ANALYTIC GEOMETRY, PROBLEM SET 1

Rectangular (cartesian) coordinates

1. Give the coordinates of the vertices of the rectangular parallelepiped whose faces are the coordinate planes and the planes $x = 1$, $y = 3$ and $z = 6$.

2. Describe the locus of points $P(x, y, z) \in \mathcal{E}_3$ if their Cartesian coordinates satisfy:
a) $xyz = 0$; b) $x^2 + y^2 + z^2 = 0$; c) $(x+1)^2 + (y-2)^2 + (z+3)^2 = 0$; d) $(x-2)(z-8) = 0$;
e) $z^2 - 25 = 0$.

3. Find $x \in \mathbb{R}$ if:
a) $P_1(x, 2, 3)$, $P_2(2, 1, 1)$ and $P_1P_2 = \sqrt{21}$; b) $Q_1(x, x, 1)$, $Q_2(0, 3, 5)$ and $Q_1Q_2 = 5$.

4. Show that the given points are collinear:
a) $P_1(1, 2, 0)$, $P_2(-2, -2, -3)$, $P_3(7, 10, 6)$; b) $Q_1(2, 3, 2)$, $Q_2(1, 4, 4)$, $Q_3(5, 0, -4)$.

5. The coordinates of the midpoint of the segment $[P_1P_2]$, determined by $P_1(x_1, y_1, z_1)$ and $P_2(2, 3, 6)$ are $(-1, -4, 8)$. Find the coordinates of P_1.

6. Let P_3 be the midpoint of the segment joining the points $P_1(-3, 4, 1)$ and $P_2(-5, 8, 3)$. Find the coordinates of the midpoint of the segment: a) joining P_1 and P_3; b) joining P_3 and P_2.

7. Compute the area of the triangle whose vertices have coordinates $P_1 = (-1, 0, 1)$, $P_2 = (0, 2, 2)$ and $P_3 = (0, -1, 2)$.

8. Consider a triangle ABC in space, whose vertices have coordinates $A(x_A, y_A, z_A)$, $B(x_B, y_B, z_B)$ and $C(x_C, y_C, z_C)$. Find, with proof, the coordinates of its centroid G.

9. Suppose a plane and a system of rectangular coordinates are fixed. Is it possible to construct an equilateral triangle ABC in this plane, such that the coordinates of the vertices are all integer numbers?

10*. Let $\mathcal{R}$ be the region consisting of the set of points in the coordinate plane that satisfy both $|8 - x| + y \leq 10$ and $3y - x \geq 15$. When $\mathcal{R}$ is revolved around the line whose equation is $3y - x = 15$, the volume of the resulting solid is $\frac{m\pi}{n\sqrt{p}}$, where m, n, and p are positive integers, m and n are relatively prime, and p is not divisible by the square of any prime. Find $m + n + p$.

Date: October 9, 2022. The starred problem is not examinable material.