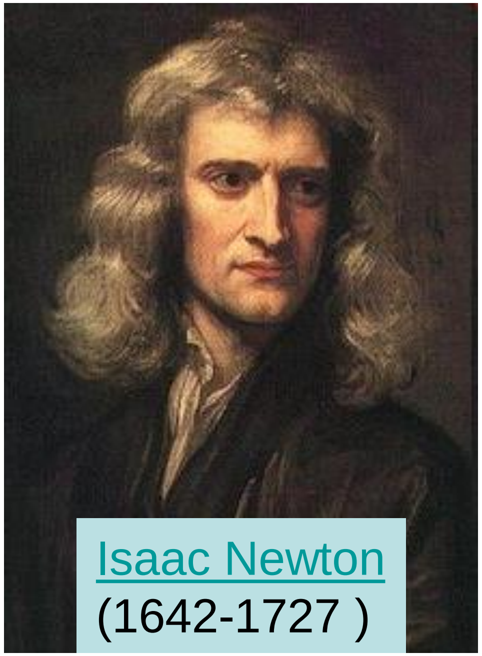
8. Dynamics. Newton's laws

Dynamics is the branch of classical mechanics concerned with the study of forces and their effects on motion. Isaac Newton defined the fundamental physical laws which govern dynamics in physics, especially his second law of motion.



Newton's laws (principles, axiomes):

I. Principle of Inertia (Galilei)

Every body continues in a state of rest or in uniform motion (constant velocity in straight line) unless acted upon by an external force.

A force is an influence that can change the motion of an object.

The tendency of an object to resist acceleration.

There are actually two aspects to the first law that may be necessary to be kept in mind.

It at once seeks to address both a stationary object and a moving one. For the stationary object it says that an object at rest continues to do so unless acted upon by an external force.

For a moving object the law holds that a particle in motion remains in motion (in a straight line) with the same speed and direction unless of course it is interacted upon by an external force.

Recall:

A frame of reference is an environment which is used to observe an event or the motion of a particle.

A coordinate system is associated with the frame to observe the event (eg the body’s location over time). The observer is equipped with measuring tools (eg rulers and clocks) to measure the positions and times of events.

Newton's first law also defines a class of reference frames which are called inertial frames.

A reference frame in which the Principle of Inertia is satisfied is called an inertial reference frame. A non-inertial frame is the one that is not inertial.

These reference frames are not supposed to undergo any type of acceleration. In other words, an inertial frame moves with a constant velocity or is at rest with respect to any other inertial frame.
(Bijan Kumar Bagchi, Classical Mechanics, CRC Press, 2017)

Example: The geocentric system of reference (The origin in the center of the Earth and the axes oriented towards three distant – motionless stars) is inertial!

A rotating frame is a common example of a non-inertial frame.
Although for short scales of time and distance we can consider the planet Earth to be an inertial frame, truly speaking, it is rotating about an axis passing through its geographical poles. Hence, technically, it does not constitute an inertial frame. A rotating frame, as we will find later, generates fictitious or pseudo forces. The Coriolis force is such a type of force and we need to carefully deal with it along with the so-called centrifugal force to make a proper use of Newton's laws in rotating frames.

Inertia is the resistance, of any physical object, to any change in its velocity.

In order to measure the inertia the inertial mass is introduced.

Inertia is one of the primary manifestations of mass, which is a quantitative property of physical systems.

An object with small inertial mass will accelerate more than an object with large inertial mass when acted upon by the same force. One says the body of greater mass has greater inertia.

The matter has the property to create an attractive field (the gravitational field) which act in different ways on different objects.

The weight is a measure for the attraction.

Galilei has shown experimentally that in one place on the Earth, all the body are falling with the same acceleration (gravitational acceleration or gravity, g) and that the ratio weight/gravity is constant.

We have

$$G = m_g g \tag{8.1}$$

where G is the weight and m_g is the gravitational mass.

Galilei, Newton, Eotvos and Dicke have experimentally shown that the inertial and the gravitational mass are numerically equal:

$$m_g = m_i := m$$

The mass has the property of additivity (the mass of o system of bodies is the sum of the body masses).

Thus, the mass is a positive scalar, it is constant during the motion of the particle and it is absolute (does not depend on the state of rest or motion and is the same in all reference systems).

The kilogram or kilogramme (symbol: kg) is the base unit of mass in the International System of Units (SI).

Force is a consequence of the principle of inertia. When a body changes his state of rest or motion and its velocity is modified and he gain acceleration we say that a force act on it.

Force is the external agent that modify the state of rest or motion of a body. It has magnitude, direction and the sense of acceleration.

$$\overrightarrow{F} = \overrightarrow{F}(\overrightarrow{r}, \overrightarrow{v}, t) \tag{8.2}$$

Examples:
- strong forces (strong nuclear forces)
- weak forces (weak nuclear forces)
- electromagnetic forces
- gravitational force

The unit of measure for the force is the Newton 1 N = (kg•m)/s².

One newton is the force needed to accelerate one kilogram of mass at the rate of one metre per second squared in the direction of the applied force.

II. Lex Secunda or Newton's Law (Galilei, Newton)

A change of motion is proportional to the motive force impressed, and takes place in the direction of the right line in which the force is impressed.

$$\mathbf{F} = m_i \mathbf{a} = m_i \frac{d\mathbf{v}}{dt} = m_i \frac{d^2\mathbf{r}}{dt^2} \,. \tag{8.3}$$

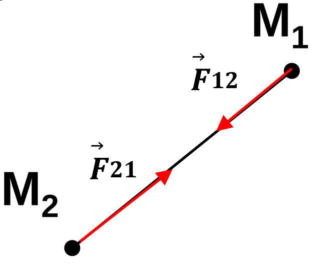
Let us now look at another aspect. The second law can be used in three main ways:

1. If we know the inertial mass of a body and all the forces acting on it, and the initial conditions, we can calculate its motion
2. If we know the motion of a body and its inertial mass, we can infer the forces acting on it.
3. A third possibility is that, if we know both forces and motion we can deduce the inertial mass of the body. To know the mass of the proton for example, we can measure how its momentum and energy vary under the action of a known force.

