

Series of real numbers - 2nd part

Exercise 1: Determine the nature (convergence or divergence) of the following series of real numbers:

$$a) \sum_{n \geq 1} \frac{3n^4 + 7}{\sqrt{n^8 + 7n^3}}, \quad b) \sum_{n \geq 1} \frac{1}{\sqrt[n]{n}}, \quad c) \sum_{n \geq 1} \frac{1}{\sqrt[n]{n!}}, \quad d) \sum_{n \geq 1} \left(1 + \frac{1}{3n}\right)^{-n}.$$

Exercise 2: Determine the nature (convergence or divergence) of the following series of real numbers:

$$a) \sum_{n \geq 1} \frac{2^n + 3^n}{5^n}, \quad b) \sum_{n \geq 1} \frac{2^n}{3^n + 5^n}.$$

Exercise 3: Determine the nature (convergence or divergence) of the following series of real numbers:

$$a) \sum_{n \geq 2} \frac{1}{3n - 4}, \quad b) \sum_{n \geq 1} \frac{1}{(4n - 1)^3}, \quad c) \sum_{n \geq 1} \frac{1}{\sqrt{4n^2 - 1}}, \quad d) \sum_{n \geq 1} \frac{\sqrt{n^2 + n}}{\sqrt[3]{n^5 - n}}.$$

Exercise 4: Determine the nature (convergence or divergence) of the following series of real numbers:

$$a) \sum_{n \geq 1} \frac{100^n}{n!}, \quad b) \sum_{n \geq 1} \frac{2^n n!}{n^n}, \quad c) \sum_{n \geq 1} \frac{3^n n!}{n^n}, \quad d) \sum_{n \geq 1} \frac{(n!)^2}{2^{n^2}}, \quad e) \sum_{n \geq 1} \frac{n^2}{\left(2 + \frac{1}{n}\right)^n}.$$

Exercise 5: Determine the nature (convergence or divergence), by discussing the value of the parameter $a > 0$, of the following series of real numbers:

$$a) \sum_{n \geq 1} \frac{a^n}{n^n}, \quad b) \sum_{n \geq 1} \left(\frac{n^2 + n + 1}{n^2} a \right)^n, \quad c) \sum_{n \geq 1} \frac{3^n}{2^n + a^n}.$$