19.10.2022
SEMINAR 3

Sets

$A = \{x \mid P(x)\}$ belongs to

$A = B \overset{def}{\iff} \forall x (x \in A \iff x \in B)$

$x \notin A \iff x \in A \iff \neg(x \in A)$

$P(A) = \{B \mid B \subseteq A\}$ - power set of A

$B \subseteq A \overset{def}{\iff} \forall x (x \in B \to x \in A)$

$A = \{1, 2, \dots, n\}$ 2 choices $\to 2^n$

$A \cap B = \{x \mid (x \in A) \wedge (x \in B)\}$

$A \cup B = \{x \mid (x \in A) \vee (x \in B)\}$

$A \setminus B = \{x \mid (x \in A) \wedge (x \notin B)\}$

$A \Delta B = (A \cup B) \setminus (A \cap B) =$

$= (A \setminus B) \cup (B \setminus A)$ - symmetric difference

$\emptyset = \{x \mid x \neq x\}$ - empty set

Exercises:

20) A, B, C sets from the universe $\mathcal{U}$.

Prove the following:

a) $A \subseteq A \iff \forall x (x \in A \to x \in A) \iff$

$\iff \forall x (\overline{(x \in A)} \vee (x \in A)) \iff$ this is true

(reflexivity)

b) if $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$

(transitivity)

Let $\left.\begin{matrix} x \in A \\ A \subseteq B \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x \in B \\ B \subseteq C \end{matrix}\right\} \Rightarrow x \in C \Rightarrow A \subseteq C$

set

$1 \notin \{1\}$

$\{1\} \subseteq \{1, 2\}$

subset

c) $A \cup (A \cap B) = A$ (absorption) (exam!!)

I: Let $x \in A \cup (A \cap B) \Leftrightarrow (x \in A) \vee (x \in (A \cap B))$

$\Leftrightarrow \underbrace{(x \in A)}_{p} \vee (\underbrace{(x \in A)}_{p} \wedge \underbrace{(x \in B)}_{q}) \overset{abs}{\Leftrightarrow} (x \in A)$

$\Rightarrow A \cup (A \cap B) \overset{\subseteq}{=} A$

II. $A \cup (A \cap B) = \{x \in \mathcal{U} \mid x \in A \cup (A \cap B)\} = \{x \in \mathcal{U} \mid (x \in A) \vee (x \in A \cap B)\}$

$= \{x \in \mathcal{U} \mid (x \in A) \vee ((x \in A) \wedge (x \in B))\} =$

$\overset{abs}{=} \{x \in \mathcal{U} \mid x \in A\} = A$

$CA = A^C = \mathcal{U} \setminus A = \{x \in \mathcal{U} \mid x \notin A\}$

the complement of A

21)

a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Let $x \in A \cap (B \cup C) \overset{def}{\Leftrightarrow} (x \in A) \wedge (x \in B \cup C)$

$\Leftrightarrow (x \in A) \wedge ((x \in B) \vee (x \in C)) \overset{distr}{\Leftrightarrow} ((x \in A) \wedge (x \in B))$

$\vee ((x \in A) \wedge (x \in C)) \Leftrightarrow (x \in ((A \cap B))) \vee (x \in (A \cap C)) \Leftrightarrow$

$x \in ((A \cap B) \cup (A \cap C))$

b) $\boxed{A \setminus B = A \cap C_B}$

Let $x \in A \setminus B \Leftrightarrow (x \in A) \wedge (x \notin B) \Leftrightarrow (x \in A) \wedge (x \in C_B) \Leftrightarrow$
$x \in A \cap C_B$

g) $C(A \cup B) = CA \cap CB$
$C(A \cap B) = CA \cup CB$ De Morgan