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$\rho = (A, B, R)$

$\rho(X) = \{ b \in B \mid \exists a \in X : a \rho b \}$ - section of ρ

$\rho\langle x \rangle = \rho(\{x\})$

33) $\delta = (\mathbb{N}, \mathbb{N}, |)$ ← divisibility

$\delta\langle 1 \rangle = ?$

$\delta^{-1}(\{4, 9\}) = ?$

$\delta^{-1}(\mathbb{N}) = ?$

$\delta(\mathbb{N}) = ?$

$\delta\langle 1 \rangle = \delta(\{1\}) = \{ b \in \mathbb{N} \mid \exists a \in \{1\} : a \delta b \} = \mathbb{N}$

all the elements divisible by 1

$\delta^{-1}(\{4, 9\}) = \{ a \in \mathbb{N} \mid \exists b \in \{4, 9\} : a \delta b \} = \underbrace{\{1, 2, 4, 3, 9\}}_{\text{divisors că avem } \delta^{-1}}$

$\delta^{-1}(\mathbb{N}) = \mathbb{N}$

$\delta(\mathbb{N}) = \mathbb{N}$ (0 divides 0) $0|0$ $1|n, n \in \mathbb{N}$

35) exam $\rho = (A, B, R)$, $\rho' = (A, B, R')$ Prove that

b) $\rho(\underbrace{X \cup X'}) = \rho(X) \cup \rho(X')$, $X, X' \subseteq A$

subset of B

Let $b \in \rho(X \cup X') \Leftrightarrow \exists a \in \underbrace{X \cup X'} : a \rho b \Leftrightarrow$

$\Leftrightarrow \exists a \Big((a \in X) \vee (a \in X') \Big) \wedge (a \rho b) \Leftrightarrow$

$\Leftrightarrow \underbrace{\exists a}_{\exists X} \Big(\underbrace{(a \in X) \wedge (a \rho b)}_{A} \Big) \underset{\vee}{\vee} \Big(\underbrace{(a \in X') \wedge (a \rho b)}_{B} \Big) \Leftrightarrow$

$\Leftrightarrow$ (tautology) $\exists x (A \vee B) \Leftrightarrow (\exists x A) \vee (\exists x B)$
2.3.9

$$\Leftrightarrow \exists a \underbrace{\left((a \in X) \wedge (a \rho b)\right)}_{A} \vee \exists a \underbrace{\left((a \in X') \wedge (a \rho b)\right)}_{B}$$

$$\left(b \in \rho(X)\right) \vee \left(b \in \rho(X')\right) \Leftrightarrow b \in \rho(X) \cup \rho(X')$$

Define the section of ρ with respect to X and prove the equivalence

$p \wedge p \Leftrightarrow p$ idempotence

$\Leftarrow$) $\rho(X \cap X') \subseteq \rho(X) \cap \rho(X')$

Let $b \in \rho(X \cap X') \Leftrightarrow \exists a \in X \cap X' : a \rho b \Leftrightarrow$

$\Leftrightarrow \exists a \ \left((a \in X) \wedge (a \in X')\right) \wedge (a \rho b) \underset{\text{com.}}{\overset{\text{idem}}{\Leftrightarrow}}$

$$\Leftrightarrow \exists a \underbrace{\left((a \in X) \wedge (a \rho b)\right)}_{A} \wedge \underbrace{\left((a \in X') \wedge (a \rho b)\right)}_{B} \Rightarrow$$

$$\exists x (A \wedge B) \Rightarrow \exists x A \wedge \exists x B$$

$$\exists a \left((a \in X) \wedge (a \rho b)\right) \wedge \exists a \left((a \in X') \wedge (a \rho b)\right) \Leftrightarrow$$

$$\Leftrightarrow b \in \rho(X) \wedge b \in \rho(X') \Leftrightarrow b \in \rho(X) \cap \rho(X')$$

$(\rho \cap \rho')(X) \subseteq \rho(X) \cap \rho'(X)$

Let $b \in (\rho \cap \rho')(X) \Leftrightarrow \exists a \in X : a (\rho \cap \rho') b \Leftrightarrow$

$\Leftrightarrow \exists a \in X$ s.t. $(a \rho b) \wedge (a \rho' b) \Leftrightarrow$

$\Leftrightarrow \exists a (a \in X) \wedge \left((a \rho b) \wedge (a \rho' b)\right)$

$\Leftrightarrow \exists a (a \in X) \wedge (a \in X) \wedge (a \rho b) \wedge (a \rho' b)$

$\Leftrightarrow \exists a \left((a \in X) \wedge (a \rho b)\right) \wedge \left((a \in X) \wedge (a \rho' b)\right) \Rightarrow$

