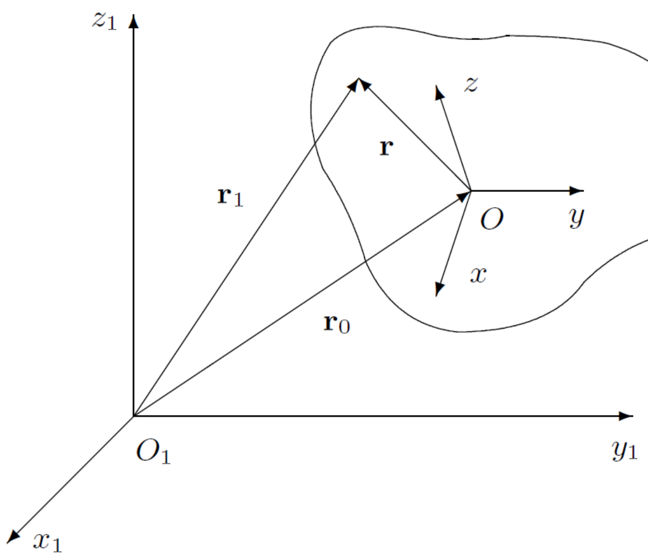
Theoretical Mechanics

4. Particular motions (continued)

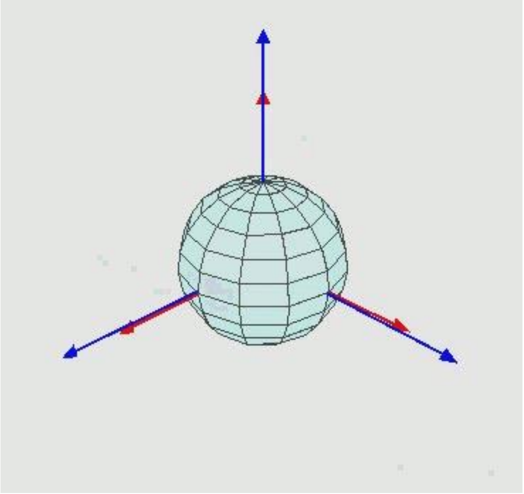
In order to study the motion of a rigid body two frame of references are required:

- $O_1x_1y_1z_1$ – fixed in space
- $O\,x\,y\,z$ – fixed on the moving rigid body



The position and motion the mobile frame of reference (i.e. the rigid body) is determined by 6 parameters (coordinates).

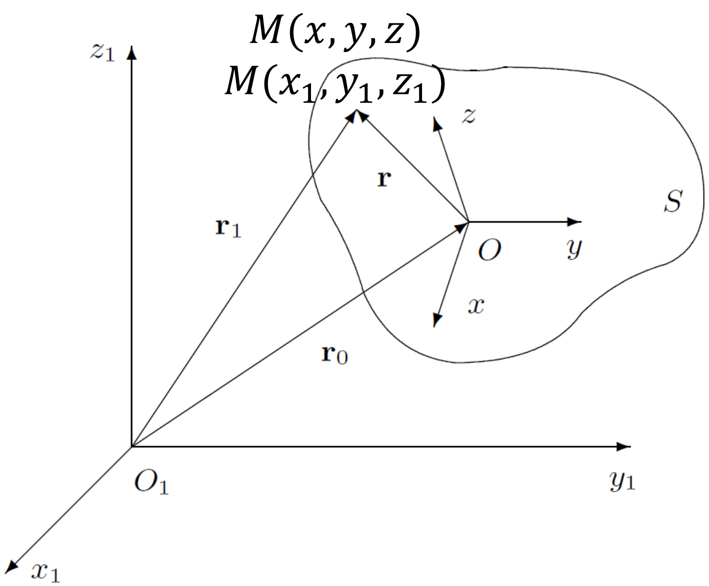
- the Euler angles $(\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\psi})$
- the coordinates of $O(\boldsymbol{x_0}, \boldsymbol{y_0}, \boldsymbol{z_0})$



The equations of motion for the rigid body are:

$$\begin{cases} x_0 = x_0(t), \quad y_0 = y_0(t), \quad z_0 = z_0(t) \\ \varphi = \varphi(t), \quad \theta = \theta(t), \quad \psi = \psi(t), \end{cases} \quad t \in [t_0, T]. \tag{3.2}$$

Poisson formulas $$\frac{d\vec{i}}{dt} = \vec{\omega} \times \vec{i}, \quad \frac{d\vec{j}}{dt} = \vec{\omega} \times \vec{j}, \quad \frac{d\vec{k}}{dt} = \vec{\omega} \times \vec{k} \tag{3.5}$$



$$\vec{r}_1 = \vec{r}_0 + \vec{r} = \vec{r}_0 + x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{v}_M = \vec{v}_0 + \vec{\omega} \times \vec{r} \tag{4.2}$$

$$\begin{aligned} \vec{a}_M &= \vec{a}_0 + \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}), \\ &= \vec{a}_0 + \dot{\vec{\omega}} \times \vec{r} - \omega^2 \vec{d}. \end{aligned} \tag{4.6}$$

Rotation about a Fixed Point

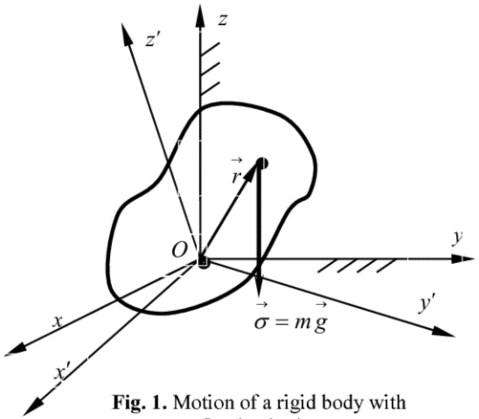
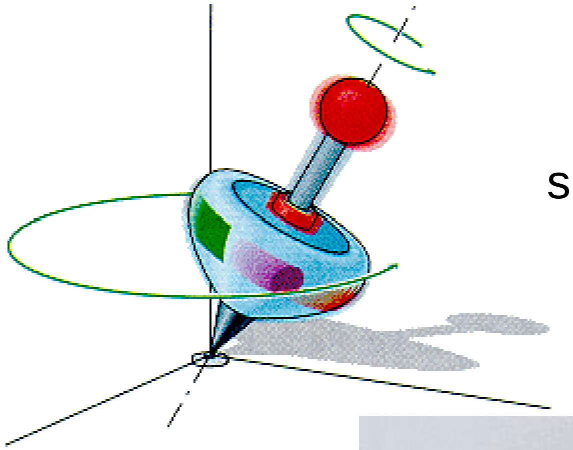


