$$f:=x->x^2-2x$$

$$f \coloneqq x \mapsto x^2 - 2 \cdot x \tag{1}$$

 \rightarrow eqd:=diff(x(t),t)=f(x(t))

$$eqd := \frac{\mathrm{d}}{\mathrm{d}t} x(t) = x(t)^2 - 2x(t)$$
 (2)

> equip := solve(f(x) = 0, x)

$$equip := 0, 2 \tag{3}$$

> equip[1]

> D(f) (equip[1])

$$-2$$
 (5)

> D(f) (equip[2])

>
$$DEplot(eqd, x(t), t = -2..2, [x(0) = -3], [x(0) = -2], [x(0) = -1], [x(0) = -\frac{1}{2}], [x(0) = -\frac{1}{3}], [x(0) = 0], [x(0) = \frac{1}{3}], [x(0) = -\frac{1}{2}], [x(0) = 1], [x(0) = 2], [x(0) = 3]])$$

$$DEplot\left(\frac{d}{dt} x(t) = x(t)^2 - 2 x(t), x(t), t = -2 ...2, \left[[x(0) = -3], [x(0) = -2], [x(0) = -1], \right]$$
 (7)

$$\left[x(0) = -\frac{1}{2} \right], \left[x(0) = -\frac{1}{3} \right], \left[x(0) = 0 \right], \left[x(0) = \frac{1}{3} \right], \left[x(0) = -\frac{1}{2} \right], \left[x(0) = 1 \right],$$

> with(plots)

[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

>
$$DEplot(eqd, x(t), t = -2..2, [x(0) = -3], [x(0) = -2], [x(0) = -1], [x(0) = -\frac{1}{2}], [x(0) = -\frac{1}{2}], [x(0) = -\frac{1}{2}], [x(0) = -\frac{1}{2}], [x(0) = -1], [x(0) = -1],$$

$$DEplot\left(\frac{d}{dt}x(t) = x(t)^2 - 2x(t), x(t), t = -2..2, \left[x(0) = -3, x(0) = -2, x(0) = -1, x(0) =$$

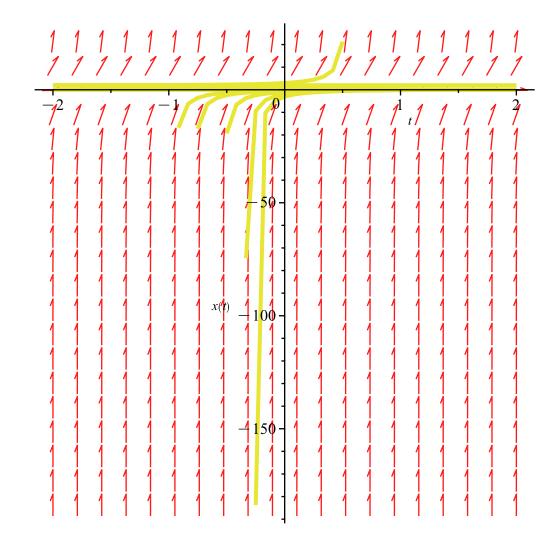
$$\left[x(0) = -\frac{1}{2}\right], \left[x(0) = -\frac{1}{3}\right], \left[x(0) = 0\right], \left[x(0) = \frac{1}{3}\right], \left[x(0) = -\frac{1}{2}\right], \left[x(0) = 1\right], \left[x(0) = \frac{1}{3}\right]$$

$$=2$$
], $[x(0)=3]$

> with(DEtools)

[AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot polygon, DFactor, DFactorLCLM, (10)DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff table, diffop2de, dperiodic sols, dpolyform, dsubs, eigenring, endomorphism charpoly, equiny, eta k, eulersols, exactsol, expsols, exterior power, firint, firtest, formal sol, gen exp, generate ic, genhomosol, gensys, hamilton eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate sols, intfactor, invariants, kovacicsols, leftdivision, liesol, line int, linearsol, matrixDE, matrix riccati, maxdimsystems, moser reduce, muchange, mult, mutest, newton polygon, normalG2, ode int y, ode y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power equivalent, rational equivalent, ratsols, redode, reduceOrder, reduce order, regular parts, regularsp, remove RootOf, riccati system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve group, super reduce, symgen, symmetric power, symmetric product, symtest, transinv, translate, untranslate, varparam, zoom

> $DEplot(eqd, x(t), t = -2..2, [x(0) = -3], [x(0) = -2], [x(0) = -1], |x(0) = -\frac{1}{2}], |x(0) = -\frac{1}{2}]$ $-\frac{1}{3}$, [x(0) = 0], $[x(0) = \frac{1}{3}]$, $[x(0) = -\frac{1}{2}]$, [x(0) = 1], [x(0) = 2], [x(0) = 3]Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further left of -.25541281, probably a singularity Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further left of -.34657359, probably a singularity Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further left of -.54930616, probably a singularity Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further left of -.97295514, probably a singularity Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further left of -.80471900, probably a singularity Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further right of .54930618, probably a singularity



$$f := x \rightarrow x \cdot (x-1) \cdot (x-2)$$

$$f := x \mapsto x \cdot (x-1) \cdot (x-2)$$
(11)

> deq := diff(x(t), t) = f(x(t))

$$deq := \frac{d}{dt} x(t) = x(t) (x(t) - 1) (x(t) - 2)$$
 (12)

 \triangleright equip := solve(f(x) = 0, x)

$$equip := 0, 1, 2 \tag{13}$$

> equip[1]

> D(f) (equip[1])

> D(f) (equip[2])

$$-1 \tag{16}$$

> D(f) (equip[3])

>
$$DEplot(deq, x(t), t = -2..2, [x(0) = -3], [x(0) = -2], [x(0) = -1], [x(0) = -\frac{1}{2}], [x(0) = -\frac{1}{2}]$$

 $-\frac{1}{3}$, [x(0) = 0], $[x(0) = \frac{1}{3}]$, $[x(0) = -\frac{1}{2}]$, [x(0) = 1], [x(0) = 2], [x(0) = 3]

Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further right of .32269256e-1, probably a singularity

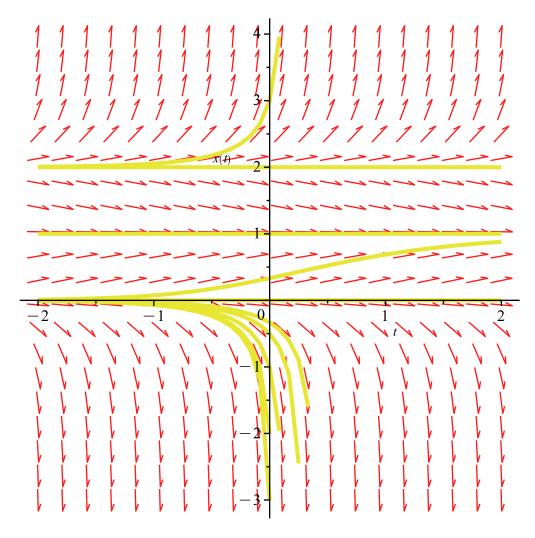
Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further right of .58891510e-1, probably a singularity

Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further right of .14384102, probably a singularity

Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further right of .41333927, probably a singularity

Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further right of .29389331, probably a singularity

Warning, plot may be incomplete, the following errors(s) were issued: cannot evaluate the solution further right of .14384100, probably a singularity



$$f := x \mapsto \sin(x) \tag{18}$$

$$eqd := \frac{\mathrm{d}}{\mathrm{d}t} \ x(t) = \sin(x(t))$$
 (19)

> equip :=
$$solve(f(x) = 0)$$

$$equip := 0 (20)$$

$$EnvAllSolutions := true$$
 (21)

> equip :=
$$solve(f(x) = 0)$$

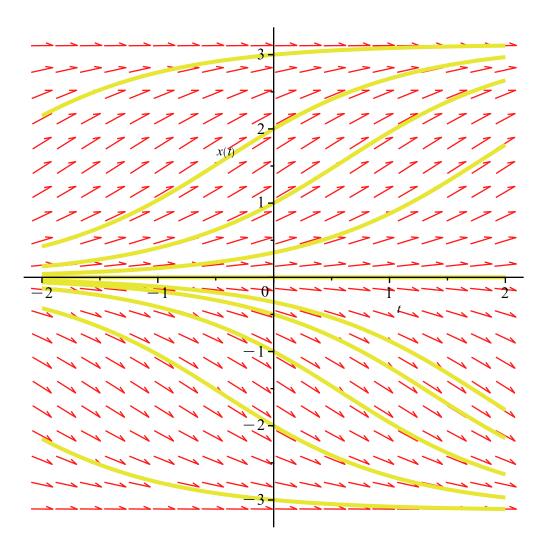
$$equip := \pi_Z I \sim \tag{22}$$

$$(\pi_{Z}I^{\sim})_{1}$$
 (23)

$$(\pi _ZI^{\sim})_{\gamma} \tag{24}$$

$$\triangleright$$
 D(f) (equip)

$$(-1)^{-ZI^{\sim}} \tag{25}$$



> with(linalg)

[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian]

> $eq1 := diff(x(t), t) = 2 \cdot x(t) + y(t)$ $eq1 := \frac{d}{dt} x(t) = 2 x(t) + y(t)$ (27)

>
$$eq2 := diff(y(t), t) = x(t) + 2 \cdot y(t)$$

 $eq2 := \frac{d}{dt} y(t) = x(t) + 2 y(t)$ (28)

 \rightarrow sist := eq1, eq2

$$sist := \frac{d}{dt} x(t) = 2 x(t) + y(t), \frac{d}{dt} y(t) = x(t) + 2 y(t)$$
 (29)

 \rightarrow $A := matrix(\lceil \lceil 2, 1 \rceil, \lceil 1, 2 \rceil \rceil)$

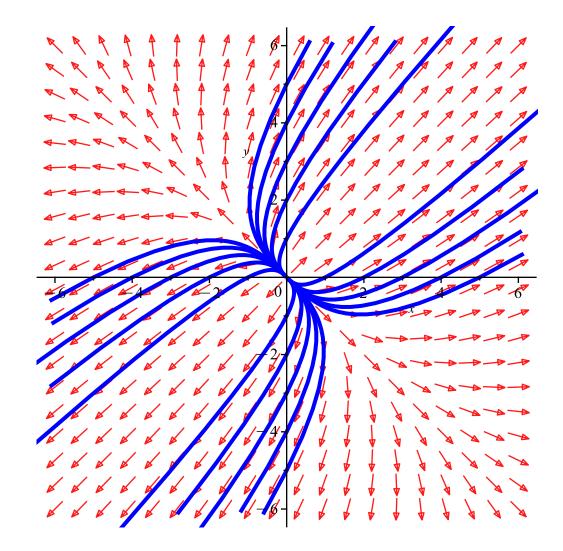
$$A := \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{30}$$

> eigenvals(A)

> $in_cond := [x(0) = 0, y(0) = i]$ \$i = 1..5, [x(0) = -i, y(0) = 0]\$i = 1..5, [x(0) = 0, y(0) = -i]\$i = 1..5, [x(0) = i, y(0) = 0]\$i = 1..5

$$in_cond := [x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 4], [x(0) = 0, y(0) = 5], [x(0) = -1, y(0) = 0], [x(0) = -2, y(0) = 0], [x(0) = -3, y(0) = 0], [x(0) = -4, y(0) = 0], [x(0) = -5, y(0) = 0], [x(0) = 0, y(0) = -1], [x(0) = 0, y(0) = -2], [x(0) = 0, y(0) = -3], [x(0) = 0, y(0) = -4], [x(0) = 0, y(0) = -5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0]$$

> $DEplot([sist], [x(t), y(t)], t=-5..5, x=-6..6, y=-6..6, [in_cond], arrows = medium, linecolor = blue, stepsize = 0.1)$



>
$$eq1 := diff(x(t), t) = -3 \cdot x(t) + 4 \cdot y(t)$$

 $eq1 := \frac{d}{dt} x(t) = -3 x(t) + 4 y(t)$ (33)

$$eq2 := diff(y(t), t) = -2 \cdot x(t) + 3 \cdot y(t)$$

$$eq2 := \frac{d}{dt} y(t) = -2 x(t) + 3 y(t)$$
(34)

 \rightarrow sist := eq1, eq2

$$sist := \frac{d}{dt} x(t) = -3 x(t) + 4 y(t), \frac{d}{dt} y(t) = -2 x(t) + 3 y(t)$$
 (35)

> A := matrix([[-3, 4], [-2, 3]])

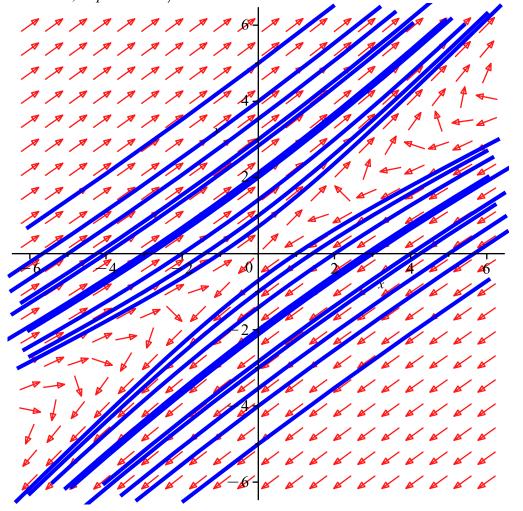
$$A := \begin{bmatrix} -3 & 4 \\ -2 & 3 \end{bmatrix} \tag{36}$$

 \Rightarrow eigenvals(A) 1, -1 (37)

> $in_cond := [x(0) = 0, y(0) = i]$ \$\(i = 1 \tdots 5, [x(0) = -i, y(0) = 0] \$\\$ i = 1 \tdots 5, [x(0) = 0, y(0) = -i] \$\\$ i = 1 \tdots 5, [x(0) = i, y(0) = 0] \$\\$ i = 1 \tdots 5 \\
 in_cond := [x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 3], (38)

$$=4], [x(0) = 0, y(0) = 5], [x(0) = -1, y(0) = 0], [x(0) = -2, y(0) = 0], [x(0) = -3, y(0) = 0], [x(0) = -4, y(0) = 0], [x(0) = -5, y(0) = 0], [x(0) = 0, y(0) = -1], [x(0) = 0, y(0) = -2], [x(0) = 0, y(0) = -3], [x(0) = 0, y(0) = -4], [x(0) = 0, y(0) = -5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0]$$