III. The principle of action and reaction

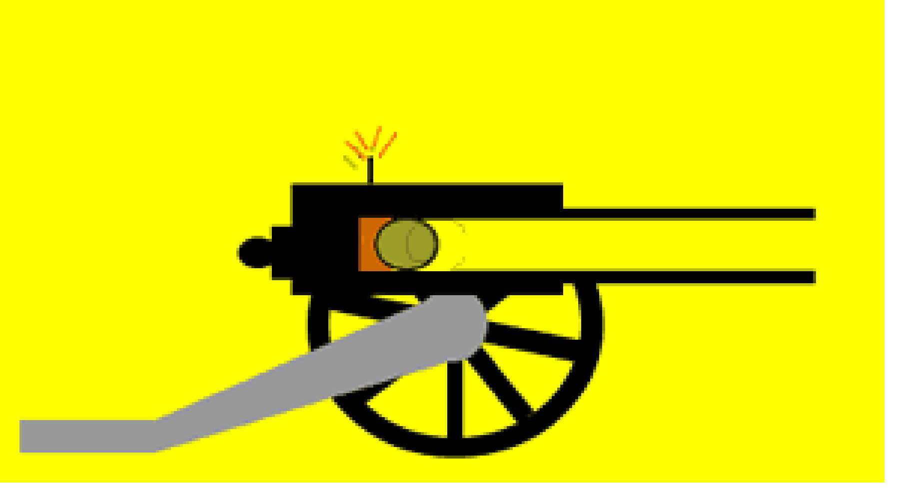
When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

To every action there is an equal and opposite reaction.



$$\overrightarrow{F}_{12} + \overrightarrow{F}_{21} = 0 \quad (8.4)$$

$$\overrightarrow{F}_{12} = \lambda \overrightarrow{M_1 M_2} \quad (8.5)$$

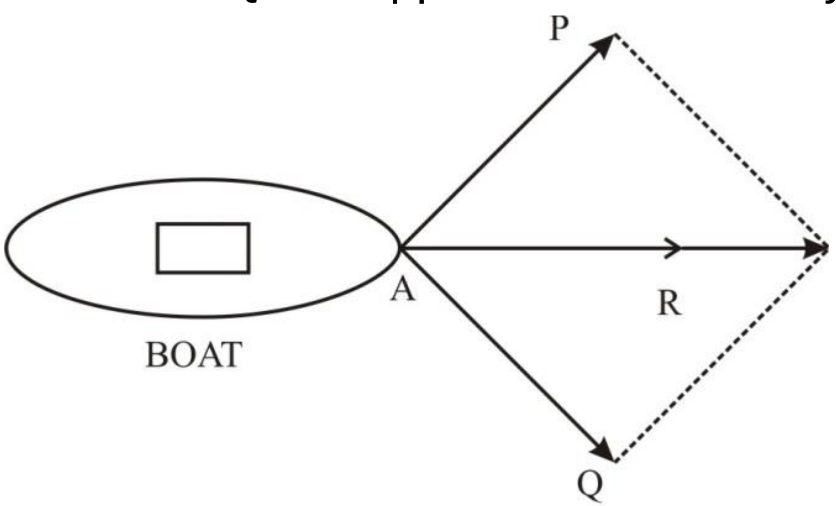


III. *The law of composition of forces*. If more than one force act at the same time on the material point we are discussing, their effect is the same as if only one force were acting, equal to the resultant of those forces.

Example

Consider two forces *P* and *Q* acting at *A* on a boat as shown in Fig.3.1. Let *R* be the resultant of these two forces *P* and *Q*. According to Newton's second law of motion, the boat will move in the direction of resultant force *R* with acceleration proportional to *R*. The same motion can be obtained when *P* and *Q* are applied simultaneously.

$$\vec{a} = \frac{1}{m}(\vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n) = \frac{\vec{F}}{m}$$



Principle of Superposition

Classical mechanics is characterized by two underlying principles governing it, namely:

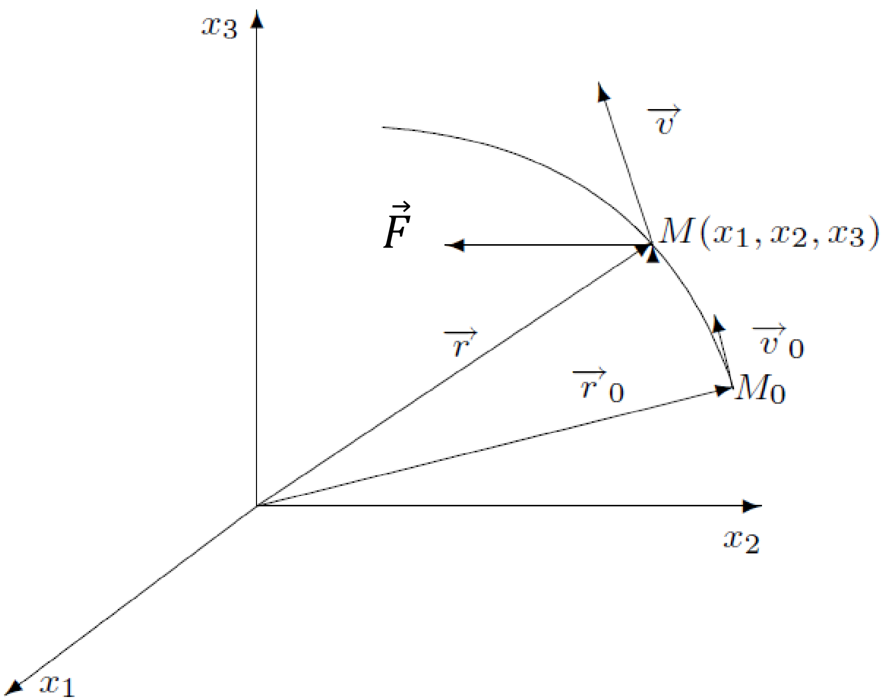
(i) Newton-Laplace principle of determinacy (or the deterministic law) which states that the state of the system (comprising the position and velocity of the particle) at time t completely determines its behaviour for all future (> t) and past (< t) times; in other words, the laws of physical phenomena are entirely deterministic

(ii) Galilean principle of relativity which states that the law of motion (8.3) has the same form in every inertial frame.

