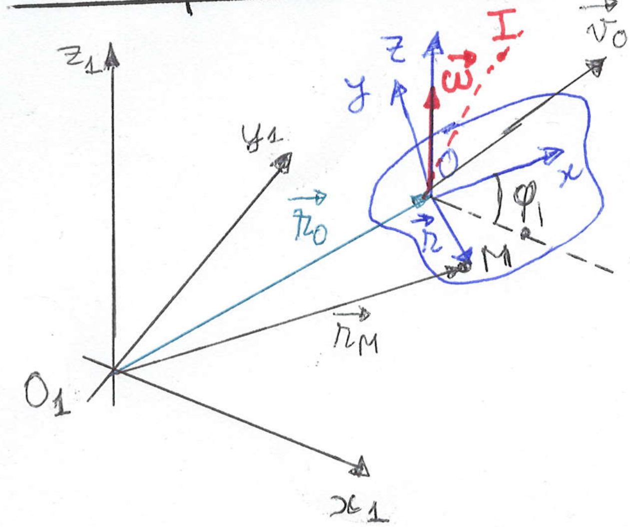
Plane parallel motion



$\vec{\omega} \perp \vec{v}_M$, $\vec{\omega}$ - angular velocity

Motion equation $\begin{cases} x_0 = x_0(t) \\ y_0 = y_0(t) \\ \varphi = \varphi(t) \end{cases}$

$$\vec{v}_M = \vec{v}_0 + \vec{\omega} \times \vec{r}$$

$$\vec{a}_M = \underbrace{\vec{a}_0}_{\text{translation}} + \underbrace{\dot{\vec{\omega}} \times \vec{r}}_{\text{rotation}} - \underbrace{\omega^2 \cdot \vec{r}}_{\text{centripetal}}$$

I - Instantaneous rotation centre (IC)

$$\vec{OI} = \frac{1}{\omega^2} \vec{\omega} \times \vec{v}_0 \ , \quad OI = \frac{v_0}{\omega} \ , \quad \vec{v}_M = \vec{\omega} \times \vec{IM}$$

Space centrode (Baza) - locus of I in $O_1 x_1 y_1$

$O(x_{10}, y_{10})$ in the frame of coordinates $O_1 x_1 y_1$

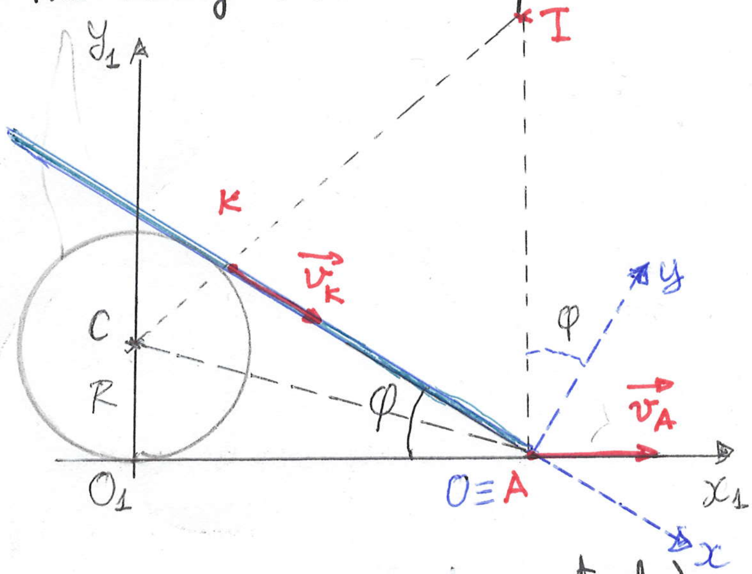
$$\begin{cases} x_1 = x_0 - \dfrac{dy_{10}}{d\varphi} \\ y_1 = y_0 + \dfrac{dx_{10}}{d\varphi} \end{cases}$$

Body centrode (l.r. Rulanta) - locus of I in Oxy

$$x = \frac{dx_{10}}{d\varphi} \sin\varphi - \frac{dy_{10}}{d\varphi} \cos\varphi$$

$$y = \frac{dx_{10}}{d\varphi} \cos\varphi + \frac{dy_{10}}{d\varphi} \sin\varphi$$

1. On a circle of radius R, tangent to the axis O_1x_1 in origin, is leaning continuously a rigid bar AB. The extremity A of the bar is moving on the horizontal axis O_1x_1.
Find the body and the space centrode.