> $DEplot([sist], [x(t), y(t)], t=-5...5, x=-6...6, y=-6...6, [in_cond], arrows = medium, linecolor = blue, stepsize = 0.1)$



>
$$eq2 := diff(y(t), t) = x(t) - 3 \cdot y(t)$$

 $eq2 := \frac{d}{dt} y(t) = x(t) - 3 y(t)$ (40)

 \Rightarrow sist := eq1, eq2

$$sist := \frac{d}{dt} x(t) = -x(t) - y(t), \frac{d}{dt} y(t) = x(t) - 3 y(t)$$
 (41)

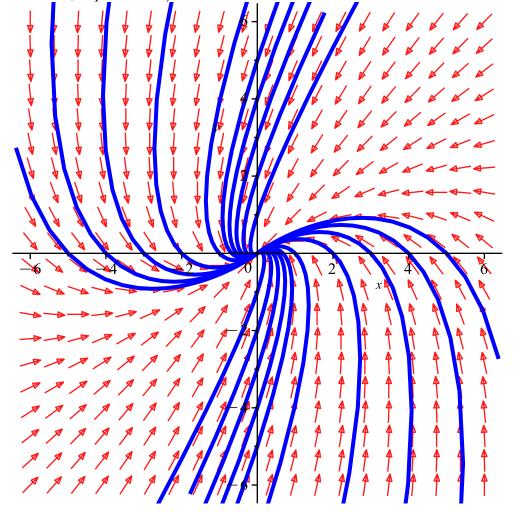
A := matrix([[-1,-1],[1,-3]])

$$A := \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix} \tag{42}$$

> eigenvals(A)

$$-2, -2$$
 (43)

> $DEplot([sist], [x(t), y(t)], t=-5...5, x=-6...6, y=-6...6, [in_cond], arrows = medium, linecolor = blue, stepsize = 0.1)$



$$eq1 := \frac{\mathrm{d}}{\mathrm{d}t} \ x(t) = -2 \ x(t) \tag{44}$$

>
$$eq2 := diff(y(t), t) = -4 \cdot x(t) - 2 \cdot y(t)$$

$$eq2 := \frac{d}{dt} y(t) = -4 x(t) - 2 y(t)$$
 (45)

 \triangleright sist := eq1, eq2

$$sist := \frac{d}{dt} x(t) = -2 x(t), \frac{d}{dt} y(t) = -4 x(t) - 2 y(t)$$
 (46)

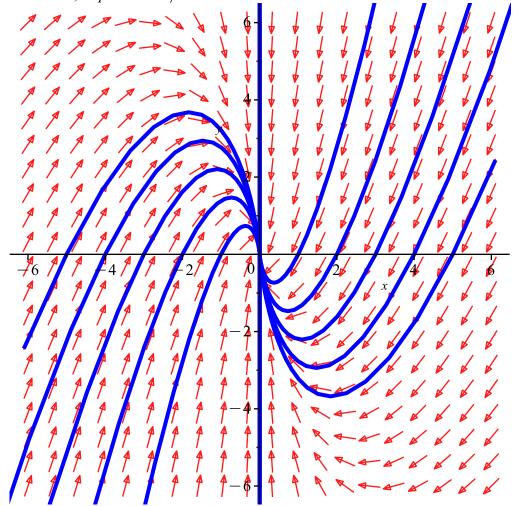
A := matrix([[-2, 0], [-4, -2]])

*(*47)

$$A := \left[\begin{array}{cc} -2 & 0 \\ -4 & -2 \end{array} \right] \tag{47}$$

 \Rightarrow eigenvals(A) -2, -2 (48)

> $DEplot([sist], [x(t), y(t)], t=-5...5, x=-6...6, y=-6...6, [in_cond], arrows = medium, linecolor = blue, stepsize = 0.1)$



>
$$eq1 := diff(x(t), t) = x(t) + 4 \cdot y(t)$$

$$eq1 := \frac{d}{dt} x(t) = x(t) + 4 y(t)$$
(49)

> eq2 := diff(y(t), t) = x(t) + y(t)

$$eq2 := \frac{d}{dt} y(t) = x(t) + y(t)$$
 (50)

 \triangleright sist := eq1, eq2

$$sist := \frac{d}{dt} x(t) = x(t) + 4 y(t), \frac{d}{dt} y(t) = x(t) + y(t)$$
 (51)

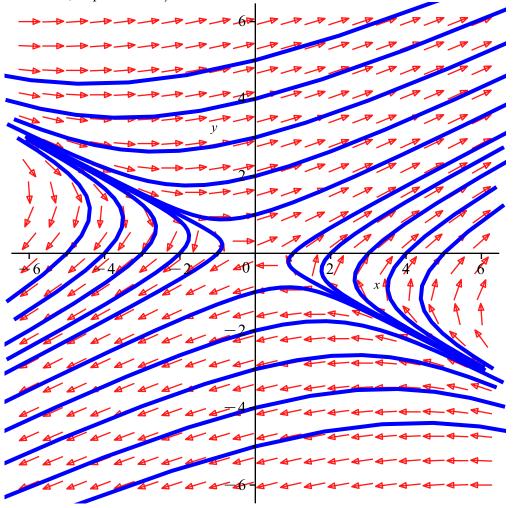
A := matrix([[1, 4], [1, 1]])

$$A := \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \tag{52}$$

> eigenvals(A)

$$3, -1$$
 (53)

> $DEplot([sist], [x(t), y(t)], t=-5...5, x=-6...6, y=-6...6, [in_cond], arrows = medium, linecolor = blue, stepsize = 0.1)$



>
$$eq1 := diff(x(t), t) = 2 \cdot x(t) - y(t)$$

$$eqI := \frac{d}{dt} x(t) = 2 x(t) - y(t)$$
 (54)

$$eq2 := \frac{d}{dt} y(t) = x(t) + 2 y(t)$$
 (55)

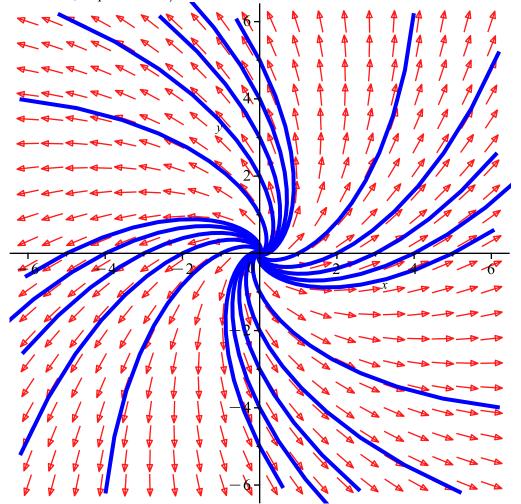
 \rightarrow sist := eq1, eq2

$$sist := \frac{d}{dt} x(t) = 2 x(t) - y(t), \frac{d}{dt} y(t) = x(t) + 2 y(t)$$
 (56)

