

ANALYTIC GEOMETRY, PROBLEM SET 7

The line in 3D. Relative positions of lines and planes
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1. Find the equation of the plane containing the points $P_1(2, -1, -3)$, $P_2(3, 1, 2)$ and parallel to the vector $\vec{a}(3, -1, -4)$.
2. Find the equation of the plane containing the perpendicular lines through $P(-2, 3, 5)$ on the planes $\pi_1 : 4x + y - 3z + 13 = 0$ and $\pi_2 : x - 2y + z - 11 = 0$.
3. Find the equation of the plane passing through the points A , B and C , where:
(a) $A(-2, 1, 1)$, $B(0, 2, 3)$ and $C(1, 0, -1)$; (b) $A(3, 2, 1)$, $B(2, 1, -1)$ and $C(-1, 3, 2)$.
4. Show that the points $A(1, 0, -1)$, $B(0, 2, 3)$, $C(-2, 1, 1)$ and $D(4, 2, 3)$ are coplanar.
5. Let d_1 and d_2 be two lines in \mathcal{E}_3 , given by $d_1 : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-5}{6}$ and $d_2 : \frac{x-1}{1} = \frac{y+1}{1} = \frac{z-5}{-3}$.
(a) Find the parametric equations of d_1 and d_2 ;
(b) Prove that they intersect and find the coordinates of their intersection point;
(c) Find the equation of the plane determined by d_1 and d_2 .
6. Given the lines $d_1 : x = 1 + t, y = 1 + 2t, z = 3 + t, t \in \mathbb{R}$ and $d_2 : x = 3 + s, y = 2s, z = -2 + s, s \in \mathbb{R}$, show that $d_1 \parallel d_2$ and find the equation of the plane determined by d_1 and d_2 .
7. Find the parametric equations of the line
$$\begin{cases} -2x + 3y + 7z + 2 = 0 \\ x + 2y - 3z + 5 = 0 \end{cases}.$$
8. Find the parametric equations of the line passing through $P_1(5, -2, 1)$ and $P_2(2, 4, 2)$. Find the equations of the line passing through $P(6, 4, -2)$ and parallel to the line $d : \frac{x}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$.
9. Given the points $A(1, 2, -7)$, $B(2, 2, -7)$ and $C(3, 4, 5)$, find the equation(s) of the internal bisector passing through the vertex A in the triangle ABC .
10. Find the equations of the line passing through the origin and parallel to the line given by the parametric equations: $x = t, y = -1 + t$ and $z = 2$.
11. Given the lines $d_1 : x = 4 - 2t, y = 1 + 2t, z = 9 + 3t$ and $d_2 : \frac{x-1}{2} = \frac{y+2}{3} = \frac{z-4}{2}$, find the intersection points between the two lines and the coordinate planes.
12. Let d_1 and d_2 be the lines given by $d_1 : x = 3 + t, y = -2 + t, z = 9 + t, t \in \mathbb{R}$ and $d_2 : x = 1 - 2s, y = 5 + s, z = -2 - 5s, s \in \mathbb{R}$.
a) Prove they are coplanar.

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- b) Find the equation of the line passing through the point $P(4, 1, 6)$ and orthogonal on the plane determined by d_1 and d_2 .
- 13.** Prove that the intersection lines of the planes $\pi_1 : 2x - y + 3z - 5 = 0$, $\pi_2 : 3x + y + 2z - 1 = 0$ and $\pi_3 : 4x + 3y + z + 2 = 0$ are parallel.
- 14.** Verify that the lines $d_1 : \frac{x-3}{1} = \frac{y-8}{3} = \frac{z-3}{4}$ and $d_2 : \frac{x-4}{1} = \frac{y-9}{2} = \frac{z-9}{5}$ are coplanar and find the equation of the plane determined by the two lines.
- 15.** Determine whether the line given by $x = 3 + 8t$, $y = 4 + 5t$, and $z = -3 - t$, $t \in \mathbb{R}$ is parallel to the plane $x - 3y + 5z - 12 = 0$.