Faculty of Mathematics and Computer Science
Babeş-Bolyai University, Cluj-Napoca
Numerical Analysis
MIE - semester IV

Laboratory 1

Deadline: 11 – 15 March 2024

Orthogonal and Taylor polynomials. Finite and divided differences

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1. The Chebyshev polynomials of the first kind are defined by

$$T_n(x) = \cos(n \arccos x), \quad x \in [-1, 1].$$

Plot (in the same figure) the polynomials T_1, T_2 and T_3.

2. Plot (in the same figure) the first ten Chebyshev polynomials of the first kind, using the following recurrence formula:

$$\begin{cases} T_0(x) = 1 \\ T_1(x) = x \\ T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad n \geq 1, \quad x \in [-1, 1]. \end{cases}$$

3. Let us consider $f : [-1, 3] \to \mathbb{R}$ be given by $f(x) = e^x$ and $x_0 = 0$. Plot (in the same figure) the function f and the first six Taylor polynomials associated to the function f and the point x_0 on the interval $[-1, 3]$. Approximate $f(2)$ by each Taylor polynomial (with 5 digits) and compute the maximum error of approximation.

4. Let us consider $f : [-1, 1] \to \mathbb{R}$ be given by $f(x) = 2x^2 + \cos(3x)$.
   - Plot the graph of the function f on the interval $[-1, 1]$;
   - Plot the first two Taylor polynomials associated to the function f and the point $x_0 = 0$ on the interval $[-1, 1]$.

5. Let $h = 0.2$ and $x_k = 1 + kh$ for every $k \in \{0, ..., 5\}$. Also, let us consider $y_k = \sqrt{2x_k^2 + 3}$.
   - Construct the finite differences table based on the points (x_k, y_k).
   - If y_4 is modified with a small value $\varepsilon = 10^{-3}$, study how the finite differences table is affected.
   - Construct the divided differences table based on the points (x_k, y_k).

Remark: 5 problems × 0.5p = 2.5p