Dynamics of the free particle (material point)

The differential equation of the motion is

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F}(\vec{r}, \vec{v}, t), \; t \in (t_0, \mathcal{T}], \tag{8.6}$$



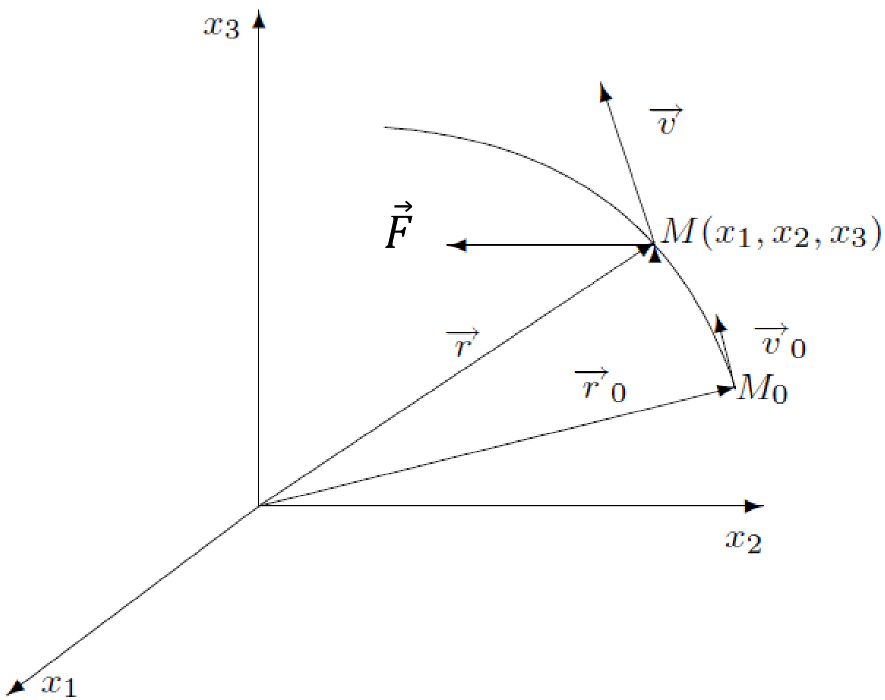
along with the initial conditions

$$\vec{r}(t_0) = \vec{r}_0, \; \vec{v}(t_0) = \vec{v}_0, \tag{8.7}$$

In the Cartesian frame $Ox_1x_2x_3$ one have

$$m\ddot{x}_i = F_i(\overrightarrow{r}, \overrightarrow{v}, t),\ t \in (t_0, \mathcal{T}],\ i = 1, 2, 3 \qquad (8.8)$$

$$x_i(t_0) = x_i^0,\ \dot{x}_i(t_0) = \dot{x}_i^0,\ i = 1, 2, 3 \qquad (8.9)$$



Consider the substitution

$$q = \begin{pmatrix} q_1 \\ \vdots \\ q_6 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} \quad Q = \begin{pmatrix} Q_1 \\ \vdots \\ Q_6 \end{pmatrix} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ F_1/m \\ F_2/m \\ F_3/m \end{pmatrix} \qquad (8.10)$$

Thus, the Cauchy problem (8.6) – (8.7) is equivalent with the first order system of equations

$$\dot{q} = Q(q,t), \quad t \in (t_0, \mathcal{T}] \tag{8.11}$$

$$q(t_0) = q^0 \tag{8.12}$$

where

$$q^0 = \begin{pmatrix} q_1^0 \\ q_2^0 \\ \vdots \\ q_6^0 \end{pmatrix} = \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \\ \dot{x}_1^0 \\ \dot{x}_2^0 \\ \dot{x}_3^0 \end{pmatrix}$$

Theorem of existence and unicity

Consider the domain $\Omega \subset \boldsymbol{R}^7$ (a vicinity of the point (q^0, t_0))

$$\Omega := \left\{(q,t) \in \boldsymbol{R}^7 \colon \left|q_k - q_k^0\right| < b_k, |t - t_0| < c, k = 1, \dots 6\right\} \tag{8.13}$$

where $b_k, c > 0$ are given constants, the functions Q_j are continous (i.e. F_i are continuous , $i = 1,2,3$) and satisfy the Lipschitz conditions (with respect to q):

$$\begin{gathered} \exists\, A_j > 0, j = 1, \dots, 6 \text{ such that } \forall (q^1, t), (q^2, t) \in Q\colon \\ |Q_k(q^1, t) - Q_k(q^2, t)| \leq \sum_{j=1}^{6} A_j \left|q_j^1 - q_j^2\right|,\ k = 1, \dots, 6. \end{gathered} \tag{8.14}$$

In these conditions a unique solution of the Cauchy (8.11)-(8.12) problem exists in Ω. Moreover, this solution is of class C^1 (functions $x_i \in C^2, i = 1,2,3$).

Let be

$$q = q\ (t, c_1, \dots, c_6) \Leftrightarrow q_j = q_j\ (t, c_1, \dots, c_6) \tag{8.15}$$

the solution of the Cauchy problem (8.11)-(8-12) where $c_1, \dots, c_6 \in \boldsymbol{R}$ (are integration constants).

Using the initial conditions we have

$$q_j^0 = q_j(t_0, c_1, \dots, c_6) \tag{8.16}$$

or

$$x_j^0 = x_j(t_0, c_1, \dots, c_6), \quad \dot{x}_j^0 = \dot{x}_j^0(t_0, c_1, \dots, c_6), \qquad j = 1,2,3. \tag{8.17}$$

Taking into account that the initial conditions (8.7) are independent then ((8.12) are independent):

$$\frac{\partial(q_1^0, \dots, q_6^0)}{\partial(c_1, \dots, c_6)} \neq 0 \tag{8.18}$$

