$ode := diff(R(t), t) = k \cdot R(t)$

$$ode := \frac{d}{dt} R(t) = k\, R(t) \quad (1)$$

$ic := R(0) = R0$

$$ic := R(0) = R0 \quad (2)$$

$sol\_a := dsolve(\{ode, ic\}, R(t))$

$$sol\_a := R(t) = R0\, e^{k t} \quad (3)$$

sol_a

$$R(t) = R0\, e^{k t} \quad (4)$$

$eq\_b := T\_half = \frac{\ln(2)}{k}$

$$eq\_b := T\_half = \frac{\ln(2)}{k} \quad (5)$$

$sol\_b := solve(eq\_b, k)$

$$sol\_b := \frac{\ln(2)}{T\_half} \quad (6)$$

sol_b

$$\frac{\ln(2)}{T\_half} \quad (7)$$

$k\_C14 := sol\_b[1]\, subs(T\_half = 5730\ )$

$$k\_C14 := \left(\frac{\ln(2)}{T\_half}\right)_1 T\_half = 5730 \left(\frac{\ln(2)}{T\_half}\right)_1 \quad (8)$$

$k\_U238 := sol\_b[1] subs\left(T\_half = 4.468 \cdot 10^9\right)$

$$k\_U238 := \left(\frac{\ln(2)}{T\_half}\right)_1 T\_half = 4.468000000 \times 10^9 \left(\frac{\ln(2)}{T\_half}\right)_1 \quad (9)$$

$k\_U235 := sol\_b[1]\, subs\left(T\_half = 706 \cdot 10^6\right)$

$$k\_U235 := \left(\frac{\ln(2)}{T\_half}\right)_1 T\_half = 706000000 \left(\frac{\ln(2)}{T\_half}\right)_1 \quad (10)$$

$ode\_d := diff(R(t\_d), t\_d) = -k\_d * R(t\_d)$

$$ode\_d := \frac{d}{dt\_d} R(t\_d) = -k\_d\, R(t\_d) \quad (11)$$

$sol\_d := dsolve(\{ode\_d, ic\_d\}, R(t\_d))$

$$sol\_d := R(t\_d) = R0\_d\, e^{-k\_d t\_d} \quad (12)$$

$R\_t\_d\_expr := rhs(sol\_d)$

$$R\_t\_d\_expr := R0\_d\, e^{-\left(-\frac{1.\ln(RootOf(10\,(R0\_d\_Z) \bullet \{R0\_d = 3,\, t\_d = 2\} - 9))}{t\_d}\right)_1 t\_d} \quad (13)$$

$k\_d := solve(R\_t\_d\_expr \bullet subs(\{t\_d = 2, R0\_d = 3\}) - 0.9, k\_d)[1]$

Warning, solving for expressions other than names or functions is not recommended.

$$k\_d := \left(-\frac{1.\ \ln(RootOf(10\ (R0\_d\ \_Z) \cdot \{R0\_d=3, t\_d=2\} - 9))}{t\_d} \right)_1 \quad (14)$$

$T\_half\_d := solve(eq\_b \cdot subs(k=k\_d), T\_half)\,[1]$

$$T\_half\_d := RootOf\left(\left(\_Z = \frac{\ln(2)}{k} \right) \cdot \left(k = \left(-\frac{1.\ \ln(RootOf(10\ (R0\_d\ \_Z) \cdot \{R0\_d=3, t\_d=2\} - 9))}{t\_d} \right)_1 \right) \right)_1 \quad (15)$$

$ode\_shroud := diff(R(t\_shroud), t\_shroud) = -k\_C14 * R(t\_shroud)$

$$ode\_shroud := \frac{d}{dt\_shroud} R(t\_shroud) = \left(-R(t\_shroud) \left(\frac{\ln(2)}{T\_half} \right)_1 T\_half = -5730\ R(t\_shroud) \left(\frac{\ln(2)}{T\_half} \right)_1 \right) \quad (16)$$

$ic\_shroud := R(0) = 0.93021 * R0\_shroud$

$$ic\_shroud := R(0) = 0.93021\ R0\_shroud \quad (17)$$

$sys\_shroud := \{ode\_shroud, ic\_shroud\}$

$$sys\_shroud := \left\{ R(0) = 0.93021\ R0\_shroud, \frac{d}{dt\_shroud} R(t\_shroud) = \left(-R(t\_shroud) \left(\frac{\ln(2)}{T\_half} \right)_1 T\_half = -5730\ R(t\_shroud) \left(\frac{\ln(2)}{T\_half} \right)_1 \right) \right\} \quad (18)$$

