

K-th Smallest Element in a BST

This problem takes advantage of the BST property:

- The inorder traversal of a BST gives sorted node values.
- The k -th smallest element is the k -th value visited in inorder traversal.

Approach 1: Simple Inorder Traversal ($O(n)$ time, $O(h)$ space):

Steps:

1. Perform inorder traversal (left \rightarrow root \rightarrow right).
2. Keep a counter while traversing.
3. When the counter equals k , return that node's value.

Complexity:

- Time: $O(k+h)$ worst-case (visiting k nodes; h is height)
- Space: $O(h)$ due to recursion stack.

Follow-Up: Optimized for Frequent Modifications

If the BST is modified frequently (insert/delete) and we need k -th smallest queries often:

Optimization:

- Augment the BST nodes with an extra field: $size$ = number of nodes in its subtree.
- While inserting/deleting, update $size$.
- To find k -th smallest:
 1. Use $size$ (left subtree) to decide:
 - If $k == size(left) + 1$, return root.
 - If $k \leq size(left)$, search left subtree.
 - Else, search right subtree with $k - size(left) - 1$.

This reduces query time to $O(h)$ and supports efficient updates.