Fig. 1. Motion of a rigid body with one fixed point in a



spinning top

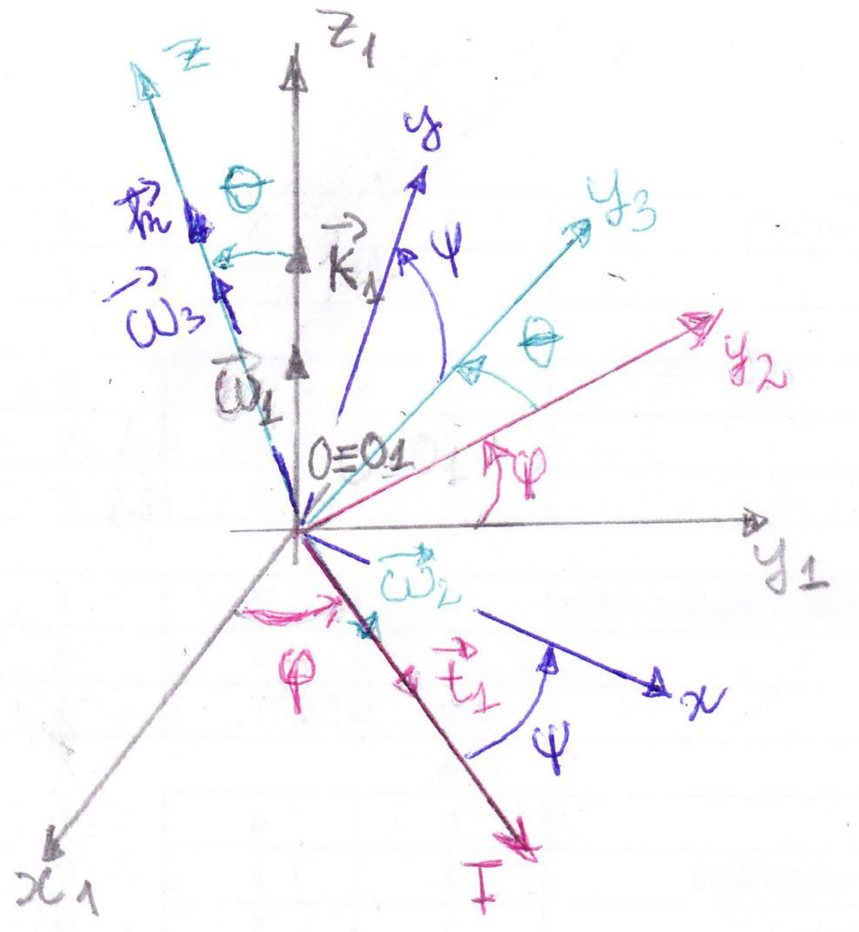
In every moment of the motion we have

$$\vec{v}_0(t) = 0, \quad \vec{\omega}(t) \neq 0 \qquad t \in [t_0, T]$$

$$(\vec{v}_0 = 0 \;\Rightarrow\; \vec{r}_0 = \text{const.} \equiv 0) \qquad (4.15)$$

The rigid body has a fixed point *O* and around this point executes rotation motions.

Consider an axis $\Delta(O;\vec{u})$ called ***rotation axis***. All the points of the rigid body describe arcs of circle in planes perpendicular on Δ.



We suppose $O_1 \equiv O$ and that at the initial moment t_0 the frames of reference $O_1x_1y_1z_1$ and $Oxyz$ coincide.

The position of the rigid body relative to the point O at a moment of time t can be obtained, in the most general case, using three successive rotations:

- precession (ϕ)
- nutation (θ)
- spin (ψ)

about the axes $O_1z_1 = Oz_1$, OI (nodal axis) and Oz with the angular velocities:

$$\vec{\omega}_1 = \dot{\varphi}\vec{k}_1, \quad \vec{\omega}_2 = \dot{\theta}\vec{t}_1, \quad \vec{\omega}_3 = \dot{\psi}\vec{k} \qquad (4.16)$$

The equations of motion are:

$$\varphi = \varphi(t), \quad \theta = \theta(t), \quad \psi = \psi(t), \quad t \in [t_0, T] \tag{4.17}$$

Velocity and acceleration:

$$\vec{v}_M = \vec{\omega} \times \vec{r}, \quad \vec{a}_M = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}). \tag{4.18}$$

Taking into account that

$$\vec{v}_M = \vec{\omega} \times \vec{r},$$

it results that at the moment t the motion of the rigid body can be considered a rotation motion around the axis $\Delta\big(O;\vec{\omega}(t)\big)$ with the angular velocity $\vec{\omega}(t)$. This is an instantaneous rotation about the axis $\Delta\big(O;\vec{\omega}(t)\big)$ which is called ***instantaneous axis of rotation***).

An arbitrary displacement of a rigid body with a fixed point O is equivalent to a rotation about a line through O.

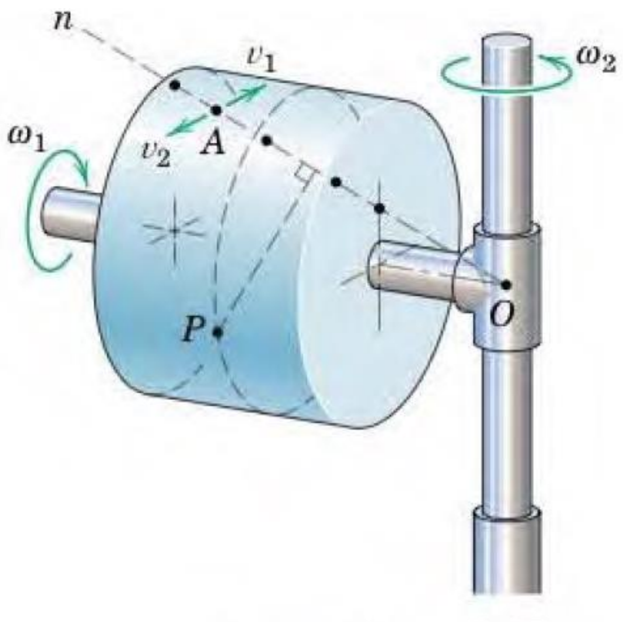


Figure 7/6

Instantaneous Axis of Rotation

To aid in visualizing the concept of the instantaneous axis of rotation, we will cite a specific example. Figure 7/6 represents a solid cylindrical rotor made of clear plastic containing many black particles embedded in the plastic. The rotor is spinning about its shaft axis at the steady rate ω_1, and its shaft, in turn, is rotating about the fixed vertical axis at the steady rate ω_2, with rotations in the directions indicated. If the rotor is photographed at a certain instant during its motion, the resulting picture would show one line of black dots sharply defined, indicating that, momentarily, their velocity was zero. This line of points with no velocity establishes the instantaneous position of the axis of rotation O-n. Any dot on this line, such as A, would have equal and opposite velocity components, v_1 due to ω_1 and v_2 due to ω_2. All other dots, such as the one at P, would appear blurred, and their movements would show as short streaks in the form of small circular arcs in planes normal to the axis O-n. Thus, all particles of the body, except those on line O-n, are momentarily rotating in circular arcs about the instantaneous axis of rotation.

If a succession of photographs were taken, we would observe in each photograph that the rotation axis would be defined by a new series of sharply-defined dots and that the axis would change position both in space and relative to the body. For rotation of a rigid body about a fixed point, then, it is seen that the rotation axis is, in general, not a line fixed in the body.

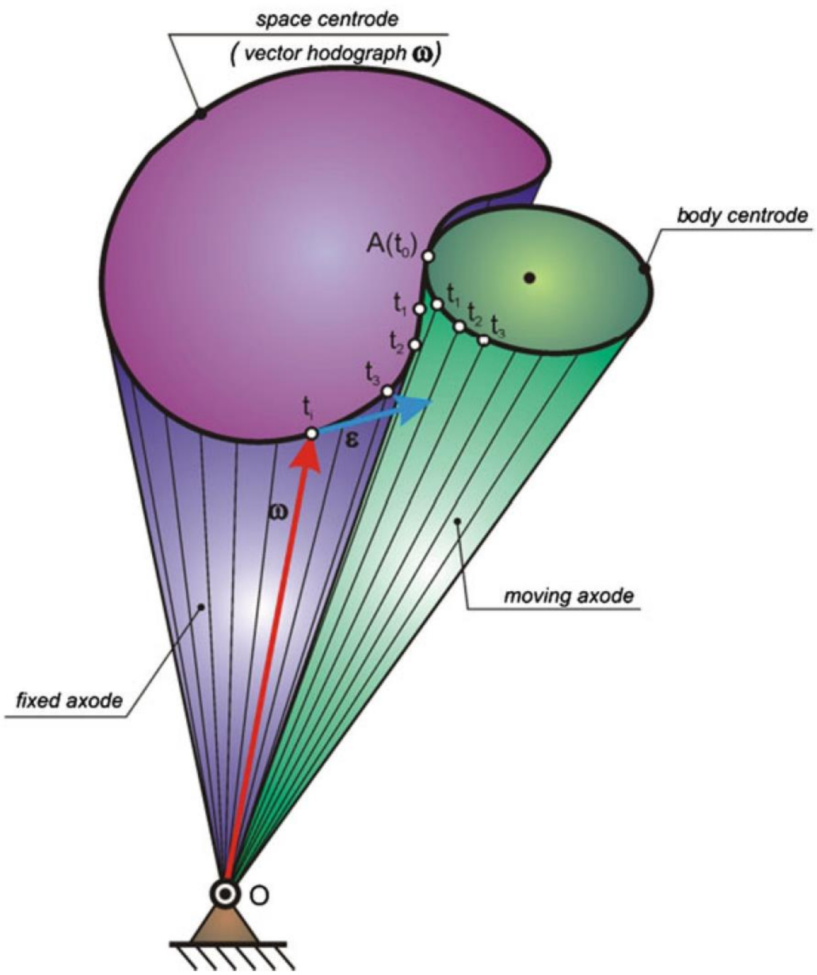
Thus, the finite motion of the rigid around the point O, from the position (A) at the time t_A in the position (B) at the time t_B is a succession of instantaneous rotations about the instantaneous axis of rotation.

