

## Best Time to Buy and Sell Stock IV

### Pick the right model first:

1. If  $k$  is large: when  $k \geq \lfloor n/2 \rfloor$ , it's equivalent to unlimited transactions.

Tips: just sum every positive price difference (sum  $\max(0, \text{prices}[i] - \text{prices}[i-1])$ ).

Why: you can buy/sell on every rise without exceeding  $k$ .

2. Otherwise ( $k$  is small): use DP with  $k$  layers.

### The classic $O(n \cdot k)$ DP:

Think in terms of  $t$  completed transactions (cell actions) up to day  $i$ .

#### A) "Hold / Not-hold" state per transaction count:

Maintain arrays of size  $k+1$ :

- $\text{hold}[t]$  = max profit after buying for the  $t$ -th transaction (i.e. currently holding a stock; you've completed  $t-1$  sells).
- $\text{cash}[t]$  = max profit after selling for the  $t$ -th time (i.e. not holding; you've completed  $t$  sells).

Initialize:

- $\text{cash}[0] = 0$ ,  $\text{hold}[0] = -\infty$  (can't hold before any buy).
- For  $t \geq 1$ :  $\text{hold}[t] = -\infty$ ,  $\text{cash}[t] = -\infty$  initially (or a very small number).

For each price  $p$ :

for  $t = 1 \dots k$ :

$$\text{hold}[t] = \max(\text{hold}[t], \text{cash}[t-1] - p)$$

$$\text{cash}[t] = \max(\text{cash}[t], \text{hold}[t] + p)$$

Answer:  $\max_{t=0 \dots k} \text{cash}[t]$  (usually  $\text{cash}[k]$  is enough since  $\text{cash}$  is nondecreasing in  $t$ ).

Why it works: each layer  $t$  represents you're working on the  $t$ -th transaction; buying uses profit from  $\text{cash}[t-1]$ , selling closes  $\text{hold}[t]$ .

#### B) "Best-so-far" trick per layer:

Let  $\text{dp}[t][i]$  = max profit up to day  $i$  with at most  $t$  transactions.

Transition when you sweep  $i$ :

$$dp[t][i] = \max(dp[t][i-1], \text{prices}[i] + \text{best})$$

$$\text{best} = \max(\text{best}, dp[t-1][i] - \text{prices}[i]).$$

Here best tracks the best "buy point with  $t-1$  transactions done" as you go.  
space-opt: keep only two 1D arrays for  $dp[t-1][*]$  and  $dp[t][*]$ .

### Complexity:

- Time:  $O(n \cdot k)$  (early-return to greedy when  $k \geq n/2$ ).
- Space:  $O(k)$  with the hold/cash form, or  $O(n) \rightarrow O(1)$  per layer with rolling arrays.