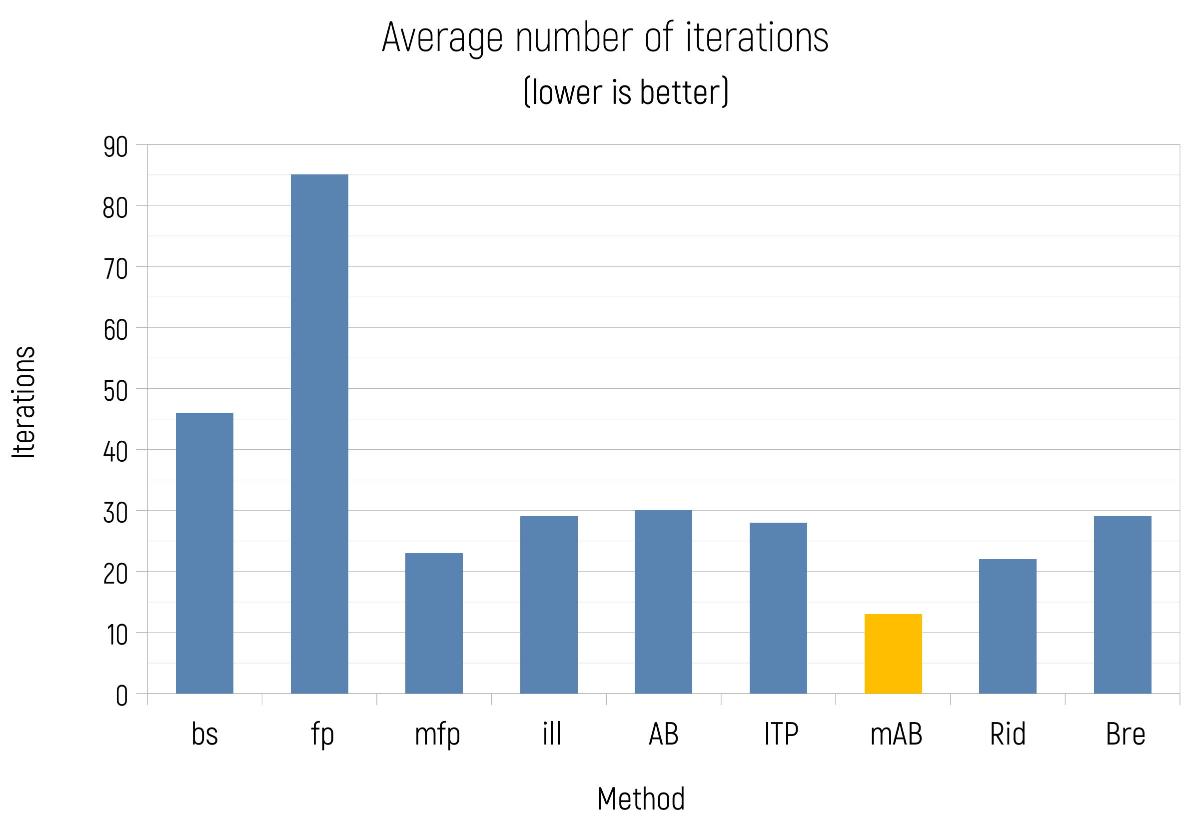
Benchmarking Root-Finding Algorithms 2

by eng. Nedelcho Ganchovski

1. Bisection method	**bs**
2. False position	**fp**
3. Modified false position	**mfp**
4. Illinois method [1]	**ill**
5. Anderson-Bjork [2]	**AB**
6. Interpolate, truncate, project [3]	**ITP**
7. Modified Anderson-Bjork [4]	**mAB** (Calcpad)
8. Ridders [5]	**Rid**
9. Brent [6]	**Bre**

Summary of the results



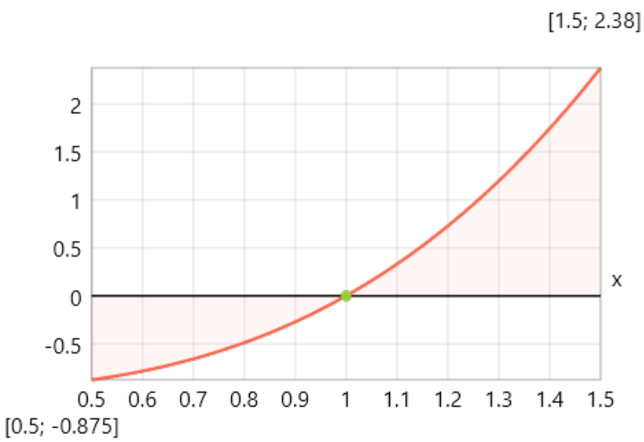
Method	bs	fp	mfp	ill	AB	ITP	mAB	Rid	Bre
SUM	4176	7709	2089	2684	2700	2581	1227	1990	2625
AVE	46	85	23	29	30	28	13	22	29
MAX	48	200	71	200	200	50	48	78	133

Root-finding Test Functions

Sérgio Galdino. *A family of regula falsi root-finding methods*. Proceedings of 2011 World Congress on Engineering and Technology. Shanghai, China. IEEE Press. ISBN 978-1-61284-365-0. p. 514-517

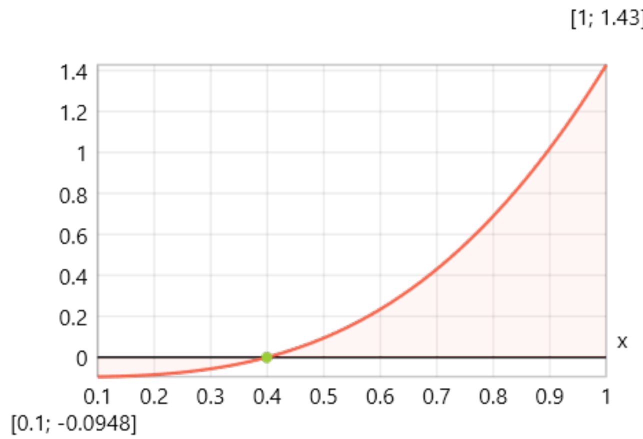
http://sergiogaldino.pbworks.com/w/file/fetch/66011429/0130-1943543

$$f_{01}(x) = x^3 - 1$$



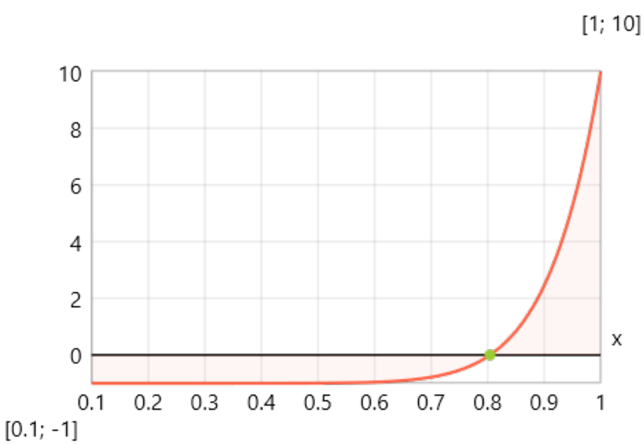
$$x_{01} = \$\mathrm{Root}\left\{ f_{01}(x) = 0\,;\ x \in [0.5\,;\ 1.5] \right\} = 1$$

$$f_{02}(x) = x^2 \cdot \left(\frac{x^2}{3} + \sqrt{2} \cdot \sin(x) \right) - \frac{\sqrt{3}}{18}$$