The equations of the instantaneous axis of rotation are:

$$A \in \Delta \;\Leftrightarrow\; \overrightarrow{\omega} \times \overrightarrow{r}_A = 0 \;\Leftrightarrow$$

$$\frac{x}{p(t)} = \frac{y}{q(t)} = \frac{z}{r(t)} \qquad \text{when } t \text{ is fixed} \tag{4.19}$$

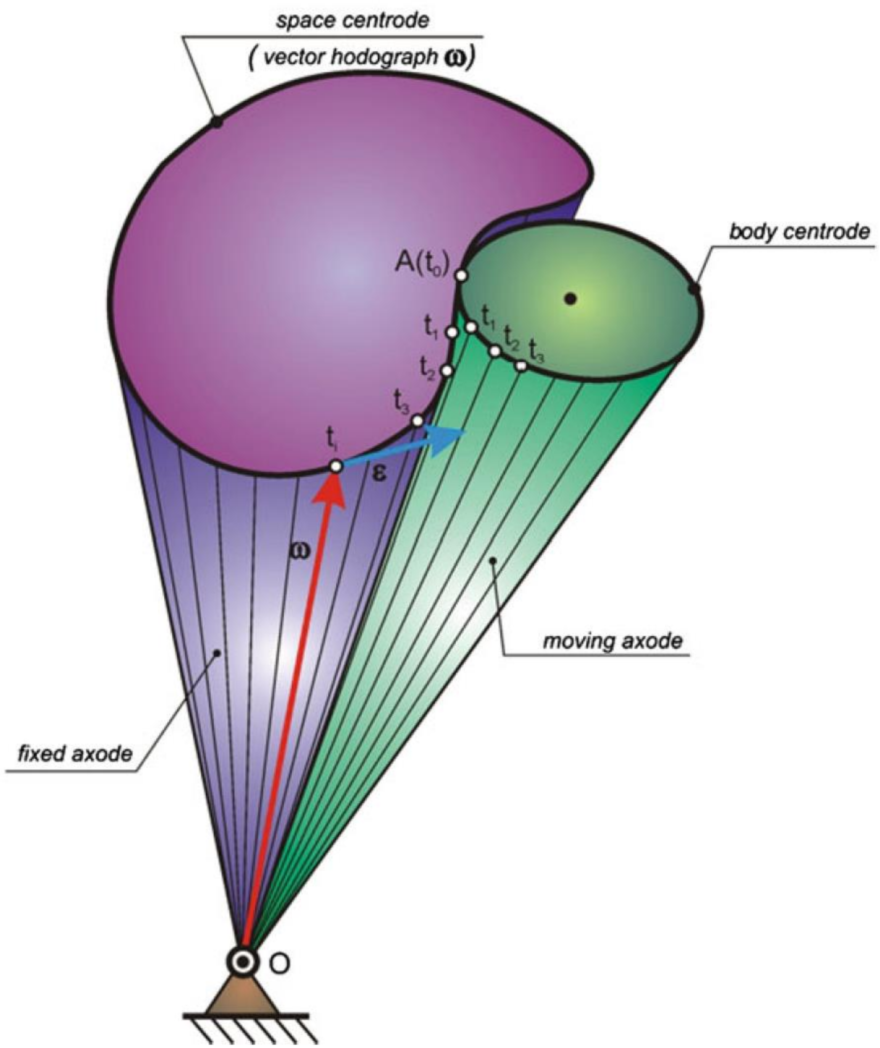
$$\overrightarrow{\omega} = \dot{\varphi}\,\overrightarrow{k}_1 + \dot{\theta}\,\overrightarrow{t}_1 + \dot{\psi}\,\overrightarrow{k}. \tag{4.20}$$



The position of the instantaneous axes of rotation in the body and space coordinate systems varies in time, but all the axes must always pass through the centre of motion about a point. Instantaneous axes of rotation intersect a sphere of radius **r** at certain points. Sets of these points in the body and space coordinate systems constitute the trajectories of motion of point A respectively in the body and space coordinate systems.

Point A belongs simultaneously to both trajectories at the given time instant. The lines passing through points A and O at the time instants t_0; t_1; t_2; … form the surface called the *stationary cone of instantaneous axes (fixed axode)* in the space coordinate system and the surface called the *moving cone of instantaneous axes (moving axode)* in the body coordinate system.

Jan Awrejcewicz, Classical Mechanics. Kinematics and Statics, Springer, 2012.



The path of point A lies on a sphere and is described by the curve called the *body centrode (non-stationary)* in the body coordinate system (non-stationary). These curves are in contact at point A since it belongs simultaneously to both of them.

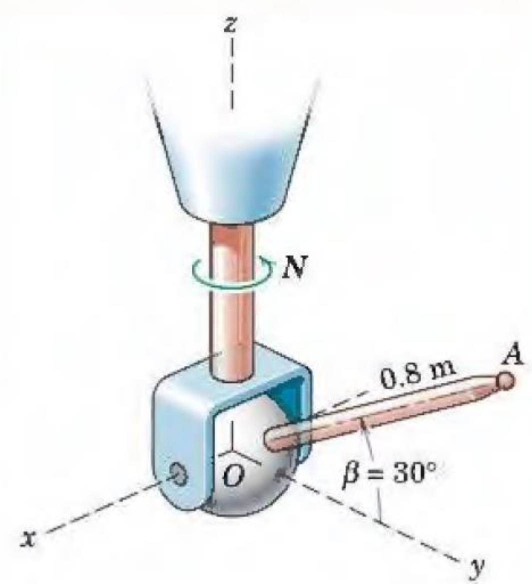
The motion about a point can be illustratively represented as the rolling of a moving axode on a fixed axode. Both axodes have contact along the generating line, which is the instantaneous axis of rotation, and do not slide with respect to one another.

The hodograph of vector $\vec{\omega}$ (the locus of the vertex of $\vec{\omega}$) lies on the fixed axode. Because $\epsilon = \dot{\vec{\omega}}$, the angular acceleration is tangent to the space centrode, and it does not necessarily have to lie on the axis of vector $\vec{\omega}$. Moving and fixed axodes can be non-closed surfaces

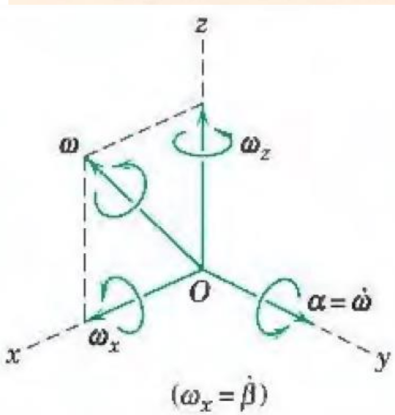
Sample Problem 7/1

(Meriam & Kraige, Engineering Mechanics, Dynamics, 2012)

The 0.8-m arm OA for a remote-control mechanism is pivoted about the horizontal x-axis of the clevis, and the entire assembly rotates about the z-axis with a constant speed $N = 60$ rev/min. Simultaneously, the arm is being raised at the constant rate $\dot{\beta} = 4$ rad/s. For the position where $\beta = 30°$, determine (*a*) the angular velocity of OA, (*b*) the angular acceleration of OA, (*c*) the velocity of point A, and (*d*) the acceleration of point A. If, in addition to the motion described, the vertical shaft and point O had a linear motion, say, in the z-direction, would that motion change the angular velocity or angular acceleration of OA?



