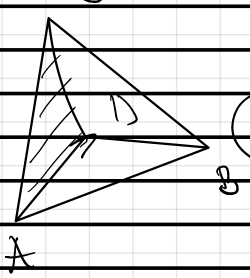
07.12.2022
SEMINAR 8

Problem set 7



(4) $A(1,0,-1)$ $\quad$ (∀) $\Leftrightarrow Vol_{Tetra\ ABCD} = 0$

$B(0,2,3)$

$Vol = \frac{1}{6} Vol_{Para\ ABCD}$

$C(-2,1,1)$ $\quad \vec{AC}(-3,1,2)$

$D(4,2,3)$ $\quad \vec{AD}(3,2,4)$ $\quad \Leftrightarrow \frac{1}{6} \left| \underbrace{(\vec{AB}, \vec{AC}, \vec{AD})} \right| = 0$

? A, B, C, D coplanar (∀) $\quad \Leftrightarrow \quad \vec{AB} \cdot (\vec{AC} \times \vec{AD}) = 0$

înlocuiesc direct cu x_B, y_B, z_B și ver dacă e 0

$$\Leftrightarrow \begin{array}{c} \vec{AB} \rightarrow \\ \vec{AC} \rightarrow \\ \vec{AD} \rightarrow \end{array} \begin{vmatrix} x_B - 1 & y_B - 0 & z_B + 1 \\ -3 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = 0$$

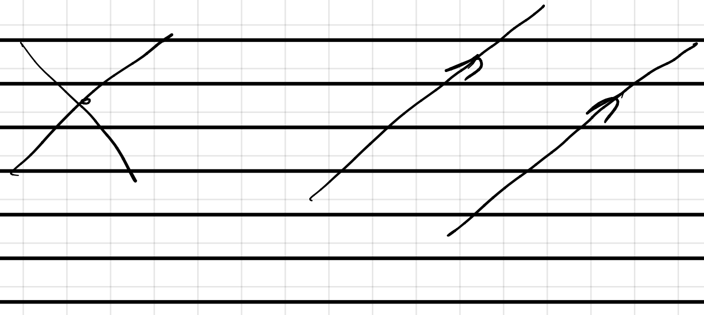
$\Leftrightarrow B \in ACD$

(6) $d_1 : \begin{cases} x = 1 + t \\ y = 1 + 2t \\ z = 3 + t \end{cases}$, $t \in \mathbb{R}$ $\quad v_1(1,2,1)$ $\quad \hookrightarrow P_1(1,1,3)$

$d_2 : \begin{cases} x = 3 + \lambda \\ y = 2\lambda \\ z = -2 + \lambda \end{cases}$ $\quad v_2(1,2,1)$ $\quad \hookrightarrow P_2(3,0,-2)$

a) ? $d_1 \parallel d_2$ (check if director vectors are parallel)

b) find plane $\pi \supseteq d_1, d_2$



a) $d_1 \parallel d_2 \Leftrightarrow$ they have the same direction vectors (DVs)

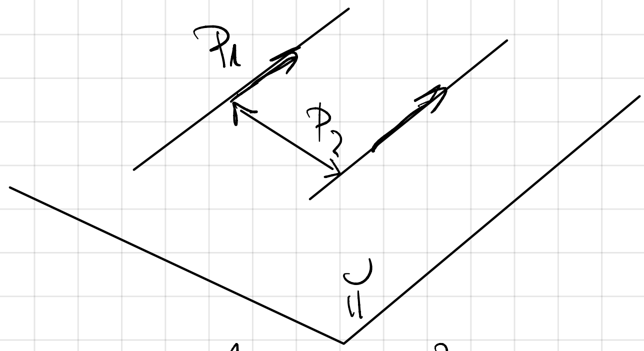
$DV(d_1) = \{ (a, 2a, a) : a \neq 0 \}$

$DV(d_2) = \{ (b, 2b, b) : b \neq 0 \}$

Since $v_1 = v_2 \Rightarrow d_1 \parallel d_2$

they can also be proportional

b)



$$\begin{array}{c|ccc|c}
 & 1 & 1 & 3 & \\
\tilde{\pi} : & x - x_p & y - y_p & z - z_p & \\
v_1 & v_x & v_y & v_z & = 0 \\
 & 1 & 2 & 1 & \\
\overrightarrow{P_1P_2} & w_x & w_y & w_z & \\
 & 2 & -1 & -5 &
\end{array}$$

