SICP Chapter 1

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HOW THIS DOCUMENT IS MADE

TODO

```
(define (foo a b)
  (+ a (* 2 b)))

(foo 5 3)
```

11

^ Dynamically evaluated when you press "enter" on the `BEGIN_SRC` block!

Also consider:

- `:results output` for what the code prints
- `:exports code` or `:exports results` to just get one or the other

$a + (\pi \times b)$ <~ inline Latex btw :)

Current command for conversion

```
pandoc --from org --to latex 1.org -o 1.tex -s; xelatex 1.tex
```

Helpers for org-mode tables

`try-these`

Takes function `f` and list `testvals` and applies `f` to each item `i`. For each `i` returns a list with `i` and the result. Useful dor making tables with a column for input and a column for output.

```
;; Surely this could be less nightmarish
(define (try-these f . testvals)
  (let ((l (if (and (= 1 (length testvals))
                    (list? (car testvals)))
               (car testvals)
               testvals)))
    (map (λ (i) (cons i
```

```
                       (cons (if (list? i)
                                 (apply f i)
                                 (f i))
                             #nil)))
         l)))
```

transpose-list

"Rotate" a list, for example from `'(1 2 3)` to `'('(1) '(2) '(3))`

```
(define (transpose-list l)
  (map (λ (i) (list i)) l))
```

print-as-rows

For manually printing items in rows to stdout. Not currently used.

```
(define (p-nl a)
  (display a)
  (newline))
(define (print-spaced args)
  (let ((a (car args))
        (d (cdr args)))
    (if (null? d)
        (p-nl a)
        (begin (display a)
               (display " ")
               (print-spaced d)))))
(define (print-as-rows . args)
  (let ((a (car args))
        (d (cdr args)))
    (if (list? a)
        (if (= 1 (length args))
            (apply print-as-rows a)
            (print-spaced a))
        (p-nl a))
    (if (null? d)
        '()
        (apply print-as-rows d))))
```

Exercise 1.1

Q

Below is a sequence of expressions. What is the result printed by the interpreter in response to each expression? Assume that the sequence is to be evaluated in the order in which it is presented.

A

```scheme
10 ;; 10
(+ 5 3 4) ;; 12
(- 9 1) ;; 8
(/ 6 2) ;; 3
(+ (* 2 4) (- 4 6)) ;; 6
(define a 3) ;; a=3
(define b (+ a 1)) ;; b=4
(+ a b (* a b)) ;; 19
(= a b) ;; false
(if (and (> b a) (< b (* a b)))
    b
    a) ;; 4
(cond ((= a 4) 6)
      ((= b 4) (+ 6 7 a))
      (else 25)) ;; 16
(+ 2 (if (> b a) b a)) ;; 6
(* (cond ((> a b) a)
         ((< a b) b)
         (else -1))
   (+ a 1)) ;; 16
```

Exercise 1.2

Q

Translate the following expression into prefix form:

$$\frac{5 + 2 + (2 - 3 - (6 + \frac{4}{5}))}{3(6-2)(2-7)}$$

A

```scheme
(/ (+ 5 2 (- 2 3 (+ 6 (/ 4 5))))
   (* 3 (- 6 2) (- 2 7)))
```

1/75

Exercise 1.3

Text

```scheme
(define (square x)
  (* x x))
```

Q

Define a procedure that takes three numbers as arguments and returns the sum of the squares of the two larger numbers.

A

```
<<square>>
(define (sum-square x y)
  (+ (square x) (square y)))
(define (square-2of3 a b c)
  (cond ((and (>= a b) (>= b c)) (sum-square a b))
        ((and (>= a b) (> c b)) (sum-square a c))
        ((and (> b a) (>= c a)) (sum-square b c))
        (else "This shouldn't happen")))
```

```
<<EX1-3>>
<<try-these>>
 (try-these square-2of3 '(7 5 3)
                        '(7 3 5)
                        '(3 5 7))
```

(7 5 3)	74
(7 3 5)	74
(3 5 7)	74

Exercise 1.4

Q

Observe that our model of evaluation allows for combinations whose operators are compound expressions. Use this observation to describe the behavior of the following procedure:

```
(define (a-plus-abs-b a b)
  ((if (> b 0) + -) a b))
```

A

This code accepts the variables `a` and `b`, and if `b` is positive, it adds `a` and `b`. However, if `b` is zero or negative, it subtracts them. This decision is made by using the `+` and `-` procedures as the results of an if expression, and then evaluating according to the results of that expression. This is in contrast to a language like Python, which would do something like this:

```
if b > 0: a + b
else: a - b
```

