

Text List

Option A: Bottom-up merge sort ($O(1)$ extra space)

Idea:

- Repeatedly merge runs (sublists) of size 1, 2, 4, 8, ... until the whole list is one run.
- You'll need three tiny helpers:
 - $\text{length}(\text{head}) \rightarrow$ number of nodes
 - $\text{split}(\text{head}, k) \rightarrow$ cut off the first k nodes and return the head of the remainder (a nullptr).
 - $\text{merge}(l_1, l_2) \rightarrow$ merge two sorted lists; return $(\text{mergedHead}, \text{mergedTail})$.

Outline:

- Compute $n = \text{length}(\text{head})$. Use a dummy prehead (0, head) to simplify links.
- For $\text{step} = 1; \text{step} < n; \text{step} <= 1$:
 - $\text{prev} = \&\text{dummy}, \text{cur} = \text{dummy}.next$
 - while cur :
 - $\text{left} = \text{cur}$
 - $\text{right} = \text{split}(\text{left}, \text{step})$ // detaches left run of size $\leq \text{step}$
 - $\text{next} = \text{split}(\text{right}, \text{step})$ // detaches right run of size $\leq \text{step}$
 - $(\text{mergedHead}, \text{mergedTail}) = \text{merge}(\text{left}, \text{right})$
 - $\text{prev} \rightarrow \text{next} = \text{mergedHead}$
 - $\text{prev} = \text{mergedTail}$
 - $\text{cur} = \text{next}$
- Return $\text{dummy}.next$.

Option B: Top-down (recursive) merge sort (simpler but $O(\log n)$ stack)

Idea:

- Use slow/fast pointers to find the middle, split into two halves, recursively sort, then merge.

Outline:

1. Base: if $head == nullptr$ || $head \rightarrow next == nullptr$ return $head$.
2. Find mid with slow/fast, split into $[head \dots mid]$ and $[midNext \dots end]$.
3. $L = \text{sort List (left)}$, $R = \text{sort List (right)}$
4. Return $\text{merge}(L, R)$.

Helper behavior (for either option):

$\text{merge}(l_1, l_2)$:

- Classic two-pointer merge for sorted list.
- Keep a dummy node, append the smaller node each time.
- Return (dummy, next, tailPointer).

$\text{split}(head, k)$ (bottom-up only):

- If $k == 0$ return $head$.
- Walk $k-1$ steps or until end; let tail be last node of first part.
- $next = tail ? tail \rightarrow next : nullptr$
- If tail exists, set $tail \rightarrow next = nullptr$
- Return $next$.

Complexity:

- Time: $O(n \log n)$
- Space: $O(1)$ extra for bottom-up; $O(\log n)$ stack for recursive.