~~Temă~~ Analiză Homework 9 — 02.01.2023

Ex1. a) $\int \frac{1}{1+\frac{1}{\sin x}}\,dx$, $\cancel{x\in(\pi,\frac{1}{4})}$ =

$$= \int \frac{1}{\frac{\sin x+1}{\sin x}}dx = \int \frac{\sin x}{\sin x+1}dx = \int \frac{\sin x}{\sin x+1}\cdot\frac{1-\sin x}{1-\sin x}dx =$$

$\cancel{\sin x = t \Rightarrow \cos x = dt}$

$$= \int \frac{\sin x-\sin^2 x}{1-\sin^2 x}dx =$$

$t=\cos x$

$dt=-\sin x\,dx$

$$= \int \frac{\sin x-(1-\cos^2 x)}{\cos^2 x} = \int\left(\frac{\sin x}{\cos^2 x}-\frac{1}{\cos^2 x}+1\right)dx =$$

$$= \int -\frac{1}{t^2}dt = -\int\frac{1}{\cos^2 x}dx+\int 1\,dx = -\frac{u^{-2+1}}{-2+1}-tg\,x+x = \frac{1}{t}-tg\,x+x =$$

$$= \frac{1}{\cos x}-tg\,x+x+C \cancel{\Rightarrow \int\frac{1}{1+\frac{1}{\sin x}}dx = 0}$$

~~b) $\int \frac{1}{3\sin x+4\cos x}dx = \int\frac{1}{3\frac{2t}{1+t^2}+4\frac{1-t^2}{1+t^2}}dx = \int\frac{1}{\frac{6t+4-4t^2}{1+t^2}}dx =$~~

$tg\frac{x}{2}=t$

~~$tg^2\frac{x}{2}\cdot\frac{1}{2}dx=dt$~~

~~$=\int\frac{1+t^2}{2(3t+2-2t^2)}dx$~~ $= \int\frac{2\,dt}{4-4t^2+6t}$

b) $\int \frac{1}{3\sin x+\cos x}dx = \int\frac{1}{3\frac{2t}{1+t^2}+\frac{4(1-t^2)}{1+t^2}}\cdot\frac{2}{1+t^2}dt =$

$tg\frac{x}{2}=t$

$$= \int\frac{2}{6t+4+4t^2}dt = \int\frac{1}{3t+2+2t^2}dt = \cancel{\int\frac{1}{(t-2)(2t+1)}dt}$$

~~$=\int\frac{1}{-2t^2+2t+t+2}dt = \int\frac{1}{-t(2t-1)+}\ \int\frac{1}{(t-2)(2t+1)}$~~

$$= \int\frac{1}{-2t^2+3t+2}dt = \int\frac{1}{-2t^2+2t+t+2}dt = \int\frac{1}{-(2t+1)(t-2)}dt + C$$

c) $\int_{-3}^{3}\frac{\sqrt{9-x^2}}{x^2}dx = \int\frac{\sqrt{9-(3\sin t)^2}}{(3\sin t)^2}\cdot 3\cos t\,dt = \int\frac{\sqrt{9-9\sin^2 t}}{9\sin^2 t}\cdot 3\cos t\,dt =$

$x=3\sin t$

$$= \int\frac{3\sqrt{1-\sin^2 t}}{9\sin^2 t}\cdot\cos t\,dt = \int\frac{\cos^2 t}{\sin^2 t}dt = \int ctg^2 t\,dt =$$

$$= \int ctg^2\left(\arcsin\frac{x}{3}\right)dx + C$$

d) $\int_{1}^{\frac{\sqrt{3}}{2}}\frac{1}{\sqrt{(x^2+1)^3}}dx = \int\frac{1}{(1+tg^2 t)^{\frac{3}{2}}}\cdot\frac{1}{\cos^2 t}dt = \int\frac{1}{\left(\frac{1}{\cos^2 t}\right)^{\frac{3}{2}}}\cdot\cos^2 t\,dt =$