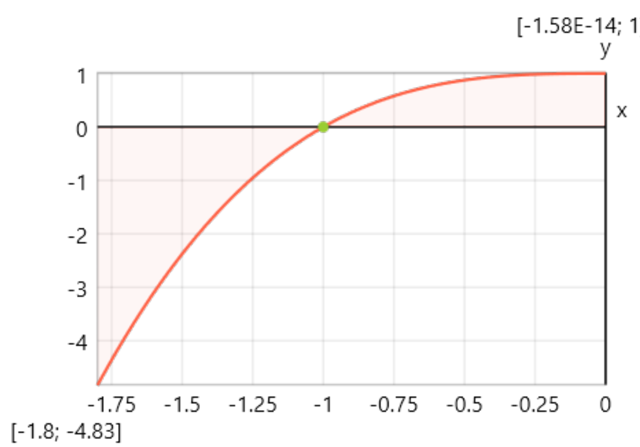
$$x_{02} = \$\mathrm{Root}\left\{ f_{02}(x) = 0\,;\ x \in [0.1\,;\ 1] \right\} = 0.399$$

$$f_{03}(x) = 11 \cdot x^{11} - 1$$



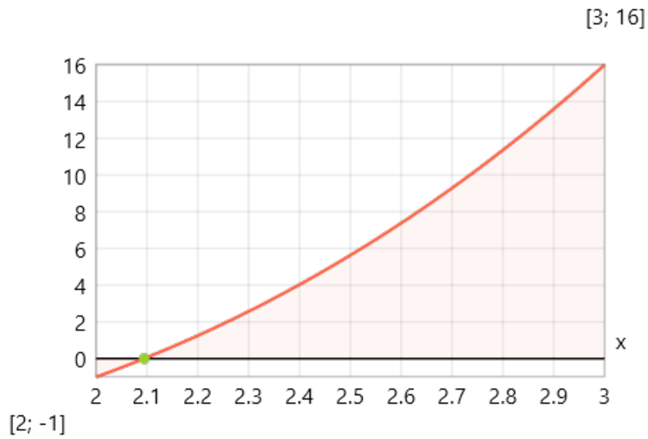
$$x_{03} = \$\mathrm{Root}\left\{ f_{03}(x) = 0\,;\ x \in [0.1\,;\ 1] \right\} = 0.804$$

$$f_{04}(x) = x^3 + 1$$



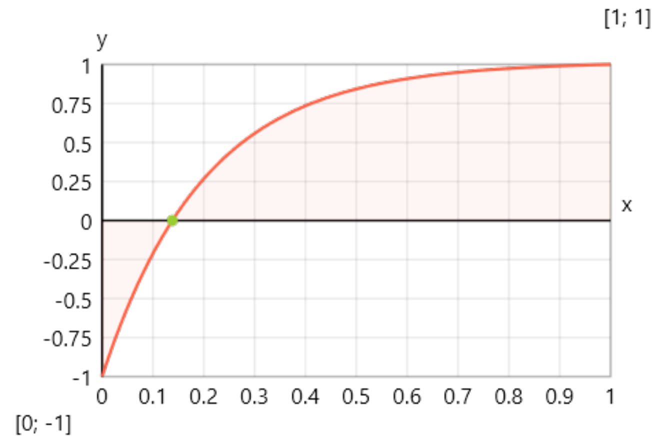
$$x_{04} = \$\mathrm{Root}\left\{ f_{04}(x) = 0\,;\ x \in [-1.8\,;\ 0] \right\} = -1$$

$$f_{05}(x)=x^3-2\cdot x-5$$



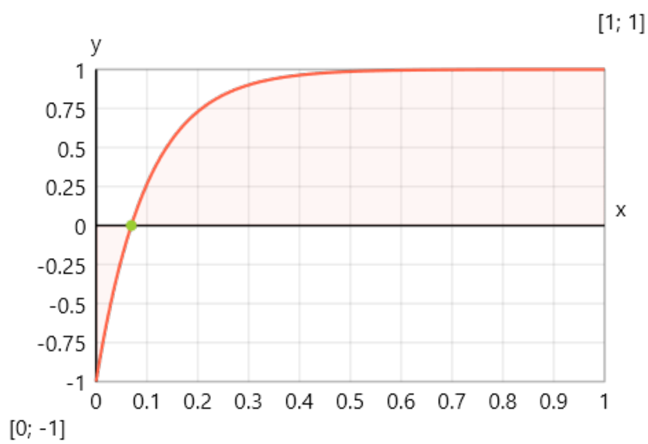
$$x_{05}=\$\text{Root}\left\{f_{05}(x)=0;\ x\in[2;\ 3]\right\}=2.09$$

$$f_{06}(x)=2\cdot x\cdot e^{-5}+1-2\cdot e^{-5\cdot x}$$



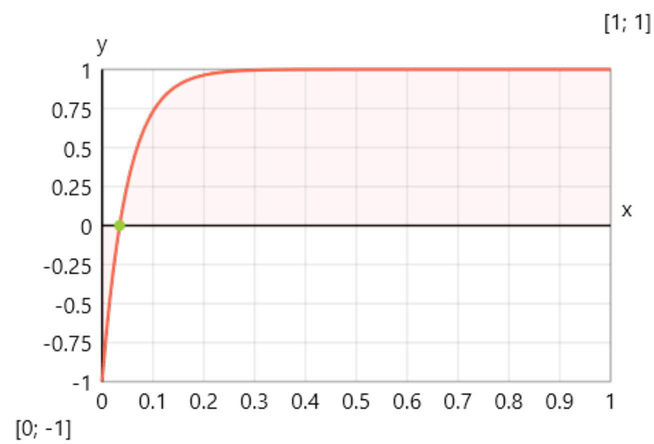
$$x_{06}=\$\text{Root}\left\{f_{06}(x)=0;\ x\in[0;\ 1]\right\}=0.138$$

