02.11.2022

② $\bar a\,(2,-3,1)$

$\bar b\,(-3,1,2)$

$\bar c\,(1,2,3)$

$(\bar a\times\bar b)\times\bar c\,,\ \bar a\times(\bar b\times\bar c)$

$\bar a\times(\beta\bar b\times\gamma\bar c)=\beta\,\bar a\times\bar b+\gamma\,\bar a\times\bar c$

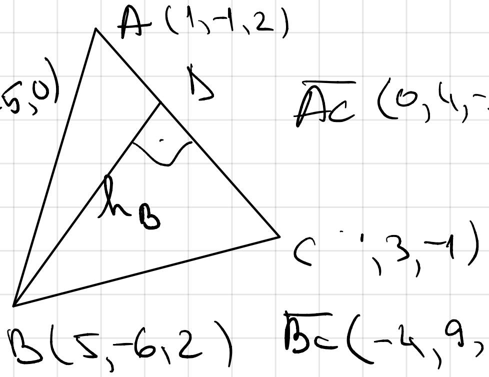
$$(\bar a\times\bar b)\times\bar c=(\bar a\cdot\bar c)\bar b-(\bar b\cdot\bar c)\bar a=\overset{-1}{(2-6+3)}\cdot\overset{\bar b}{\begin{bmatrix}-3\\1\\2\end{bmatrix}}-\overset{5}{(-3+2+6)}\overset{\bar a}{\begin{bmatrix}2\\-3\\1\end{bmatrix}}$$

$$=\begin{bmatrix}3\\-1\\-2\end{bmatrix}-\begin{bmatrix}10\\-15\\5\end{bmatrix}=\begin{bmatrix}-7\\14\\-7\end{bmatrix}$$

④ $A(1,-1,2)\quad B(5,-6,2)\quad C(1,3,-1)\quad BD=?$

$$=\frac{\|\overline{AB}\times\overline{BC}\|}{2}=A_{\Delta ABC}=\frac{h_B\|\overline{AC}\|}{2}$$

$\overline{AB}\,(4,-5,0)$



$\overline{AC}\,(0,4,-3)$

$\overline{BC}\,(-4,9,-3)$

$$\overline{AB}\times\overline{BC}=\begin{vmatrix}i&j&k\\4&-5&0\\-4&9&-3\end{vmatrix}=$$

$$=15i+36k-20k+12j=15i+12j+16k$$

$$\|\overline{AB}\times\overline{BC}\|=\sqrt{225+144+256}=\sqrt{625}=25$$

⑤ 1) $\bar p\perp\bar a\ (2,3,-1)\iff\bar p\cdot\bar a=0\iff$

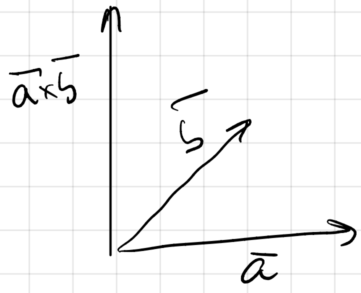
2) $\bar p\perp\bar b\ (1,-1,3)\qquad\bar p\cdot\bar b=0\iff$

3) $\bar p\cdot\bar c=51\qquad\iff$

$\bar c\,(2,-3,4)$

$$\begin{cases}2x+3y-z=0\\x-y+3z=0\\2x-3y+4z=51\end{cases}$$

$\bar p\,(x,y,z)$



(1) and (2) $\Rightarrow \exists\, \lambda \in \mathbb{R}$ $\quad \bar{p} = \lambda\, \bar{a} \times \bar{b}$

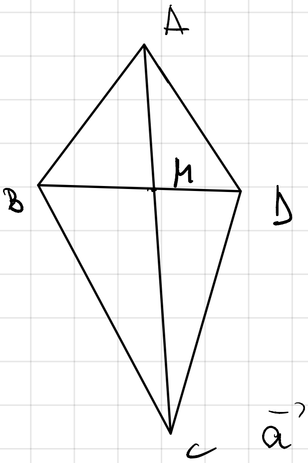
(3) $(\lambda \cdot \bar{a} \times \bar{b}) \cdot \bar{c} = 51 \qquad (\bar{a}, \bar{b}, \bar{c}) = (\bar{a} \times \bar{b}) \cdot \bar{c}$

$$\lambda = \frac{51}{(\bar{a}, \bar{b}, \bar{c})}$$

? norm of the cross product

$$\Rightarrow \bar{p} = \frac{51\, \bar{a} \times \bar{b}}{(\bar{a}, \bar{b}, \bar{c})}$$

(6) $\mathcal{A}_{ACB} = \mathcal{A}_{ACD}$



M = mid point of BD

$\mathcal{A}_{ACB} = \mathcal{A}_{ACD}$

$$\frac{\|\overline{BA} \times \overline{BC}\|}{2} = \frac{\|\overline{DA} \times \overline{DC}\|}{2}$$

$\left.\begin{aligned} \overline{BA} &= \overline{BM} + \overline{MA} \\ \overline{AD} &= \overline{AM} + \overline{MD} \end{aligned}\right\} \Rightarrow$

$\left.\begin{aligned} \overline{CB} &= \overline{CM} + \overline{MB} \\ \overline{CD} &= \overline{CM} + \overline{MD} \end{aligned}\right\} \Rightarrow$

$\bar{a}$ } fixat, are origine fixa

$\bar{a}$ – are origine aleatorie

↳ repr. vect a in spatiu

$\|\bar{a}\|$ – lungimea vectorului

$\|\bar{a}\|$ –

$\bar{a}$

$\vec{a} \leq \bar{a}$

$|\bar{a}| = \vec{a}$

$|\vec{a}| = a$

$\mathcal{A}_{ACB} = \mathcal{A}_{ACD}$

$\Updownarrow$?

