COMPLEMENTS OF GEOMETRY - SEMINAR 1

1. Prove that the perpendicular bisectors of a triangle are concurrent.

2. Prove that the internal bisectors of a triangle are concurrent.

3. Prove the bisector theorem in $\triangle ABC$: If D is the intersection of the bisector of $\angle A$ with (BC), then show that

$$\frac{AB}{AC} = \frac{BD}{CD}.$$

4. In triangle ABC, denote by C' and respectively B' the midpoints of the sides AB and AC. Let G be the intersections of the lines BB' and CC'. Show that $BG = 2GB'$ and $CG = 2GC'$.

5. Prove that the medians of a triangle are concurrent.

6. Let $\triangle ABC$ be a triangle in which $\angle BAC$ is a right angle and let $[AD]$ with $D \in (BC)$ be an altitude in this triangle. Show that $AB^2 = BD \cdot BC$, $AC^2 = CD \cdot BC$ and $AD^2 = BD \cdot CD$.

7. Prove Pytagora's theorem.

For presentation next time

Presentation 1. Let P be a point lying inside the square $ABCD$. Denote by M, N the feet of the perpendiculars from P to BC and CD, respectively. If $AP \perp MN$, show that either $AP = MN$, or $AP \perp BD$.

Presentation 2. In $\triangle ABC$, we consider the points $M \in (AB)$ and $N \in (BC)$ such that

$$2 \cdot \frac{CN}{BC} = \frac{AM}{AB}.$$

Let P be a point on AC. Show that the lines MN and NP are perpendicular if and only if PN is the internal angle bisector of the angle $\angle MPC$.

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