$$f_{07}(x)=2\cdot x\cdot e^{-10}+1-2\cdot e^{-10\cdot x}$$



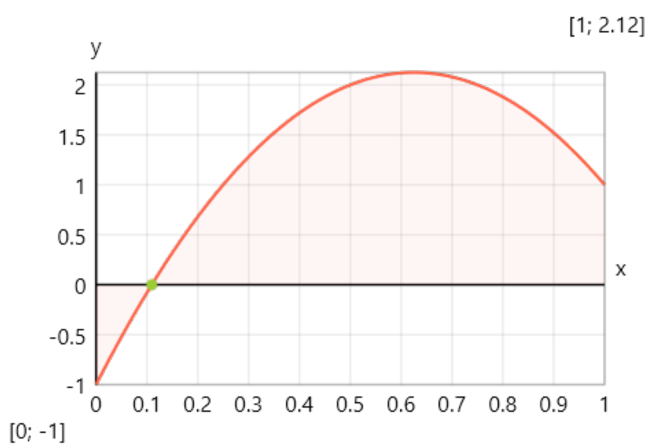
$$x_{07}=\$\text{Root}\left\{f_{07}(x)=0;\ x\in[0;\ 1]\right\}=0.0693$$

$$f_{08}(x)=2\cdot x\cdot e^{-20}+1-2\cdot e^{-20\cdot x}$$



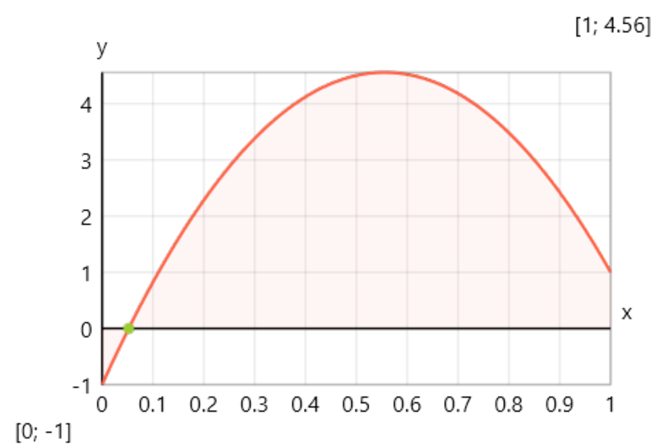
$$x_{08}=\$\text{Root}\left\{f_{08}(x)=0;\ x\in[0;\ 1]\right\}=0.0347$$

$$f_{09}(x)=\left(1+(1-5)^2\right)\cdot x^2-(1-5\cdot x)^2$$



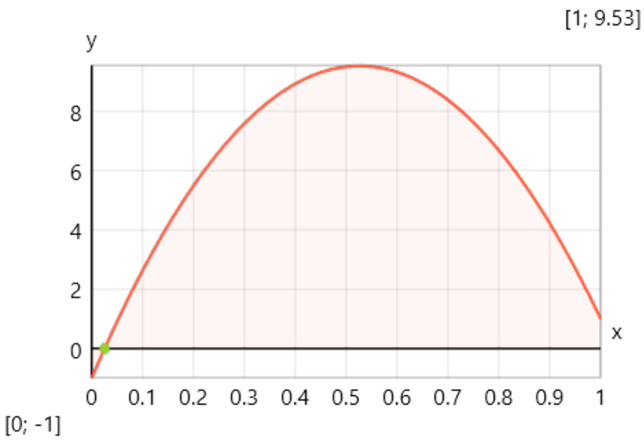
$$x_{09}=\$\text{Root}\left\{f_{09}(x)=0;\ x\in[0;\ 1]\right\}=0.11$$

$$f_{10}(x)=\left(1+(1-10)^2\right)\cdot x^2-(1-10\cdot x)^2$$



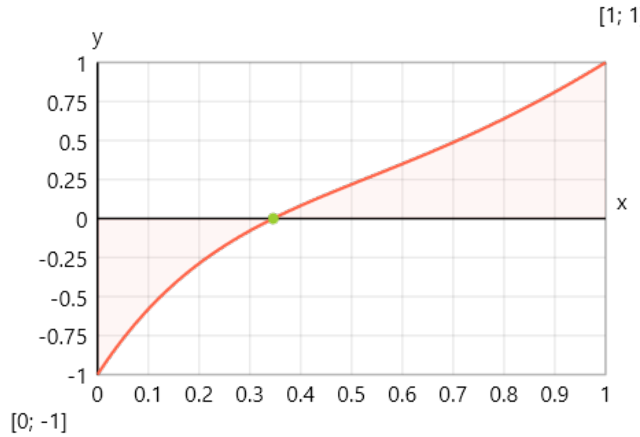
$$x_{10}=\$\text{Root}\left\{f_{10}(x)=0;\ x\in[0;\ 1]\right\}=0.0525$$

$$f_{11}(x)=\left(1+(1-20)^2\right)\cdot x^2-(1-20\cdot x)^2$$



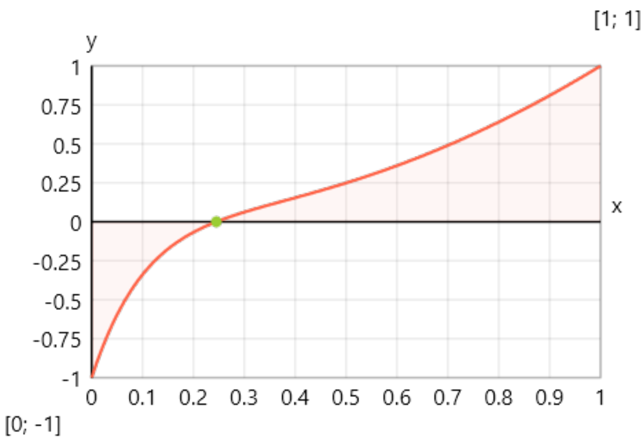
$$x_{11}=\$\text{Root}\left\{f_{11}(x)=0\,;\ x\in[0\,;\ 1]\right\}=0.0256$$