<u>Geometrical</u> (for space centrode)

Let be $\varphi = (O_1x_1, Ox) \Rightarrow \widehat{O_1OC} = \varphi/2$.

$$\left.\begin{array}{l} \widehat{IOC} = \frac{\pi}{2} - \varphi/2 \\ \widehat{KCO} = \frac{\pi}{2} - \frac{\varphi}{2} \end{array}\right\} \Rightarrow \Delta ICO \text{ - isosceles triangle} \Rightarrow d(I, O_1x_1) = d(I, C) \Rightarrow$$

$\Rightarrow$ the space centrode is a parabola with the focus C and the directrix O_1x_1

<u>Analytical</u>

$$\begin{cases} x_{10} = R\,\mathrm{ctg}\,\frac{\varphi}{2} \\ y_{10} = 0 \end{cases} \xrightarrow[\text{Centrode}]{\text{Space}} \begin{cases} x_1 = x_{10} - \frac{dy_{10}}{d\varphi} \\ y_1 = y_{10} + \frac{dx_{10}}{d\varphi} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x_1 = R\,\mathrm{ctg}\,\frac{\varphi}{2} \\ y_1 = -\frac{R}{2}\frac{1}{\sin^2\frac{\varphi}{2}} \end{cases} \Rightarrow y_1 = -\frac{R}{2}\frac{\sin^2\frac{\varphi}{2} + \cos^2\frac{\varphi}{2}}{\sin^2\frac{\varphi}{2}} =$$

$$= -\frac{R}{2}\left(1 + \mathrm{ctg}^2\frac{\varphi}{2}\right).$$

We have: $y_1 = -\frac{R}{2}\left(1 + \frac{x_1^2}{R^2}\right) \Rightarrow \boxed{y_1 = -\frac{1}{2R}(R^2 + x_1^2)}$ - parabola

Body centrode

$$\begin{cases} x = \dfrac{dx_{10}}{d\varphi}\sin\varphi - \dfrac{dy_{10}}{d\varphi}\cos\varphi \\ y = \dfrac{dx_{10}}{d\varphi}\cos\varphi + \dfrac{dy_{10}}{d\varphi}\sin\varphi \end{cases} \Leftrightarrow \begin{cases} x = -R\dfrac{1}{2\sin^2\varphi}\cdot\sin\varphi \\ y = -R\dfrac{1}{2\sin^2\varphi}\cdot\cos\varphi \end{cases}$$

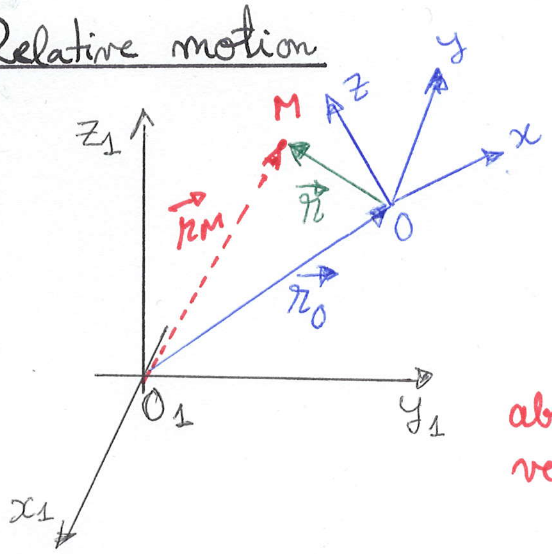
$$\Rightarrow \begin{cases} x = -\dfrac{R}{2}\,\dfrac{2\sin\frac{\varphi}{2}\cdot\cos\frac{\varphi}{2}}{\sin^2\frac{\varphi}{2}} \\ y = -\dfrac{R}{2}\,\dfrac{\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2}}{\sin^2\frac{\varphi}{2}} \end{cases} \Rightarrow \begin{cases} x = -R\,\mathrm{ctg}\,\dfrac{\varphi}{2} \\ y = -\dfrac{R}{2}\left(\mathrm{ctg}^2\dfrac{\varphi}{2}-1\right) \end{cases}$$