Solution. ***(a)*** Since the arm OA is rotating about both the x- and the z-axes, it has the components $\omega_x = \dot{\beta} = 4$ rad/s and $\omega_z = 2\pi N/60 = 2\pi(60)/60 = 6.28$ rad/s. The angular velocity is



$$\boldsymbol{\omega} = \boldsymbol{\omega}_x + \boldsymbol{\omega}_z = 4\mathbf{i} + 6.28\mathbf{k} \text{ rad/s} \qquad \textit{Ans.}$$

(b) The angular acceleration of OA is

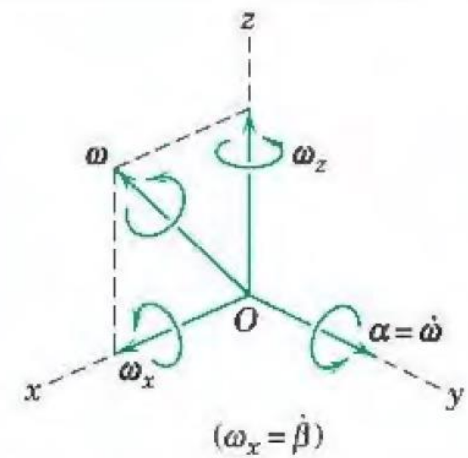
$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}_x + \dot{\boldsymbol{\omega}}_z$$

Since $\boldsymbol{\omega}_z$ is not changing in magnitude or direction, $\dot{\boldsymbol{\omega}}_z = \mathbf{0}$. But $\boldsymbol{\omega}_x$ is changing direction and thus has a derivative which, from Eq. 7/3, is

$$\dot{\boldsymbol{\omega}}_x = \boldsymbol{\omega}_z \times \boldsymbol{\omega}_x = 6.28\mathbf{k} \times 4\mathbf{i} = 25.1\mathbf{j}\ \text{rad/s}^2$$

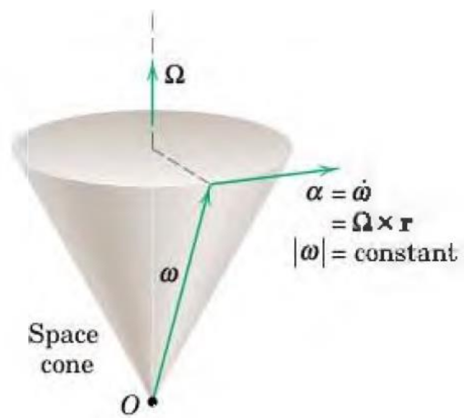
Therefore,

$$\boldsymbol{\alpha} = 25.1\mathbf{j} + \mathbf{0} = 25.1\mathbf{j}\ \text{rad/s}^2 \qquad \textit{Ans.}$$



When the magnitude of $\boldsymbol{\omega}$ remains constant, the angular acceleration $\boldsymbol{\alpha}$ is normal to $\boldsymbol{\omega}$. For this case, if we let $\boldsymbol{\Omega}$ stand for the angular velocity with which the vector $\boldsymbol{\omega}$ itself rotates (*precesses*) as it forms the space cone, the angular acceleration may be written

$$\boxed{\boldsymbol{\alpha} = \boldsymbol{\Omega} \times \boldsymbol{\omega}} \qquad \textbf{(7/3}$$



$$\vec{\boldsymbol{\omega}}_x = \omega_x \vec{\imath} \Rightarrow \dot{\vec{\boldsymbol{\omega}}}_x = \omega_x \frac{d\vec{\imath}}{dt} = \omega_x (\vec{\boldsymbol{\omega}}_z \times \vec{\imath}) = \vec{\boldsymbol{\omega}}_z \times \omega_x \vec{\imath}$$
$$= \vec{\boldsymbol{\omega}}_z \times \vec{\boldsymbol{\omega}}_x = \omega_z \vec{k} \times \omega_x \vec{\imath}$$

Remark: The unit vector $\vec{\imath}$ rotates about Oz axis with the angular velocity $\vec{\boldsymbol{\omega}}_z$.

(c) With the position vector of A given by $\mathbf{r} = 0.693\mathbf{j} + 0.4\mathbf{k}$ m, the velocity of A from Eq. 7/1 becomes

$$\boxed{\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}} \qquad (7/1)$$

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 6.28 \\ 0 & 0.693 & 0.4 \end{vmatrix} = -4.35\mathbf{i} - 1.60\mathbf{j} + 2.77\mathbf{k} \text{ m/s} \qquad \textit{Ans.}$$

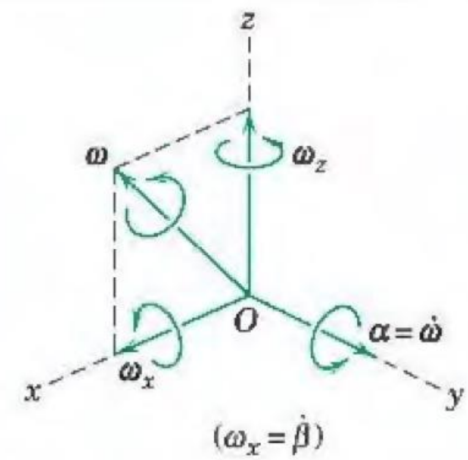
(d) The acceleration of A from Eq. 7/2 is

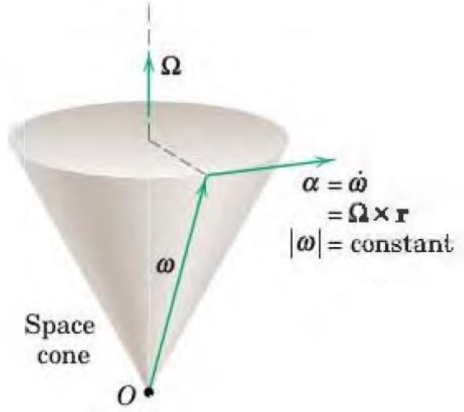
$$\boxed{\mathbf{a} = \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})} \qquad (7/2)$$

$$\begin{aligned} \mathbf{a} &= \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \\ &= \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times \mathbf{v} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 25.1 & 0 \\ 0 & 0.693 & 0.4 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 6.28 \\ -4.35 & -1.60 & 2.77 \end{vmatrix} \\ &= (10.05\mathbf{i}) + (10.05\mathbf{i} - 38.4\mathbf{j} - 6.40\mathbf{k}) \\ &= 20.1\mathbf{i} - 38.4\mathbf{j} - 6.40\mathbf{k} \text{ m/s}^2 \qquad \textit{Ans.} \end{aligned}$$

2

The angular motion of OA depends only on the angular changes N and $\dot{\beta}$, so any linear motion of O does not affect $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$.