Exercise 1.5

Q

Ben Bitdiddle has invented a test to determine whether the interpreter he is faced with is using applicative-order evaluation or normal-order evaluation. He defines the following two procedures:

```scheme
(define (p) (p))

(define (test x y)
  (if (= x 0)
      0
      y))
```

Then he evaluates the expression

```scheme
(test 0 (p))
```

What behavior will Ben observe with an interpreter that uses applicative-order evaluation? What behavior will he observe with an interpreter that uses normal-order evaluation? Explain your answer. (Assume that the evaluation rule for the special form if is the same whether the interpreter is using normal or applicative order: The predicate expression is evaluated first, and the result determines whether to evaluate the consequent or the alternative expression.)

A

In either type of language, `(define (p) (p))` is an infinite loop. However, a normal-order language will encounter the special form, return `0`, and never evaluate `(p)`. An applicative-order language evaluates the arguments to `(test 0 (p))`, thus triggering the infinite loop.

Exercise 1.6

Text code

```scheme
(define (abs x)
  (if (< x 0)
      (- x)
      x))
```

```scheme
(define (average x y)
  (/ (+ x y) 2))
```

```scheme
<<average>>
(define (improve guess x)
  (average guess (/ x guess)))

```

```scheme
<<square>>
<<abs>>
(define (good-enough? guess x)
  (< (abs (- (square guess) x)) 0.001))

(define (sqrt-iter guess x)
  (if (good-enough? guess x)
      guess
      (sqrt-iter (improve guess x) x)))

(define (sqrt x)
  (sqrt-iter 1.0 x))
```

Q

Exercise 1.6: Alyssa P. Hacker doesn't see why if needs to be provided as a special form. "Why can't I just define it as an ordinary procedure in terms of cond?" she asks. Alyssa's friend Eva Lu Ator claims this can indeed be done, and she defines a new version of if:

```scheme
(define (new-if predicate
                then-clause
                else-clause)
  (cond (predicate then-clause)
        (else else-clause)))
```

Eva demonstrates the program for Alyssa:

```scheme
(new-if (= 2 3) 0 5)
;; => 5

(new-if (= 1 1) 0 5)
;; => 0
```

Delighted, Alyssa uses new-if to rewrite the square-root program:

```scheme
(define (sqrt-iter guess x)
  (new-if (good-enough? guess x)
          guess
          (sqrt-iter (improve guess x) x)))
```

What happens when Alyssa attempts to use this to compute square roots? Explain.

A

Using Alyssa's `new-if` leads to an infinite loop because the recursive call to `sqrt-iter` is evaluated before the actual call to `new-if`. This is because `if` and `cond` are special forms that change the way evaluation is handled; whichever branch is chosen leaves the other branches unevaluated.

Exercise 1.7

Text

```
(define (mean-square x y)
  (average (square x) (square y)))
```

Q

The good-enough? test used in computing square roots will not be very effective for finding the square roots of very small numbers. Also, in real computers, arithmetic operations are almost always performed with limited precision. This makes our test inadequate for very large numbers. Explain these statements, with examples showing how the test fails for small and large numbers. An alternative strategy for implementing good-enough? is to watch how guess changes from one iteration to the next and to stop when the change is a very small fraction of the guess. Design a square-root procedure that uses this kind of end test. Does this work better for small and large numbers?

A

The current method has decreasing accuracy with smaller numbers. Notice the steady divergence from correct answers here (should be decreasing powers of 0.1):

```
<<txt-sqrt>>
<<try-these>>
(try-these sqrt 0.01 0.0001 0.000001 0.00000001 0.0000000001)
```

0.01	0.10032578510960605
0.0001	0.03230844833048122
1e-06	0.031260655525445276
1e-08	0.03125010656242753
1e-10	0.03125000106562499

And for larger numbers, an infinite loop will eventually be reached. 10^{12} can resolve, but 10^{13} cannot.

```
<<txt-sqrt>>
(sqrt 1000000000000)
```

1000000.0

My original answer was this, which compares the previous iteration until the new and old are within an arbitrary dx.

```
<<txt-sqrt>>
(define (inferior-good-enough? guess lastguess)
  (<=
   (abs (-
         (/ lastguess guess)
         1))
   0.0000000000001)) ; dx
(define (new-sqrt-iter guess x lastguess) ;; Memory of previous value
  (if (inferior-good-enough? guess lastguess)
      guess
      (new-sqrt-iter (improve guess x) x guess)))
(define (new-sqrt x)
  (new-sqrt-iter 1.0 x 0))
```

