

Maximum Sum Circular Subarray

This is the classic maximum subarray sum on a circular array. The key is to combine two cases.

Core Idea (Kadane twice):

For any array, the maximum circular subarray sum is either:

1. Non-wrapping: entirely inside the array (no wrap) \rightarrow just the standard Kadane maximum subarray.
2. Wrapping: it wraps around the end to the start. If a subarray wraps, it's equivalent to $\text{total sum} - (\text{minimum subarray sum})$, because the wrap includes "everything except" some middle chunk; that excluded chunk must be the minimum subarray.

So: $\text{answer} = \max(\text{max_subarray_sum}, \text{total_sum} - \text{min_subarray_sum})$.

Important edge case:

If all numbers are negative, then:

- max_subarray_sum is the largest (least negative) element,
- min_subarray_sum equals total_sum ,
- $\text{total_sum} - \text{min_subarray_sum}$ becomes 0, which is invalid because the subarray must be non-empty.

So in that case, return max_subarray_sum only.

How to compute the pieces:

• Kadane's max sum in $O(n)$:

• Track $\text{curMax} = \max(\text{num}, \text{curMax} + \text{num})$ and
 $\text{bestMax} = \max(\text{bestMax}, \text{curMax})$.

• Kadane's min sum (same trick but flipped):

• Track $\text{curMin} = \min(\text{num}, \text{curMin} + \text{num})$ and
 $\text{bestMin} = \min(\text{bestMin}, \text{curMin})$.

• total_sum : accumulate in the same pass.

Do it all in one pass:

• initialize $\text{curMax} = \text{curMin} = 0$, $\text{bestMax} = -\infty$, $\text{bestMin} = +\infty$,

$total = 0$

For each x :

$curMax = \max(x, curMax + x)$, $bestMax = \max(bestMax, curMax)$.

$curMin = \min(x, curMin + x)$, $bestMin = \min(bestMin, curMin)$.

$total += x$

If $bestMax < 0$ (all negative), return $bestMax$. Else return $\max(bestMax, total - bestMin)$.

Why this works (intuition):

A wrapping subarray looks like: suffix + prefix. Its sum equals the total array minus a contiguous middle block that's not taken; to maximize the wrap; minimize the middle block $\rightarrow total - minSubarray$