$n_1(-2, 3, 7)$

(7) $l: \begin{cases} \pi_1: -2x + 3y + 7z + 2 = 0 \\ \pi_2: x + 2y - 3z + 5 = 0 \end{cases}$ $\quad l: \begin{cases} x = \\ y = \\ z = \end{cases}$ We solve the system

$$\left(\begin{array}{ccc|c} -2 & 3 & 7 & -2 \\ 1 & 2 & -3 & -5 \end{array}\right) \overset{r_1 \leftrightarrow r_2}{\sim} \left(\begin{array}{cccc} 1 & 2 & -3 & -5 \\ -2 & 3 & 7 & -2 \end{array}\right) \overset{r_2 + 2r_1}{\sim}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -5 \\ 0 & 7 & 1 & -12 \end{array}\right) \underset{\frac{r_2}{7}}{\overset{r_1 - \frac{2}{7} r_2}{\sim}} \left(\begin{array}{ccc|c} 1 & 0 & -\frac{23}{7} & -\frac{11}{7} \\ 0 & 1 & \frac{1}{7} & -\frac{12}{7} \end{array}\right)$$

1st method

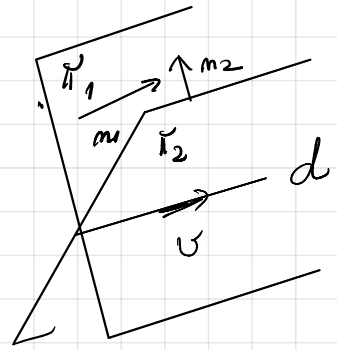
$l: \begin{cases} x = -\frac{51}{7} + \frac{23}{7} \\ y = -\frac{12}{7} - \frac{1}{7} \\ z = \lambda \end{cases} \quad \lambda \in \mathbb{R}$

2nd method:

$$\begin{cases} x = x_0 + v_x t \\ y = y_0 + v_y t \\ z = z_0 + v_z t \end{cases}, \quad t \in \mathbb{R}$$

$\uparrow$ a particular sol to (*)

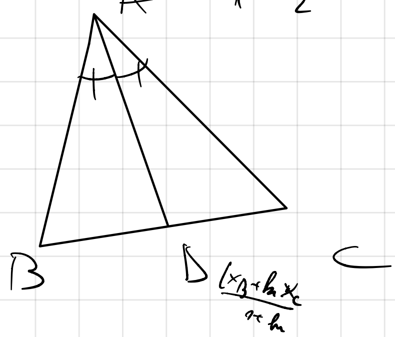
$\uparrow v = n_1 \times n_2$



$n_1 \perp v$

$n_2 \perp v$ $\forall v$ a DV of d

$n_1 \times n_2$ is a DV for d



(9) $A(1,2,-7)$ $\vec{AB}(1,0,0)$

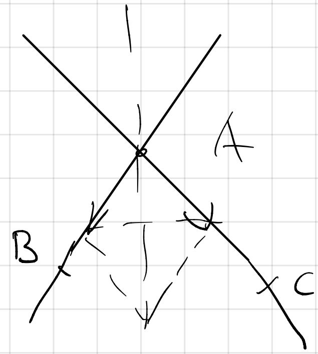
$B(2,2,-7)$ $\vec{AC}(2,2,+12)$

$C(3,4,5)$ $2(1,1,6)$

? internal angle bisector of $\hat{A}$ in ΔABC

If $AB = AC$ then $\vec{AB} + \vec{AC}$ is DV

$\dfrac{\vec{AB}}{\|\vec{AB}\|}$



$$\left\| \frac{\vec{AB}}{\|\vec{AB}\|} \right\| = \frac{1}{\|\vec{AB}\|} \cdot \|\vec{AB}\| = 1$$