This solution can correctly find small and large numbers:

```
<<inferior-good-enough>>
(new-sqrt 10000000000000)
```

3162277.6601683795

```
<<try-these>>
<<inferior-good-enough>>
(try-these new-sqrt '(0.01 0.0001 0.000001 0.00000001 0.0000000001))
```

0.01	0.1
0.0001	0.01
1e-06	0.001
1e-08	9.999999999999999e-05
1e-10	9.999999999999999e-06

However, I found this solution online that isn't just simpler but automatically reaches the precision limit of the system:

```
<<txt-sqrt>>
(define (best-good-enough? guess x)
   (= (improve guess x) guess))
```

So, my final definition of `sqrt`:

```
<<average>>
(define (improve guess x)
  (average guess (/ x guess)))
(define (good-enough? guess x)
   (= (improve guess x) guess))
(define (sqrt-iter guess x)
```

```
  (if (good-enough? guess x)
      guess
      (sqrt-iter (improve guess x) x)))
(define (sqrt x)
  (sqrt-iter 1.0 x))
```

```
<<try-these>>
<<sqrt>>
(try-these sqrt '(0.01 0.0001 0.000001 0.00000001 0.0000000001))
```

0.01	0.1
0.0001	0.01
1e-06	0.001
1e-08	9.999999999999999e-05
1e-10	9.999999999999999e-06

Exercise 1.8

Q

Newton's method for cube roots is based on the fact that if y is an approximation to the cube root of x, then a better approximation is given by the value:

$$\frac{\frac{x}{y^2} + 2y}{3} \tag{1}$$

Use this formula to implement a cube-root procedure analogous to the square-root procedure. (In 1.3.4 we will see how to implement Newton's method in general as an abstraction of these square-root and cube-root procedures.)

A1

My first attempt works, but needs an arbitrary limit to stop infinite loops:

```
<<square>>
(define (cb-good-enough? guess x)
  (= (cb-improve guess x) guess))
(define (cb-improve guess x)
  (/
   (+
    (/ x (square guess))
    (* guess 2))
   3))
(define (cbrt-iter guess x counter)
  (if (or (cb-good-enough? guess x) (> counter 100))
      guess
```

```
      (begin
        (cbrt-iter (cb-improve guess x) x (+ 1 counter)))))
(define (cbrt x)
  (cbrt-iter 1.0 x 0))

(try-these cbrt 7 32 56 100)
```

7	1.912931182772389
32	3.174802103936399
56	3.825862365544778
100	4.641588833612779

However, this will hang on an infinite loop when trying to run `(cbrt 100)`. I speculate it's a floating point precision issue with the "improve" algorithm. So to avoid it I'll just keep track of the last guess and stop improving when there's no more change occurring. Also while researching I discovered that (again due to floating point) `(cbrt -2)` loops forever unless you initialize your guess with a slightly different value, so let's do 1.1 instead.

A2

```
<<square>>
(define (cb-good-enough? nextguess guess lastguess x)
  (or (= nextguess guess)
      (= nextguess lastguess)))
(define (cb-improve guess x)
  (/
   (+
    (/ x (square guess))
    (* guess 2))
   3))
(define (cbrt-iter guess lastguess x)
  (define nextguess (cb-improve guess x))
  (if (cb-good-enough? nextguess guess lastguess x)
      nextguess
      (cbrt-iter nextguess guess x)))
(define (cbrt x)
  (cbrt-iter 1.1 9999 x))
```

```
<<cbrt>>
<<try-these>>
(try-these cbrt 7 32 56 100 -2)
```

7	1.912931182772389
32	3.174802103936399

56	3.825862365544778
100	4.641588833612779
-2	-1.2599210498948732

Exercise 1.9

Q

Each of the following two procedures defines a method for adding two positive integers in terms of the procedures inc, which increments its argument by 1, and dec, which decrements its argument by 1.

```
(define (+ a b)
  (if (= a 0)
      b
      (inc (+ (dec a) b))))

(define (+ a b)
  (if (= a 0)
      b
      (+ (dec a) (inc b))))
```

Using the substitution model, illustrate the process generated by each procedure in evaluating `(+ 4 5)`. Are these processes iterative or recursive?