$x=tg\,t$

$dx=\frac{1}{\cos^2 x}dt$

$$= \int\frac{\cos^3 t}{\cos^2 t}dt = \int\cos t\,dt = \sin t = \sin(\operatorname{arctg} x) =$$

$$= \frac{x}{\sqrt{1+x^2}}+C$$

e) $\int \frac{1}{\sqrt{(x^2-8)^3}}dx = \int \frac{1}{(8tg^2t-8)^{\frac{3}{2}}}dt = \int \frac{1}{\sqrt{8(tg^2t-1)^3}}dt = 2\sqrt{2}\int \frac{ctg\,t\ arcctg\,t}{16\sqrt{2}}dt$

$x = 2\sqrt{2}\,tg\,t$
$dx = 2\sqrt{2}\,tg^2t$

f) $\int_2^0 \sqrt{2x-x^2}\,dt = \int_2^0 \sqrt{x(2-x)}\,dx = \int_0^2 \sqrt{-(x-1)^2+1}\,dx =$ $\qquad x-1 = \sin t$

$= \int_{+\frac{\pi}{2}}^{\sin\frac{-\pi}{2}} \sqrt{-(\sin^2t)+1}\cdot\cos t\cdot dt = \int_{\frac{\pi}{2}}^{\frac{-\pi}{4}} \sqrt{\cos^2t}\cdot\cos t\,dt =$ $\qquad x = \sin t+1$

$\qquad dx = \cos t$

$= \int_{\frac{\pi}{2}}^{\frac{-\pi}{2}} \cos^2t\,dt = \left(\frac{1}{4}\sin 2t + \frac{1}{2}t\right)\Big|_{\frac{\pi}{4}}^{\frac{-\pi}{2}} + C$

g) $\int \sqrt{4-x^2}\,dx = \int\left(\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\arcsin\frac{x}{2}\right)\Big|_{\frac{\pi}{2}}^{-2} + C =$

h) $\int x\sqrt{1+x^2}\,dx = \cancel{\frac{1}{2}x^2\sqrt{1+x^2}} - \int \frac{2}{3}(1+x^2)^{\frac{2}{3}} + C$

$f(x) = \sqrt{1+x^2} \rightarrow f'(x) = \cancel{\frac{1}{2\sqrt{1+x^2}}\cdot 2x = x\sqrt{1+x^2}}\ \frac{1}{2\sqrt{1+x^2}}\cdot 2x$

$g'(x) = x \rightarrow g(x) = \frac{1}{2}x^2$

<u>Ex 2</u> a) $\int \frac{2x-1}{x^2-3x+2}dx = \int \frac{2x-1}{x(x-1)-2(x-1)}dx = \int \frac{2x-1}{(x-1)(x-2)}dx =$

$= \int\left(\frac{2x}{(x-1)(x-2)} - \frac{1}{(x-1)(x-2)}\right)dx = \int \frac{2x}{(x-1)(x-2)}dx - \int \frac{1}{(x-1)(x-2)}dx =$

$= \ln|4x^2-12x+8| - 3\ln|2x-2| + 3\ln|2x-4| + \ln|2x-2| - \ln|2x-4| =$

$= \ln|4x^2-12x+8| - 2\ln|2x-2| + 2\ln|2x-4| + C$

b) $\int \frac{4}{(x-1)(x+1)^2}dx = 4\int \frac{1}{(x-1)(x+1)^2}dx = 4\int \frac{1}{(t-2)t^2}dt = 4\int\left(-\frac{1}{4t} - \frac{1}{2t^2} + \frac{1}{4(t-2)}\right)dt$

$x+1 = t \qquad = \int\left(-\frac{1}{t} - \frac{2}{t^2} + \frac{1}{t-2}\right)dt = -\int \frac{1}{t}dt - \int \frac{2}{t^2}dt +$

$+ \int \frac{1}{t-2}dt = -\ln|t| + \frac{2}{t} + \ln|t-2| = -\ln|x+1| + \frac{2}{x+1} + \ln|x-1| + C$

c) $\int \frac{1}{x^3-x^4}dx = \int \frac{1}{x^3(1-x)}dx = \int -\frac{x^2+x+1}{x^3} - \frac{1}{x-1}dx =$