We have: $\mathrm{ctg}\,\dfrac{\varphi}{2} = -\dfrac{x}{R}$

Thus, $y = -\dfrac{R}{2}\left(\dfrac{x^2}{R^2}-1\right) \Rightarrow \boxed{y = -\dfrac{1}{2R}(x^2-R^2)}$ - parabola

We used $\left(\mathrm{ctg}\,\dfrac{\varphi}{2}\right)' = -\dfrac{1}{2}\,\dfrac{1}{\sin^2\frac{\varphi}{2}}$

Relative motion



$$\vec{r}_M = \vec{r}_0 + x(t)\cdot\vec{i} + y(t)\cdot\vec{j} + z(t)\cdot\vec{k}$$

$$\underbrace{\vec{v}_M}_{\text{absolute velocity}} = \underbrace{\vec{v}_0 + \vec{\omega} \times \vec{r}}_{\text{transport velocity}} + \frac{\partial \vec{r}}{\partial t}$$

$$\frac{\partial \vec{r}}{\partial t} \Rightarrow \vec{v}_r = \dot{x}\cdot\vec{i} + \dot{y}\,\vec{j} + \dot{z}\cdot\vec{k}$$

relative velocity

$$\vec{a}_M = \underbrace{\vec{a}_0 + \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})}_{\vec{a}_t} + \underbrace{2\cdot\vec{\omega} \times \frac{\partial \vec{r}}{\partial t}}_{\vec{a}_c} + \underbrace{\frac{\partial^2 \vec{r}}{\partial t^2}}_{\vec{a}_r}$$

absolute acceleration

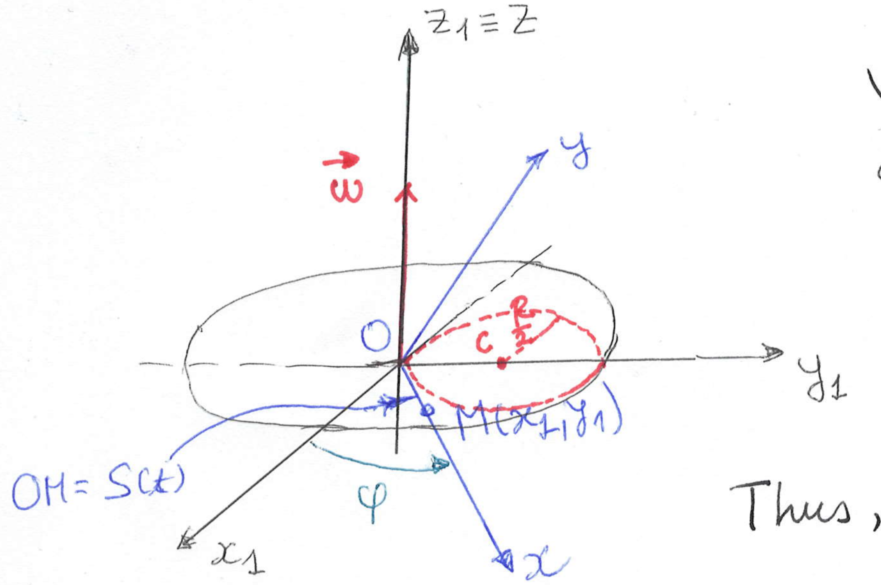
transport acceleration

Coriolis acceleration (due to the rotation of Oxyz)

relative acceleration (due to the motion of M in Oxyz)

① A disk of radius R has a rotation motion with the constant angular velocity $\vec{\omega}$ about an axis $Oz_1 = Oz$ passing through the centre of the disk. On the disk's diameter moves a point M that starts from the centre O with the motion equation $S = R\sin\omega t$.

