Faculty of Mathematics and Computer Science
Babeş-Bolyai University, Cluj-Napoca
Numerical Analysis
MIE - semester IV

Laboratory 12

Deadline: 3−7 June 2024

Numerical methods for solving nonlinear equations

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1. Let us consider the equation
$$\sin x = x + 1, \quad x \in \mathbb{R}$$
and $x_0 = \frac{\pi}{4}$. Solve the equation using Newton's method with precision $\varepsilon = 10^{-4}$ and a maximum number of iterations $N = 100$.

2. Let us consider the following form of the Kepler's equation
$$\Phi(x) = x - \frac{4\sin x - \pi}{5}, \quad x \in \mathbb{R}.$$
For $x_0 = 1$, apply six iterations of the Newton's method in order to find the position $\Phi(x)$ of a satellite in $\Delta t = 9'$.

3. Let us consider the equation $x^3 - 2x^2 - 1 = 0$ and the initial points $x_0 = 1$ and $x_1 = 2$. Solve the equation using the secant method with precision $\varepsilon = 10^{-4}$ and a maximum number of iterations $N = 100$. Determine the necessary numbers of iterations to reach the precision ε.

4. Let $f : [1, 2] \to \mathbb{R}$ be given by $f(x) = (x - 2)^2 - 2\ln x$. Let $\varepsilon = 10^{-4}$ and $N = 100$. Solve the equation $f(x) = 0$ using bisection and false position methods with precision ε and maximum number of iterations N. Determine in each case the necessary numbers of iterations to reach the precision ε.

5. Let $f : [0, 1] \to \mathbb{R}$ be given by $f(x) = x - \frac{1}{5}\sin x - \frac{1}{2}$. Knowing that f has exactly one zero between $x_0 = 0.5$ and $x_1 =$, determine the solution of the equation $f(x) = 0$ using
   - Newton's method
   - secant method
   - bisection method

   Determine also how many steps each method would require to produce six digits and compute the error at each step (we know that the exact solution is 0.61546850).

6. Let us consider the function $f : (0, \infty) \to \mathbb{R}$ be given by $f(x) = (x^2 - 1)^\alpha \log x$, where $\alpha \geq 1$. Consider the particular case $\alpha = 2$, $x_0 = 0.8$ and $\varepsilon = 10^{-10}$.

- Use the Newton's method in the following two forms

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \quad \text{(standard)},$$

respectively

$$x_{k+1} = x_k - m\frac{f(x_k)}{f'(x_k)} \quad \text{(root of multiplicity } m\text{)}$$

to approximate the multiple zero $x^* = 1$ of $f(x)$. In this case, $m = \alpha + 1$.

- Determine also the number of iterations required to converge for each case.

7. Let us consider the function $f : \mathbb{R} \to \mathbb{R}$ be given by $f(z) = xe^x - 1$ and the point $x_0 = 0.5$. Consider the following fixed point iterations

   (a) $x_{k+1} = e^{-x_k}$

   (b) $x_{k+1} = \frac{1+x_k}{e^{x_k}+1}$

   (c) $x_{k+1} = x_k + 1 - x_k e^{x_k}$

   For each case, compute the approximation of the solution of $f(x) = 0$ and the number of iterations necessary to reach the precision $\varepsilon = 10^{-10}$.

Remark: 1-5 (0.5p), 6-7 (1p) - bonus points