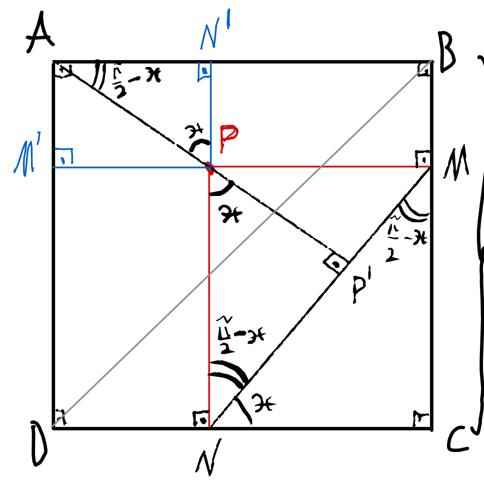

ABCD a square
$PM \perp BM$, $M \in BC$
$PN \perp DC$, $N \in DC$

$AP \perp MN \overset{?}{\Rightarrow} MN = AP$ or $AP \perp DB$

• Let $N' \in AD$, $PN' \perp AD$ | $P' \in MN$, $AP' \perp MN$
$N' \in AB$, $PN' \perp AB$

• We denote $DC = BC = AB = AD = l$ (ABCD sqr.) (1)
$m(\sphericalangle MNC) = \varkappa$

Solution: 1. We will try to get that:
$\Delta MNC \overset{?}{\sim} \Delta APN'$

$NP \perp NC \Rightarrow m(\sphericalangle PNP') = \frac{\pi}{2} - \varkappa$

$PP' \perp P'N \Rightarrow \Delta NPP$ right triangle $\Rightarrow$

$\Rightarrow m(\sphericalangle NPP') = \frac{\pi}{2} - \frac{\pi}{2} + \varkappa = \varkappa$

$\sphericalangle NPA$ and $\sphericalangle NPP'$ opposite angles $\Rightarrow m(\sphericalangle NPA) = \varkappa$
$m(\sphericalangle MNC) = \varkappa$

$\Rightarrow m(\sphericalangle NPA) = m(\sphericalangle MNC)$
$m(\sphericalangle NCM) = m(\sphericalangle PN'A)$ $\Big\} \overset{A.A.}{\Longrightarrow} \Delta MNC \sim \Delta APN'$

2. We have that $m(\sphericalangle M'DN) = m(\sphericalangle DM'P) = m(\sphericalangle DNP) = \frac{\pi}{2}$
$\Rightarrow DM'PN$ is a rectangle
Similar for $AM'PN'$ and $N'PMB$, which are rectangles too.

From 1. $\Rightarrow \frac{MC}{AN'} = \frac{NC}{N'P} = \frac{MN}{AP}$ (4)

From 2. we get: $AN' = M'P$, $M'P = DN$ $\Big\} \Rightarrow AN' = DN$; $N'P = MB$ $\Bigg\} \Leftrightarrow$

$\Leftrightarrow \frac{MC}{DN} = \frac{NC}{MB}$ (2)

(1) $\Rightarrow DN = DC - NC = l - NC$
$BM = BC - MC = l - MC$ $\Bigg\} \Leftrightarrow \frac{MC}{l - NC} = \frac{NC}{l - MC}$ (3) $\Leftrightarrow$

$\Leftrightarrow lNC - NC^2 = lMC - MC^2 \Leftrightarrow$

$\Leftrightarrow MC^2 - lMC + (lNC - NC^2) = 0$

$\Delta = l^2 - 4lNC + 4NC^2 = (l - 2NC)^2$

$\sqrt{\Delta} = |l - 2NC| \Rightarrow$ there are 4 result.

i) $l > 2NC$: $MC = \frac{l - l + 2NC}{2} = NC$; $MC = \frac{l + l - 2NC}{2} = l - NC$

ii) $l < 2NC$: $MC = \frac{l + 2NC - l}{2} = NC$; $MC = \frac{l - 2NC + l}{2} = l - NC$ $\Bigg\} \Rightarrow$

$\Rightarrow$ We can conclude that there are only 2 sol.

I $MC = NC \overset{(2)}{\Rightarrow} \frac{NC}{DN} = \frac{MC}{MB} \Rightarrow MN \parallel DB$, $AP \perp MN$ $\Big\} \Rightarrow AP \perp DB$

II $MC = l - NC \overset{(2)(3)(4)}{\Longrightarrow} \frac{NC}{MB} = \frac{MN}{AP} = \frac{l - NC}{l - NC} = 1 \Rightarrow$

$\Rightarrow \frac{MN}{AP} = 1 \Leftrightarrow MN = AP$

And we are done.