COMPLEMENTS OF GEOMETRY - SEMINAR 3

1. In a triangle the *simedians* are the cevians which are izogonal to the corresponding *medians*. Show that in any triangle the simedians are concurrent.

2.(USAMO '96) Let $\triangle ABC$ be a triangle and M a point in its interior such that $\angle MAB = 10°$, $\angle MBA = 20°$, $\angle MAC = 40°$ and $\angle MCA = 30°$. Prove that $\triangle ABC$ is isosceles.

3.(OJM 2024) Let $ABCD$ be a parallelogram. Let M be a point on the side DC and E, N points on the diagonal AC such that $BE \perp AC$ and $\frac{CM}{CD} = \frac{EN}{EA}$. Show that if MN and NB are perpendicular, then $ABCD$ is a rectangle.

Problems for presentation

Presentation 1. Let $ABCD$ be a parallelogram. Let X and Y be points on the sides AB and BC, respectively, such that $AX = CY$. Prove that the intersection of lines AY and CX lies on the angle bisector of $\angle ADC$.

Presentation 2. Let $ABCDE$ be a convex pentagon such that $AB + CD = BC + DE$ and a circle ω with center O on the side AE is tangent to the sides AB, BC, CD and DE at points P, Q, R and S, respectively. Prove that the lines PS and AE are parallel.

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