$sys\_shroud := \{ode\_shroud, ic\_shroud\}$

$$sys\_shroud := \left\{ R(0) = 0.93021\ R0\_shroud, \frac{d}{dt\_shroud} R(t\_shroud) = \left(-R(t\_shroud) \left(\frac{\ln(2)}{T\_half} \right)_1 T\_half = -5730\ R(t\_shroud) \left(\frac{\ln(2)}{T\_half} \right)_1 \right) \right\} \quad (19)$$

$sol\_shroud := dsolve(sys\_shroud, numeric)$

Warning, The use of global variables in numerical ODE problems is deprecated, and will be removed in a future release. Use the 'parameters' argument instead (see ?dsolve,numeric,parameters)

$$sol\_shroud := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \quad (20)$$

$sol\_shroud;$

$$sol\_shroud \quad (21)$$

$restart$

$ode\_thermal := diff(T(t), t) = k * (T(t) - TA)$

$$ode\_thermal := \frac{d}{dt} T(t) = k\ (T(t) - TA) \quad (22)$$

$ic\_thermal := T(0) = T0$

$$ic\_thermal := T(0) = T0 \quad (23)$$

$sol\_thermal := dsolve(\{ode\_thermal, ic\_thermal\}, T(t))$

$$sol\_thermal := T(t) = TA + e^{kt}\,(T0 - TA) \tag{24}$$

sol_thermal;

$$T(t) = TA + e^{kt}\,(T0 - TA) \tag{25}$$

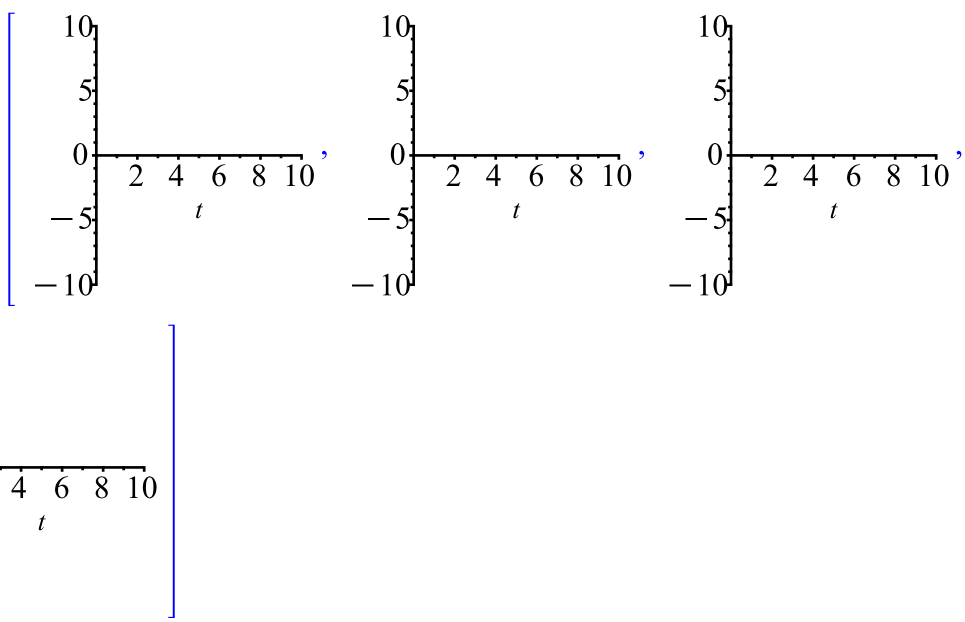
$T0\_values := [20, 25, 30, 35]$

$$T0\_values := [20, 25, 30, 35] \tag{26}$$

$local\_plots := [seq(plot(sol\_thermal.subs(T0 = val), t = 0 .. 10, legend = [\text{"T0 = "} + String(val)]), val = T0\_values)];$

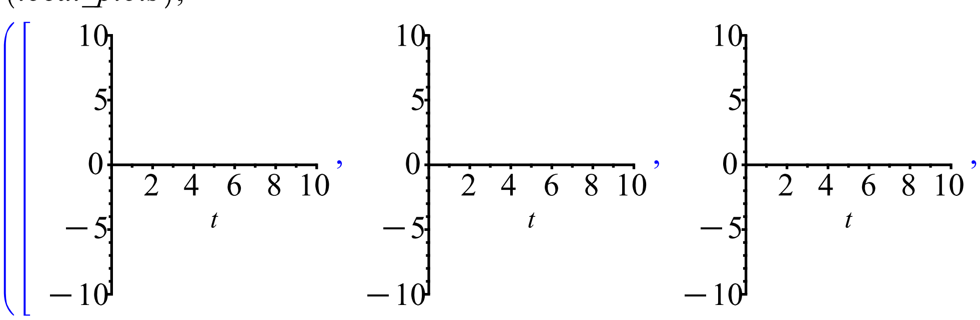
```
Warning, expecting only range variable t in expression (T(t) = TA+exp
(k*t)*(T0-TA)) . (T0 = 20) to be plotted but found names [T, T0, TA,
k]
Warning, expecting only range variable t in expression (T(t) = TA+exp
(k*t)*(T0-TA)) . (T0 = 25) to be plotted but found names [T, T0, TA,
k]
Warning, expecting only range variable t in expression (T(t) = TA+exp
(k*t)*(T0-TA)) . (T0 = 30) to be plotted but found names [T, T0, TA,
k]
Warning, expecting only range variable t in expression (T(t) = TA+exp
(k*t)*(T0-TA)) . (T0 = 35) to be plotted but found names [T, T0, TA,
k]
```

