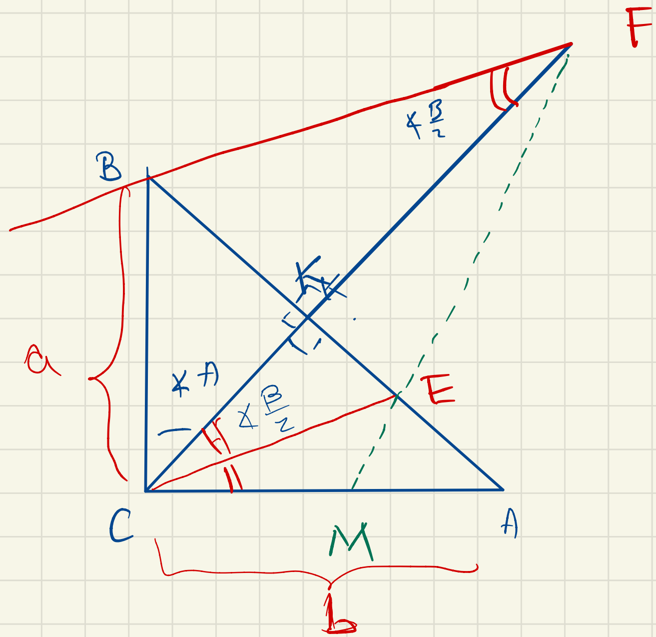
2. Let CK be an altitude in $\triangle ABC$ where $m(\angle BCA) = 90°$. In $\triangle ACK$, we construct $(CE$ the angle bisector of $\angle ACK$ with $E \in (AK)$. The line that passes through B and is parallel to CE meets CK in the point F. Show that the line EF meets the segment AC in its midpoint.

3. In a triangle the *simedians* are the cevians which are izogonal to the corresponding



Fie M - mij. lui [CA]. Vreau să arăt că

$$\frac{AM}{MC} \cdot \frac{FC}{FK} \cdot \frac{EK}{EA} = 1$$

$$\frac{AM}{MC} = 1. \qquad (1)$$

$$\frac{FC}{FK} = 1 + \frac{CK}{FK} = 1 + \frac{EK}{BK} \qquad (2)$$

Din (1) și (2) ne rămâne de

arătat că:

$$\left(1+\frac{EK}{BK}\right)\cdot\frac{EK}{AE}=1.$$

Fie $a=BC$, $b=CA$.

Atunci $CK=\dfrac{ab}{\sqrt{a^2+b^2}}$ şi din Pitagora

în ΔACK se află $AK=\dfrac{b^2}{\sqrt{a^2+b^2}}$.

Acum $BK=\sqrt{a^2+b^2}-AK=\dfrac{a^2}{\sqrt{a^2+b^2}}$

Acum din T. bisectoarei,

$$\frac{AE}{EK}=\frac{b}{CK}=\frac{b}{ab}\cdot\sqrt{a^2+b^2}=\frac{\sqrt{a^2+b^2}}{a}.$$

Ne mai trebuie EK.

$$\frac{AE+EK}{EK}=\frac{\sqrt{a^2+b^2}+a}{a}\Rightarrow\frac{EK}{AK}=\frac{a}{a+\sqrt{a^2+b^2}}$$

De unde se află

$$EK = \frac{a b^2}{a^2+b^2+a\sqrt{a^2+b^2}}$$

Acum calculăm :

$$\left(1+\frac{EK}{BK}\right)\cdot\frac{EK}{AE} =$$

Input

$$\left(1+\frac{a\,b^2}{a^2+b^2+a\sqrt{a^2+b^2}}\times\frac{\sqrt{a^2+b^2}}{a^2}\right)\times\frac{a}{\sqrt{a^2+b^2}}$$

Result

1