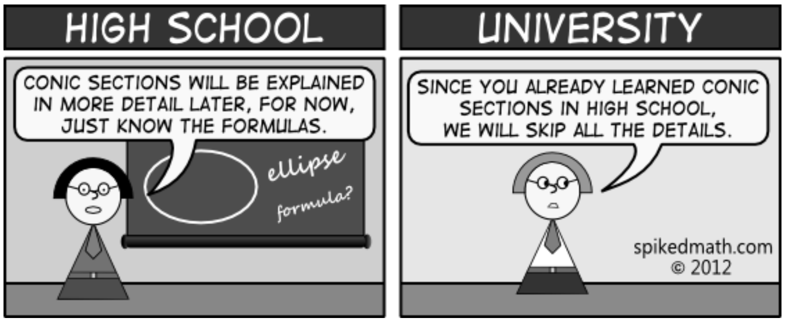
ANALYTIC GEOMETRY, SEMINAR 10



1. Problems

1. Find the equation of the circle:
a) of diameter $[AB]$, where $A(1,2)$ and $B(-3,-1)$; b) of center $I(2,-3)$ and radius $R=7$;
c) of center $I(-1,2)$ and which passes through $A(2,6)$;
d) centered at the origin and tangent to $d: 3x-4y+20=0$; e) passing through $A(3,1)$ and $B(-1,3)$ and having the center on the line $d: 3x-y-2=0$;
f) determined by $A(1,1)$, $B(1,-1)$ and $C(2,0)$; g) tangent to both $d_1: 2x+y-5=0$ and $d_2: 2x+y+15=0$, if the tangency point with d_1 is $M(3,1)$.

2. a) Determine the position of the point $A(1,-2)$ relative to the circle $C: x^2+y^2-8x-4y-5=0$;
b) Find the intersection between the line $d: 7x-y+12=0$ and the circle $C: (x-2)^2+(y-1)^2-25=0$;
c) Determine the position of the line $d: 2x-y-3=0$ relative to the circle $C: x^2+y^2-3x+2y-3=0$.

3. Find the equation of
a) the tangent line to $C: x^2+y^2-5=0$ at the point $A(-1,2)$;
b) the tangent lines to $C: x^2+y^2+10x-2y+6=0$, parallel to $d: 2x+y-7=0$;
c) the tangent lines to $C: x^2+y^2-2x+4y=0$, orthogonal on $d: x-2y+9=0$.

4. Find the foci of the ellipse $\mathcal{E}: 9x^2+25y^2-225=0$.

5. Find the intersection points between the line $d_1: x+2y-7=0$ and the ellipse given by the equation $\mathcal{E}: x^2+3y^2-25=0$.

6. Find the position of the line $d: 2x+y-10=0$ relative to the ellipse $\mathcal{E}: \frac{x^2}{9}+\frac{y^2}{4}-1=0$.

7. Find the equation of a line which is orthogonal on $d_1: 2x-2y-13=0$ and tangent to the ellipse $\mathcal{E}: x^2+4y^2-20=0$.

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8. Consider the ellipse $x^2 + 4y^2 = 25$. Find the chords on the ellipse which have the point $A(7/2, 7/4)$ as their midpoint.

9. Consider the ellipse $\frac{x^2}{4} + y^2 = 1$ with F_1, F_2 as foci. Find the points M, situated on the ellipse for which the angle $\angle F_1MF_2$ is right.

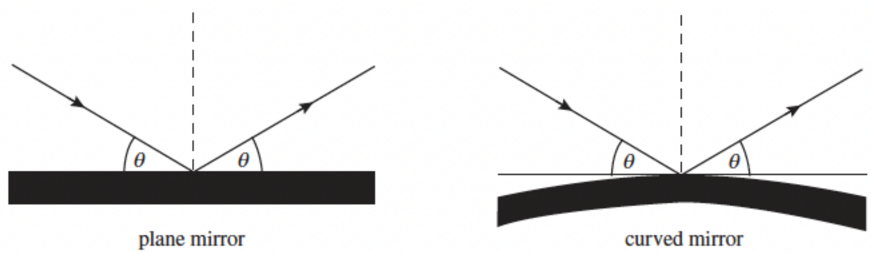
10. Consider the ellipse $\frac{x^2}{4} + y^2 = 1$ with F_1, F_2 as foci. Find the point M on the ellipse for which $\angle F_1MF_2$ is maximal.

11. Consider the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Determine the geometric locus of the midpoints of the chords on the ellipse which are parallel to the line $x + 2y = 1$.

2. The reflection (optical) property of the ellipse

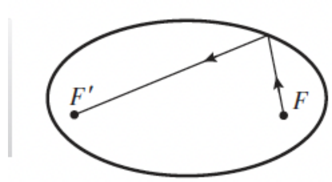
All reflecting surfaces - mirrors, for example - obey the same Reflection Law. The Reflection Law is often expressed in terms of angles made with the normal to the surface rather than the surface itself. However, here we give the following version:

The Reflection Law. The angle that incoming light makes with the tangent to a surface is the same as the angle that the reflected light makes with the tangent.



This law applies to all mirrors, no matter whether the reflecting surface is plane or curved. Indeed, in many practical applications the mirror is designed to have a cross-section that is a conic curve - for example, the Lovell radio-telescope (in England) uses a parabolic reflector to focus parallel radio waves from space onto a receiver (more on that in the last seminar).

Theorem 1 (Reflection Property of the Ellipse)**.** *Light which comes from one focus of an elliptical mirror is reflected at the ellipse to pass through the second focus.*



Prove this theorem using the Reflection Law and your analytic geometry skills.