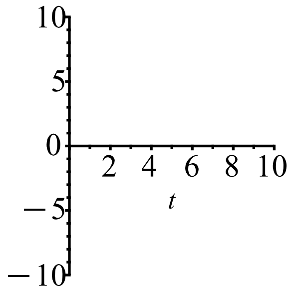
local_plots :=



display(*local_plots*);

display





restart

$ode\_malthus := diff(x(t\_malthus), t\_malthus) = r\_malthus * x(t\_malthus)$

$$ode\_malthus := \frac{d}{dt\_malthus}\, x(t\_malthus) = r\_malthus\, x(t\_malthus) \quad \textbf{(27)}$$

$ic\_malthus := x(0) = x0\_malthus$

$$ic\_malthus := x(0) = x0\_malthus \quad \textbf{(28)}$$

$sol\_malthus := dsolve(\{ode\_malthus, ic\_malthus\}, x(t\_malthus))$

$$sol\_malthus := x(t\_malthus) = x0\_malthus\, e^{r\_malthus\, t\_malthus} \quad \textbf{(29)}$$

$sol\_malthus;$

$$x(t\_malthus) = x0\_malthus\, e^{r\_malthus\, t\_malthus} \quad \textbf{(30)}$$

$ode\_verhulst := diff(x(t\_verhulst), t\_verhulst) = r0\_verhulst * x(t\_verhulst) * (1 - x(t\_verhulst) / K\_verhulst)$

$$ode\_verhulst := \frac{d}{dt\_verhulst}\, x(t\_verhulst) = r0\_verhulst\, x(t\_verhulst) \left(1 - \frac{x(t\_verhulst)}{K\_verhulst}\right) \quad \textbf{(31)}$$

$ic\_verhulst := x(0) = x0\_verhulst$

$$ic\_verhulst := x(0) = x0\_verhulst \quad \textbf{(32)}$$

$sol\_verhulst := dsolve(\{ode\_verhulst, ic\_verhulst\}, x(t\_verhulst))$

$$sol\_verhulst := x(t\_verhulst) = \frac{K\_verhulst\, x0\_verhulst}{e^{-r0\_verhulst\, t\_verhulst}\, K\_verhulst - e^{-r0\_verhulst\, t\_verhulst}\, x0\_verhulst + x0\_verhulst} \quad \textbf{(33)}$$

$sol\_verhulst;$

$$x(t\_verhulst) = \frac{K\_verhulst\, x0\_verhulst}{e^{-r0\_verhulst\, t\_verhulst}\, K\_verhulst - e^{-r0\_verhulst\, t\_verhulst}\, x0\_verhulst + x0\_verhulst} \quad \textbf{(34)}$$