$$\|\overline{BA} \times \overline{BC}\| = \|\overline{DA} \times \overline{DC}\|$$

$\overline{MA} \times \overline{MC} = \bar{0}$

$$\overline{BA} \times \overline{BC} = \pm\, \overline{DA} \times \overline{DC}$$

$\Downarrow$

$$(\overline{BM} + \overline{MA}) \times (\overline{BM} + \overline{MC}) = \pm (\overline{DM} + \overline{MA}) \times (\overline{DM} + \overline{MC})$$

$$\overline{BM} \times \overline{MC} + \overline{MA} \times \overline{BM} + \underbrace{\overline{MA} \times \overline{MC}}_{0} = -(\overline{DM} \times \overline{MC} + \overline{MA} \times \overline{DM})$$

$$-\overline{MB} \times \overline{MC} - \overline{MA} \times \overline{MB} = -(\overline{DM} \times \overline{MC} + \overline{MA} \times \overline{DM})$$

$$-\overline{MB}\times\overline{MC}-\overline{MA}\times\overline{MD}=-(\overline{MB}\times\overline{MC}+\overline{MA}\times\overline{MB})$$

$$\mathcal{A}_{ABC}=\mathcal{A}_{ACD}$$
$$\Updownarrow$$
$$\|\overline{AB}\times\overline{AC}\|=\|\overline{AD}\times\overline{AC}\|$$
$$\Updownarrow$$
$$\overline{AB}\times\overline{AC}=-\overline{AD}\times\overline{AC}$$
$$\Updownarrow$$
$$(\overline{AB}+\overline{AD})\times\overline{AC}=0$$
$$\Updownarrow$$
$$2\,\overline{AM}\times\overline{AC}=0 \quad \text{true since } A, M, C \text{ colinear}$$

7. Prove that the points A(1, 2, –1), B(0, 1, 5), C(–1, 2, 1) and D(2, 1, 3) are situated in the same plane.

$$\text{Volume}(ABCD)=\frac{1}{6}\left|(\overline{AB},\overline{AC},\overline{AD})\right|$$

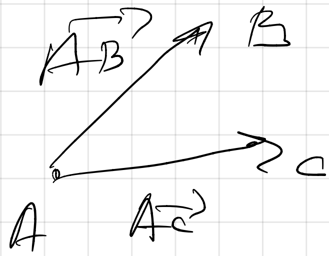
Points in the same plane $\Leftrightarrow$ we calculate volume and has to be 0

$\overline{AB}=(-1,-1,6)$

$\overline{AC}=(-2,0,2)$

$\overline{AD}=(1,-1,4)$

$$(\overline{AB},\overline{AC},\overline{AD})=\begin{vmatrix}-1&-1&6\\-2&0&2\\1&-1&4\end{vmatrix}=12-2-2-8=0$$



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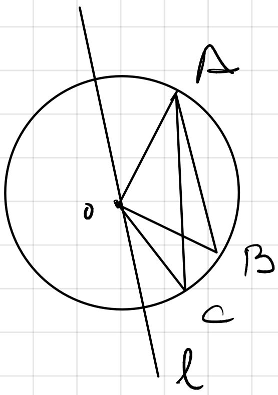
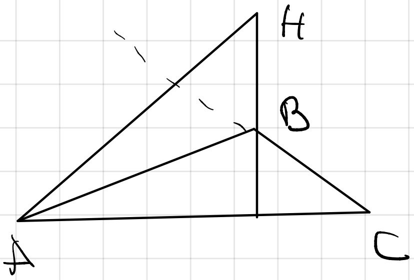


$\|\overline{OA}\| = \|\overline{OB}\| = \|\overline{OC}\| = 1$

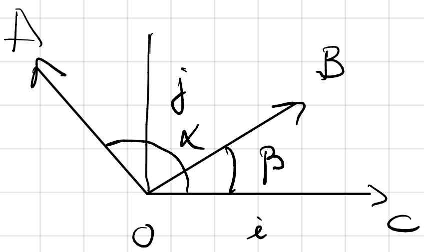
$\underbrace{\|\overline{OA}+\overline{OB}+\overline{OC}\|} \geq 1$

$\|\overline{OH}\| \geq 1 \Leftrightarrow H$ outside of the circle

H orthocenter of ΔABC



We can choose coordinates



$\overrightarrow{OC}\,(1,0)$ – the components of $\vec{i}$

$\overrightarrow{OA}\,(\cos\alpha, \sin\alpha)$

$\overrightarrow{OB}\,(\cos\beta, \sin\beta)$

$+$ ―――――――――

$(1+\cos\alpha+\cos\beta,\ \sin\alpha+\sin\beta)$

$$\|\overline{OA}+\overline{OB}+\overline{OC}\| = (1+\cos\alpha+\cos\beta)^2 + (\sin\alpha+\sin\beta)^2$$

$$= 3 + 2\cos\alpha\cos\beta + 2\cos\alpha + 2\underbrace{\cos\beta}_{\geq 0} + 2\underbrace{\sin\alpha\sin\beta}_{\geq 0}$$

$$= 3 + \underbrace{2\underbrace{\cos\alpha}_{\geq -1}\underbrace{(\cos\beta+1)}_{\geq 0}}_{\geq -2} + \ldots$$

≥ 1

(10) to write them in coord (with i, j, k)

$a(a_1, a_2, a_3)$ calc cross product

$\Rightarrow$ expression of components of vectors

[calc pentru din stg și din dreapta și compar]