 $\overline{\triangleright}$ A := matrix([[2,-1],[1,2]])

$$A := \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \tag{57}$$

> $DEplot([sist], [x(t), y(t)], t=-5...5, x=-6..6, y=-6...6, [in_cond], arrows = medium, linecolor = blue, stepsize = 0.1)$



$$eq1 := \frac{\mathrm{d}}{\mathrm{d}t} \ x(t) = -y(t)$$
 (59)

$$eq2 := \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = x(t)$$
 (60)

$$\Rightarrow$$
 sist := eq1, eq2

$$sist := \frac{\mathrm{d}}{\mathrm{d}t} \ x(t) = -y(t), \ \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = x(t)$$

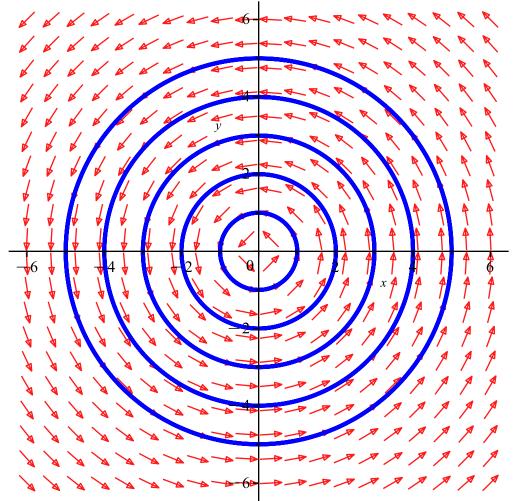
 $\overline{\triangleright}$ A := matrix([[0,-1],[1,0]])

.

$$A := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{62}$$

 $\rightarrow eigenvals(A)$ I, -I (63)

> $DEplot([sist], [x(t), y(t)], t=-5..5, x=-6..6, y=-6..6, [in_cond], arrows = medium, linecolor = blue, stepsize = 0.1)$



>
$$eq1 := diff(x(t), t) = x(t) - 4 \cdot y(t)$$

$$eq1 := \frac{d}{dt} x(t) = x(t) - 4 y(t)$$
(64)

$$eq2 := diff(y(t), t) = 5 \cdot x(t) - 3 \cdot y(t)$$

$$eq2 := \frac{d}{dt} y(t) = 5 x(t) - 3 y(t)$$
(65)

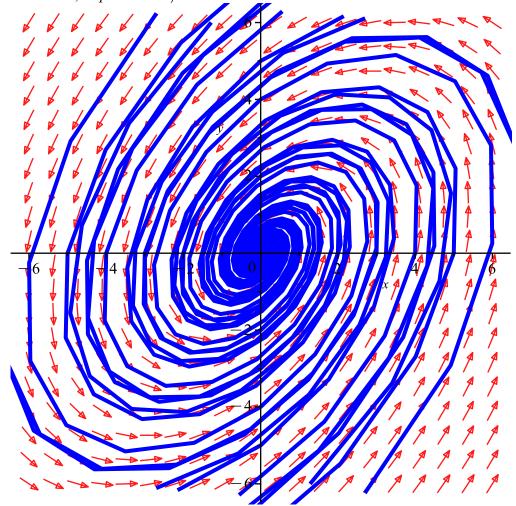
> sist := eq1, eq2 $sist := \frac{d}{dt} x(t) = x(t) - 4 y(t), \frac{d}{dt} y(t) = 5 x(t) - 3 y(t)$ (66)

> A := matrix([[1,-4],[5,-3]])

. .___

$$A := \begin{bmatrix} 1 & -4 \\ 5 & -3 \end{bmatrix} \tag{67}$$

> $DEplot([sist], [x(t), y(t)], t = -5...5, x = -6...6, y = -6...6, [in_cond], arrows = medium, linecolor = blue, stepsize = 0.1)$



$$fl := (x, y) \mapsto y \tag{69}$$

$$eq1 := diff(x(t), t) = fI(x(t), y(t))$$

$$eq1 := \frac{d}{dt} x(t) = y(t)$$

$$(71)$$

$$eq2 := diff(y(t), t) = f2(x(t), y(t))$$

$$eq2 := \frac{d}{dt} y(t) = x(t) (1 - x(t)^2) + y(t)$$
(72)

$$\Rightarrow sist2 := eq1, eq2$$

$$sist2 := \frac{d}{dt} x(t) = y(t), \frac{d}{dt} y(t) = x(t) (1 - x(t)^2) + y(t)$$
 (73)

>
$$EquiP := solve(\{fI(x, y) = 0, f2(x, y) = 0\}, \{x, y\})$$

 $EquiP := \{x = 0, y = 0\}, \{x = 1, y = 0\}, \{x = -1, y = 0\}$ (74)

> *EquiP*[1, 1]

$$x = 0 \tag{75}$$

> *EquiP*[1, 2]

$$y = 0 (76)$$

 $\rightarrow J := jacobian([fl(x,y),f2(x,y)],[x,y])$

$$J := \begin{bmatrix} 0 & 1 \\ -3x^2 + 1 & 1 \end{bmatrix}$$
 (77)

> A1 := subs(EquiP[1, 1], EquiP[1, 2], y = 0, eval(J))

$$AI := \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \tag{78}$$

> eigenvals(A1)

$$\frac{1}{2} + \frac{\sqrt{5}}{2}, \frac{1}{2} - \frac{\sqrt{5}}{2} \tag{79}$$

 \rightarrow A2 := subs(EquiP[2, 1], EquiP[2, 2], eval(J))

$$A2 := \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} \tag{80}$$

> eigenvals(A2)

$$\frac{1}{2} + \frac{I\sqrt{7}}{2}, \frac{1}{2} - \frac{I\sqrt{7}}{2}$$
 (81)

 \rightarrow A3 := subs(EquiP[3, 1], EquiP[3, 2], eval(J))

$$A3 := \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} \tag{82}$$

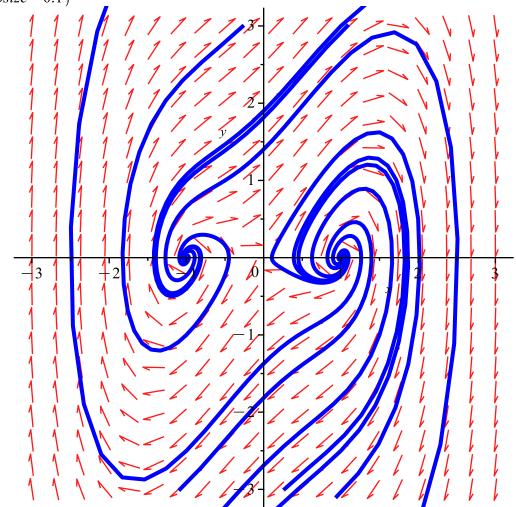
> eigenvals(A3)

$$\frac{1}{2} + \frac{I\sqrt{7}}{2}, \frac{1}{2} - \frac{I\sqrt{7}}{2}$$
 (83)