Thus, a solution of the system (7.16) exists

$$c_k = c_k\,(t_0, q^0), \qquad k = 1, \dots, 6 \tag{8.19}$$

and the solution of the Cauchy problem (8.11)-(8-12) is given by

$$q_j = q_j(t, t_0, q^0), \qquad j = 1, \dots, 6 \tag{8.20}$$

Then, we have

$$x_i = x_j(t, t_0, \vec{r}_0, \vec{v}_0), \qquad i = 1,2,3 \tag{8.21}$$

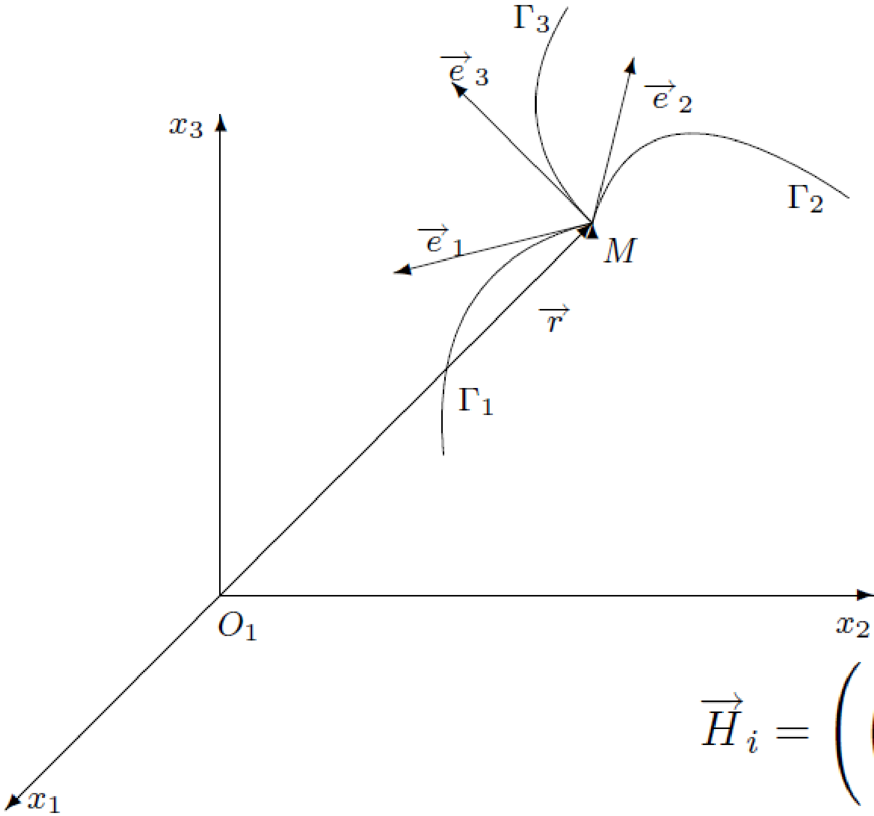
or

$$\vec{r} = \vec{r}(t, t_0, \vec{r}_0, \vec{v}_0) \tag{8.22}$$

Equations (8.21) or (8.22) represent the law of the motion of the material point in the physical space in finite interval of time.

Curvilinear coordinates(q_1, q_2, q_3)

Eq. (8.6) is projected on the curvilinear coordinates q_i:



$$\frac{m}{2H_i}\left[\frac{d}{dt}\left(\frac{\partial v^2}{\partial \dot{q}_i}\right)-\frac{\partial v^2}{\partial q_i}\right]=F_{q_i}$$

$$i = 1, 2, 3, \tag{8.23}$$

where

$$F_{q_i} = pr_{\vec{e}_i}\vec{F} = \vec{F}\cdot\vec{e}_i$$

$$\vec{H}_i = \left(\left(\frac{\partial x_1}{\partial q_i}\right)^2+\left(\frac{\partial x_2}{\partial q_i}\right)^2+\left(\frac{\partial x_3}{\partial q_i}\right)^2\right)^{1/2} = \left|\frac{\partial \vec{r}}{\partial q_i}\right|$$

The initial conditions are:

$$q_i(t_0) = q_i^0, \quad \dot{q}_i(t_0) = \dot{q}_i^0, \quad i = 1, 2, 3 \qquad (8.24)$$

where $q_i^0, q_j^0 \in \mathbb{R}$ are given constants.

Curvilinear coordinates(q_1, q_2, q_3)

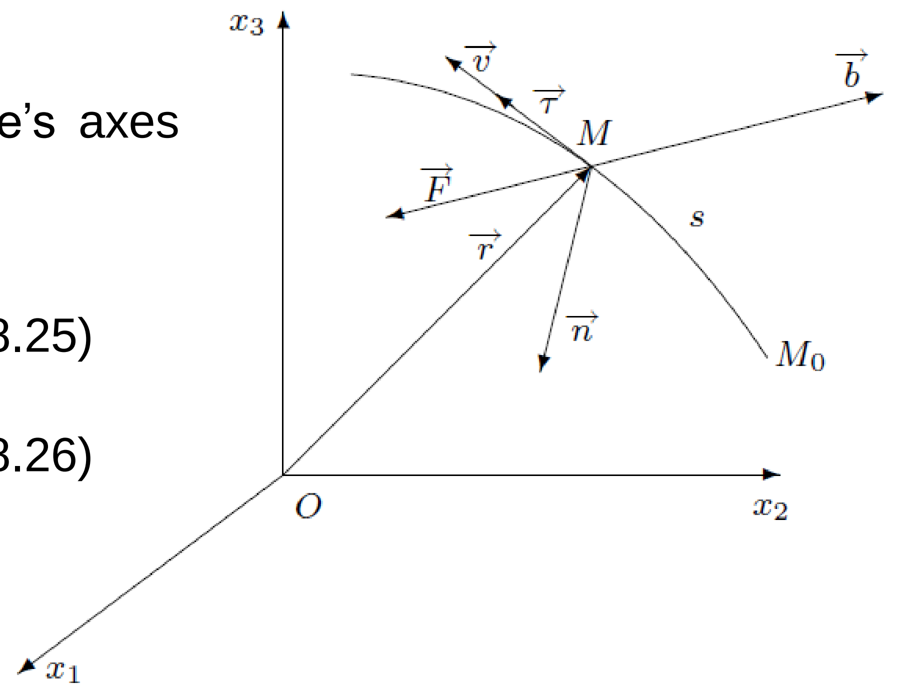
Eq. (8.6) is projected on the Frenet frame's axes $(\vec{\tau}, \vec{n}, \vec{b})$