$$f_{12}(x)=x^2-(1-x)^5$$



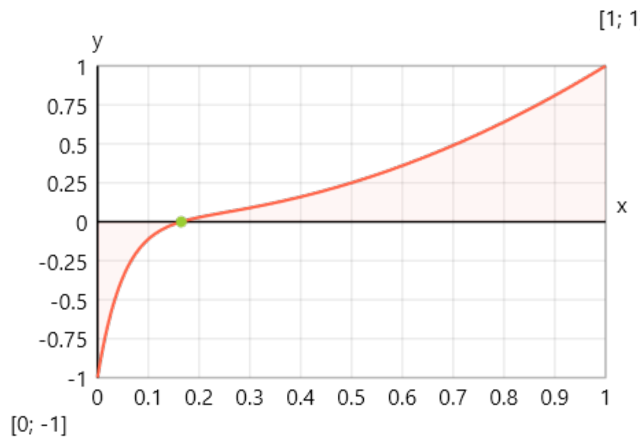
$$x_{12}=\$\text{Root}\left\{f_{12}(x)=0\,;\ x\in[0\,;\ 1]\right\}=0.346$$

$$f_{13}(x)=x^2-(1-x)^{10}$$



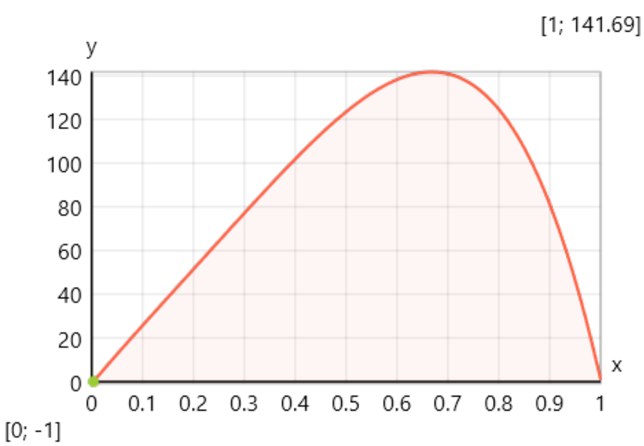
$$x_{13}=\$\text{Root}\left\{f_{13}(x)=0\,;\ x\in[0\,;\ 1]\right\}=0.245$$

$$f_{14}(x)=x^2-(1-x)^{20}$$



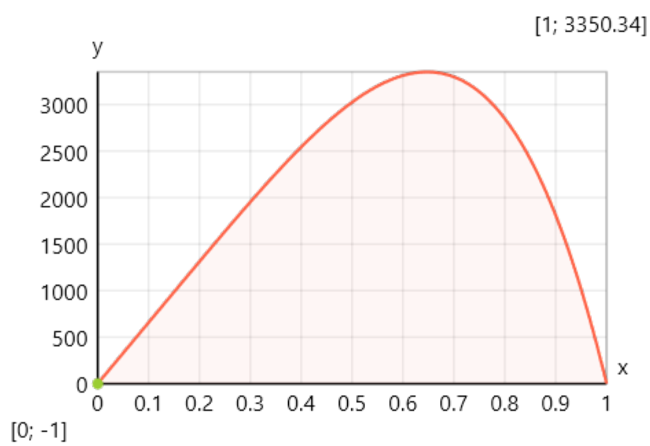
$$x_{14}=\$\text{Root}\left\{f_{14}(x)=0\,;\ x\in[0\,;\ 1]\right\}=0.165$$

$$f_{15}(x)=\left(1+(1-5)^4\right)\cdot x-(1-5\cdot x)^4$$



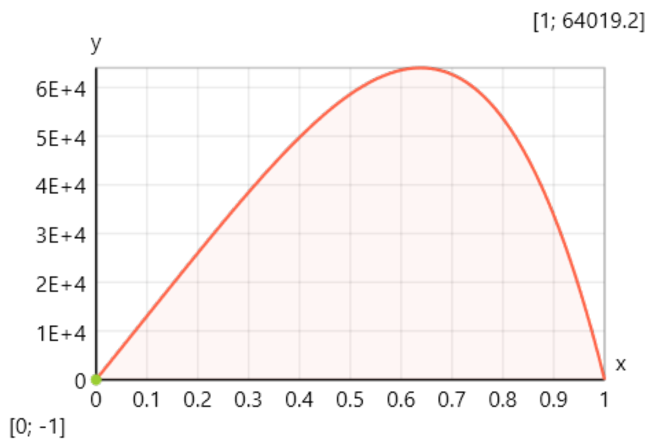
$$x_{15}=\$\text{Root}\left\{f_{15}(x)=0\,;\ x\in[0\,;\ 1]\right\}=0.00362$$

$$f_{16}(x)=\left(1+(1-10)^4\right)\cdot x-(1-10\cdot x)^4$$



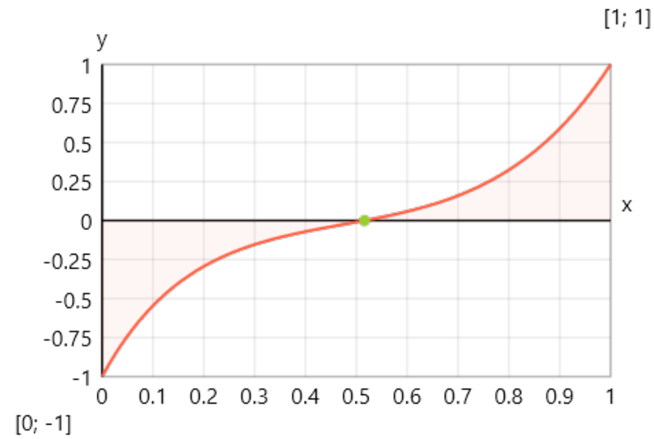
$$x_{16}=\$\text{Root}\left\{f_{16}(x)=0\,;\ x\in[0\,;\ 1]\right\}=0.000151$$

$$f_{17}(x)=\left(1+(1-20)^4\right)\cdot x-(1-20\cdot x)^4$$



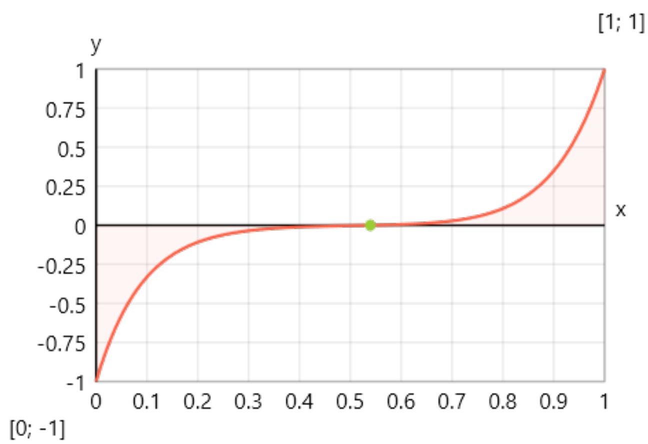
$$x_{17}=\$\mathrm{Root}\left\{f_{17}(x)=0;\ x\in[0;\ 1]\right\}=7.67\times10^{-6}$$