> condin :=
$$[x(0) = -1, y(0) = 1]$$
, $[x(0) = -0.5, y(0) = 1]$, $[x(0) = 1, y(0) = 1]$, $[x(0) = 1, y(0) = 1]$, $[x(0) = 1, y(0) = 1]$, $[x(0) = -0.5, y(0) = 1]$, $[x(0) = -0.5, y(0) = 1]$, $[x(0) = -1, y(0) = -2.5]$, $[x(0) = 1, y(0) = -1]$, $[x(0) = 1.5, y(0) = -1]$, $[x(0) = 1, y(0) = -2.5]$;

condin :=
$$[x(0) = -1, y(0) = 1]$$
, $[x(0) = -0.5, y(0) = 1]$, $[x(0) = 1, y(0) = 1]$, $[x(0) = -0.5, y(0) = 1]$, $[x(0) = -1, y(0) = -1]$, $[x(0) = -1, y(0) = -1]$, $[x(0) = 1, y(0) = -1]$, $[x(0) =$

> DEplot([sist2], [x(t), y(t)], t = -10..10, x = -3..3, y = -3..3, [condin], linecolor = blue,stepsize = 0.1)



$$f2 := (x, y) \mapsto y \cdot x \tag{86}$$

ightharpoonup eq1 := diff(x(t), t) = fI(x(t), y(t))

$$eq1 := \frac{d}{dt} x(t) = -2 x(t) + y(t) + 2$$
 (87)

 $\overline{\hspace{-1em}}$ > eq2 := diff(y(t), t) = f2(x(t), y(t))

$$eq2 := \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = y(t) \ x(t)$$
 (88)

 \rightarrow sist2 := eq1, eq2

$$sist2 := \frac{d}{dt} x(t) = -2 x(t) + y(t) + 2, \frac{d}{dt} y(t) = y(t) x(t)$$
 (89)

From EquiP :=
$$solve(\{fl(x, y) = 0, f2(x, y) = 0\}, \{x, y\})$$

$$EquiP := \{x = 0, y = 0\}$$
(90)

>
$$J := jacobian([f1(x, y), f2(x, y)], [x, y])$$

$$J := \begin{bmatrix} 0 & 2y \\ 1 & 0 \end{bmatrix}$$
(91)

 \rightarrow A1 := subs(EquiP[1], EquiP[2], eval(J))

$$A1 := \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{92}$$

>
$$f1 := (x, y) \rightarrow y^2$$

$$fI := (x, y) \mapsto y^2 \tag{93}$$

$$> f2 := (x, y) \rightarrow x$$

$$f2 := (x, y) \mapsto x \tag{94}$$

$$eq1 := \frac{\mathrm{d}}{\mathrm{d}t} x(t) = y(t)^2$$
 (95)

$$eq2 := \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = x(t)$$
 (96)

$$sist2 := \frac{d}{dt} x(t) = y(t)^2, \frac{d}{dt} y(t) = x(t)$$
 (97)

FequiP := $solve(\{f1(x, y) = 0, f2(x, y) = 0\}, \{x, y\})$ $EquiP := \{x = 0, y = 0\}$

$$EquiP := \{x = 0, y = 0\}$$
 (98)

J := jacobian([f1(x, y), f2(x, y)], [x, y])

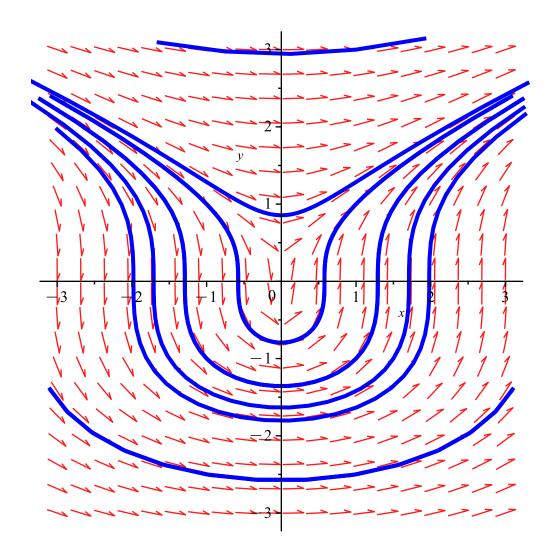
$$J := \begin{bmatrix} 0 & 2y \\ 1 & 0 \end{bmatrix} \tag{99}$$

 \rightarrow A1 := subs(EquiP[1], EquiP[2], eval(J))

$$AI := \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{100}$$

> eigenvals(A1)

> DEplot([sist2], [x(t), y(t)], t = -10..10, x = -3..3, y = -3..3, [condin], linecolor = blue,stepsize = 0.1)



$$f2 := (x, y) \mapsto y \cdot x \tag{103}$$

>
$$eq1 := diff(x(t), t) = fI(x(t), y(t))$$

$$eq1 := \frac{d}{dt} x(t) = -2 x(t) + y(t) + 2$$
 (104)

>
$$eq2 := diff(y(t), t) = f2(x(t), y(t))$$

$$eq2 := \frac{\mathrm{d}}{\mathrm{d}t} \ y(t) = y(t) \ x(t)$$
 (105)

$$\rightarrow sist2 := eq1, eq2$$

$$sist2 := \frac{d}{dt} x(t) = -2 x(t) + y(t) + 2, \frac{d}{dt} y(t) = y(t) x(t)$$
 (106)

>
$$EquiP := solve(\{f1(x, y) = 0, f2(x, y) = 0\}, \{x, y\})$$

 $EquiP := \{x = 1, y = 0\}, \{x = 0, y = -2\}$ (107)

>
$$J := jacobian([f1(x, y), f2(x, y)], [x, y])$$

$$J := \begin{bmatrix} -2 & 1 \\ y & x \end{bmatrix} \tag{108}$$

A1 := subs(EquiP[1, 1], EquiP[1, 2], eval(J))

$$AI := \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} \tag{109}$$

> eigenvals(A1)

$$-2, 1$$
 (110)

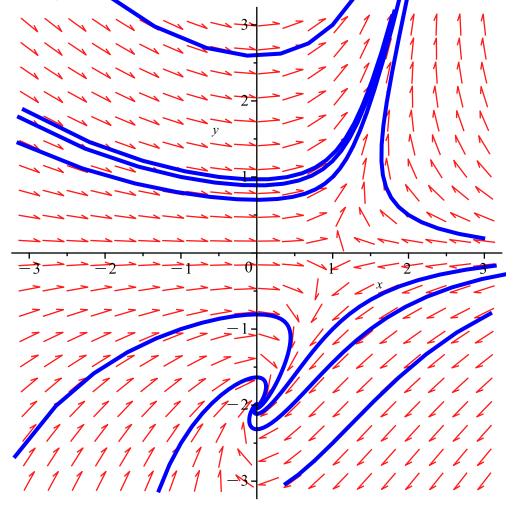
 \rightarrow A2 := subs(EquiP[2, 1], EquiP[2, 2], eval(J))

$$A2 := \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix} \tag{111}$$

> eigenvals(A2)

$$-1 + I, -1 - I$$
 (112)

> DEplot([sist2], [x(t), y(t)], t = -10..10, x = -3..3, y = -3..3, [condin], linecolor = blue, stepsize = 0.1)