$$m\frac{dv}{dt} = F_\tau, \quad m\frac{v^2}{R} = F_n, \quad 0 = F_b \qquad (8.25)$$

$$s(t_0) = s_0, \quad \dot{s}(t_0) = v_0 \qquad (8.26)$$

where $v = \dfrac{ds}{dt}$ $(v(t_0) = v_0)$; $F_b = 0 \Rightarrow$

$$\vec{a} \in (\vec{\tau}, \vec{n})$$



Example

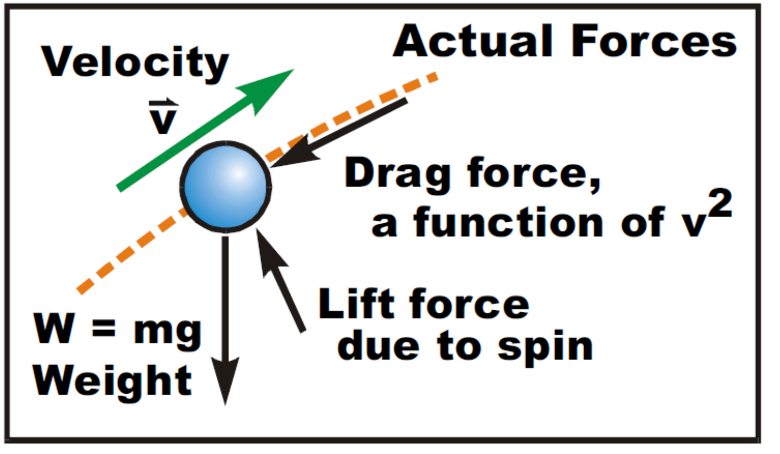
Projectile Motion

1. Definition of a Projectile: An object that is "projected" or thrown, which has no capacity for self-propulsion.

2. Actual forces on a Projectile: Drag, lift due to spin, weight, wind.

3. Are the forces on a projectile (other than weight) significant?

In other words, does the ideal projectile model "fit" or not?



Motion in a resisting medium

In studying the motion of a body in a resisting medium, we assume that the resistive (drag) force on a body, and hence its deceleration, is some function of its speed.

For slow, laminar, nonturbulent motion through a viscous fluid, the resistance is indeed simply proportional to the speed, as can be shown at least by dimensional arguments. One thinks, for example, of Stokes's Law for the motion of a sphere through a viscous fluid.

For faster motion, when laminar flow breaks up and the flow becomes turbulent, a resistive force that is proportional to the square of the speed may represent the actual physical situation better.

$$\vec{R} = -R(v) \cdot \frac{\vec{v}}{v} \text{ sau } \vec{R} = -m \cdot g \cdot \varphi(v) \cdot \frac{\vec{v}}{v}$$

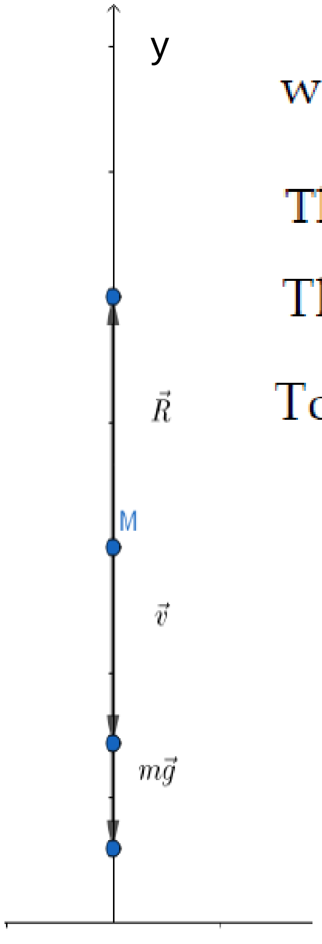
where $\phi(v)$ is a positive increasing function depending on v.

Usually,

$$R = \frac{1}{2} \cdot \rho \cdot A \cdot C_D \cdot v^2$$

where, $A = \pi r^2$, the cross sectional area of the projectile exposed to the air
(ping pong ball diameter is 3.8 cm = 0.038 m)
ρ = density of the air (about 1.21 kg/m^3 at 20° C)
C_d = drag coefficient (typically taken to be 1/2 at speeds your projectiles will go)
v = speed of fluid (in this case, air) relative to the sphere

Body falling under gravity in a resisting medium, resistive force proportional to the speed (1D)



$$\ddot{y} = g - \gamma v,$$

where g is the gravitational acceleration.

The body reaches a constant speed when $\ddot{y}$ becomes zero.

This occurs at a speed $\hat{v} = \frac{g}{\gamma}$, which is called the *terminal speed.*