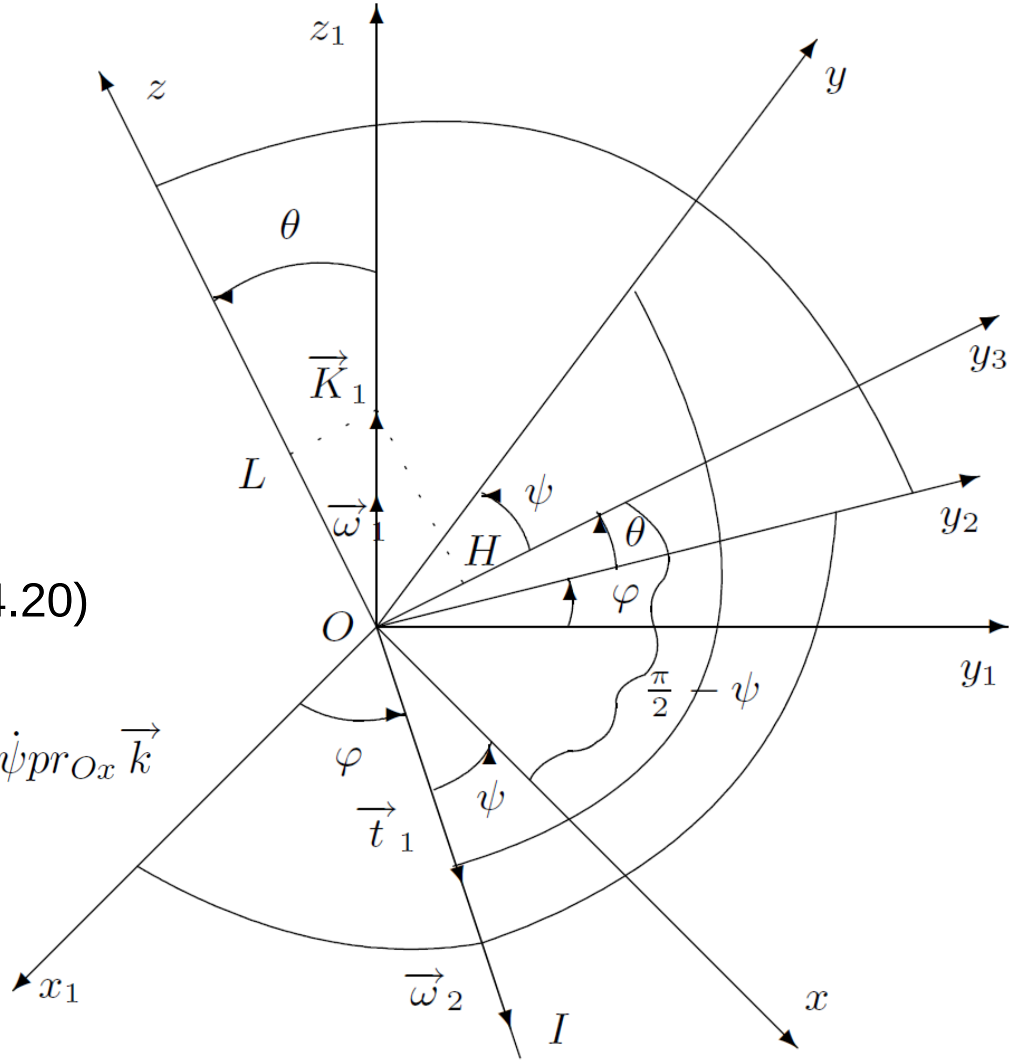


Euler kinematics formulas

In order to find the components of $\vec{\omega}$ we have to obtain the versors $\vec{k}_1$ and $\vec{t}_1$ as function of $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$:

$$\vec{\omega} = \mathrm{p}\,\vec{\mathrm{i}} + \mathrm{q}\,\vec{\mathrm{j}} + \mathrm{r}\,\vec{\mathrm{k}}$$

$$\overrightarrow{\omega} = \dot{\varphi}\,\overrightarrow{k}_1 + \dot{\theta}\,\overrightarrow{t}_1 + \dot{\psi}\,\overrightarrow{k}. \qquad (4.20)$$



$$p = \mathrm{pr}_{Ox}\overrightarrow{\omega} \overset{(4.20)}{=} \dot{\varphi} pr_{Ox}\overrightarrow{k}_1 + \dot{\theta} pr_{Ox}\overrightarrow{t}_1 + \dot{\psi} pr_{Ox}\overrightarrow{k}$$

$$\overrightarrow{k}_1 = \overrightarrow{OH} + \overrightarrow{OL}, \quad \overrightarrow{OH} \parallel Oy_3, \quad \overrightarrow{OL} \parallel Oz$$

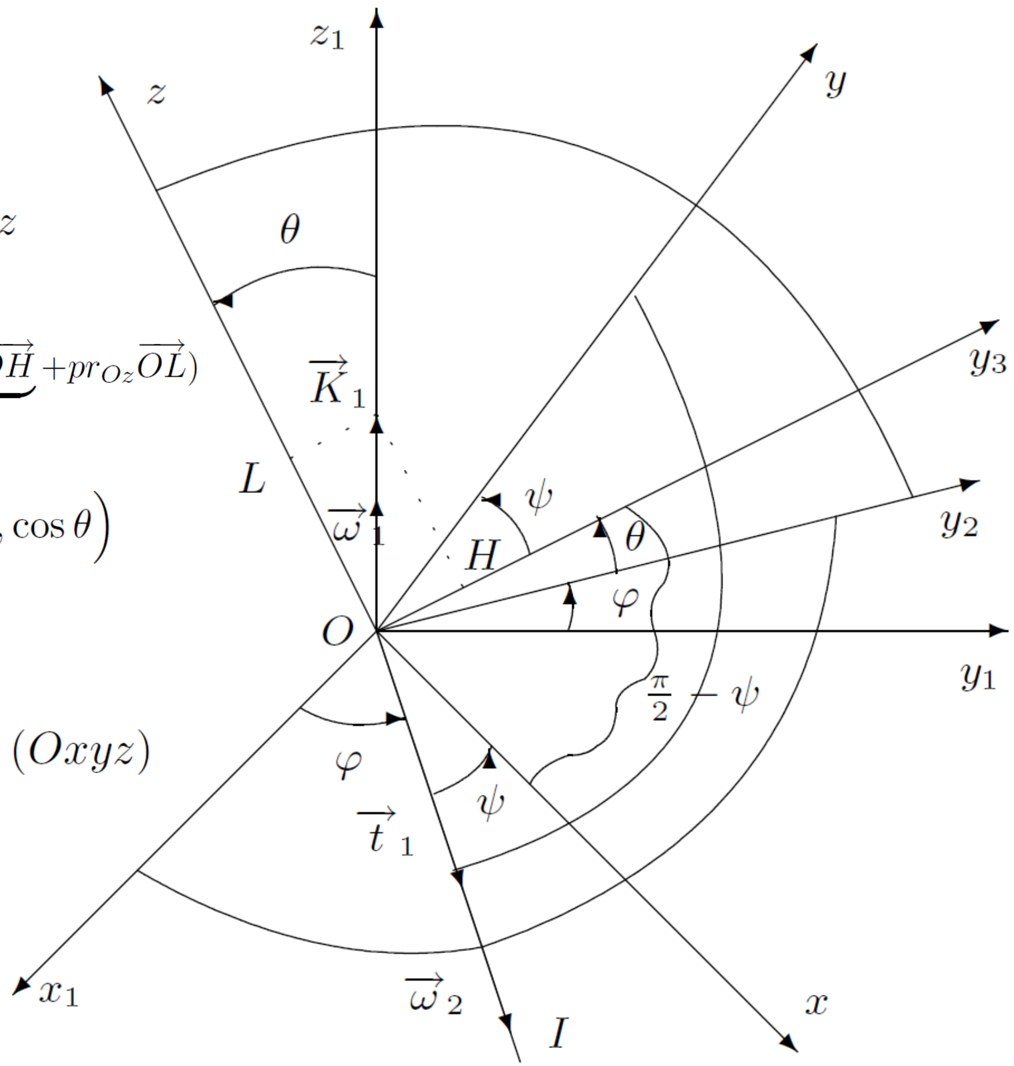
$$\overrightarrow{k}_1 = \overrightarrow{OH} + \overrightarrow{OL}, \quad \overrightarrow{OH} \| \ Oy_3, \quad \overrightarrow{OL} \| \ Oz$$

$$\overrightarrow{k}_1 = (pr_{Ox}\overrightarrow{OH} + \underbrace{pr_{Ox}\overrightarrow{OL}}_{=0}, pr_{Oy}\overrightarrow{OH} + \underbrace{pr_{Oy}\overrightarrow{OL}}_{=0}, \underbrace{pr_{Oz}\overrightarrow{OH}}_{=0} + pr_{Oz}\overrightarrow{OL})$$