 $fl := (x, y) \rightarrow x^2 - y^2$

$$fI := (x, y) \mapsto x^2 - y^2$$
 (113)

$$f2 := (x, y) \mapsto y \cdot x - 1 \tag{114}$$

$$eq1 := \frac{d}{dt} x(t) = x(t)^2 - y(t)^2$$
 (115)

$$eq2 := \frac{d}{dt} y(t) = y(t) x(t) - 1$$
 (116)

 $\Rightarrow sist2 := eq1, eq2$

$$sist2 := \frac{d}{dt} x(t) = x(t)^2 - y(t)^2, \frac{d}{dt} y(t) = y(t) x(t) - 1$$
 (117)

> EquiP := solve($\{fI(x, y) = 0, f2(x, y) = 0\}, \{x, y\}$)

$$EquiP := \{x = -RootOf(Z^2 + 1), y = RootOf(Z^2 + 1)\}, \{x = 1, y = 1\}, \{x = -1, y = -1\}$$
 (118)

 $\rightarrow J := jacobian([f1(x,y), f2(x,y)], [x,y])$

$$J \coloneqq \begin{bmatrix} 2x & -2y \\ y & x \end{bmatrix} \tag{119}$$

 \rightarrow A1 := subs(EquiP[1, 1], EquiP[1, 2], eval(J))

$$A1 := \begin{bmatrix} -2 \operatorname{RootOf}(\underline{Z}^2 + 1) & -2 \operatorname{RootOf}(\underline{Z}^2 + 1) \\ \operatorname{RootOf}(\underline{Z}^2 + 1) & -\operatorname{RootOf}(\underline{Z}^2 + 1) \end{bmatrix}$$
(120)

> eigenvals(A1)

$$RootOf(3 RootOf(_Z^2 + 1) _Z + _Z^2 - 4)$$
 (121)

A2 := subs(EquiP[2, 1], EquiP[2, 2], eval(J))

$$A2 := \begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix} \tag{122}$$

 \rightarrow eigenvals (A2)

$$\frac{3}{2} + \frac{I\sqrt{7}}{2}, \frac{3}{2} - \frac{I\sqrt{7}}{2}$$
 (123)

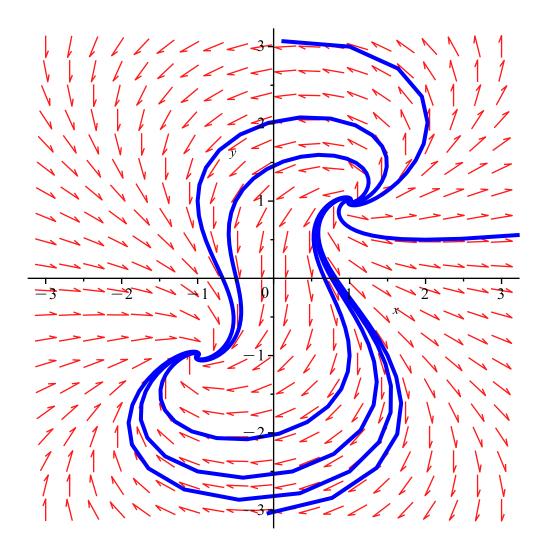
> A3 := subs(EquiP[3, 1], EquiP[3, 2], eval(J))

$$A3 := \begin{bmatrix} -2 & 2 \\ -1 & -1 \end{bmatrix} \tag{124}$$

> eigenvals(A3)

$$-\frac{3}{2} + \frac{I\sqrt{7}}{2}, -\frac{3}{2} - \frac{I\sqrt{7}}{2}$$
 (125)

DEplot([sist2], [x(t), y(t)], t = -10..10, x = -3..3, y = -3..3, [condin], linecolor = blue,



>
$$fI := (x, y) \rightarrow 2 \cdot x - 1.2 \cdot x \cdot y$$

$$fI := (x, y) \mapsto 2 \cdot x + (-1) \cdot 1.2 \cdot x \cdot y$$
 (126)

>
$$f2 := (x, y) \rightarrow -y + 0.9 \cdot x \cdot y$$

$$f2 := (x, y) \mapsto -y + 0.9 \cdot x \cdot y$$
 (127)

$$\overline{}$$
 > $eq1 := diff(x(t), t) = fI(x(t), y(t))$

$$eq1 := \frac{d}{dt} x(t) = 2 x(t) - 1.2 y(t) x(t)$$
 (128)

>
$$eq2 := diff(y(t), t) = f2(x(t), y(t))$$

$$eq2 := \frac{d}{dt} y(t) = -y(t) + 0.9 y(t) x(t)$$
 (129)

$$\rightarrow$$
 sist := eq1, eq2

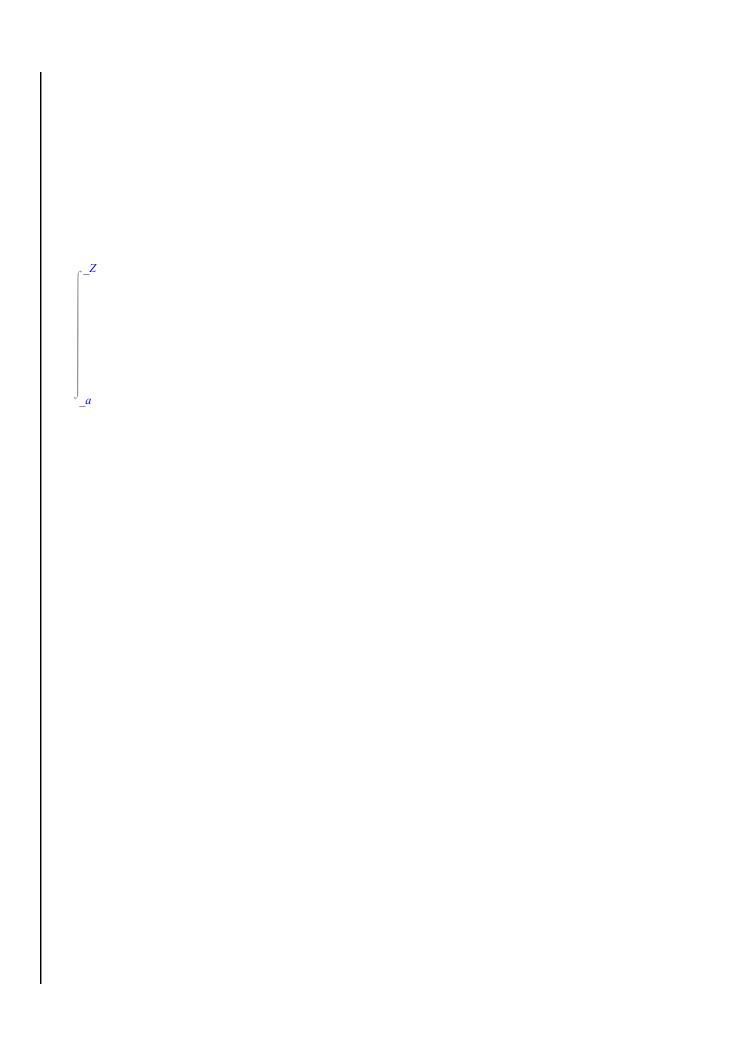
$$sist := \frac{d}{dt} x(t) = 2 x(t) - 1.2 y(t) x(t), \frac{d}{dt} y(t) = -y(t) + 0.9 y(t) x(t)$$
 (130)