To obtain the *first time integral,* we write the equation of motion as

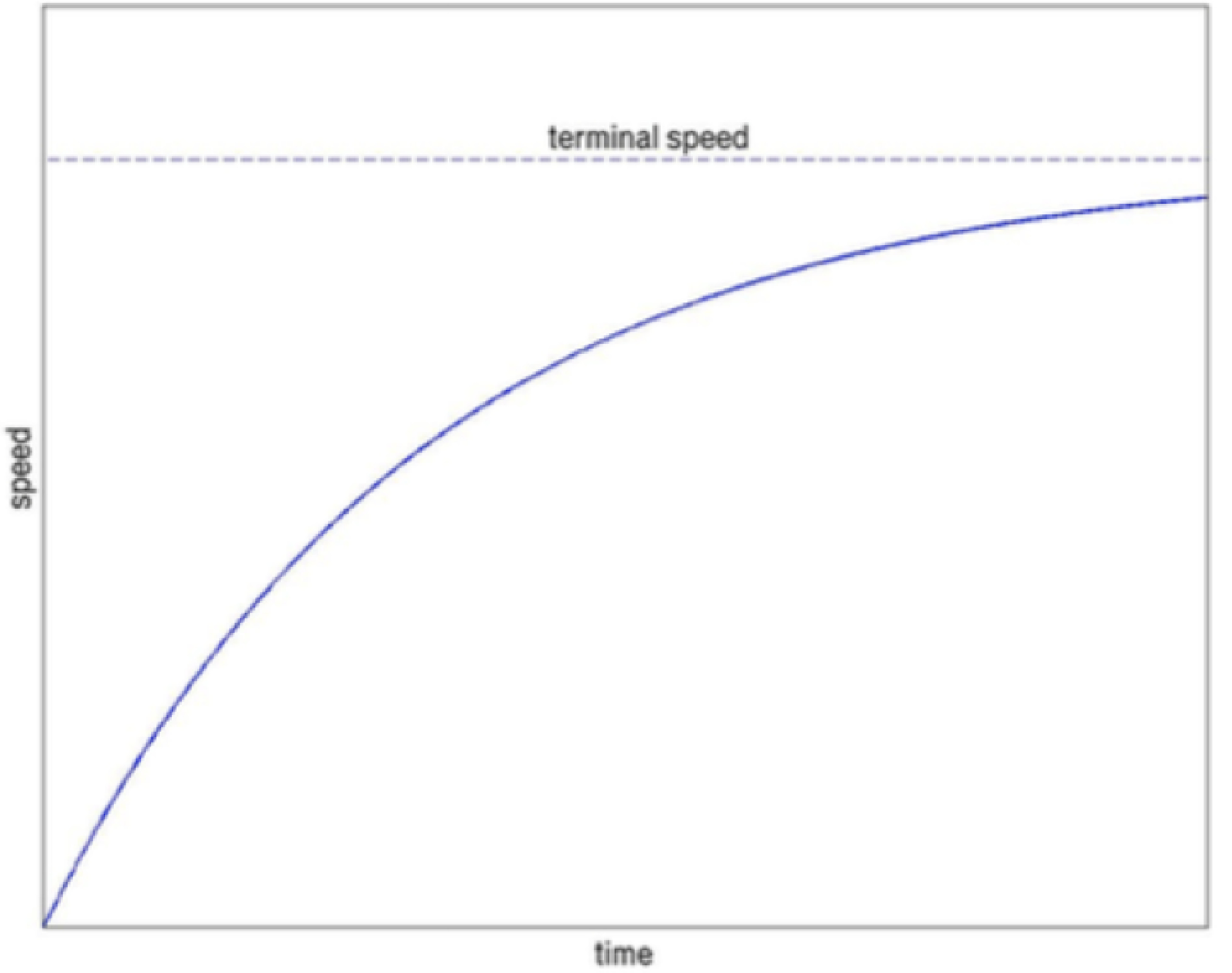
$$\frac{dv}{dt} = \gamma(\hat{v} - v)$$

$$\frac{dv}{\hat{v} - v} = \gamma dt.$$

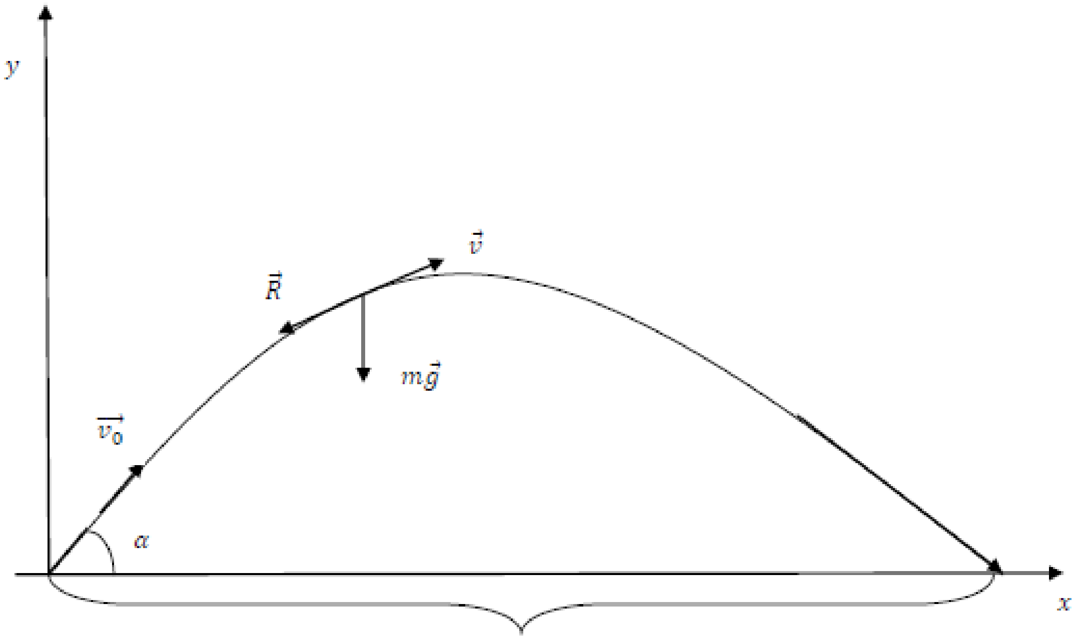
$$\frac{dv}{v - \hat{v}} = -\gamma dt.$$

With initial condition $v = 0$ when $t = 0$, the *first time integral* becomes

$$v = \hat{v}(1 - e^{\gamma t}).$$



2D motion. Effect of the resisting medium (Popescu Claudia).



$$m\vec{a} = m\vec{g} + \vec{R}$$

$$\vec{R} = -mg\lambda v^2 \left|\frac{\vec{v}}{v}\right|$$

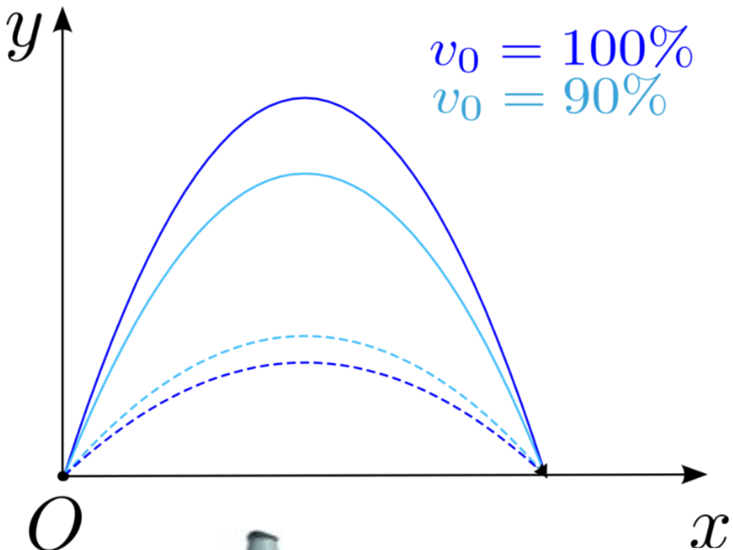
$$\begin{cases} x_0 = 0, v_{ox} = v_0 cos\alpha \\ y_0 = 0, v_{oy} = v_0 sin\alpha \end{cases}$$