A

The first procedure is recursive, while the second is iterative though tail-recursion.

recursive procedure

```
(+ 4 5)
(inc (+ 3 5))
(inc (inc (+ 2 5)))
(inc (inc (inc (+ 1 5))))
(inc (inc (inc (inc (+ 0 5)))))
(inc (inc (inc (inc 5))))
(inc (inc (inc 6)))
(inc (inc 7))
(inc 8)
9
```

iterative procedure

```
(+ 4 5)
(+ 3 6)
(+ 2 7)
(+ 1 8)
(+ 0 9)
9
```

Exercise 1.10

Q

The following procedure computes a mathematical function called Ackermann's function.

```
(define (A x y)
  (cond ((= y 0) 0)
        ((= x 0) (* 2 y))
        ((= y 1) 2)
        (else (A (- x 1)
                 (A x (- y 1))))))
```

What are the values of the following expressions?

```
(A 1 10)
(A 2 4)
(A 3 3)
```

(1 10)	1024
(2 4)	65536
(3 3)	65536

```
<<ackermann>>
(define (f n) (A 0 n))
(define (g n) (A 1 n))
(define (h n) (A 2 n))
(define (k n) (* 5 n n))
```

Give concise mathematical definitions for the functions computed by the procedures `f`, `g`, and `h` for positive integer values of n. For example, `(k n)` computes $5n^2$.

A

`f`

```
<<try-these>>
<<EX1-10-defs>>
(try-these f 1 2 3 10 15 20)
```

1	2
2	4
3	6
10	20
15	30
20	40

$$f(n) = 2n$$

`g`

```
<<try-these>>
<<EX1-10-defs>>
(try-these g 1 2 3 4 5 6 7 8)
```

1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256

$$g(n) = 2^n$$

`h`

```
<<try-these>>
<<EX1-10-defs>>
(try-these h 1 2 3 4)
```

1	2
2	4
3	16

4	65536

It took a while to figure this one out, just because I didn't know the term. This is repeated exponentiation. This operation is to exponentiation, what exponentiation is to multiplication. It's called either *tetration* or *hyper-4* and has no formal notation, but two common ways would be these:

$$h(n) = 2 \uparrow\uparrow n$$

$$h(n) = {}^{n}2$$

Exercise 1.11

Q

A function f is defined by the rule that:

$$f(n) = n \text{ if } n < 3$$

and

$$f(n) = f(n-1) + 2f(n-2) + 3f(n-3) \text{ if } n \geq 3$$

Write a procedure that computes f by means of a recursive process. Write a procedure that computes f by means of an iterative process.

A

Recursive

```
(define (fr n)
  (if (< n 3)
      n
      (+      (fr (- n 1))
         (* 2 (fr (- n 2)))
         (* 3 (fr (- n 3))))))
```

```
<<try-these>>
<<EX1-11-fr>>
(try-these fr 1 3 5 10)
```

1	1
3	4
5	25
10	1892

Iterative

1. Attempt 1

```scheme
;; This seems like it could be better
(define (fi n)
  (define (formula l)
    (let ((a (car l))
           (b (cadr l))
           (c (caddr l)))
      (+ a
         (* 2 b)
         (* 3 c))))
  (define (iter l i)
    (if (= i n)
        (car l)
        (iter (cons (formula l) l)
              (+ 1 i))))
  (if (< n 3)
      n
      (iter '(2 1 0) 2)))
```

```
<<try-these>>
<<EX1-11-fi>>
(try-these fi 1 3 5 10)
```

1	1
3	4
5	25
10	1892

It works but it seems wasteful.

2. Attempt 2

```scheme
(define (fi2 n)
  (define (formula a b c)
      (+ a
         (* 2 b)
         (* 3 c)))
  (define (iter a b c i)
    (if (= i n)
        a
        (iter (formula a b c)
              a
              b
              (+ 1 i))))
```

```
  (if (< n 3)
      n
      (iter 2 1 0 2)))
```

```
<<try-these>>
<<EX1-11-fi2>>
(try-these fi2 1 3 5 10)
```

1	1
3	4
5	25
10	1892

I like that better.

Exercise 1.12

Q

The following pattern of numbers is called Pascal's triangle.

Pretend there's a Pascal's triangle here.

The numbers at the edge of the triangle are all 1, and each number inside the triangle is the sum of the two numbers above it. Write a procedure that computes elements of Pascal's triangle by means of a recursive process.