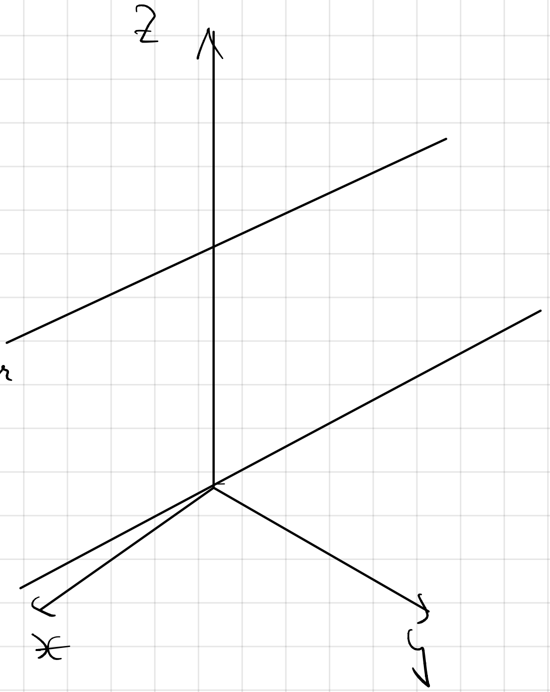
$\dfrac{\vec{AB}}{\|\vec{AB}\|} + \dfrac{\vec{AC}}{\|\vec{AC}\|}$ is DV

$$\frac{(1,0,0)}{1} + \frac{\cancel{2}(1,1,6)}{\cancel{2}\sqrt{38}}$$

$$l = \begin{cases} x = 1 + \lambda(\sqrt{38} + 1) \\ y = 2 + \lambda \\ z = -7 + \lambda 6 \end{cases}$$

sau: $D\left(\frac{x_B + k x_C}{1+k}, y, \ldots\right)$

$k = \frac{BD}{DC} = \frac{BA}{CA} = \frac{1}{2\sqrt{38}}$



(10) $l \begin{cases} x = t \\ y = -1 + t \\ z = 2 \end{cases}$

? $l' \ni 0$, $l' \parallel l$

have the same director vectors

$l': \begin{cases} x = t \\ y = t \\ z = 0 \end{cases}$

(11) $d_1: \begin{cases} x = 4 - 2t \\ y = 1 + 2t \\ z = 9 + 3t \end{cases}$ $\qquad d_2: \frac{x-1}{2} = \frac{y+2}{3} = \frac{z-4}{2} = s$

? $d_1 \cap d_2$

$d_2: \begin{cases} x = 1 + 2s \\ y = -2 + 3s \\ z = 4 + 2s \end{cases}$

$d_1 \cap d_2 \Rightarrow \begin{cases} 4 - 2t = 1 + 2s \\ 1 + 2t = -2 + 3s \\ 9 + 3t = 4 + 2s \end{cases} \Rightarrow \begin{cases} 4 - 2t - 1 - 2s = 0 \\ 1 + 2t + 2 - 3s = 0 \\ 9 + 3t - 4 - 2s = 0 \end{cases} \overset{+}{\Leftrightarrow} 5 + 1 - 5s = 0$

$5s = 6 \Rightarrow \boxed{s = \frac{6}{5}}$

$4 - 2t - 1 - 2 \cdot \frac{4}{5} = 0$

$4 - 2t - 1 - 2 \cdot \frac{6}{5} = 0$

$2t = 3 - \frac{12}{5} = \frac{3}{5} \Rightarrow \boxed{t = \frac{3}{10}}$

$$\frac{9}{10} - \frac{12}{5} = \overset{10}{\cancel{-5}} \iff 9 - 2\cdot 4 = -50$$

$\Downarrow$

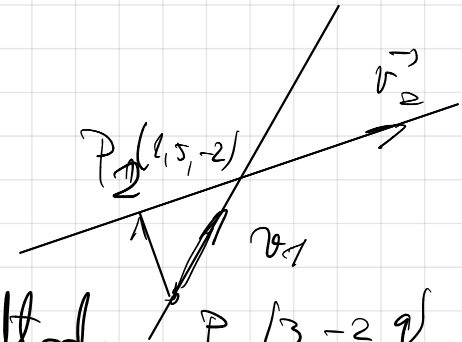
incompatible $\Rightarrow d_1 \cap d_2 = \emptyset$

(12) $d_1 : \begin{cases} x = 3 + t \\ y = -2 + t \\ z = 9 + t \end{cases} \quad t \in \mathbb{R} \quad \hookrightarrow v_1(1,1,1)$

