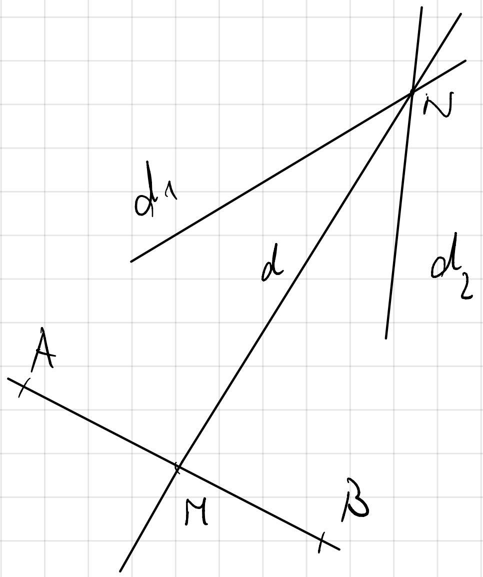
16.11.2022

SEMINAR 7

1



$d_1: 2x - 5y - 1 = 0$

$d_2: x + 4y - 7 = 0$

$N = d_1 \cap d_2$

M divides $[AB]$ in ratio $k = \frac{2}{3}$

$A(4,-3)$

$B(-1,2)$

$? d \ni M, N$

$$\begin{cases} 2x - 5y - 1 = 0 \\ x + 4y - 7 = 0 \quad / \cdot (-2) \end{cases}$$

$-13y + 13 = 0$

$y = 1$

$x_N + 4 - 7 = 0 \Rightarrow x_N = 3$

$\Rightarrow N(3,1)$

$$M\left(\frac{x_A + k x_B}{1+k}, \frac{y_A + k y_B}{1+k}\right) \Rightarrow M\left(\frac{4 + \frac{2}{3}(-1)}{1 + \frac{2}{3}}, \frac{-3 + \frac{2}{3} \cdot 2}{1 + \frac{2}{3}}\right)$$

$$\Rightarrow M\left(\frac{\frac{10}{3}}{\frac{5}{3}}, \frac{-\frac{5}{3}}{\frac{5}{3}}\right) \Rightarrow M(2,-1)$$

$l \ni P(x_P, y_P), Q(x_Q, y_Q)$

$l: \frac{x - x_Q}{x_P - x_Q} = \frac{y - y_Q}{y_P - y_Q}$

$\frac{x-3}{3-2} = \frac{y-1}{1+1}$

$x - 3 = \frac{y-1}{2} \Rightarrow 2x - 6 - y + 1 = 0$

$d: 2x - y - 5 = 0$

second method: (line boundness)

$$l_{r,s} : r(2x-5y-1) + s(x+4y-7) = 0$$

$$M \in l_{r,s} \Rightarrow r(4+5-1) + s(2-4-7) = 0 \Leftrightarrow 8r - 9s = 0 \Rightarrow r = \frac{9}{8}s$$

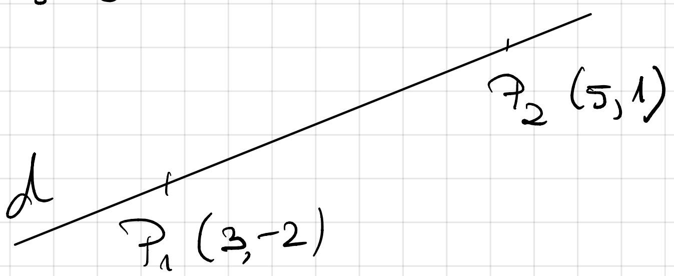
$$\Rightarrow M \in l_{\frac{9}{8}s,s} : s\left(\frac{9}{8}(2x-5y-1) + x+4y-7\right) = 0$$

$$\Leftrightarrow \frac{18x-45y-9 + 8x+32y-56}{8} = 0$$

$$\Leftrightarrow 26x - 13y - 65 = 0 \; / :13$$

$$\Leftrightarrow 2x - y - 5 = 0$$

(3)



find parametric equations

$$d : \begin{cases} x = x_0 + a \cdot t \\ y = y_0 + bt \end{cases}$$

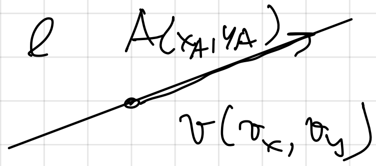
$$\overline{P_1P_2}(x_2 - x_1, y_2 - y_1)$$

$$\overline{P_1P_2}(2,3)$$

$$d : \begin{cases} x = 5 + 2s \\ y = 1 + 3s \end{cases} \quad (s \in \mathbb{R})$$

← putem pune și x_1

↓ putem pune și y_1



$$l : \begin{cases} x = x_A + \lambda v_x \\ y = y_A + \lambda v_y \end{cases}, \lambda \in \mathbb{R}$$

(5) $d_1 : x = 3 - t \quad y = 1 + 2t$

$d_2 : x = -1 + 3t \quad y = 9 - 6t$

$$d_1 \begin{cases} x_1 = 3 - t \\ y_1 = 1 + 2t \end{cases} \quad t \in \mathbb{R} \qquad d_2 \begin{cases} x_2 = -1 + 3s \\ y_2 = 9 - 6s \end{cases} \quad s \in \mathbb{R}$$