$$= \left(\cos\left(\frac{\pi}{2} - \psi\right)\sin\theta, \sin\theta\sin\left(\frac{\pi}{2} - \psi\right), \cos\theta\right)$$

$$= (\sin\theta\sin\psi, \sin\theta\cos\psi, \cos\theta)$$

$$\Rightarrow \ \overrightarrow{k}_1 = (\sin\theta\sin\psi, \sin\theta\cos\psi, \cos\theta) \ (Oxyz)$$



$$\vec{t}_1 = (\cos\psi, -\sin\psi, 0)\ (Oxyz)$$

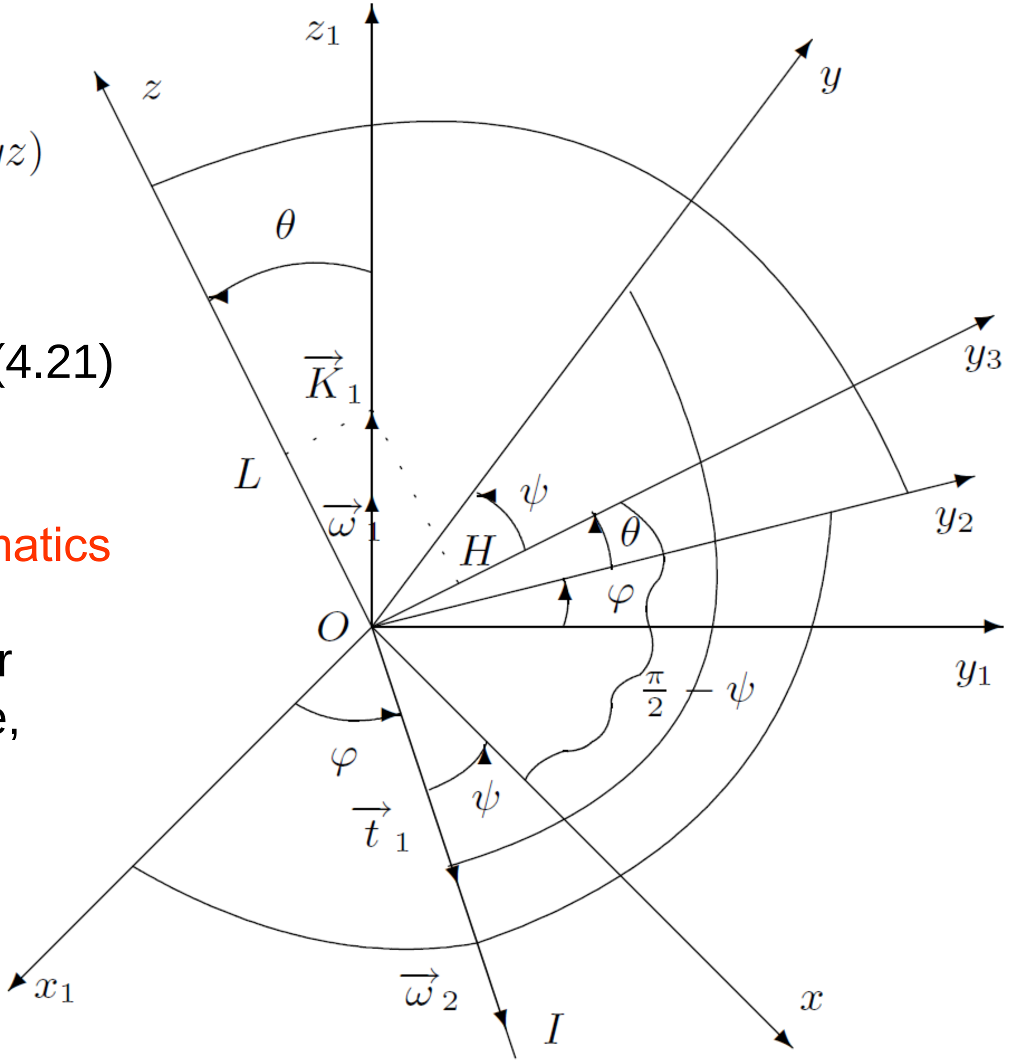
$$\vec{k}_1 = (\sin\theta\sin\psi, \sin\theta\cos\psi, \cos\theta)\ (Oxyz)$$

$$\overset{(4.20)}{\Rightarrow} \begin{cases} p = \dot{\varphi}\sin\theta\sin\psi + \dot{\theta}\cos\psi \\ q = \dot{\varphi}\sin\theta\cos\psi - \dot{\theta}\sin\psi \\ r = \dot{\varphi}\cos\theta + \dot{\psi} \end{cases} \quad (4.21)$$

Equations (4.21) are the Euler's kinematics formulas.

It is also possible to obtain the angular velocity in the fixed frame of reference, $O_1x_1y_1z_1$: $\vec{\omega} = (p_1, q_1, r_1)$

$$\begin{cases} p_1 = \dot{\psi}\sin\theta\sin\varphi + \dot{\theta}\cos\varphi \\ q_1 = -\dot{\psi}\sin\theta\cos\varphi + \dot{\theta}\sin\varphi \\ r_1 = \dot{\psi}\cos\theta + \dot{\varphi}. \end{cases} \quad (4.22)$$



5. The (most) general motion of the rigid body

The rigid body has a (most) general motion if

$$\vec{v}_0(t) \neq 0, \; \vec{\omega}(t) \neq 0, \quad \forall\, t \in [t_0, T] \tag{5.1}$$

(there is no restriction on $\vec{v}_0$ and $\vec{\omega}$).
The (most) general motion is the motion with 6 degree of fredom.

The equations of motion for the rigid body are:

$$\begin{cases} x_0 = x_0(t), \quad y_0 = y_0(t), \quad z_0 = z_0(t) \\ \varphi = \varphi(t), \quad \theta = \theta(t), \quad \psi = \psi(t), \end{cases} \qquad t \in [t_0, T]. \tag{5.2}$$

Velocity and acceleration of a point *M* of the rigid body:

$$\overrightarrow{v}_M = \overrightarrow{v}_0 + \overrightarrow{\omega} \times \overrightarrow{r} \tag{5.3}$$

$$\overrightarrow{a}_M = \overrightarrow{a}_0 + \overrightarrow{\omega} \times \overrightarrow{r} + \overrightarrow{\omega} \times (\overrightarrow{\omega} \times \overrightarrow{r}) \tag{5.4}$$

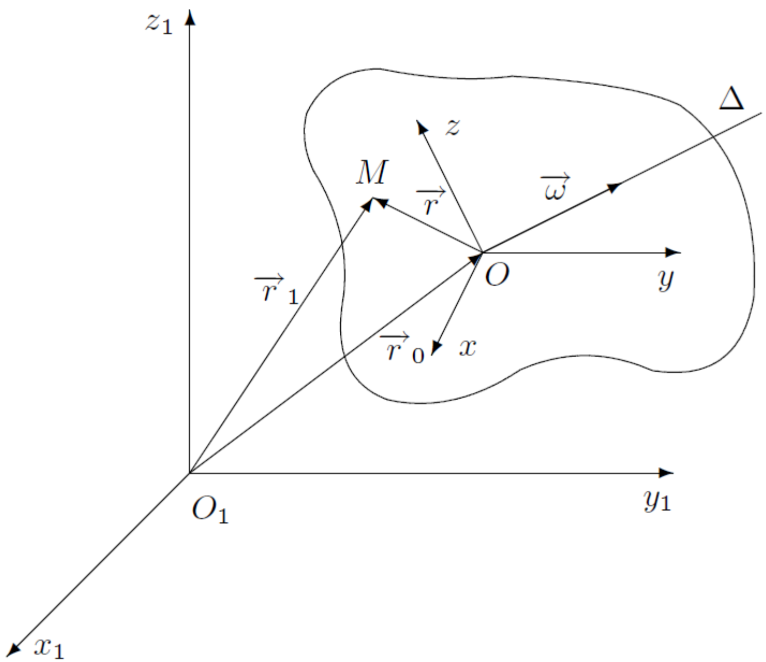
The roto-translation motion of the rigid body

Considering Eqs. (5.3) and (5.4) we deduce that the (most) general motion of the rigid body at the moment t is the composition of two motions:

- An instantaneous translation with the velocity $\vec{v}_O(t)$
- An instantaneous rotation with the velocity $\vec{\omega} \times \vec{r}$ about the axis $\Delta\big(O, \vec{\omega}(t)\big)$, the support of the angular velocity at the moment t.