>
$$in_cond := x(0) = 0.5, y(0) = 2$$

$$in_cond := x(0) = 0.5, y(0) = 2$$
 (131)

 $\gt sol := dsolve(\{sist, in_cond\}, \{x(t), y(t)\})$

sol := x(t) = RootOf -2(132)



$$\begin{pmatrix}
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) - Z2 \sim}{\frac{31}{2 e^{40}}} \right) + 6 \right) e^{\frac{9 f}{20}} \\
e^{-1} RootOf \left(Z4 \sim, \frac{24 \sim, \frac{31}{2 e^{40}} - 24 \sim}{\frac{31}{2 e^{40}}} \right) + 6 \right) e^{\frac{9 f}{20}}
\end{pmatrix}$$

$$\left(-1 \right)^{-Z2 \sim}$$

$$\left(e^{-1} RootOf \left(Z \left(5 \text{ LambertW} \left(Z4 \sim, \frac{\sqrt{2} Z(-1)^{-Z2 \sim}}{2 e^{\frac{31}{40}}} \right) + 6 \right) \right) e^{\frac{9 f}{20}} \sqrt{f} \right)$$

$$d_{f} = \int_{a}^{\frac{1}{2}} \left(\int_{a}^{d} \left(\int_$$

$$\frac{RootOf\left(Z\left(5 \text{ LambertW}\left(Z4\sim, \frac{\sqrt{2} Z(-1)^{-Z2\sim}}{2 e^{\frac{31}{40}}} \right) + 6\right) e^{\frac{9 f}{20}}}{2 e \sqrt{f} (-1)^{-Z2\sim}} + 1\right)}{2 e \sqrt{f} (-1)^{-Z2\sim}} + 1$$

$$y(t) = - \left[5 e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1)^{-Z2 \sim}}{2 e^{\frac{31}{40}}} \right) \right) \right]$$

+ 6

$$e^{\frac{1}{20}\left[9RootOf-2\right]}$$

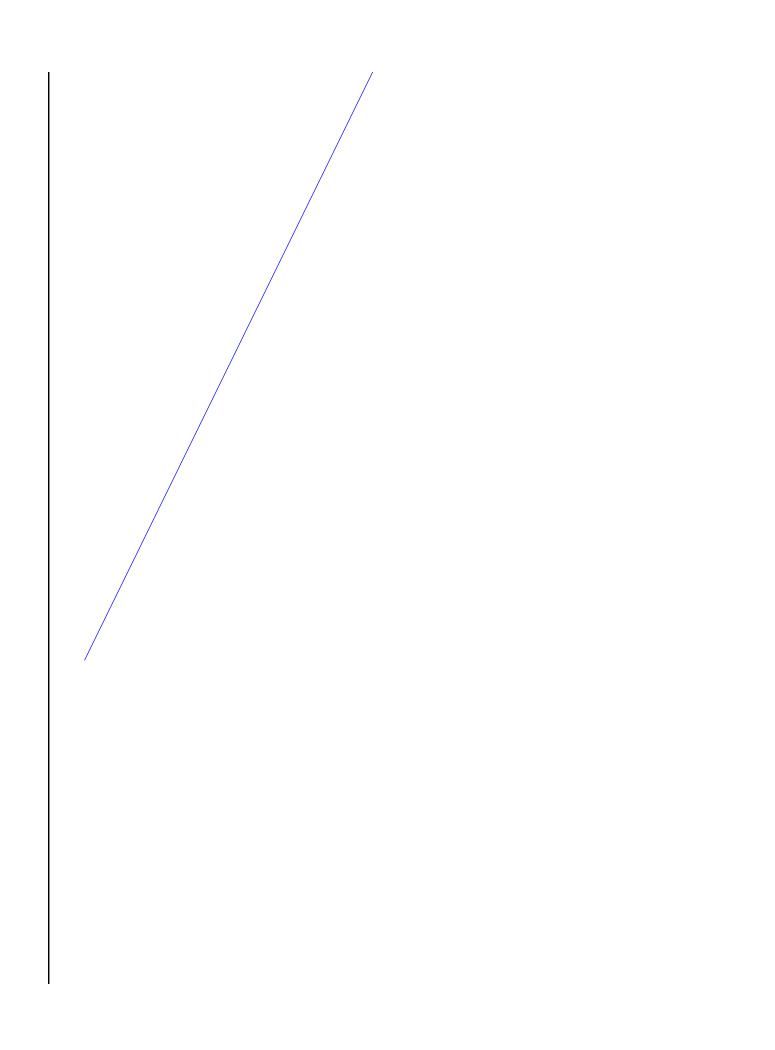
$$\int_{a}^{Z} \left(\operatorname{LambertW} \left(\underbrace{Z4\sim, \frac{\sqrt{2} Z(-1) Z2\sim}{31}}_{2 e^{40}} \right) + 6 \right) e^{\frac{9 f}{20}}$$

$$= \frac{2 \left(\operatorname{LambertW} \left(\underbrace{Z4\sim, \frac{\sqrt{2} Z(-1) Z2\sim}{31}}_{2 e^{40}} \right) + 6 \right) e^{\frac{9 f}{20}}$$

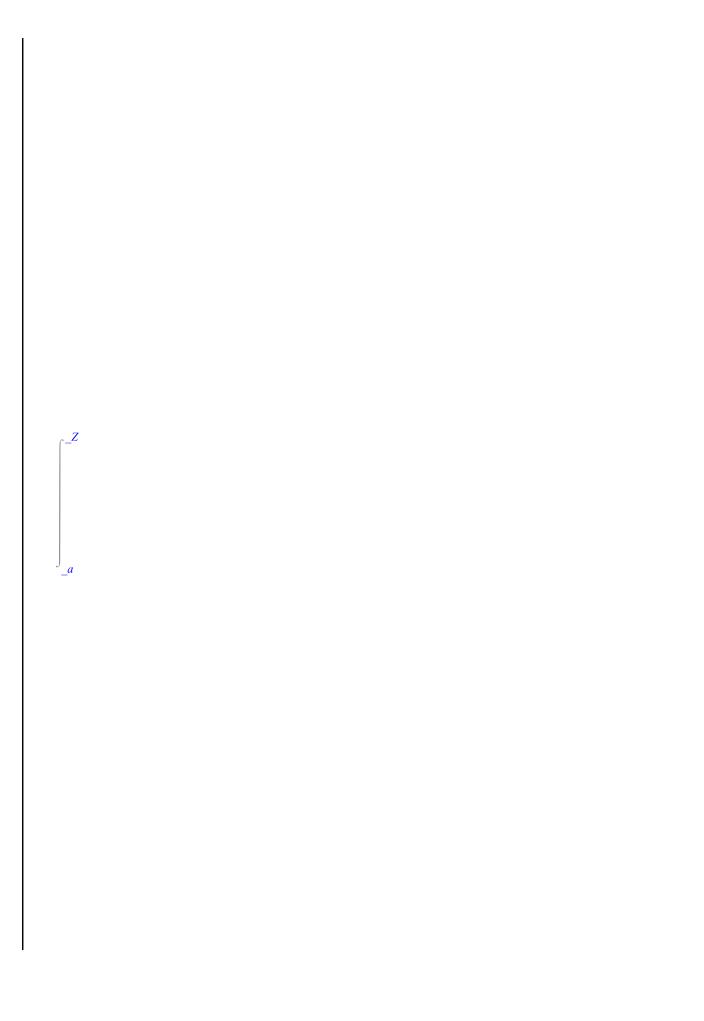
$$= \frac{2 \sqrt{f} (-1) Z2\sim}{2 \sqrt{f} (-1) Z2\sim}$$

