COMPLEMENTS OF GEOMETRY - SEMINAR 6

1. Let $\triangle ABC$ be an equilateral triangle with side length l. Consider M a point inside the triangle $\triangle ABC$ and let D, E, F be the projections of M on the sides BC, CA and AB, respectively. Prove that

a) $MF + MD + ME = \frac{l\sqrt{3}}{2}$;

b) $\frac{1}{MD} + \frac{1}{ME} + \frac{1}{MF} \geq \frac{6\sqrt{3}}{l}$;

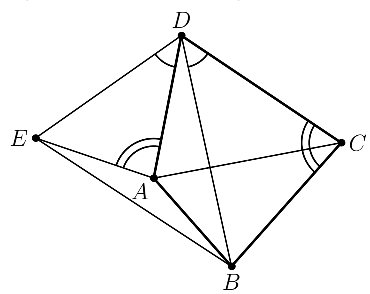
c) $\frac{1}{MD+ME} + \frac{1}{ME+MF} + \frac{1}{MF+MD} \geq \frac{3\sqrt{3}}{l}$.

2.

Show that in every convex quadrilateral $ABCD$ the following inequality holds:

$$AC \cdot BD \leq AB \cdot CD + AD \cdot BC$$

and that equality holds if and only if $ABCD$ is cyclic.



Problems for presentation

Presentation 1. Let ABC be a triangle and let $\mathcal{C}$ be a circle intersecting the sides BC, CA, AB in the points A_1, A_2, B_1, B_2, C_1, C_2. Show that the lines AA_1, BB_1, CC_1 are concurrent if and only if AA_2, BB_2, CC_2 are concurrent.

Presentation 2.

(a) Let $\triangle ABC$ be an equilateral triangle. Let P be a point on minor arc AB of its circumcircle. Prove that $PC = PA + PB$.

(b) In a regular heptagon $ABCDEFG$, prove that: $\frac{1}{AB} = \frac{1}{AC} + \frac{1}{AD}$.

Date: April 14, 2024.