$\Rightarrow \exists a \left((a \in X) \wedge (a \rho b)\right) \wedge \exists a \left((a \in X) \wedge (a \rho' b)\right)$

$\Leftrightarrow b \in \rho(X) \wedge b \in \rho'(X) \Leftrightarrow b \in \left(\rho(X) \cap \rho'(X)\right)$

$f=(A,B,F)$ function $\Leftrightarrow \forall a\in A \quad |f\langle a\rangle|=1$

$f: A \to B \quad A \xrightarrow{f} B$

$x\, 1_A\, x \qquad x=x \Rightarrow x(\rho^{-1}\circ\rho)x$

relation

(41)) $\rho=(A,B,R)$ relation

Prove that ρ is a function $\Leftrightarrow 1_A \subseteq \rho^{-1}\circ\rho$ and $\rho\circ\rho^{-1}\subseteq 1_B$

For equivalence, we have to prove both ways

"$\Rightarrow$" ρ is a function $\Rightarrow \forall a\in A \quad \exists! b\in B : a\rho b \Rightarrow$

$\Rightarrow \forall a\in A \quad \exists! b\in B : (a\rho b)\wedge(b\rho^{-1}a) \xRightarrow[\text{of 2 relation}]{\text{composition}}$

$\Rightarrow a\in(\rho^{-1}\circ\rho)a \Rightarrow 1_A \subseteq \rho^{-1}\circ\rho$

Let $b_1, b_2 \in B$

$b_1(\rho\circ\rho^{-1})b_2 \Rightarrow \exists a\in A : (b_1\rho^{-1}a)\wedge(a\rho b_2)$

$\Leftrightarrow \exists a\in A : (a\rho b_1)\wedge(a\rho b_2) \xRightarrow{\rho \text{ is a function}} b_1=b_2$

$f(a)=b_1 \qquad f(a)=b_2$

function $\Rightarrow b_2=b_1$

$\Rightarrow \rho\circ\rho^{-1}\subseteq 1_B$

"$\Leftarrow$" $1_A\subseteq\rho^{-1}\circ\rho \Rightarrow \forall a\in A \quad a(\rho^{-1}\circ\rho)a \Rightarrow$

$\Rightarrow \forall a\in A \;\exists b\in B : (a\rho b)\wedge(b\rho^{-1}a)$

$\Rightarrow \forall a\in A \;\exists b\in B : a\rho b$

$\rho\circ\rho^{-1}\subseteq 1_B$

$\forall a\in A \;\exists$

$b_1(\rho\circ\rho^{-1})b_2 \Leftrightarrow \exists a\in A : (b_1\rho^{-1}a)\wedge(a\rho b_2) \Leftrightarrow$

$\Leftrightarrow \exists a\in A : (a\rho b_1)\wedge(a\rho b_2)$

If $(\forall a \in A) \exists b_1, b_2 \in B : (a \rho b_1) \wedge (a \rho b_2) \Rightarrow$
$\forall a \in A \; \exists \cancel{b_1, b_2 \in B} (b_1 \rho^{-1} a) \wedge (a \rho b_2) \Rightarrow \forall a \in A \; b_1 (\rho \circ \rho^{-1}) b_2$

$\rho \circ \rho^{-1} \subseteq 1_B$

$\Rightarrow b_1 = b_2$

$\neg(\forall x A) \Leftrightarrow \exists x \neg A$

43) exam

d) $C\left(\bigcap_{i \in I} A_i\right) = \bigcup_{i \in I} C(A_i)$, $A_i \in P(U)$

Let $x \in C\left(\bigcap_{i \in I} A_i\right) \Leftrightarrow \overline{x \in \bigcap_{i \in I} A_i} \Leftrightarrow \overline{\forall i \in I \; x \in A_i} \Leftrightarrow$

$\exists i \in I, \overline{x \in A_i} \Leftrightarrow \exists i \in I \; x \in C(A_i) \Leftrightarrow x \in \bigcup_{i \in I} C(A_i)$

$\boxed{\exists x \forall y A \Rightarrow \forall y \exists x A}$

46) exam CHECK LECT 4

b) $f\left(\bigcap_{i \in I} X_i\right) \subseteq \bigcap_{i \in I} f(X_i)$

Let $y \in f\left(\bigcap_{i \in I} X_i\right) \Leftrightarrow \exists x \in \bigcap_{i \in I} X_i : f(x) = y \Leftrightarrow$

$\Leftrightarrow \exists x \; \forall i \in I \; (x \in X_i) \wedge (f(x) = y) \Rightarrow$

$\forall i \in I \; \exists x (x \in X_i) \wedge (f(x) = y)$

$\Leftrightarrow \forall i \in I \; y \in f(X_i) \Leftrightarrow y \in \bigcap_{i \in I} f(X_i)$

$f^{-1}(Y)$ - preimage