

Permutation Sequence

For this problem the goal is to return the k -th lexicographical permutation of numbers $[1 \dots n]$.

Key Idea: Factorial Number System:

Instead of generating all permutations ($O(n!)$), we compute the k -th permutation directly using factorials.

Steps:

1. Precompute factorials for numbers $1 \dots n$.
2. Create a list of numbers $[1, 2, \dots, n]$.
3. Convert k to 0-based index $(k-1)$.
4. For each position:
 - Determine which number to pick using $\text{index} = k / \text{factorial}[n-1]$
 - Append number at index to result.
 - Remove that number from list.
 - Update $k = k \% \text{factorial}[n-1]$
 - Decrease n and repeat.

Complexity:

- Time: $O(n^2)$ (due to erase from vector; negligible for $n \leq 9$)
- Space: $O(n)$ for factorial and numbers list.