Show $d_1 = d_2$

dir vectors

$$d_1 : \frac{x - x_1}{a} = \frac{y - y_1}{b} \Rightarrow \frac{x-3}{-1} = \frac{y-1}{2} \Rightarrow 2(x-3) = -y + 1$$

$$2x - 6 = -y + 1$$

$$d_1 : 2x + y - 7 = 0$$

$d_2: \frac{x - x_2}{a} = \frac{y - y_2}{b} \Rightarrow \frac{x+1}{3} = \frac{y-9}{-6} \Rightarrow -6x - 6 = 3y + 27 \; |:3$

$d_2: -2x - 2 - y + 9 = 0 \; | \cdot (-1)$

$d_2: 2x + y + 7 = 0$

$\Rightarrow d_1 = d_2$

II

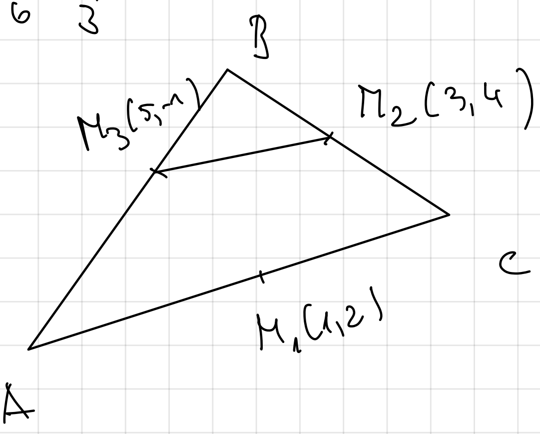
$v_2 = -3 v_1 \Rightarrow$ they have the same dir. vectors

$\Rightarrow d_1 \parallel d_2$

$P_1 \in d_2$ if $\exists s$ s.t. $\begin{cases} 3 = -1 + 3s \\ 1 = 9 - 6s \end{cases} \Rightarrow \begin{cases} 4 = 3s \\ 1 = 9 - 6s \end{cases} \Rightarrow \begin{cases} 4 = 3s \\ -8 = -6s \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} s = \frac{4}{3} \\ s = \frac{8}{6} = \frac{4}{3} \end{cases} \Rightarrow \exists s$

(6)



I

$\frac{x_A + x_C}{2} = 1$

$\frac{y_A + y_C}{2} = 2$

⋮ system...

$\underline{\overline{II}}$ $M_3M_2(-2,5) \parallel M_1(1,2)$

$AC \begin{cases} x = 1 + \lambda(-2) \\ y = 2 + 5\lambda \end{cases}$ $\qquad \frac{x-1}{-2} = \frac{y-2}{5} = 1 \ldots$

⑦ $d: 2x+3y+4=0$

$M_0(2,1) \in d_1$

$? d_1$

a) $d_1 \parallel d$

b) $d_1 \perp d$

c) angle between d and d_1 is $\frac{\pi}{4}$

a) $\underline{I}$ $y = -\frac{2}{3}x - \frac{4}{3}$
↓ slope
$= \ldots$

b) $d: a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$\parallel \qquad \parallel$

$\alpha a_1 \qquad \alpha b_1$

$d_1: 2x+3y+c_2=0$

$l: \frac{x-x_0}{v_x} = \frac{y-y_0}{v_y}$

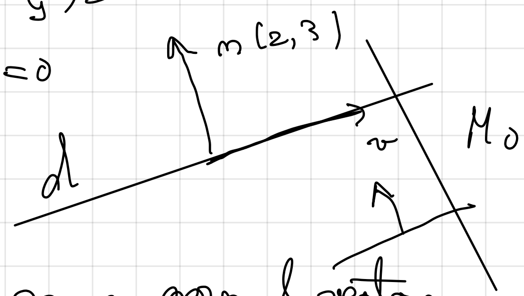
$v_y(x-x_0) = v_x(y-y_0) = 0$

$(v_y, -v_x)$

$n(v_y, -v_x) \cdot v(v_x, v_y) =$

$= v_y \cdot v_x - v_x v_y = 0$

$n \perp v$



$\overline{\underline{III}}$ $d_1 \parallel d_2 \Leftrightarrow$ have the same normal vectors

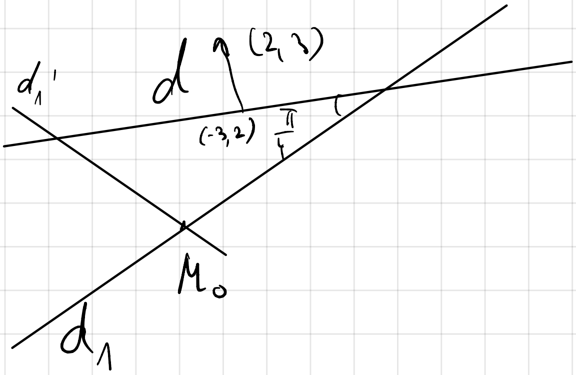
$\Leftarrow d_1: 2x+3y+c_2=0$

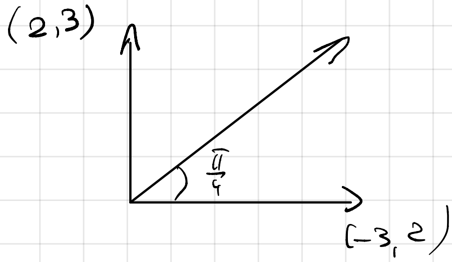
b) $(2,3)$ a vector orthogonal to d_1

$\underline{I}$ $d_1: \frac{x-2}{2} = \frac{y-1}{3}$

$\underline{II}$: slopes

c)





slope ?