$$f_{18}(x)=e^{-5\cdot x}\cdot(x-1)+x^5$$



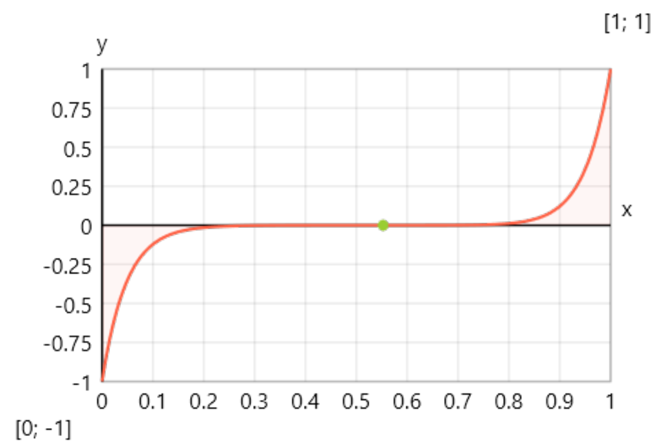
$$x_{18}=\$\mathrm{Root}\left\{f_{18}(x)=0;\ x\in[0;\ 1]\right\}=0.516$$

$$f_{19}(x)=e^{-10\cdot x}\cdot(x-1)+x^{10}$$



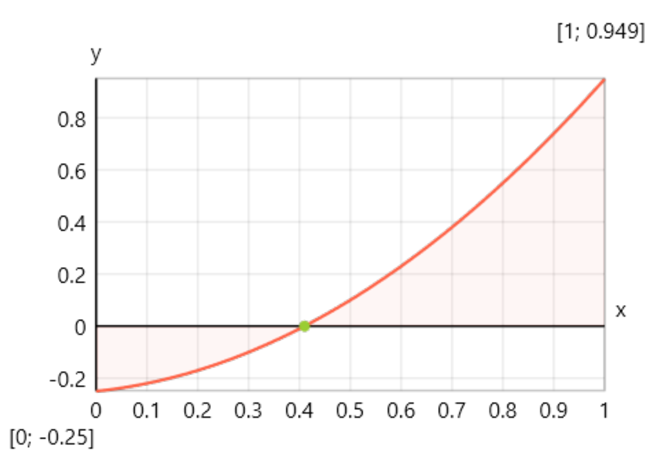
$$x_{19}=\$\mathrm{Root}\left\{f_{19}(x)=0;\ x\in[0;\ 1]\right\}=0.54$$

$$f_{20}(x)=e^{-20\cdot x}\cdot(x-1)+x^{20}$$



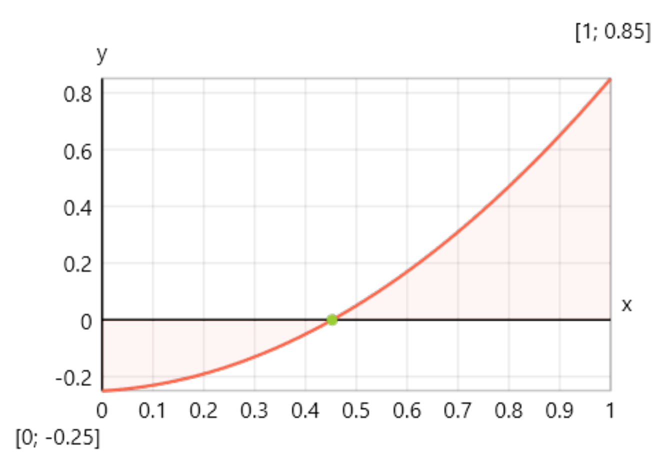
$$x_{20}=\$\mathrm{Root}\left\{f_{20}(x)=0;\ x\in[0;\ 1]\right\}=0.553$$

$$f_{21}(x)=x^2+\sin\left(\frac{x}{5}\right)-\frac{1}{4}$$



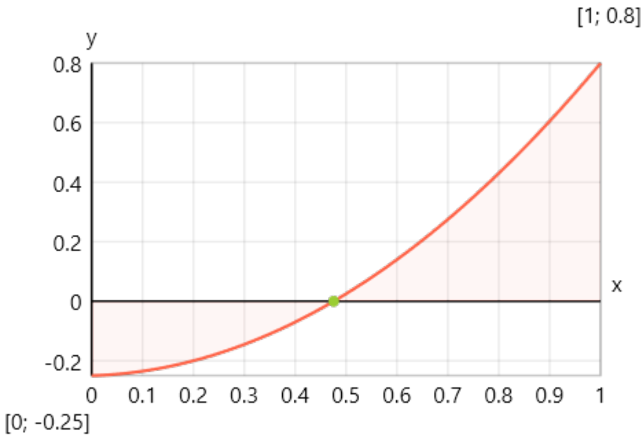
$$x_{21}=\$\mathrm{Root}\left\{f_{21}(x)=0;\ x\in[0;\ 1]\right\}=0.41$$

$$f_{22}(x)=x^2+\sin\left(\frac{x}{10}\right)-\frac{1}{4}$$



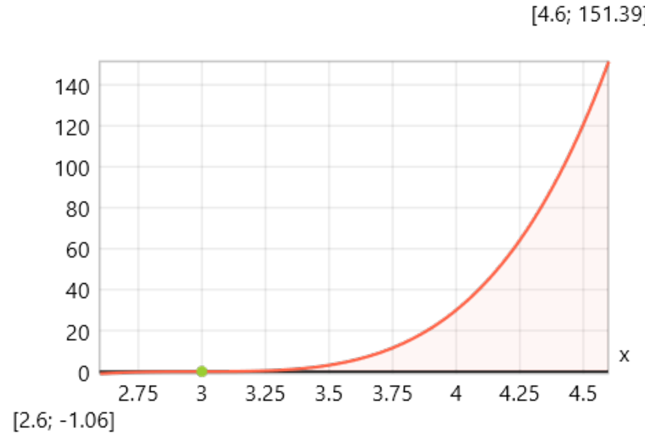
$$x_{22}=\$\mathrm{Root}\left\{f_{22}(x)=0;\ x\in[0;\ 1]\right\}=0.453$$