$= \ln|x - \frac{1}{x} - \frac{1}{2x^2} - \ln|x-1| + C\ \cancel{\ln|x| - \frac{1}{2x}}$

d) $\int \frac{2x+5}{x^2+5x+10}dx = \int \frac{1}{t}dt = \ln|t| = \ln|x^2+5x+10| + C$

$x^2+5x+10 = t$
$(2x+5)dx = dt$

e) $\int \frac{1}{x^2+x+1}\,dx = \int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx = \int \frac{4}{4t^2+3}\,dt = 4\cdot\int \frac{1}{4t^2+3}\,dt =$

$= 4\int \frac{1}{2\sqrt{3}(v^2+1)}\,dv = 4\cdot\frac{1}{2\sqrt{3}}\cdot\int \frac{1}{v^2+1}\,dv = 4\cdot\frac{1}{2\sqrt{3}}\operatorname{arctg} v =$

$= 4\cdot\frac{1}{2\sqrt{3}}\operatorname{arctg}\left(\frac{2}{\sqrt{3}}(x+\frac{1}{2})\right) = \frac{2}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}+C$

Ex3 a) $I = \int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx = \int \frac{\sqrt{x+1}-\sqrt{x}}{x+1-x}\,dx = \int \frac{\sqrt{x+1}-\sqrt{x}}{1}\,dx =$

$= \int (\sqrt{x+1}-\sqrt{x})\,dx = \int \sqrt{x+1}\,dx - \int \sqrt{x}\,dx = \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x\sqrt{x}+C$

b) $I = \int \frac{1}{x+\sqrt{x-1}}\,dx = \cancel{\int \frac{x-\sqrt{x-1}}{x^2-x+1}\,dx} = \int \frac{1}{t+1+\sqrt{t}}\,dt = \int \frac{2v}{v^2+1+v}\,dv =$

$= 2\int \frac{v}{(v+\frac{1}{2})^2+\frac{3}{4}}\,dv = 4\cdot\int \frac{2w-1}{4w^2+3}\,dw = 4\left(\int \frac{2w}{4w^2+3}\,dw - \int \frac{1}{4w^2+3}\,dw\right) =$

$= 4\left(\frac{1}{4}\ln|4w^2+3| - \frac{1}{2\sqrt{3}}\operatorname{arctg}\left(\frac{2}{\sqrt{3}}w\right)\right) =$

$= \ln|4x+4\sqrt{x-1}| - \frac{2}{\sqrt{3}}\operatorname{arctg}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)+C$

Ex4. a) $I = \int \frac{1}{1+\sqrt{x^2+2x-2}}\,dx = \cancel{\int \frac{1-\sqrt{x^2+2x-2}}{1-x^2-2x+2}\,dx} = \cancel{\int \frac{1-\sqrt{x^2+2x-2}}{3-x^2-2x}\,dx} =$

$\cancel{= \int \frac{\sqrt{x^2+2x+2}-1}{x^2+2x+2-1}\,dx} = \int \frac{\sqrt{x^2+2x+2}-1}{(x+1)^2}\,dx = \int \frac{\sqrt{(x+1)^2+1}-1}{(x+1)^2}\,dx$

$= \int \frac{\sqrt{x^2+2x-2}-1}{x^2+2x-2-1}\,dx = \int \frac{\sqrt{x^2+2x+1-3}-1}{x^2+2x+1-3}\,dx = \int \frac{\sqrt{(x+1)^2-3}-1}{(x+1)^2-3}\,dx =$

$(x+1)^2-3=t$ | $2(x+1)\,dx=dt$

$(x+1)^2=t+3$

$x=\sqrt{t+3}-1$

$= \int \frac{\sqrt{t}-1}{t}\,dt = \int \frac{1}{\sqrt{t}} - \frac{1}{t}\,dt = 2\sqrt{t} - \ln|t| = 2\sqrt{(x+1)^2-3} - \ln|(x+1)^2-3|+C$

b) $\int \frac{1}{(x+1)\sqrt{-4x^2-x+1}}\,dx = \int \frac{\sqrt{-4x^2-x+1}}{(x+1)(-4x^2-x+1)}\,dx = \int \frac{\sqrt{t}}{\frac{6}{8}t}\,dt = \int \frac{1}{\frac{6}{8}\sqrt{t}}\,dt$