$$\begin{cases} m\ddot{x} = -mg\lambda(\dot{x}^2 + \dot{y}^2)\dfrac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} \\ m\ddot{y} = -mg - mg\lambda(\dot{x}^2 + \dot{y}^2)\dfrac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} \end{cases}$$

$$\begin{cases} \ddot{x} = -g\lambda(\dot{x}^2 + \dot{y}^2)\dfrac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} \\ \ddot{y} = -g - g\lambda(\dot{x}^2 + \dot{y}^2)\dfrac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} \end{cases}$$

MRSI (Multiple Rounds Simultaneous Impact)

It is to fire several shells per gun at a target and have all of them arrive simultaneously, which is called MRSI (Multiple Rounds Simultaneous Impact).

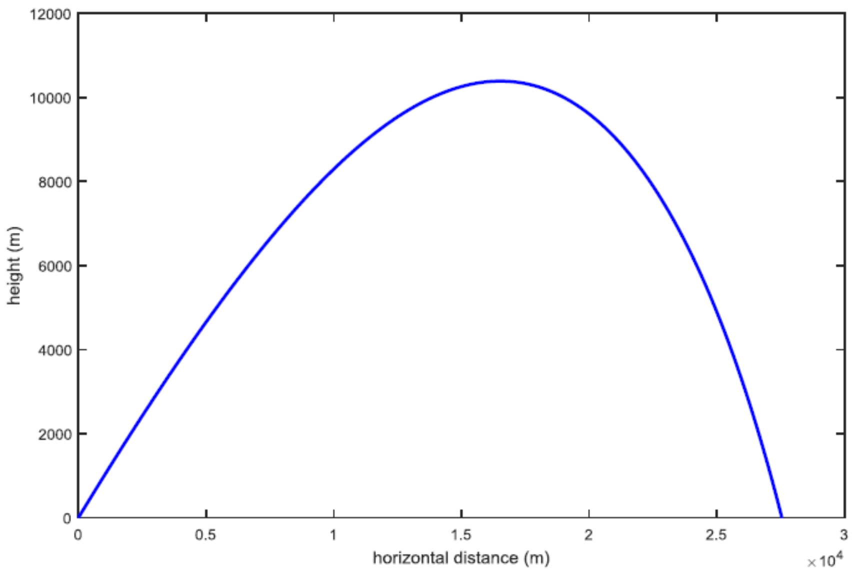


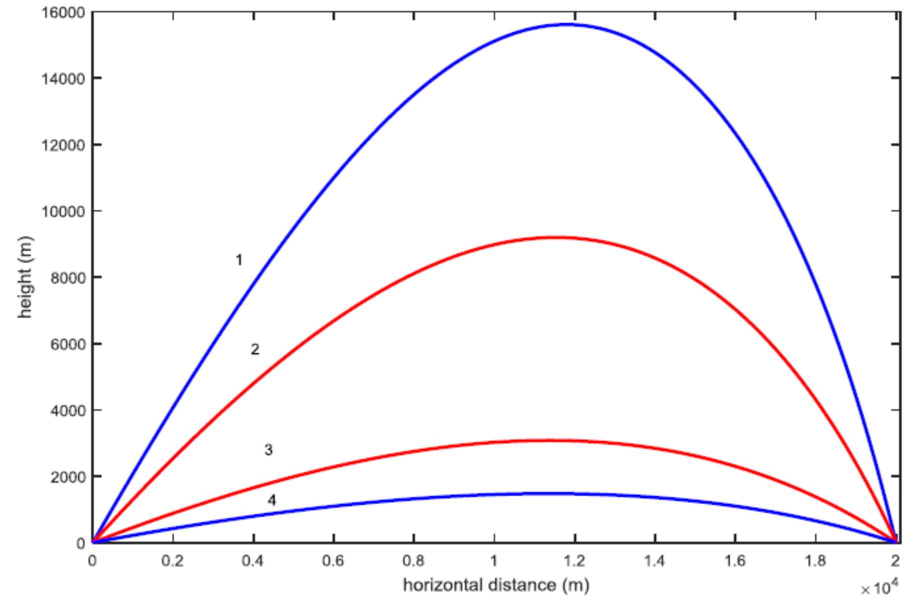
A412 Model was manufactured by Arsenal Resita under the designation 1982 between 1982 and 1989. A maximum range of 33 kilometers was reached Using NORINCO's Base Bleed ammunition. The A412 cannon can fire a 7-8 rounds per minute.

Type : High Explosive, Fragmentation
Code : OF 18 & OF 482

Total weight : 59,0 kg Complete round
Diam : 130,0 mm
Muzzle velocity : 930 m/s – Full charge
705 m/s reduced charge
Max range : 27980 m – Full charge
19130 m – Reduced charge

MRSI (Multiple Rounds Simultaneous Impact)






```
Flight time: 91.026230 s
Final velocity: 387.867286 m/s
Range: 27547.298431 m
```