$$f_{23}(x)=x^2+\sin\left(\frac{x}{20}\right)-\frac{1}{4}$$



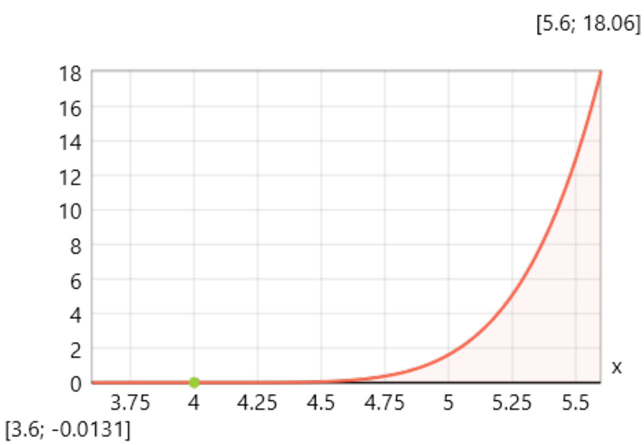
$$x_{23}=\$\mathrm{Root}\left\{f_{23}(x)=0\,;\ x\in[0\,;\ 1]\right\}=0.476$$

$$f_{24}(x)=(x+2)\cdot(x+1)\cdot(x-3)^3$$



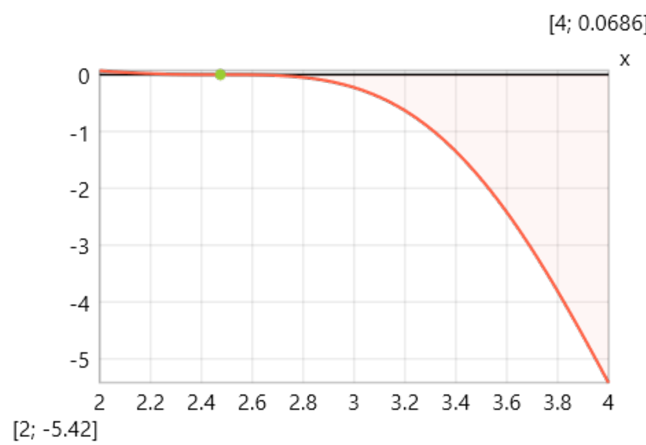
$$x_{24}=\$\mathrm{Root}\left\{f_{24}(x)=0\,;\ x\in[2.6\,;\ 4.6]\right\}=3$$

$$f_{25}(x)=(x-4)^5\cdot\ln(x)$$



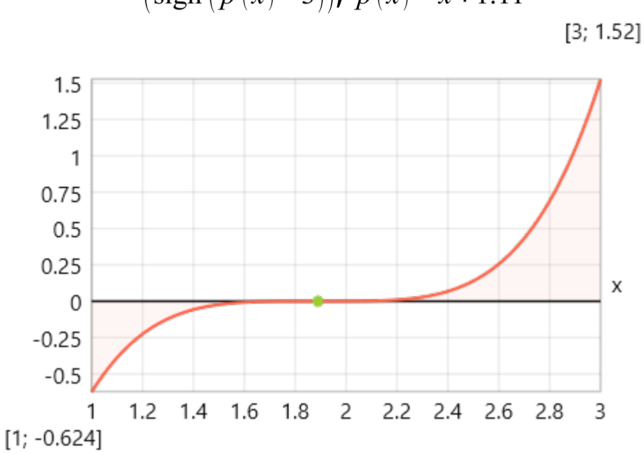
$$x_{25}=\$\mathrm{Root}\left\{f_{25}(x)=0\,;\ x\in[3.6\,;\ 5.6]\right\}=4$$

$$f_{26}(x)=\left(\sin(x)-\frac{x}{4}\right)^3$$



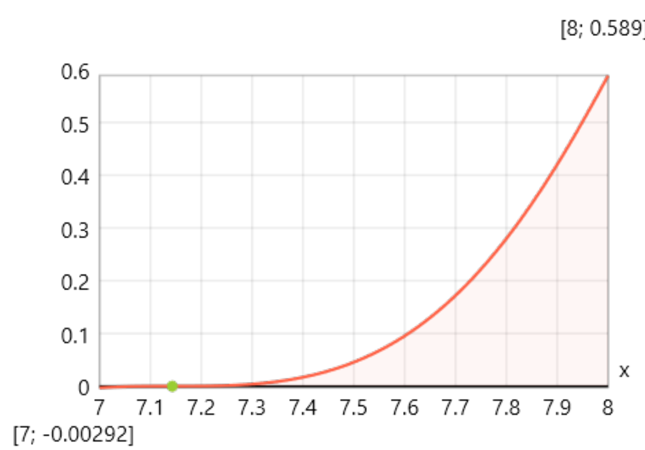
$$x_{26}=\$\mathrm{Root}\left\{f_{26}(x)=0\,;\ x\in[2\,;\ 4]\right\}=2.47$$

$$f_{27}(x)=\left(81-p(x)\cdot\left(108-p(x)\cdot\left(54-p(x)\cdot\left(12-p(x)\right)\right)\right)\right)\cdot\left(\mathrm{sign}\left(p(x)-3\right)\right),\ p(x)=x+1.11$$



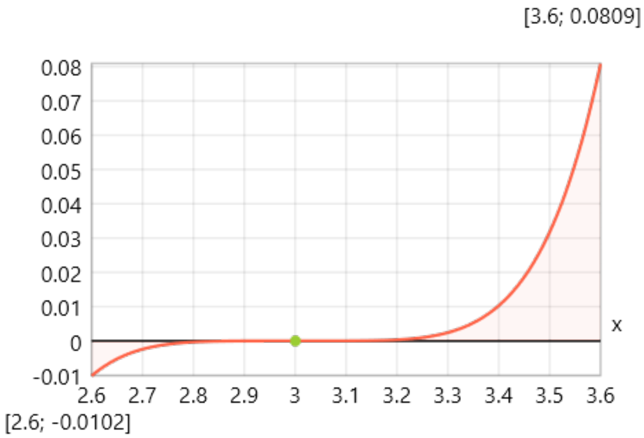
$$x_{27}=\$\mathrm{Root}\left\{f_{27}(x)=0\,;\ x\in[1\,;\ 3]\right\}=1.89$$

