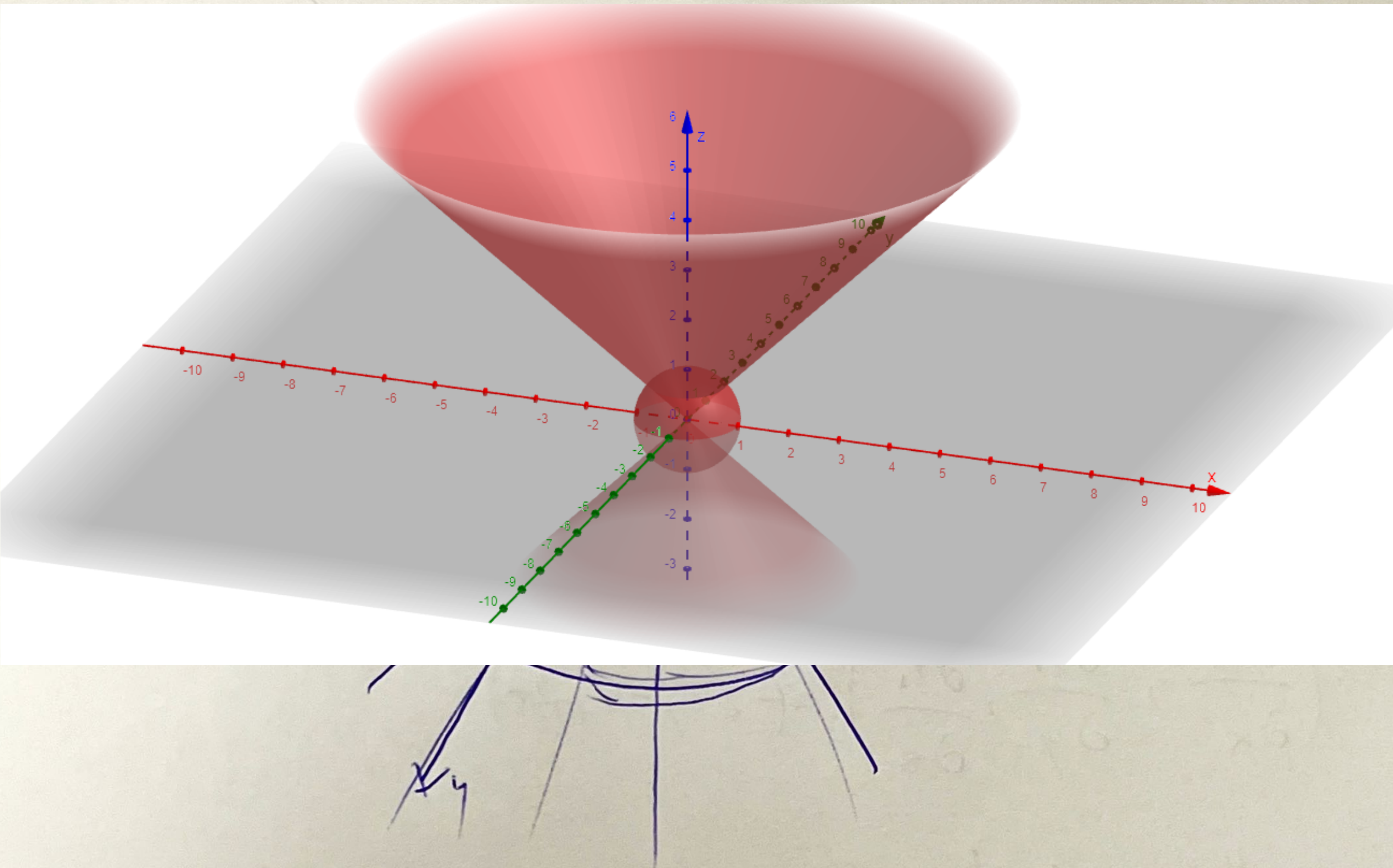
At these points, we need to check $\nabla F_1 \cdot \nabla F_2 = 0$

$$\nabla F_1 \cdot \nabla F_2 = (-2x, -2y, 2z) \cdot (2x, 2y, 2z)$$

$$= -4x^2 - 4y^2 + 4z^2$$

Since $z^2 = x^2 + y^2 \Rightarrow -4x^2 - 4y^2 + 4(x^2 + y^2) = -4(x^2 + y^2) + 4(x^2 + y^2) = 0$

$\Rightarrow$ the dot product is $0 \Rightarrow$ normals are perpendicular $\Rightarrow$ surfaces are orthogonal


z
y
x