

ANALYTIC GEOMETRY, PROBLEM SET 6

Line bundles, angle between two lines (in 2D) and the equation of the plane (3D)

1. Given the bundle of lines of equations $(1 - t)x + (2 - t)y + t - 3 = 0, t \in \mathbb{R}$ and $x + y - 1 = 0$, find:
 - (1) the coordinates of the vertex of the bundle;
 - (2) the equation of the line in the bundle which cuts Ox and Oy in M , respectively N , such that $OM^2 \cdot ON^2 = 4(OM^2 + ON^2)$;
2. Let B be the bundle of vertex $M_0(5, 0)$. An arbitrary line from B intersects the lines $d_1 : y - 2 = 0$ and $d_2 : y - 3 = 0$ in M_1 respectively M_2 . Prove that the line passing through M_1 and parallel to OM_2 passes through a fixed point.
3. Determine the angle between the lines:
 - (1) $y = 2x + 1$ and $y = -x + 2$;
 - (2) $y = 3x - 4$ and $x = 3 + t, y = -1 - 2t$ for $t \in \mathbb{R}$.
 - (3) $y = \frac{2}{5}x + 1$ and $4x + 3y - 12 = 0$.
4. Determine the equation of the line which passes through $A(3, 1)$ and makes an angle of 45° with the line $2x + 3y - 1 = 0$.
5. Consider the triangle given by the points $A(1, -2)$, $B(5, 4)$ and $C(-2, 0)$. Find the equations of the internal, respectively external bisectors corresponding to the vertex A of this triangle.
6. The points of intersection of the lines $d_1 : x + 2y - 1 = 0$, $d_2 : 5x + 4y - 17 = 0$ and $d_3 : x - 4y + 11 = 0$ determine a triangle. Find the equations of the altitudes of these triangles without determining the coordinates of the vertices of the triangle!
7. Let $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ be distinct points in space. Prove that the equation of the plane containing P_1 and P_2 that is parallel to a vector $\bar{a} = (l, m, n)$ is
$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l & m & n \end{vmatrix} = 0.$$
8. Find the equation of the plane passing through $P(7, -5, 1)$ which determines on the positive half-axes three segments of the same length.
9. Find the equation for each of the following planes:
 - (a) containing $P(2, 1, -1)$ and perpendicular to the vector $\bar{n} = (1, -2, 3)$;

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- (b) determined by $O(0, 0, 0)$, $P_1(3, -1, 2)$ and $P_2(4, -2, -1)$;
- (c) containing $P(3, 4, -5)$ and parallel to both $\overline{a_1}(1, -2, 4)$ and $\overline{a_2}(2, 1, 1)$;
- (d) containing the points $P_1(2, -1, -3)$ and $P_2(3, 1, 2)$ and parallel to the vector $\overline{a}(3, -1, -4)$.

10. Find the equation of the plane containing the perpendicular lines through $P(-2, 3, 5)$ on the planes $\pi_1 : 4x + y - 3z + 13 = 0$ and $\pi_2 : x - 2y + z - 11 = 0$.