$$f_{28}(x)=\sin\left((x-7.14)^3\right)$$



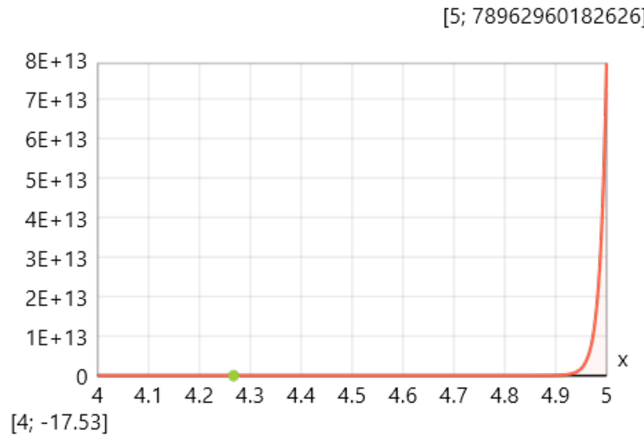
$$x_{28}=\$\mathrm{Root}\left\{f_{28}(x)=0\,;\ x\in[7\,;\ 8]\right\}=7.14$$

$$f_{29}(x) = e^{(x-3)^5} - 1$$



$$x_{29} = \$\text{Root}\left\{f_{29}(x) = 0;\ x \in [2.6;\ 3.6]\right\} = 3$$

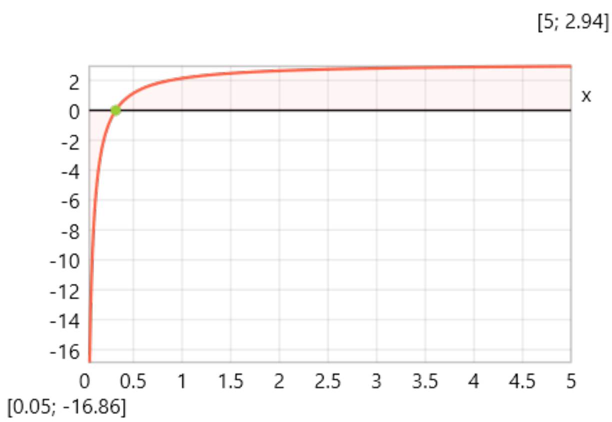
$$f_{30}(x) = e^{(x-3)^5} - e^{x-1}$$



$$x_{30} = \$\text{Root}\left\{f_{30}(x) = 0;\ x \in [4;\ 5]\right\} = 4.27$$

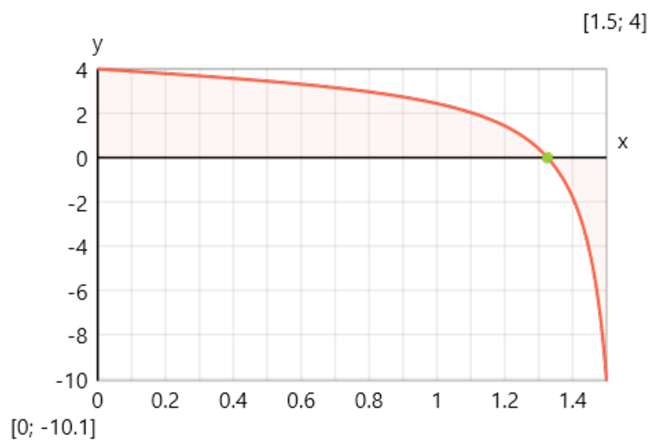
My functions

$$f_{31}(x) = \pi - \frac{1}{x}$$



$$x_{31} = \$\text{Root}\left\{f_{31}(x) = 0;\ x \in [0.05\ v\ 5]\right\} = 0.318$$

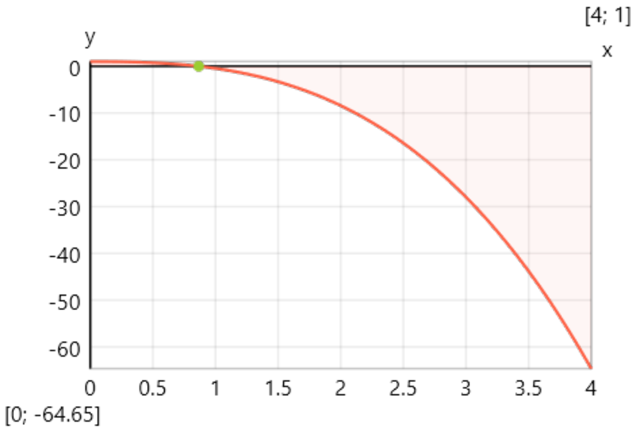
$$f_{32}(x) = 4 - \tan(x)$$



$$x_{32} = \$\text{Root}\left\{f_{32}(x) = 0;\ x \in [0;\ 1.5]\right\} = 1.33$$

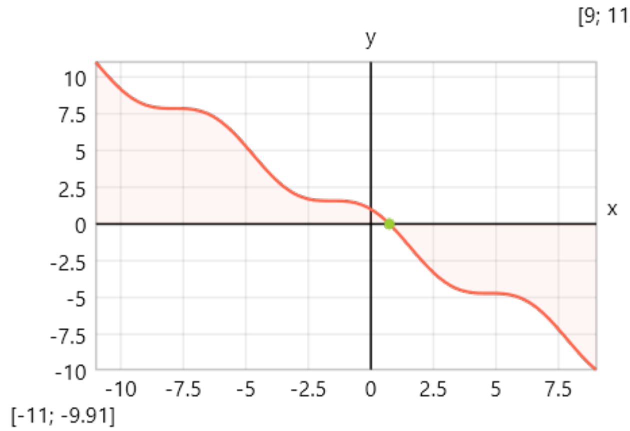
Steven A. Stage. *Comments on An Improvement to the Brent's Method*. International Journal of Experimental Algorithms (IJEA), Volume (4) : Issue (1) : 2013

$$f_{33}(x) = \cos(x) - x^3$$



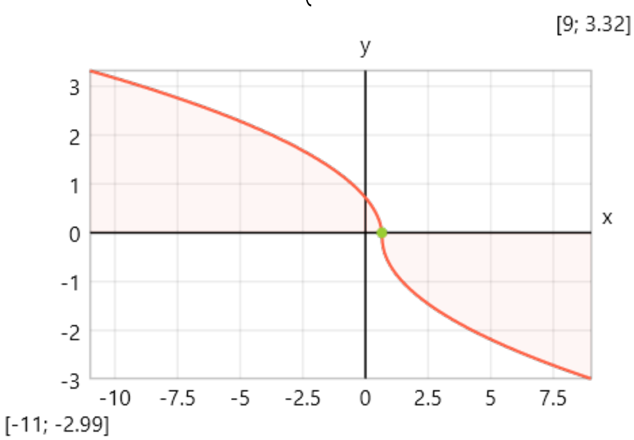
$$x_{33} = \$\text{Root}\{f_{33}(x) = 0;\ x \in [0;\ 4]\} = 0.865$$