$x\_malthus := x0\_malthus * \exp(r\_malthus * t\_malthus);$

$$x\_malthus := x0\_malthus\, e^{r\_malthus\, t\_malthus} \quad \textbf{(35)}$$

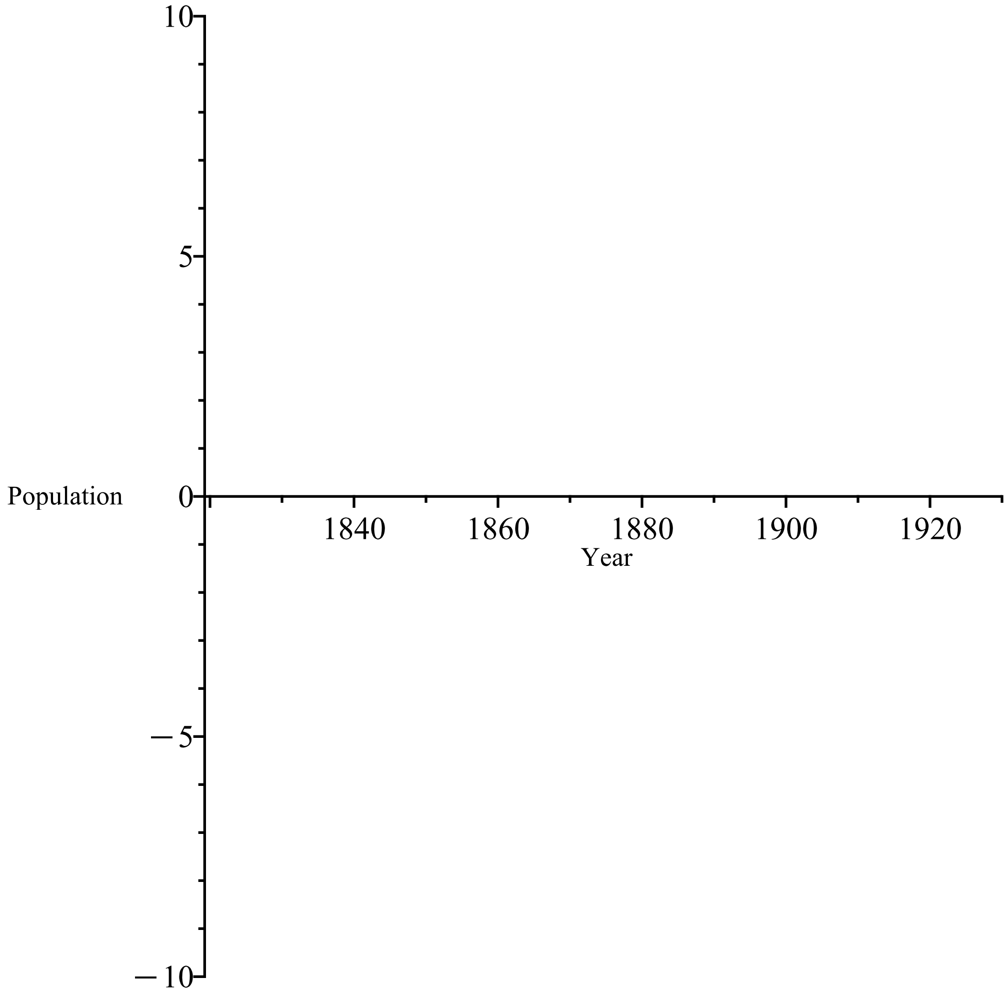
$x\_verhulst := K\_verhulst * x0\_verhulst / (\exp(-r0\_verhulst * t\_malthus) * K\_verhulst - \exp(-r0\_verhulst * t\_malthus) * x0\_verhulst + x0\_verhulst);$

$$x\_verhulst := \frac{K\_verhulst\, x0\_verhulst}{e^{-r0\_verhulst\, t\_malthus}\, K\_verhulst - e^{-r0\_verhulst\, t\_malthus}\, x0\_verhulst + x0\_verhulst} \quad \textbf{(36)}$$

plot([*x_malthus*, *x_verhulst*], *t_malthus* = 1820 .. 1930, *legend* = ["Malthus", "Verhulst"], *color* = [*blue*, *red*], *labels* = ["Year", "Population"]);

```
Warning, expecting only range variable t_malthus in expression
x0_malthus*exp(r_malthus*t_malthus) to be plotted but found names
```

[r_malthus, x0_malthus]



> $population\_data\_malthus := [1820 = 9.6e6, 1830 = 12.9e6, 1840 = 17.1e6, 1850 = 23.2e6, 1860 = 31.4e6, 1870 = 38.6e6, 1880 = 50.2e6, 1890 = 62.9e6, 1900 = 76e6, 1910 = 92e6, 1920 = 106.5e6, 1930 = 123.2e6]$

$$population\_data\_malthus := [1820 = 9.6 \times 10^6, 1830 = 1.29 \times 10^7, 1840 = 1.71 \times 10^7, 1850 = 2.32 \times 10^7, 1860 = 3.14 \times 10^7, 1870 = 3.86 \times 10^7, 1880 = 5.02 \times 10^7, 1890 = 6.29 \times 10^7, 1900 = 7.6 \times 10^7, 1910 = 9.2 \times 10^7, 1920 = 1.065 \times 10^8, 1930 = 1.232 \times 10^8] \quad \textbf{(37)}$$

$eq\_malthus := sol\_malthus \cdot rhs$

$$eq\_malthus := \left(x(t\_malthus) = x0\_malthus\ \mathrm{e}^{r\_malthus\ t\_malthus}\right) \cdot rhs \quad \textbf{(38)}$$

$params\_malthus := solve(\{seq(eq\_malthus(subs(t\_malthus = year)) = population, year, population$ **in** $population\_data\_malthus)\}, r\_malthus)$

Error, invalid input: seq expects its 3rd argument, step, to be of type numeric, but received population in [1820 = .96e7, 1830 = .129e8, 1840 = .171e8, 1850 = .232e8, 1860 = .314e8, 1870 = .386e8, 1880 = .502e8, 1890 = .629e8, 1900 = .76e8, 1910 = .92e8, 1920 = .1065e9, 1930 = .1232e9]