$with\cdot(DEtools):with\cdot(plots):$

$deq := x\cdot diff(y(x),x\$2) - (x+3)\cdot diff(y(x),x) + 2\cdot y(x) = 0$

$$deq := x\left(\frac{d^2}{dx^2}\,y(x)\right) - (x+3)\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = 0 \quad (1)$$

varphi $:= x \to x^2 + 4\cdot x + 6$

$$\varphi := x \mapsto x^2 + 4\cdot x + 6 \quad (2)$$

$subs(y(x) = \text{varphi}(x), deq)$

$$x\left(\frac{d^2}{dx^2}\,\left(x^2+4\,x+6\right)\right) - (x+3)\left(\frac{d}{dx}\,\left(x^2+4\,x+6\right)\right) + 2\,x^2 + 8\,x + 12 = 0 \quad (3)$$

$simplify(\%)$

$$0 = 0 \quad (4)$$

$restart$

$deq := x\cdot diff(y(x),x\$2) - (x+1)\cdot diff(y(x),x) - 2\cdot(x-1)\cdot y(x) = 0$

$$deq := x\left(\frac{d^2}{dx^2}\,y(x)\right) - (x+1)\left(\frac{d}{dx}\,y(x)\right) - 2\,(x-1)\,y(x) = 0 \quad (5)$$

varphi $:= x \to e^{2\cdot x}$

$$\varphi := x \mapsto e^{2\cdot x} \quad (6)$$

$varphi2 := x \to x^2 + 1$

$$\varphi 2 := x \mapsto x^2 + 1 \quad (7)$$

$subs(y(x) = \text{varphi}(x), deq)$

$$x\left(\frac{d^2}{dx^2}\,e^{2x}\right) - (x+1)\left(\frac{d}{dx}\,e^{2x}\right) - 2\,(x-1)\,e^{2x} = 0 \quad (8)$$

$simplify(\%)$

$$0 = 0 \quad (9)$$

$subs(y(x) = varphi2(x), deq)$

$$x\left(\frac{d^2}{dx^2}\,\left(x^2+1\right)\right) - (x+1)\left(\frac{d}{dx}\,\left(x^2+1\right)\right) - 2\,(x-1)\,\left(x^2+1\right) = 0 \quad (10)$$

$simplify(\%)$

$$-2\,x^3 - 2\,x + 2 = 0 \quad (11)$$

$restart'$

$$\frac{d}{dx}\,restart(x) \quad (12)$$

$restart$

$deq := x^3\cdot diff(y(x),x\$3) - 3\cdot x^2\cdot diff(y(x),x\$2) + 6\cdot x\cdot diff(y(x),x) - 6\cdot y(x) = 0$

$$deq := x^3 \left(\frac{d^3}{dx^3} y(x) \right) - 3x^2 \left(\frac{d^2}{dx^2} y(x) \right) + 6x \left(\frac{d}{dx} y(x) \right) - 6y(x) = 0 \quad (13)$$

varphi := x→x

$$\varphi := x \mapsto x \quad (14)$$

varphi2 := x→x^2

$$\varphi 2 := x \mapsto x^2 \quad (15)$$

subs(y(x) = varphi(x), deq)

$$x^3 \left(\frac{d^3}{dx^3} x \right) - 3x^2 \left(\frac{d^2}{dx^2} x \right) + 6x \left(\frac{d}{dx} x \right) - 6x = 0 \quad (16)$$

simplify(%)

$$0 = 0 \quad (17)$$

subs(y(x) = varphi2(x), deq)

$$x^3 \left(\frac{d^3}{dx^3} (x^2) \right) - 3x^2 \left(\frac{d^2}{dx^2} (x^2) \right) + 6x \left(\frac{d}{dx} (x^2) \right) - 6x^2 = 0 \quad (18)$$

simplify(%)

$$0 = 0 \quad (19)$$

restart

deq := x·diff(y(x), x$2) − (x + 3)·diff(y(x), x) + 2·y(x) = 0

$$deq := x \left(\frac{d^2}{dx^2} y(x) \right) - (x+3) \left(\frac{d}{dx} y(x) \right) + 2y(x) = 0 \quad (20)$$

varphi := x→a·x^2 + b·x + c

$$\varphi := x \mapsto a \cdot x^2 + b \cdot x + c \quad (21)$$

expr := eval(deq, y(x) = varphi(x))

$$expr := 2xa - (x+3)(2xa + b) + 2ax^2 + 2bx + 2c = 0 \quad (22)$$

simplify(expr)

$$(x-3)b - 4xa + 2c = 0 \quad (23)$$

expr := collect(lhs(expr), x)

$$expr := (-4a + b)x - 3b + 2c \quad (24)$$

c1 := coeff(expr, x, 1)

$$c1 := -4a + b \quad (25)$$

c2 := coeff(expr, x, 0)