$d_2 : \begin{cases} x = 1 - 2s \\ y = 5 + s \\ z = -2 - 5s \end{cases} \quad s \in \mathbb{R} \quad \hookrightarrow v_2(-2, 1, -5)$

a) ? $\exists$ plane $\pi \supseteq d_1, d_2$

b) ? line $\ell \ni P(4,1,6)$, $\ell \perp \pi$

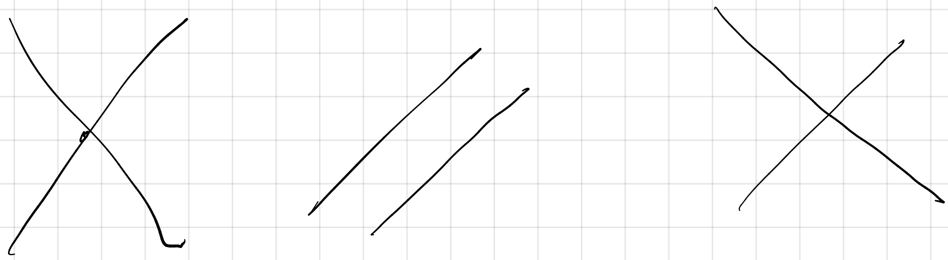


1st method: $P_1(3,-2,9)$

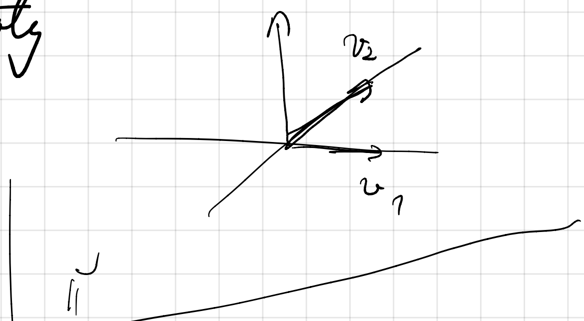
$(\overrightarrow{P_1P_2}, v_1, v_2) = 0$

$\iff d_1, d_2$ are coplanar

2nd method:



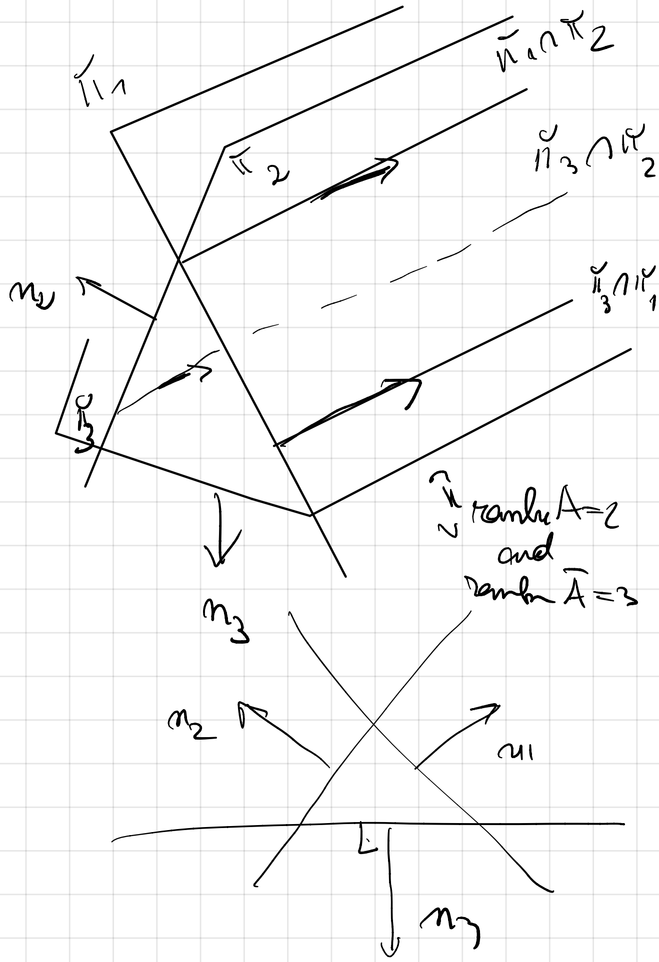
$d_1 \cap d_2 \neq \emptyset$ — see it's not empty



(13) $\pi_1: 2x - y + 3z = 0$
$n_1(2, -1, 3)$

$\pi_2: 3x + y + 2z - 1 = 0$
$n_2(3, 1, 2)$

$\pi_3 = 4x + 3y + z + 2 = 0$
$n_3(4, 3, 1)$



Finding DV for $\pi_1 \cap \pi_2$, $\pi_3 \cap \pi_2$, $\pi_3 \cap \pi_1$

$\begin{cases} \pi_1 \\ \pi_2 \end{cases}$ ... param z ... DV

coplanaritatea celor 3 vect $(n_1, n_2, n_3) = 0 \Leftrightarrow n_1, n_2, n_3$ coplanar
$\parallel$
det mat rays (*) $= 0$