

# Max Points on a Line

## Core Idea:

Fix each point  $i$  as an anchor, then count how many other points share the same slope with it. The most frequent slope through  $i$  gives the maximum number of collinear points that pass through  $i$ . Take the max over all anchors.

## How to represent a slope robustly (no floating-point!):

Use a reduced rational  $(dy, dx)$ :

- Let  $dy = y_j - y_i$ ,  $dx = x_j - x_i$ .
- Reduce by  $g = \gcd(|dy|, |dx|)$ :  $dy /= g$ ,  $dx /= g$ .
- Normalize sign so each geometric slope has a unique key:
  - Force  $dx > 0$ . If  $dx < 0$ , flip both signs.
  - For vertical lines, set to  $(1, 0)$ .
  - For horizontal lines, set to  $(0, 1)$ .
  - For the zero vector (shouldn't occur since points are unique), you'd normally count duplicates - here constraints say all points are unique.

Use a hash map from  $(dy, dx) \rightarrow \text{count}$ .

## Per-anchor procedure:

For each anchor  $i$ :

1. Clear the slope counter map.
2. For each other point  $j$ :
  - Compute  $(dy, dx)$ , reduce & normalize.
  - Increment the count for that slope.
3. The best through  $i$  is  $1 + \text{max count on any slope}$  (the  $+1$  for the anchor itself).
4. Track a global maximum.

## Minor optimizations (optional):

- Early pruning: after processing anchors  $0 \dots i$ , if  $\text{global-max} \geq n - i$ , you can stop (can't beat it).
- Use a compact key for the slope (e.g. pair of ints) with a custom hash; avoid

strings.