This motion of the rigid body is called the ***instantaneous roto - translation motion.***

Thus, the finite motion of the rigid between the moments the time t_A iand t_B is a succession of instantaneous roto – translations at the intermediary moments of time around the axis $\Delta(t)$ and along the direction of $\vec{v}_O(t)$.



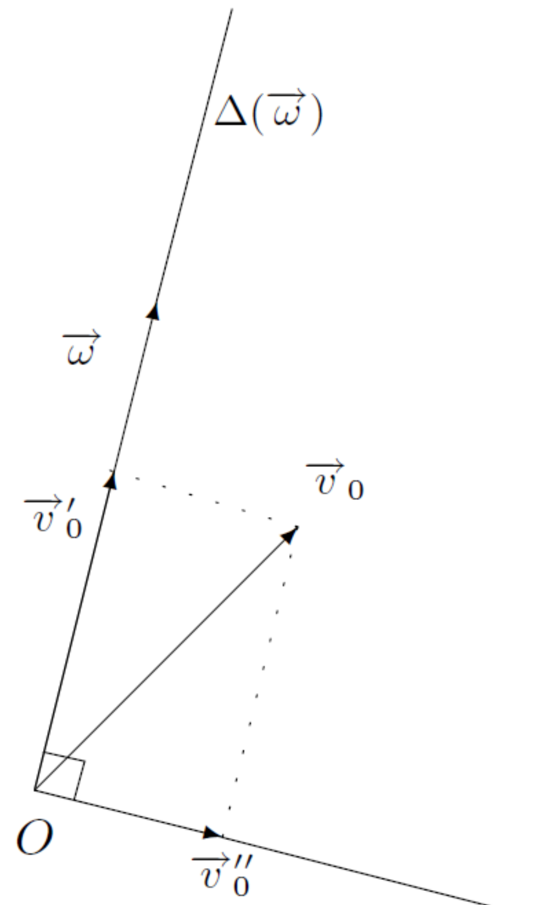
The helical (screw) motion of the rigid body

The motion of a rigid body may be the combination of rotation about an axis (the screw axis) and a translation along that axis. This screw move is characterized by the velocity vector for the translation and the angular velocity vector in the same or opposite direction.

Indeed, consider the decomposition of $\vec{v}_O(t)$

$$\overrightarrow{v}_0 = \overrightarrow{v}'_0 + \overrightarrow{v}''_0, \qquad (5.5)$$

where $\overrightarrow{v}'_0 \| \overrightarrow{\omega}$, $\vec{v}_0{}'' \in \{\overrightarrow{v_0}, \vec{\omega}\}$, $\vec{v}_0{}'' \perp \vec{\omega}$



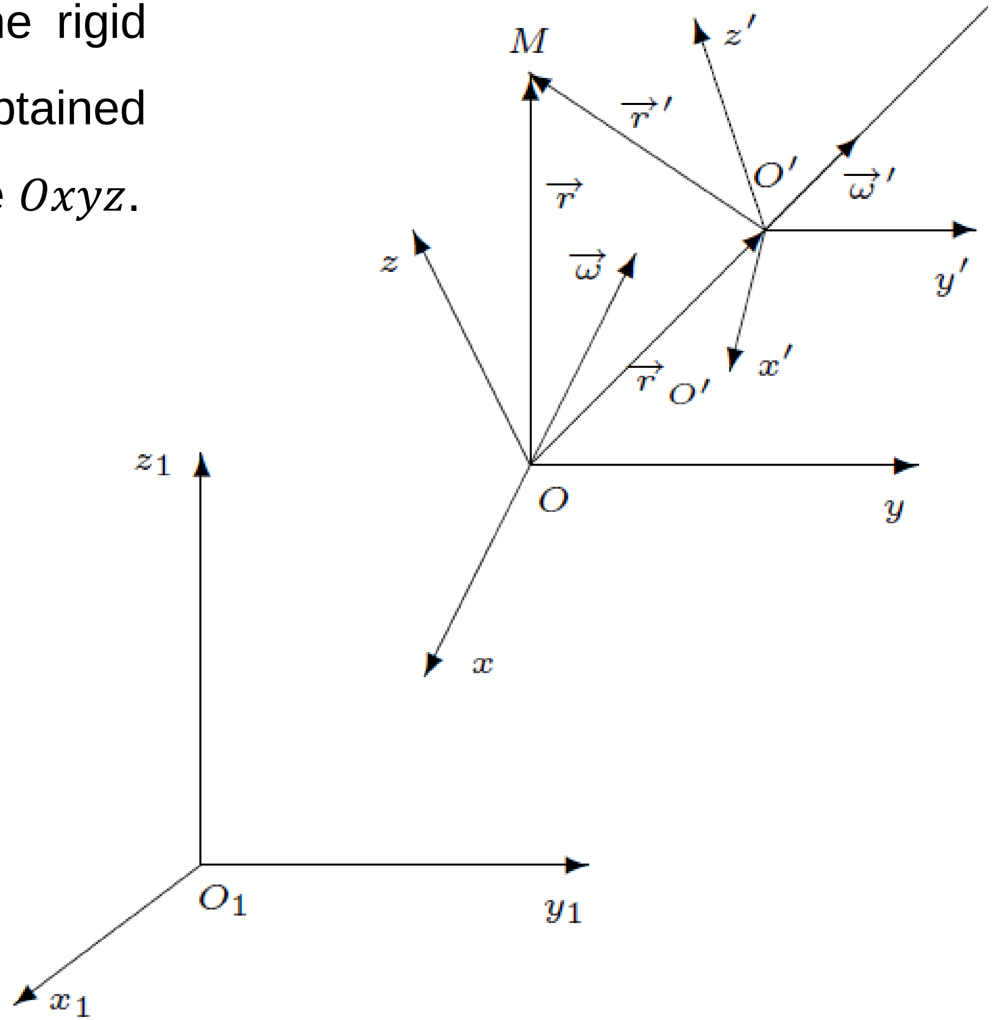
Consider O' an arbitrary point of the rigid and $O'xyz$ the origin of the frame obtained after a translation of the mobile frame $Oxyz$.

We use the notations

$$\overrightarrow{r} = \overrightarrow{OM}, \ \overrightarrow{r}' = \overrightarrow{O'M}$$

and we have

$$\overrightarrow{r} = \overrightarrow{OO'} + \overrightarrow{r}'. \qquad (5.6)$$



Using the velocity formula for a rigid body and (5.6) we deduce:

