

Interleaving String

Think dynamic programming with a "how much of s_1 and s_2 can build a prefix of s_3 " mindset.

Core checks:

If $|s_1| + |s_2| \neq |s_3|$, answer is false immediately.

2D DP idea (then we'll trim to 1D):

Let $dp[i][j]$ be true iff the first i chars of s_1 and the first j chars of s_2 can interleave to form the first $i+j$ chars of s_3 .

Base:

$dp[0][0] = \text{true}$.

First row: $dp[0][j] = dp[0][j-1] \ \&\& \ s_2[j-1] == s_3[j-1]$.

First column: $dp[i][0] = dp[i-1][0] \ \&\& \ s_1[i-1] == s_3[i-1]$.

Transition for general $i, j \geq 1$:

You can come from top if $dp[i-1][j]$ is true and $s_1[i-1] == s_3[i+j-1]$.

Or from left if $dp[i][j-1]$ is true and $s_2[j-1] == s_3[i+j-1]$.

So: $dp[i][j] = (dp[i-1][j] \ \&\& \ s_1[i-1] == s_3[i+j-1]) \ \vee$
 $(dp[i][j-1] \ \&\& \ s_2[j-1] == s_3[i+j-1])$.

The answer is $dp[|s_1|][|s_2|]$.

Space-optimized ($O(|s_2|)$):

Keep a 1D array $dp[j]$ representing the current row's $dp[i][j]$.

Initialization ($i=0$ row):

$dp[0] = \text{true}$ and for $j = 1 \dots |s_2|$: $dp[j] = dp[j-1] \ \&\& \ (s_2[j-1] == s_3[j-1])$.

For each i from $1 \dots |s_1|$:

Update the first column:

$dp[0] = dp[0] \ \&\& \ (s_1[i-1] == s_3[i-1])$.

For each j from $1 \dots |s_2|$:

Use the same logic but carefully with overwritten values:

"From top" uses the current $dp[j]$ (which still holds the previous row's value before you overwrite it) and checks

$$S1[i-1] == S3[i+j-1].$$

· "From left" uses $dp[j-1]$ (already updated for this row) and checks $S2[j-1] == S3[i+j-1]$.

· Combine with OR.

Why this works:

You're deciding for each i, j , whether the last character of the $S3$ -prefix come from $S1[i-1]$ or $S2[j-1]$. If either source makes the rest consistent, mark true.