COMPLEMENTS OF GEOMETRY - SEMINAR 4

1. In triangle ABC, denote by M the midpoint of the segment $[BC]$ and let $(AD$ be the interior bisector of the angle A, where $D \in BC$. Show that

$$m_a^2 := AM^2 = \frac{b^2+c^2}{2} - \frac{a^2}{4} \text{ and } l_a^2 := AD^2 = bc\left(1 - \left(\frac{a}{b+c}\right)^2\right).$$

2. In an acute triangle ABC, AD, BE and CF are heights. Show that the line AD is the angle bisector of $\angle EDF$

3. Let $ABCD$ be a cyclic quadrilateral. A line parallel to BC intersects AB, CD at E, F respectively. Show that A, D, F, E are concyclic.

Problems for presentation

Presentation 1. (a) Let ABC be a triangle inscribed in a circle ω. Let l be a line passing through A different from AB. Let L be a point on l such that AB separates the points C and L. Show that AL is tangent to ω if and only if $\angle LAB \equiv \angle ACB$.

(b) Let ABC be a triangle. Denote by ω_a the circle tangent to AB at A and passing through C. Similarly, denote by ω_b the circle tangent to BC at B and passing through A, and ω_c the circle tangent to CA at C and passing through B. Prove that the circles ω_a, ω_b and ω_c intersect at one point. (This point is called the "Brocard point" of the triangle $\triangle ABC$)

Presentation 2. Let ABC be a triangle which is not right. Denote by H the orthocenter of this triangle and let M and N be points on its sides AB and AC. Prove that the common chord of circles with diameters CM and BN passes through H.

Date: March 19, 2024.