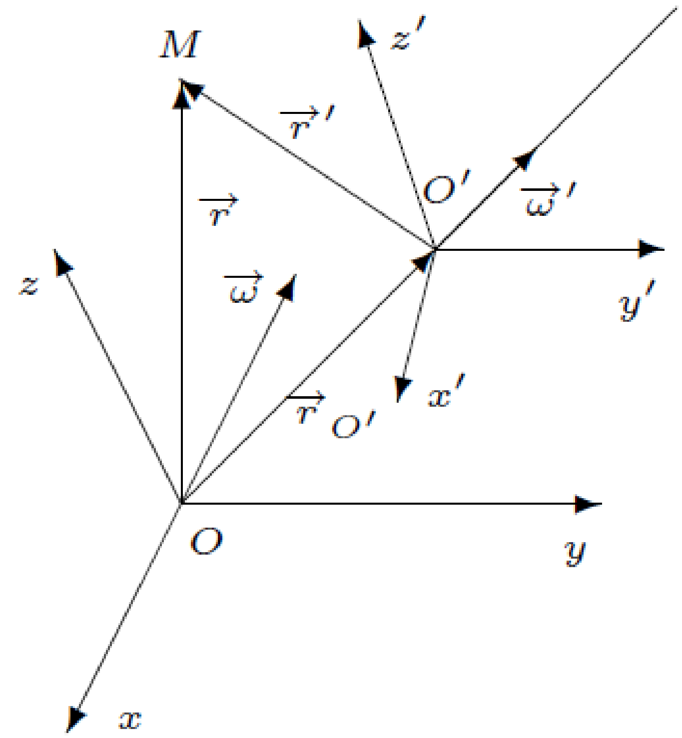
$$\vec{v}_M = \vec{v}_0 + \vec{\omega} \times \vec{r} = \vec{v}_0 + \vec{\omega} \times \overrightarrow{OO}' + \vec{\omega} \times \vec{r}\,',$$

$$\vec{v}_M = \vec{v}_{O'} + \overrightarrow{\omega'} \times \vec{r}\,' = \vec{v}_O + \vec{\omega} \times \overrightarrow{OO'} + \overrightarrow{\omega'} \times \vec{r}\,'$$

$$\vec{v}_{O'} = \vec{v}_O + \vec{\omega} \times \overrightarrow{OO'}$$

$$\vec{v}_0 + \vec{\omega} \times \overrightarrow{OO}' + \vec{\omega} \times \vec{r}\,' = \vec{v}_0 + \vec{\omega} \times \overrightarrow{OO}' + \vec{\omega}\,' \times \vec{r}\,'$$

$$\vec{\omega} \times \vec{r}\,' = \vec{\omega}\,' \times \vec{r}\,' \;\Leftrightarrow\; \vec{\omega}\,' = \vec{\omega} + \lambda \vec{r}\,',\; \lambda \in \mathbb{R}. \qquad (5.7)$$



But all the rigid points, at a moment t move around a unique axis passing through the point O' ($\vec{\omega}$ and $\vec{\omega}'$ depend on t and not on $\vec{r}$) and thus, $\lambda = 0$:

$$\vec{\omega}' = \vec{\omega} \qquad (5.6)$$

The angular velocities in O and O' ($\vec{\omega}' = \vec{\omega}$) are equal, thus the angular velocity does not depend on the origin O.

Next we choose the point O' such that $\overrightarrow{OO'} \perp (\vec{v}_O, \vec{\omega})$ and

$$\vec{v}_O'' + \vec{\omega} \times \overrightarrow{OO'} = 0 \quad | \, \vec{\omega} \times$$

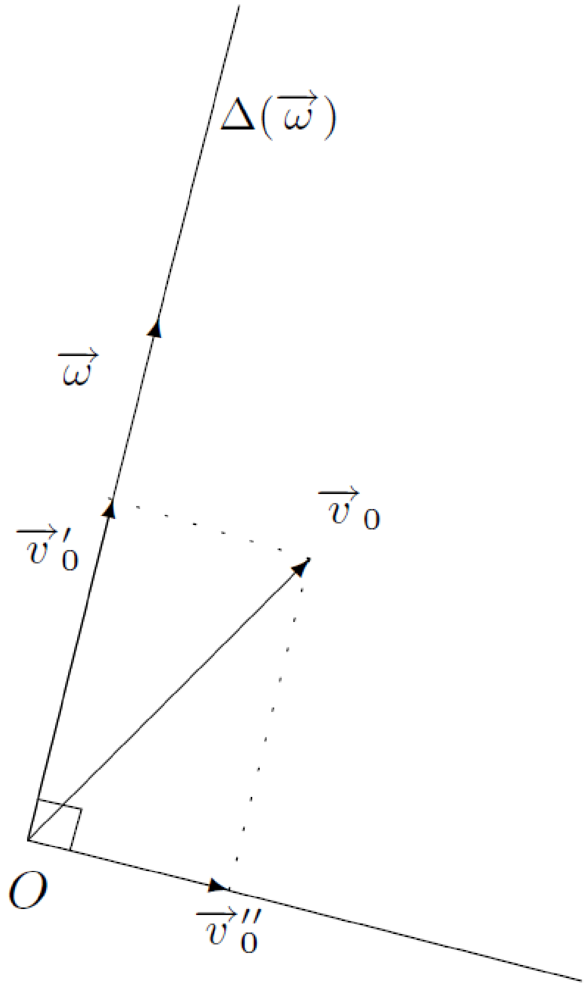
we obtain

$$\vec{\omega} \times \vec{v}_0'' + \vec{\omega} \times (\vec{\omega} \times \overrightarrow{OO'}) = 0$$

$$\Rightarrow \vec{\omega} \times \vec{v}_0'' + (\underbrace{\vec{\omega} \cdot \overrightarrow{OO'}}_{=0})\vec{\omega} - \omega^2 \overrightarrow{OO'} = 0$$

$$(\vec{\omega} \perp \overrightarrow{OO'})$$

$$\overrightarrow{OO'} = \frac{1}{\omega^2} \vec{\omega} \times \vec{v}_0''. \qquad (5.6)$$



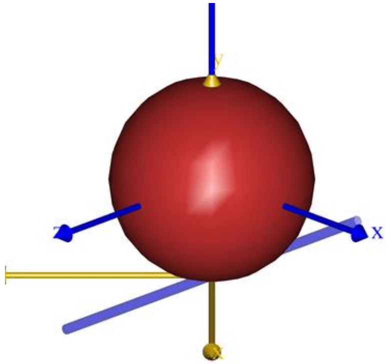
Using equations

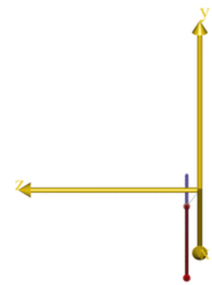
$$\overrightarrow{v}_M = \overrightarrow{v}_0 + \overrightarrow{\omega} \times \overrightarrow{r} = \overrightarrow{v}'_0 + \underbrace{\overrightarrow{v}''_0 + \overrightarrow{\omega} \times \overrightarrow{OO'}}_{=0} + \overrightarrow{\omega} \times \overrightarrow{r}''$$

and $\vec{\omega}' = \vec{\omega}$ we have

$$\overrightarrow{v}_M = \overrightarrow{v}'_0 + \overrightarrow{\omega}' \times \overrightarrow{r}', \qquad (5.10)$$

Eq. (5.10) demonstrates that the motion of the rigid body is a rotation around a single axis $\Delta'(O', \vec{\omega}')$ with the angular velocity $\vec{\omega}' = \vec{\omega}$ and a translation along this axis $\Delta'(O', \vec{\omega}')$ with the velocity $\vec{v}'_O$.





(https://anyscript.org/tools/instantaneous-axis-of-rotation/#)

The axis $\Delta'(O', \vec{\omega}')$ passing through O' and having the direction of the angular velocity $\vec{\omega}' = \vec{\omega}$ about which takes place the instantaneous helical motion at the moment t is called ***instantaneous helical (screw) axis***.

Let be

$$A \in \Delta'(O'; \overrightarrow{\omega}') \;\Leftrightarrow\; \overrightarrow{v}_A \parallel \overrightarrow{\omega}.$$

Considering in the mobile frame of reference

$$\overrightarrow{v}_A = (v_x, v_y, v_z),\; \overrightarrow{v}_0 = (v_{Ox}, v_{Oy}, v_{Oz}),\; \overrightarrow{\omega} = (p, q, r),\; A(x, y, z),$$

The equations of the axis $\Delta'(O', \vec{\omega}')$ are obtained

$$\boxed{\frac{v_{Ox} + qz - ry}{p(t)} = \frac{v_{Oy} + rx - pz}{q(t)} = \frac{v_{Oz} + py - qx}{r(t)} \;(t \text{ fixat}).}$$