$-4x^2-x+1=t$ $\quad \cancel{-8x-1=t-8x}$ $\quad -(8x-1)dx=dt \quad -(8x+1)dx=dt$

$x+1-8x-2 \quad (8x+1)dx=-dt \quad \cancel{(x+1)dx=}$

$x+\frac{1}{8}dx=-\frac{1}{8}dt \quad x+1\,dx=\frac{6}{8}dt$

$= \int \frac{8}{6\sqrt{t}}\,dt = \frac{8}{6}2\sqrt{-4x^2-x+1} = \int \frac{8}{3}\sqrt{-4x^2-x+1} =$

Ex 5: a) $\int_1^2 \frac{1}{x^3+x^2+x+1}dx = \int_1^2 \frac{1}{x^2(x+1)+x+1}dx = \int_1^2 \frac{1}{(x+1)(x^2+1)}dx =$

$= \int_1^2 \frac{1}{2(x+1)}dx + \int_1^2 \frac{-x+1}{2(x^2+1)}dx = \frac{1}{2}\ln|x+1| + \frac{1}{2}\left(-\frac{1}{2}\ln|x^2+1| + \operatorname{arctg} x\right) + C =$

$\cancel{\frac{A}{x+1} + \frac{B}{x^2+1} = \frac{Ax^2+Bx+A+B}{(x+1)(x^2+1)}}$

$= \left(\frac{1}{2}\ln|x+1| - \frac{1}{4}\ln|x^2+1| + \frac{1}{2}\operatorname{arctg} x\right)\Big|_1^2$

$= \frac{1}{2}\ln 3 - \frac{1}{4}\ln 5 + \frac{1}{2}\operatorname{arctg} 2 - \frac{1}{2}\ln 2 + \frac{1}{4}\ln 2 - \frac{1}{2}\operatorname{arctg} 1 =$

$= \frac{1}{2}\ln 3 - \frac{1}{4}\ln 5 + \frac{1}{2}\operatorname{arctg} 2 - \frac{1}{4}\ln 2 - \frac{\pi}{8}$

b) $\int_1^3 \frac{1}{x(x^2+9)}dx = \int_1^3 \frac{1}{9x} - \frac{x}{9(x^2+9)}dx = \left(\frac{1}{9}\ln|x| - \frac{1}{18}\ln|x^2+9|\right)\Big|_1^3 =$

$= \frac{1}{9}\ln 3 - \frac{1}{18}\ln 18 - \frac{1}{9}\ln 1 + \frac{1}{18}\ln 10 = \frac{1}{9}\ln 3 - \frac{1}{18}\ln\frac{18}{10}$

