

Happy Number

For this number we must repeatedly replace the number with the sum of squares of its digits and check if we eventually reach 1 (happy) or fall into a cycle (unhappy).

[Key Insight:]

- If the process enters a cycle, it will never reach 1.
- Known property: The cycle for unhappy numbers always includes 4 (but we can detect cycles generally).
- Two main methods:
 1. Hash set to track visited numbers.
 2. Floyd's Cycle Detection (fast & slow pointers) to detect loops.

[Approach 1:] Hash Set (simple)

1. While n is not 1 and not seen before:
 - Add n to visited set.
 - Replace n with sum of squares of its digits.
2. If $n == 1$, return true; else false

[Approach 2:] Floyd's Cycle Detection ($O(1)$ space)

- Use two pointers:
 - $slow = \text{sumOfSquares}(n)$
 - $fast = \text{sumOfSquares}(\text{sumOfSquares}(n))$
- Loop until $slow == fast$:
 - If $slow == 1$ or $fast == 1 \rightarrow$ happy number.
 - Else continue.
- If loop ends and value isn't 1 \rightarrow not happy.

[Complexity:]

- Time: $O(\log n)$ per iteration (digits processing), total bounded since numbers decrease fast.
- Space: $O(1)$ for Floyd's method, $O(k)$ for hash set method ($k =$ number of iterations).