Faculty of Mathematics and Computer Science
Babeş-Bolyai University, Cluj-Napoca
Numerical Analysis
MIE - semester IV

Laboratory 10

Deadline: 20 – 24 May 2024

Direct and Iterative methods for solving linear systems I

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1. Implement Gauss method for solving linear systems, using partial elimination. Solve the following system

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 10 \\ 2x_1 + 3x_2 + x_3 + 5x_4 = 31 \\ -x_1 + x_2 - 5x_3 + 3x_4 = -2 \\ 3x_1 + x_2 + 7x_3 - 2x_4 = 18 \end{cases}$$

Hint: Input: n (order of the system), A (matrix of coefficients), b (vector of free terms)
Output: x (vector of the solutions or a message in case of incompatibility of the system)

Algorithm:

- for $p = 1, ..., n-1$
  - let $abs(a(p,q)) = max(abs(A(p:n,p)))$
  - if $A(q,p) = 0$, then "Message" and Exit
  - if $q \neq p$, interchange the lines p and q from A and b
  - perform the necessary operations for obtaining zeros on the column p, below $A(p,p)$
  - apply the transformations also to the vector b
- if $A(n,n) = 0$, then "Message" and Exit
- for $i = n : -1 : 1$ compute $x(i)$
- display x

2. Find the LU decomposition of the following matrix (using Doolittle method):

$$M = \begin{pmatrix} 6 & 2 & 1 & -1 \\ 2 & 4 & 1 & 0 \\ 1 & 1 & 4 & -1 \\ -1 & 0 & -1 & 3 \end{pmatrix}.$$

If $b = \begin{pmatrix} 8 & 7 & 5 & 1 \end{pmatrix}$, solve the system $Mx = b$.

3. Let us consider the following family of hyperplanes in $\mathbb{R}^6$:

$$H_k \, : \, a_k x_1 + b_k x_2 + c_k x_3 + d_k x_4 + e_k x_5 + f_k x_6 = z_k, \quad k \in \{1, ..., 6\},$$

where $x = (x_1, ..., x_6) \in \mathbb{R}^6$ and $a_k, b_k, c_k, d_k, e_k, f_k, z_k \in \mathbb{R}$, for all $k \in \{1, ..., 6\}$.

- If we intersect the hyperplanes $H_1, ..., H_6$, then we obtain a system of the form $Ax = b$. Determine A and b knowing that
  - $z_1 = z_6 = 2$ and $z_k = 1$, for all $k = \overline{2,5}$;
  - $a_1 = b_2 = c_3 = d_4 = e_5 = f_6 = 3$;
  - $b_1 = c_2 = d_3 = e_4 = f_5 = -1$;
  - $a_2 = b_3 = c_4 = d_5 = e_6 = -1$;
  - all other coefficients are zero.
- Determine the intersection point of the hyperplanes $H_1, ..., H_6$ using the classical formulas (not matriceal) for
  - Jacobi iterative method;
  - Gauss-Seidel iterative method;
  - relaxation iterative method.

  In each case, determine also the number of iterations needed for finding the intersection point if we consider precision $\varepsilon = 10^{-3}$.

Remark: 1,3 (1p), 2 (0.5p)