c) $\int_{-1}^1 \frac{x^2+1}{x^4+1}dx = \int_{-1}^1 \frac{x^2}{x^4+1}dx - \int_{-1}^1 \frac{1}{x^4+1}dx = -\frac{1}{4\sqrt{2}}(\ln|2x^2+2\sqrt{2}x+2| -$
$- 2\operatorname{arctg}(\sqrt{2}x+1)) + \frac{1}{4\sqrt{2}}(+\ln|2x^2+2\sqrt{2}x+2| + 2\operatorname{arctg}(\sqrt{2}x+1)) +$
$+ \frac{1}{4\sqrt{2}}(\ln|2x^2+2\sqrt{2}x+2| + 2\operatorname{arctg}(\sqrt{2}x+1)) + \frac{1}{4\sqrt{2}}(-\ln|2x^2-2\sqrt{2}x+2| +$
$+ 2\operatorname{arctg}(\sqrt{2}x-1))\Big|_{-1}^{1} =$
$= -\frac{1}{4\sqrt{2}}(\ln|2+2\sqrt{2}+2| - 2\operatorname{arctg}(\sqrt{2}+1)) + \frac{1}{4\sqrt{2}}(\ln|2+2\sqrt{2}+2| +$
$+ 2\operatorname{arctg}(\sqrt{2}+1)) + \frac{1}{4\sqrt{2}}(\ln|2+2\sqrt{2}+2| + 2\operatorname{arctg}(\sqrt{2}+1)) + \frac{1}{4\sqrt{2}}(-\ln|2-2\sqrt{2}+2| +$
$+ 2\operatorname{arctg}(\sqrt{2}-1)) + \frac{1}{4\sqrt{2}}(\ln|2-2\sqrt{2}+2| - 2\operatorname{arctg}(-\sqrt{2}+1)) -$
$- \frac{1}{4\sqrt{2}}(\ln|2-2\sqrt{2}+2| + 2\operatorname{arctg}(-\sqrt{2}-1)) - \frac{1}{4\sqrt{2}}(\ln|2-2\sqrt{2}+2| +$
$+ 2\operatorname{arctg}(1-\sqrt{2})) - \frac{1}{4\sqrt{2}}(-\ln|2+2\sqrt{2}+2| + 2\operatorname{arctg}(-\sqrt{2}-1)) =$
$= -\frac{1}{4\sqrt{2}}\ln(4+2\sqrt{2}) + \frac{1}{2\sqrt{2}}\operatorname{arctg}(\sqrt{2}+1) + \frac{1}{4\sqrt{2}}\ln(4+2\sqrt{2}) + \frac{1}{2\sqrt{2}}\operatorname{arctg}(\sqrt{2}+1) +$
$+ \frac{1}{4\sqrt{2}}\ln(4+2\sqrt{2}) + \frac{1}{2\sqrt{2}}\operatorname{arctg}(\sqrt{2}+1) + \frac{1}{4\sqrt{2}}\ln(4-2\sqrt{2}) + \frac{1}{2\sqrt{2}}\operatorname{arctg}(\sqrt{2}-1)$

d) $\int_{-1}^1 \frac{x}{x^2+x+1}dx = \int_{-1}^1 \frac{x}{(x+\frac{1}{2})^2+\frac{3}{4}}dx = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{\frac{1}{2}(2t-1)}{4t^2+3}dt =$

$= \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{2t-1}{4t^2+3}dt = 2\int_{-\frac{1}{2}}^{\frac{3}{2}} \left(\frac{2t}{4t^2+3} - \frac{1}{4t^2+3}\right)dt =$

$= 2\left(\frac{1}{4}\ln|4t^2+3| - \frac{1}{2\sqrt{3}}\operatorname{arctg}\left(\frac{2}{\sqrt{3}}t\right)\right)\Big|_{-\frac{1}{2}}^{\frac{3}{2}} =$

$= \left(\frac{1}{2}\ln|4x^2+4x+4| - \frac{1}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)\right)\Big|_{-1}^{1} =$

$= \frac{1}{2}\ln|4+4+4| - \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2+1}{\sqrt{3}} - \frac{1}{2}\ln|4-4+4| + \frac{1}{\sqrt{3}}\operatorname{arctg}\left(\frac{1-2}{\sqrt{3}}\right) =$

$= \frac{1}{2}\ln 12 - \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{3}{\sqrt{3}} - \frac{1}{2}\ln 4 + \frac{1}{\sqrt{3}}\operatorname{arctg}\left(-\frac{1}{\sqrt{3}}\right) =$

$= \frac{1}{2}\ln 3 - \frac{1}{\sqrt{3}}\frac{\pi}{3} + \frac{1}{\sqrt{3}}\frac{\pi}{6} - \frac{1}{2}\ln 3$

Ex 6. a) $\int_{-3}^{-2} \frac{x}{(x+1)(x^2+3)}\,dx = \int_{-3}^{-2} -\frac{1}{4(x+1)} + \frac{x+3}{4(x^2+3)}\,dx =$