A

I guess I'll rotate the triangle 45 degrees to make it the top-left corner of an infinite spreadsheet.

```
(define (pascal x y)
  (if (or (= x 0)
          (= y 0))
      1
      (+ (pascal (- x 1) y)
         (pascal x (- y 1)))))
```

```
<<try-these>>
<<pascal-rec>>
(let ((l (iota 8)))
  (map (λ (row)
         (map (λ (xy)
                (apply pascal xy))
              row))
       (map (λ (x)
```

```
            (map (λ (y)
                   (list x y))
                 l))
          l)))
```

1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8
1	3	6	10	15	21	28	36
1	4	10	20	35	56	84	120
1	5	15	35	70	126	210	330
1	6	21	56	126	252	462	792
1	7	28	84	210	462	924	1716
1	8	36	120	330	792	1716	3432

The test code was much harder to write than the actual solution.

Exercise 1.13

Q

Prove that $\text{Fib}(n)$ is the closest integer to $\frac{\phi^n}{\sqrt{5}}$ where Phi is $\frac{1+\sqrt{5}}{2}$. Hint: let $\Upsilon = \frac{1-\sqrt{5}}{2}$. Use induction and the definition of the Fibonacci numbers to prove that

$$\text{Fib}(n) = \frac{\phi^n - \Upsilon^n}{\sqrt{5}}$$

A

I don't know how to write a proof yet, but I can make functions to demonstrate it.

Fibonacci number generator

```
(define (fib-iter n)
  (define (iter i a b)
    (if (= i n)
        b
    (iter (+ i 1)
          b
          (+ a b))))
  (if (<= n 2)
      1
      (iter 2 1 1)))
```

Various algorithms relating to the question

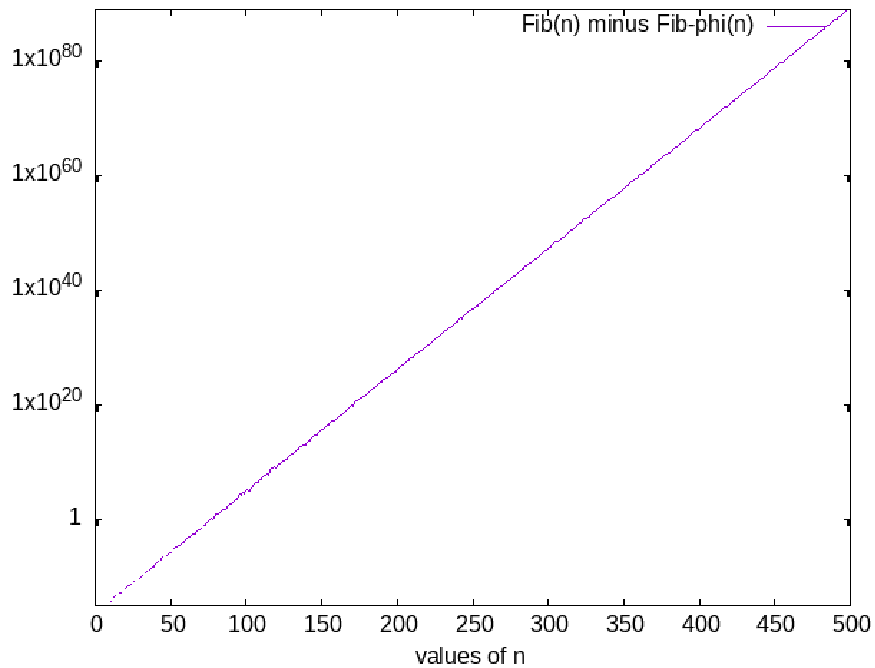
```
<<sqrt>>
(define sqrt5
  (sqrt 5))
(define phi
  (/ (+ 1 sqrt5) 2))
(define upsilon
  (/ (- 1 sqrt5) 2))
(define (fib-phi n)
  (/ (- (expt phi n)
        (expt upsilon n))
     sqrt5))
```

```
(use-srfis '(1))
<<fib-iter>>
<<fib-phi>>
<<try-these>>

(let* ((vals (drop (iota 21) 10))
       (fibs (map fib-iter vals))
       (approx (map fib-phi vals)))
  (zip vals fibs approx))
```

10	55	54.99999999999999
11	89	89.0
12	144	143.99999999999997
13	233	232.99999999999994
14	377	377.00000000000006
15	610	610.0
16	987	986.9999999999998
17	1597	1596.9999999999998
18	2584	2584.0
19	4181	4181.0
20	6765	6764.999999999999

You can see they follow extremely closely. Graphing the differences:


Fib(n) minus Fib-phi(n)
1x10^80
1x10^60
1x10^40
1x10^20
1
0
50
100
150
200
250
300
350
400
450
500
values of n