$$\left(e^{-1} RootOf\left(Z\left(5 LambertW\left(Z4\sim, \frac{\sqrt{2} Z(-1)Z^{2}\sim}{\frac{31}{2e^{40}}}\right) + 6\right)\right)e^{\frac{9}{20}}\sqrt{f}$$

$$+ \int_{\underline{a}} \frac{1}{\int_{\underline{a}} \left(\operatorname{LambertW} \left(\underbrace{Z4\sim, \frac{\sqrt{2} \underline{Z(-1)} \underline{Z2\sim}}{\frac{31}{2 e^{\frac{31}{40}}}} + 6 \right) e^{\frac{9\underline{f}}{20}} \right) + 1} d\underline{f} + 2t \right) \right)}$$



$$6 \left| RootOf \right| -2$$



$$\begin{pmatrix}
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{9 f}{20}} \\
LambertW \left(Z4 \sim, \frac{31}{2 e^{40}} \right) + 6 \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) e^{\frac{2 f}{20}} \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) e^{\frac{2 f}{20}} \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) e^{\frac{2 f}{20}} \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) e^{\frac{2 f}{20}} \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(2 LambertW \left(Z4 \sim, \frac{\sqrt{2} Z(-1) Z2 \sim}{31} \right) + 6 \right) e^{\frac{2 f}{20}} \\
e^{-1} RootOf \left(2 LambertW \left(Z4$$

$$\left| \left(e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4\sim, \frac{\sqrt{2} Z(-1)^{-Z2\sim}}{2 e^{\frac{31}{40}}} \right) + 6 \right) \right) e^{\frac{9 f}{20}} \sqrt{f} \right| \right|$$

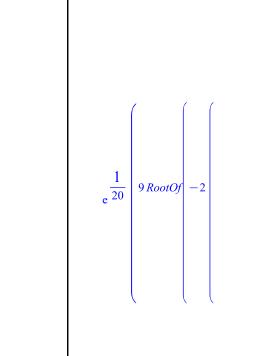
$$d_{f} = \int_{a}^{\frac{1}{2}} \left(\int_{a}^{d} \left(\int_$$

$$\frac{RootOf\left(Z\left(5 \text{ LambertW}\left(Z4\sim, \frac{\sqrt{2} Z(-1)^{-Z2\sim}}{2 e^{\frac{31}{40}}} \right) + 6\right)\right) e^{\frac{9 f}{20}}}{2 e \sqrt{f} (-1)^{-Z2\sim}} + 1\right) d_f f + 2 t$$

LambertW
$$Z4\sim$$
, $e^{-1}RootOf\left(Z\left(5 \text{ LambertW}\left(Z4\sim, \frac{\sqrt{2} Z(-1)Z2\sim}{\frac{31}{2}e^{\frac{31}{40}}}\right)\right)\right)$

e

+ 6



$$\int_{e}^{Z} \left(\operatorname{LambertW} \left(\underbrace{Z4\sim, \frac{\sqrt{2} Z(-1) Z2\sim}{2 e^{40}}} \right) + 6 \right) e^{\frac{9 f}{20}}$$

$$= 2\sqrt{f} (-1) Z2\sim$$

$$= 2\sqrt{f} (-1) Z2\sim$$

$$\left(e^{-1} RootOf\left(Z\left(5 LambertW\left(Z4\sim, \frac{\sqrt{2} Z(-1)Z2\sim}{\frac{31}{2e^{40}}}\right) + 6\right)\right)e^{\frac{9 f}{20}}\sqrt{f}\right)$$

$$+ \int_{-a}^{1} \frac{1}{\int_{-a}^{1} \left[LambertW \left(\frac{Z4\sim, \frac{\sqrt{2} - Z(-1) - Z2\sim}{31}}{2 e^{\frac{1}{40}}} \right) + 6 \right] e^{\frac{9 - f}{20}}} df + 2t} df + 2t$$



$$\left(\begin{array}{c} \\ 2 \\ RootOf \end{array} \right. - 2$$

$$\int_{a}^{-Z} \left(\underset{e}{\text{LambertW}} \left(\underbrace{Z4\sim, \frac{\sqrt{2} Z(-1) Z2\sim}{2 e^{\frac{31}{40}}} + 6}_{\text{LambertW}} \right) + \frac{9 J}{20} \right)$$

$$= \frac{2 \left(\underset{e}{\text{LambertW}} \left(\underbrace{Z4\sim, \frac{\sqrt{2} Z(-1) Z2\sim}{3 e^{\frac{31}{40}}} + 6}_{\text{20}} \right) \right) + \frac{9 J}{20}$$

$$= \frac{2 \sqrt{J} (-1) Z2\sim}{2 \sqrt{J} (-1) Z2\sim}$$

$$e^{-1} RootOf \left(Z \left(5 LambertW \left(Z4\sim, \frac{\sqrt{2} Z(-1) Z2\sim}{\frac{31}{2 e^{40}}} \right) + 6 \right) \right) e^{\frac{9}{20}} \sqrt{f}$$

$$+ \int_{-a}^{1} \frac{1}{\int_{-a}^{1} \left(\operatorname{LambertW} \left(\underbrace{Z4\sim, \frac{1}{2} \underbrace{Z(-1) - Z2\sim}_{2 e \sqrt{f}} + 6}_{2 e \sqrt{f}} \right) + 6 \right) e^{\frac{g-f}{20}} \right) + 1} df + 2t$$

$$(-1)^{-Z2^{\sim}}$$

$$(-1)^{-Z2^{\sim}}$$

$$J := \begin{bmatrix} 2 - 1.2 \, y & -1.2 \, x \\ 0.9 \, y & -1 + 0.9 \, x \end{bmatrix}$$
 (134)

 \rightarrow A1 := subs(EquiP[1, 1], EquiP[1, 2], eval(J))

$$A1 := \begin{bmatrix} 2. & -0. \\ 0. & -1. \end{bmatrix}$$
 (135)

> eigenvals(A1)

$$-1., 2.$$
 (136)

 \rightarrow A2 := subs(EquiP[2, 1], EquiP[2, 2], eval(J))

$$A2 := \begin{bmatrix} 0. & -1.3333333333\\ 1.500000000 & -1. \times 10^{-10} \end{bmatrix}$$
 (137)

> eigenvals(A2)

$$-5.0000000000000000 \times 10^{-11} + 1.41421356219632 \text{ I, } -5.00000000000000 \times 10^{-11}$$
 (138)

- 1.41421356219632 I

> DEplot([sist], [x(t), y(t)], t = -10..10, x = -3..3, y = -3..3, [condin], linecolor = blue, stepsize = 0.1)

