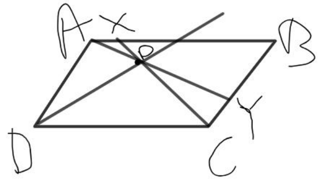
Presentation 1 - Seminar 3

Presentation 1. Let $ABCD$ be a parallelogram. Let X and Y be points on the sides AB and BC, respectively, such that $AX = CY$. Prove that the intersection of lines AY and CX lies on the angle bisector of $\angle ADC$.



Law of sines:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(c)} \quad <-> \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

1.Using parallel lines in AB and DC => <DCP=<PXB
=><PXB=180-<AXP
and the same for AD and BC => <PYC=180-<DAP
We can deduce that:
<DAP = 180-<PYC
<DCP=180-<AXP

2.Since we know those two angles we can use that to our advantage and apply the Law of sines as following:

$$\frac{\sin(<ADP)}{AP} = \frac{\sin(<DAP)}{DP} => \sin(<ADP) = \frac{\sin(<DAP)}{DP} \cdot AP$$

$$\frac{\sin(<CDP)}{PC} = \frac{\sin(<DCP)}{DP} => \sin(<CDP) = \frac{\sin(<DCP)}{DP} \cdot PC$$

3.We know from the hypothesis that AX and CY are equal so now is the time to use that and change AP and PC in terms of AX and CY using the law of sines:

$$AP = \frac{\sin(<AXP)}{\sin(<APX)} \cdot AX$$

$$PC = \frac{\sin(<PYC)}{\sin(<CPY)} \cdot CY$$

AX and CY equal so we might cancel them out so we only remain with: 4.

We also know that $<APX = <CPY$ from the interseCtion of AY witH CY

If two angles add up to 180 $^\circ$ it means their sin are equal.

Or they can have the same angle.

$$4.\ \sin(<ADP) = \frac{\sin(<DAP)}{DP} \cdot \frac{\sin(<AXP)}{\sin(<APX)}$$

$$\sin(<CDP) = \frac{\sin(<DCP)}{DP} \cdot \frac{\sin(<PYC)}{\sin(<CPY)}$$

Using:
Sin(<DAP)=sin(<PYC)
sin(<DCP)=sin(<AXP)
sin(<APX)=sin(<CPY) (equal angles)

We proved that sin<ADP=sin<CDP
We know that ABCD is a parallelogram and the angle ADC is smaller than 180.

=> It results that the line that passes through DP, P being the intersection of XC with AY, is the angle bisector
<ADP=<CDP