19.10.2022
SEMINAR 3

Sets

$A = \{x \mid P(x)\}$ belongs to

$A = B \overset{def}{\iff} \forall x (x \in A \iff x \in B)$

$x \notin A \iff x \in A \iff \neg(x \in A)$

$P(A) = \{B \mid B \subseteq A\}$ - power set of A

$B \subseteq A \overset{def}{\iff} \forall x (x \in B \to x \in A)$

$A = \{1, 2, \dots, n\}$ 2 choices $\to 2^n$

$A \cap B = \{x \mid (x \in A) \wedge (x \in B)\}$

$A \cup B = \{x \mid (x \in A) \vee (x \in B)\}$

$A \setminus B = \{x \mid (x \in A) \wedge (x \notin B)\}$

$A \Delta B = (A \cup B) \setminus (A \cap B) =$

$= (A \setminus B) \cup (B \setminus A)$ - symmetric difference

$\emptyset = \{x \mid x \neq x\}$ - empty set

Exercises:

20) A, B, C sets from the universe $\mathcal{U}$.

Prove the following:

a) $A \subseteq A \iff \forall x (x \in A \to x \in A) \iff$

$\iff \forall x (\overline{(x \in A)} \vee (x \in A)) \iff$ this is true

(reflexivity)

b) if $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$

(transivity)

Let $x \in A$
$\left.\begin{matrix} \\ A \subseteq B \end{matrix}\right\} \Rightarrow x \in B$
$\left.\begin{matrix} \\ B \subseteq C \end{matrix}\right\} \Rightarrow x \in C \Rightarrow A \subseteq C$

set

$1 \notin \{1\}$

$\{1\} \subseteq \{1, 2\}$

subset

c) $A \cup (A \cap B) = A$ (absorption) (exam!!)

I: Let $x \in A \cup (A \cap B) \Leftrightarrow (x \in A) \vee (x \in (A \cap B))$

$\Leftrightarrow \underbrace{(x \in A)}_{p} \vee (\underbrace{(x \in A)}_{p} \wedge \underbrace{(x \in B)}_{q}) \overset{abs}{\Leftrightarrow} (x \in A)$

$\Rightarrow A \cup (A \cap B) \overset{\Sigma}{=} A$

II. $A \cup (A \cap B) = \{x \in \mathcal{U} \mid x \in A \cup (A \cap B)\} = \{x \in \mathcal{U} \mid (x \in A) \vee (x \in A \cap B)\}$

$= \{x \in \mathcal{U} \mid (x \in A) \vee ((x \in A) \wedge (x \in B))\} =$

$\overset{abs}{=} \{x \in \mathcal{U} \mid x \in A\} = A$

$CA = A^C = \mathcal{U} \setminus A = \{x \in \mathcal{U} \mid x \notin A\}$

the complement of A

21)

a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Let $x \in A \cap (B \cup C) \overset{def}{\Leftrightarrow} (x \in A) \wedge (x \in B \cup C)$

$\Leftrightarrow (x \in A) \wedge ((x \in B) \vee (x \in C)) \overset{distr}{\Leftrightarrow} ((x \in A) \wedge (x \in B))$

$\vee ((x \in A) \wedge (x \in C)) \Leftrightarrow (x \in ((A \cap B)) \vee (x \in (A \cap C)) \Leftrightarrow$

$x \in ((A \cap B) \cup (A \cap C))$

b) $\boxed{A \setminus B = A \cap C_B}$

Let $x \in A \setminus B \Leftrightarrow (x \in A) \wedge (x \notin B) \Leftrightarrow (x \in A) \wedge (x \in C_B) \Leftrightarrow$
$x \in A \cap C_B$

g) $C(A \cup B) = CA \cap CB$
$C(A \cap B) = CA \cup CB$ De Morgan

Let $x \in C(A\cup B) \iff \overline{x\in A\cup B} \iff \overline{(x\in A)\vee(x\in B)}$

de M

$\iff \overline{(x\in A)} \wedge \overline{(x\in B)} \iff (x\in CA)\wedge(x\in CB) \iff x\in (CA\cap CB)$

22) c) $(A\Delta B)\Delta C = A\Delta(B\Delta C)$

$A\Delta B = (A\cup B)\setminus(A\cap B)$

a) $A\Delta B = (A\cap CB)\cup(B\cap CA)$

a) $A\Delta B = (A\cup B)\setminus(A\cap B) \iff (A\cup B)\cap(A\cap B)$

$=(A\cup B)\cap(CA\cup CB) = (A\cap CA)\cup(A\cap CB)\cup(B\cap CA)\cup(B\cap CB)$

$=(A\cap CB)\cup(B\cap CA)$

compl $\quad = \emptyset$

c) $(A\Delta B)\Delta C = ((A\Delta B)\cap CC)\cup(C\cap C(A\Delta B)) =$

$=(((A\cap CB)\cup(B\cap CA))\cap CC)\cup(C\cap C((A\cap CB)\cup(B\cap CA)))$

$=((A\cap CB\cap CC)\cup(B\cap CA\cap CC))\cup(C\cap(CA\cup B))\cap(CB\cup A)) =$

$=(A\cap CB\cap CC)\cup(B\cap(A\cap C)\cup(C\cap((CA\cap CB)\cup\underbrace{(CA\cap A)}_{=\emptyset}\cup\underbrace{(B\cap CB)}_{=\emptyset}\cap(B\cap A)))$

$=((A\cap CB\cap CC)\cup(B\cap CA\cap CC)\cup((C\cap CA)\cap CB)\cup(C\cap(B\cap A))$

Tranchech $\rightarrow = A\Delta(B\Delta C)$

23) If $\left.\begin{aligned} A\cap C &= B\cap C \\ A\cup C &= B\cup C \end{aligned}\right\} \Rightarrow A=B$

$A = A\cup(A\cap C) = A\cup(B\cap C) \overset{distr}{=} (A\cup B)\cap(A\cup C)$

$=(A\cup B)\cap(B\cup C) \overset{Comm.}{=} (B\cup A)\cap(B\cup C) \overset{distr}{=} B\cup(A\cap C) = B\cup(B\cap C)$

$\overset{abs}{=} B$

24) Determina $X = ?$ $\begin{cases} A\cap X = B \\ A\cup X = C \end{cases}$

$X = ((A\cup X)\setminus A)\cup(A\cap X)$

$= (C\setminus A)\cup B$