$$c2 := -3b + 2c \quad (26)$$

solve({c1 = 0, c2 = 0}, {a, b, c})

$$\{a = a, b = 4a, c = 6a\} \quad (27)$$

$varphi2 := x \to \exp(a \cdot x) \cdot (b \cdot x + c)$

$$\varphi 2 := x \mapsto e^{a \cdot x} \cdot (b \cdot x + c) \tag{28}$$

$expr := eval(deq, y(x) = varphi2(x))$

$$expr := x \left(a^2 e^{x a} (b x + c) + 2 a e^{x a} b\right) - (x + 3) \left(a e^{x a} (b x + c) + e^{x a} b\right) + 2 e^{x a} (b x + c) = 0 \tag{29}$$

$simplify(expr)$

$$e^{x a} \left(x (b x + c) a^2 + \left(-b x^2 + (-b - c) x - 3 c\right) a + b x - 3 b + 2 c\right) = 0 \tag{30}$$

$expr := \exp(-a \cdot x) \cdot expr$

$$expr := e^{-x a} \left(x \left(a^2 e^{x a} (b x + c) + 2 a e^{x a} b\right) - (x + 3) \left(a e^{x a} (b x + c) + e^{x a} b\right) + 2 e^{x a} (b x + c)\right) = 0 \tag{31}$$

$expr := simplify(expr)$

$$expr := x (b x + c) a^2 + \left(-b x^2 + (-b - c) x - 3 c\right) a + b x - 3 b + 2 c = 0 \tag{32}$$

$collect(expr, x)$

$$\left(b a^2 - b a\right) x^2 + \left(c a^2 + (-b - c) a + b\right) x - 3 a c - 3 b + 2 c = 0 \tag{33}$$

$c1 := coeff(lhs(expr), x, 2)$

$$c1 := b a^2 - b a \tag{34}$$

$c2 := coeff(lhs(expr), x, 1)$

$$c2 := c a^2 + (-b - c) a + b \tag{35}$$

$c3 := coeff(lhs(expr), x, 0)$

$$c3 := -3 a c - 3 b + 2 c \tag{36}$$

$solve(\{c1 = 0, c2 = 0, c3 = 0\}, \{a, b, c\})$

$$\{a = a, b = 0, c = 0\}, \{a = 1, b = b, c = -3 b\} \tag{37}$$

$a := 1$

$$a := 1 \tag{38}$$

$b := 1$

$$b := 1 \tag{39}$$

$c := -3 \cdot b$

$$c := -3 \tag{40}$$

$varphi2(x)$

$$e^{x} (x - 3) \tag{41}$$

$odetest(y(x) = varphi2(x), deq)$

$$0 \tag{42}$$

restart

$deq := x^2 \cdot diff(y(x), x\$2) - 2 \cdot x \cdot diff(y(x), x) + 2 \cdot y(x) = 0$

$$deq := x^2 \left(\frac{d^2}{dx^2} y(x) \right) - 2x \left(\frac{d}{dx} y(x) \right) + 2y(x) = 0 \quad (43)$$

varphi[1] := $x \to x$

$$\varphi_1 := x \mapsto x \quad (44)$$

varphi[2] := $x \to x^2$

$$\varphi_2 := x \mapsto x^2 \quad (45)$$

$odetest(y(x) = \text{varphi}[1](x), deq)$

$$0 \quad (46)$$

$odetest(y(x) = \text{varphi}[2](x), deq)$

$$0 \quad (47)$$

$with(linalg):$
$A := wronskian([\text{varphi}[1](x), \text{varphi}[2](x)], x)$

$$A := \begin{bmatrix} x & x^2 \\ 1 & 2x \end{bmatrix} \quad (48)$$

$det(A)$

$$x^2 \quad (49)$$

restart

varphi[1] := $x \to x$

$$\varphi_1 := x \mapsto x \quad (50)$$

varphi[2] := $x \to e^x$

$$\varphi_2 := x \mapsto e^x \quad (51)$$

$deq := (x-1) \cdot diff(y(x), x\$2) - x \cdot diff(y(x), x) + y(x) = 0$

$$deq := (x-1) \left(\frac{d^2}{dx^2} y(x) \right) - x \left(\frac{d}{dx} y(x) \right) + y(x) = 0 \quad (52)$$

$odetest(y(x) = \text{varphi}[1](x), deq)$

$$0 \quad (53)$$

$$odetest(y(x) = \text{varphi}[2](x), deq)$$

$$0 \tag{54}$$

$$with(linalg):$$
$$A := wronskian([\text{varphi}[1](x), \text{varphi}[2](x)], x)$$

$$A := \begin{bmatrix} x & e^{x} \\ 1 & e^{x} \end{bmatrix} \tag{55}$$

$$det(A)$$

$$x\,e^{x} - e^{x} \tag{56}$$