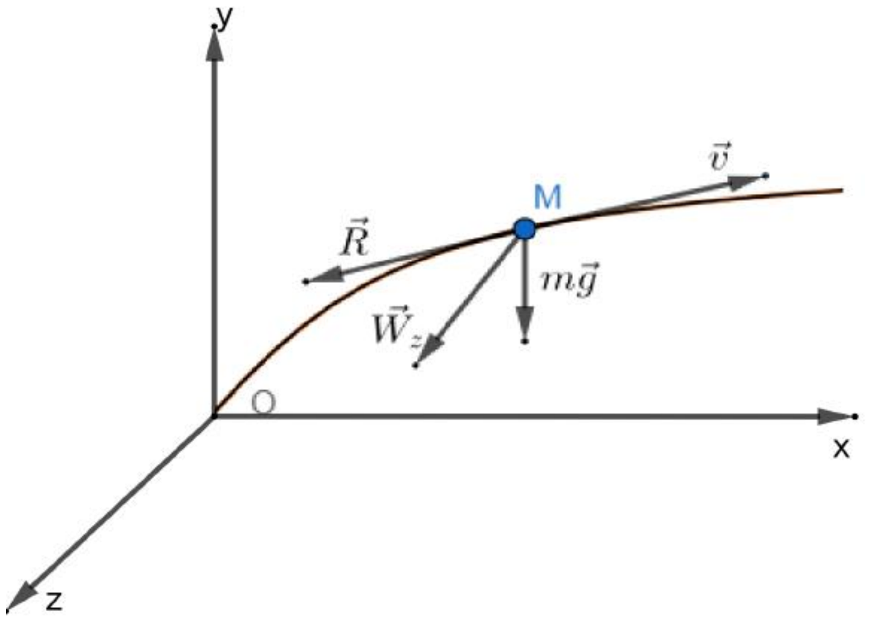
LOVITURA	α_1	α_2	v_0	Unghi (grade)	Timp zbor (secunde)	Timp lansare (secunde)
1	30	80	930	64.62	112.65	0
2	30	80	705	53.05	86.05	26.6
3	5	40	705	25.03	49.70	62.95
4	5	40	930	12.69	34.39	78.26

Tabel.III.3. Caracteristicile tragerilor de tip MRSI la o distanţă de 20 km

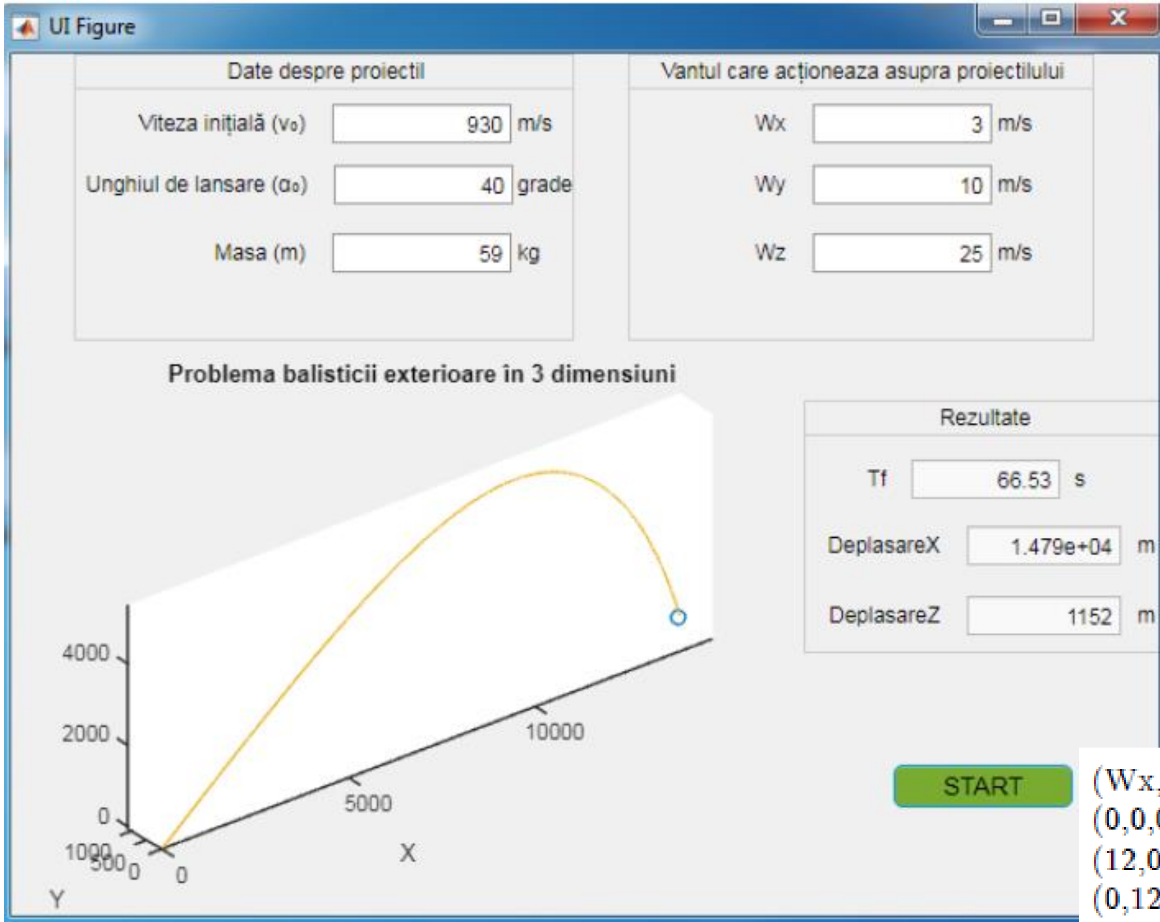
Wind effect on the projectile trajectory

Mathematical model
Laszlo Rebeka

$$\frac{dv_x}{dt} = -\frac{\rho \cdot A \cdot C_D}{2 \cdot m} \cdot \sqrt{v_x^2 + v_y^2 + (v_z - W_z)^2} \cdot v_x$$
$$\frac{dv_y}{dt} = -g - \frac{\rho \cdot A \cdot C_D}{2 \cdot m} \cdot \sqrt{v_x^2 + v_y^2 + (v_z - W_z)^2} \cdot v_y$$
$$\frac{dv_z}{dt} = -\frac{\rho \cdot A \cdot C_D}{2 \cdot m} \cdot \sqrt{v_x^2 + v_y^2 + (v_z - W_z)^2} \cdot (v_z - W z).$$



W_z is the lateral wind force



(Wx,Wy,Wz)	Deplasarea pe Ox	Deplasarea pe Oz
(0,0,0)	15,43 km	0
(12,0,0)	15,97 km	0
(0,12,0)	15,69 km	0
(0,0,12)	15,43 km	432,2 m
(5,5,5)	15,76 km	184,6 m
(12,12,12)	16,25 km	458,8 m
(16,16,16)	16,54 km	624,4 m