Find the path, the absolute velocity and acceleration of M.



We choose Ox - the direction of the diameter on wich the point M moves.

Consider $\varphi = (\widehat{Ox_1, Ox})$

Thus, $\dot{\varphi} = \omega \Rightarrow \varphi = \omega \cdot t + \underbrace{\cancel{\varphi_0}}_{=0}$

We have $\begin{cases} x_1 = S\cos\varphi \\ y_1 = S\sin\varphi \end{cases} \Rightarrow \begin{cases} x_1 = R\sin(\omega t)\cos(\omega t) \\ y_1 = R\sin(\omega t)\cdot\sin(\omega t) \end{cases}$

$$\Rightarrow \begin{cases} x_1^2 = R^2\sin^2\omega t\,(1 - \sin^2\omega t) \\ y_1 = R\sin^2\omega t \end{cases}$$

$$\Rightarrow x_1^2 = R^2\frac{y_1}{R}\left(1 - \frac{y_1}{R}\right) \Rightarrow x_1^2 = Ry_1 - y_1^2 \Rightarrow$$

$$\Rightarrow x_1^2 + y_1^2 - Ry_1 = 0$$

$$\Rightarrow \boxed{x_1^2 + \left(y_1 - \frac{R}{2}\right)^2 = \frac{R^2}{4}}$$

- the equation of the circle $\mathcal{C}\left(\underbrace{\left(0, \frac{R}{2}\right)}_{C}, \frac{R}{2}\right)$

Velocity

$$\vec{v}_M = \vec{v}_t + \vec{v}_r$$

$$\vec{v}_t = \vec{\omega} \times \vec{r} = \omega . \vec{k} \times \underbrace{R \sin \omega t}_{S} \cdot \vec{i} = R\omega \sin \omega t \cdot \vec{j}$$

$$(\vec{k} \times \vec{i} = \vec{j})$$

$$\vec{v}_r = \frac{ds}{dt} \cdot \vec{i} = R\omega \cos \omega t \cdot \vec{i}$$

Thus, $\vec{v}_M = R\omega \left[\cos(\omega t) \cdot \vec{i} + \sin(\omega t) \cdot \vec{j} \right]$

$$v_M = R\omega$$

Acceleration

$$\vec{a}_M = \vec{a}_t + \vec{a}_c + \vec{a}_r$$

$$\vec{a}_t = \vec{\omega} \times (\vec{\omega} \times \vec{r}) = -\omega^2 . \underset{\vec{r}}{\overset{\shortparallel}{\vec{d}}} = -\omega^2 . \vec{r} = -\omega^2 R \sin \omega t \cdot \vec{i}$$

$\vec{d}$ is the distance from O to M

$$\vec{a}_c = 2 \left(\vec{\omega} \times \frac{\partial \vec{r}}{\partial t} \right) = 2 (\vec{\omega} \times \vec{v}_r) = 2\, \omega \vec{k} \times R\omega \cos \omega t \cdot \vec{i}$$

$$= 2 \omega^2 R \cos \omega t \cdot \vec{j}$$

$$\vec{a}_r = \frac{\partial \vec{v}_r}{\partial t} = -R\omega^2 \sin \omega t \cdot \vec{i}$$

Thus, $\vec{a}_M = 2R\omega^2 \left(-\sin \omega t \cdot \vec{i} + \cos \omega t \cdot \vec{j} \right)$

$$a_M = 2R\omega^2 .$$