$= (-\frac{1}{4}\ln|x+1| + \frac{1}{4}(\frac{1}{2}\ln|\frac{x^2}{3}+1| + \sqrt{3}\,\text{arctg}(\frac{x}{\sqrt{3}})))\Big|_{-3}^{-2} =$

$= -\frac{1}{4}\ln|-1| + \frac{1}{8}\ln|\frac{4}{3}+1| + \frac{\sqrt{3}}{4}\text{arctg}\frac{-2}{\sqrt{3}} +$

$+ \frac{1}{4}\ln|-2| - \frac{1}{8}\ln|\frac{9}{3}+1| - \frac{\sqrt{3}}{4}\text{arctg}\frac{-3}{\sqrt{3}} =$

$= \frac{1}{8}\ln\frac{7}{3} - \frac{\sqrt{3}}{4}\text{arctg}\frac{2}{\sqrt{3}} + \frac{1}{4}\ln 2 - \frac{1}{8}\ln 4 + \frac{\sqrt{3}}{4}\text{arctg}\sqrt{3} =$

$\cancel{= \frac{1}{8}\ln\frac{7}{12} + \frac{1}{4}\ln 2 - \frac{\sqrt{3}}{4}\text{arctg}\frac{2}{\sqrt{3}}}$

b) $\int_0^1 \frac{x+1}{(x^2+4x+5)^2}\,dx = \int_0^1 \frac{x+2-1}{[(x+2)^2+1]^2}\,dx = \int_2^{-1} \frac{t-1}{(t^2+1)^2}\,dt =$

$\cancel{f(x) = x+1 \quad f(x) = \frac{1}{(x^2+4x+5)^2} = (x^2+4x+5)^{-2}}$

$\cancel{f'(x) = -2(x^2+4x+5)^{-3}\cdot(2x+4) = \frac{-4x-8}{(x^2+4x+5)^3}}$

$\cancel{= \frac{1}{2}(x+1)(\text{arctg}(x+2) + \frac{1}{2}\sin}$

$= \frac{1}{2}\sin(\text{arctg}(x+2)) - \frac{1}{2}\text{arctg}(x+1) + \frac{1}{4}\sin(2\,\text{arctg}(x+2))\Big|_0^1 = \ldots$

c) $\int_1^2 \frac{1}{x^3+x}\,dx = \int_1^2 \frac{1}{x} - \frac{x}{x^2+1}\,dx = (\ln|x| - \frac{1}{2}\ln|x^2+1|)\Big|_1^2 =$

$= \ln 2 - \frac{1}{2}\ln 5 - \ln 1 + \frac{1}{2}\ln 2 = \frac{3}{2}\ln 2 - \frac{1}{2}\ln 5$

d) $\int_0^2 \frac{x^3+2x^2+x+4}{(x+1)^2}\,dx = \int_0^2 x + \frac{4}{(x+1)^2}\,dx = (\frac{x^2}{2} - \frac{4}{x+1})\Big|_0^2 =$

$= \frac{4}{2} - \frac{4}{3} + 4 = \frac{12-8+24}{6} = \frac{14}{3}$

e) $\int_0^1 \frac{1}{(x+1)(x^2+4)}\,dx = \int_0^1 \frac{1}{5(x+1)} + \frac{-x+1}{5(x^2+4)}\,dx = (\frac{1}{5}\ln|x+1| + \frac{1}{5}(-\frac{1}{2}\ln|x^2+4| +$

$+ \frac{1}{2}\text{arctg}(\frac{x}{2})))\Big|_0^1 = \frac{1}{5}\ln 2 - \frac{1}{10}\ln 5 + \frac{1}{10}\text{arctg}\frac{1}{2} + \frac{1}{10}\ln 4 =$

f) $\int_2^3 \frac{2x^3+x^2+2x-1}{x^4-1}\,dx = \int_2^3 \frac{1}{x^2+1} + \frac{1}{x+1} + \frac{1}{x-1}\,dx = (\text{arctg}(x) + \ln|x+1| +$

$+ \ln|x-1|)\Big|_2^3 = \text{arctg}\,3 + \ln 4 + \ln 2 - \text{arctg}\,2 - \ln 3 - \ln 1 =$

$= \text{arctg}\,3 - \text{arctg}\,2 + \ln\frac{8}{3}$

g) $\int_0^1 \frac{x^3+2}{(x+1)^3}\,dx = \int_0^1 \frac{x^3}{(x+1)^3} + \frac{2}{(x+1)^3}\,dx = (x+1 - 3\ln|x+1| - \frac{3}{x+1} + \frac{1}{2(x+1)^2} - \frac{1}{(x+1)^2})\Big|_0^1 =$

$= (2 - 3\ln 2 - \frac{11}{8}) + 1 = 3 - 3\ln 2 - \frac{11}{8}$

Ex 7: a) $\int_{-1}^1 \frac{1}{\sqrt{4-x^2}}\,dx = \arcsin\frac{x}{2}\Big|_{-1}^1 = \ldots$

b) $\int_0^1 \frac{1}{\sqrt{x^2+x+1}}\,dx = \int_0^1 \frac{1}{\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}}\,dx = \cancel{\frac{1}{2}\ln|\text{tg}}$

$= \ln\left|\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \sqrt{\frac{1}{3}(4x^2+4x+4)}\right|\Big|_0^1$

c) $\int_{-1}^{1} \frac{1}{\sqrt{4x^2+x4}} dx = \frac{1}{2} \ln \left| \frac{8}{\sqrt{15}} x + \frac{1}{\sqrt{15}} + \sqrt{\frac{64x^2+16x+16}{15}} \right| \Big|_{-1}^{1}$

d) $\int_2^3 \frac{x^2}{(x^2-1)\sqrt{x^2-1}} dx = \left(-\frac{x}{\sqrt{x^2-1}} + \ln\left|\sqrt{x^2-1}+x\right|\right)\Big|_2^3$

Ex8: a) $\int_2^3 \sqrt{x^2+2x-7}\, dx = \int_2^3 \sqrt{(x+1)^2-8}\, dx =$
$= 8\left(-\ln\left|\sqrt{x^2+2x-7}+x+1\right| + \frac{3}{2}\ln(2)\right) + \frac{1}{16}(x+1)\sqrt{x^2+2x-7} +$
$+ \frac{1}{4}\left(2\ln\left|\sqrt{x^2+2x-7}+x+1\right| - 3\ln 2\right)\Big|_2^3$

b) $\int_0^1 \sqrt{6+4x-2x^2}\, dx = \sqrt{2}\int_0^1 \sqrt{-(x-1)^2+4}\, dx = \sqrt{2}\int_{-1}^{0}\sqrt{-u^2+4}\, du =$
$= \sqrt{2}\int \cos^2(v)\, dv = \sqrt{2}\cdot 4\cdot \frac{1}{2}\int_0^1 1+\cos(2v)\, dv =$
$= \sqrt{2}\cdot 4\cdot \frac{1}{2}\left(v + \frac{1}{2}\sin 2v\right) = 2\sqrt{2}\arcsin\left(\frac{1}{2}(x-1)\right) + \sqrt{2}\sin$
$2\arcsin\left(\frac{1}{2}(x-1)\right)\Big|_0^1$

c) $\int_0^{3/4} \frac{1}{(x+1)\sqrt{x^2+1}} dx = \left(\sqrt{2}\left(\frac{1}{2}\ln\left|\frac{\operatorname{tg}\left(\frac{\operatorname{arctg}(x)}{2}\right)-1}{\sqrt{2}}+1\right| -\right.\right.$
$\left.\left. -\frac{1}{2}\ln\left|\frac{\operatorname{tg}\left(\frac{\operatorname{arctg}(x)}{2}\right)-1}{\sqrt{2}}-1\right|\right)\right)\Big|_0^{3/4}$

d) $\int_2^3 \frac{1}{x\sqrt{x^2-1}} dx = \operatorname{arctg}\left(\sqrt{x^2-1}\right)\Big|_2^3$