$$f_{34}(x) = \cos(x) - x$$



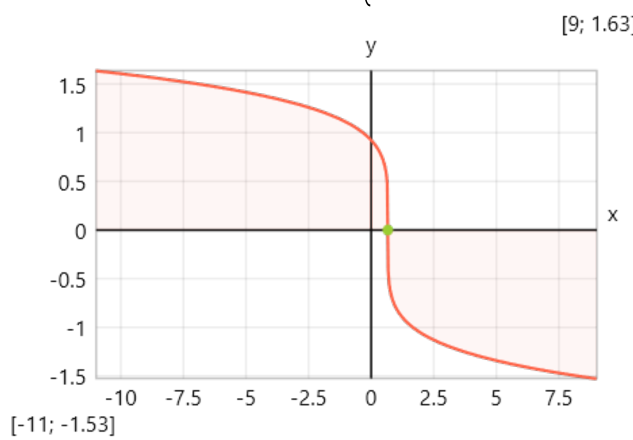
$$x_{34} = \$\text{Root}\{f_{34}(x) = 0;\ x \in [-11;\ 9]\} = 0.739$$

$$f_{35}(x) = \sqrt{\left|x - \frac{2}{3}\right|} \cdot \begin{cases} \text{if } x \le \frac{2}{3}: & 1 \\ \text{else:} & -1 \end{cases} - 0.1$$



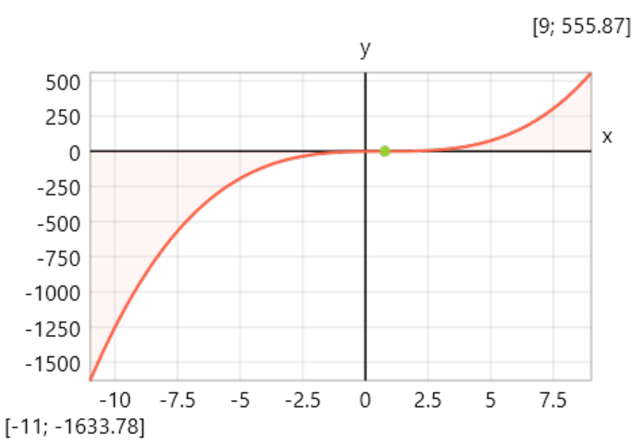
$$x_{35} = \$\text{Root}\{f_{35}(x) = 0;\ x \in [-11;\ 9]\} = 0.657$$

$$f_{36}(x) = \left(\left|x - \frac{2}{3}\right|\right)^{0.2} \cdot \begin{cases} \text{if } x \le \frac{2}{3}: & 1 \\ \text{else:} & -1 \end{cases}$$



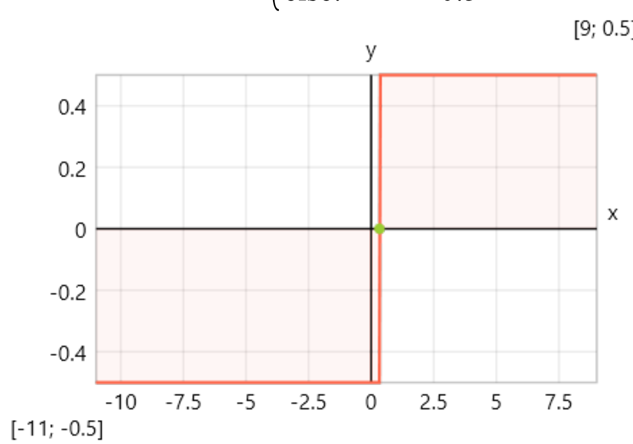
$$x_{36} = \$\text{Find}\{f_{36}(x);\ x \in [-11;\ 9]\} = 0.667$$

$$f_{37}(x) = \left(x - \frac{7}{9}\right)^3 + \left(x - \frac{7}{9}\right) \cdot 10^{-3}$$



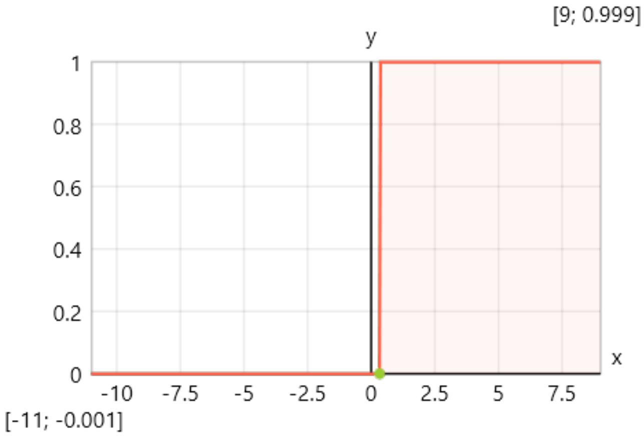
$$x_{37} = \$\text{Root}\{f_{37}(x) = 0;\ x \in [-11;\ 9]\} = 0.778$$

$$f_{38}(x) = \begin{cases} \text{if } x \le \frac{1}{3}: & -0.5 \\ \text{else:} & 0.5 \end{cases}$$



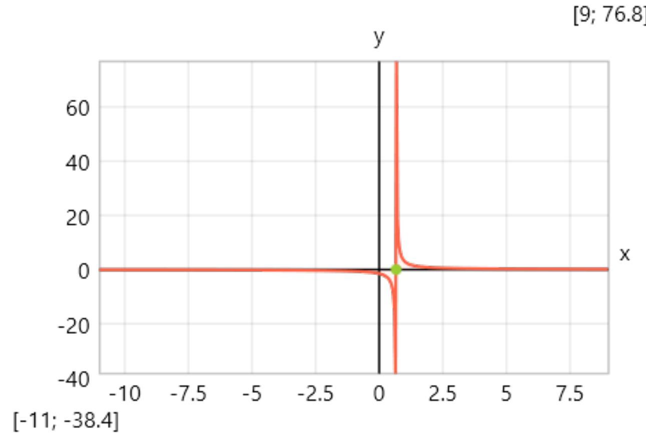
$$x_{38} = \$\text{Find}\{f_{38}(x);\ x \in [-11;\ 9]\} = 0.333$$

$$f_{39}(x)=\begin{cases}\text{if } x\le\frac{1}{3}: & -\left(10^{-3}\right)\\ \text{else:} & 1-10^{-3}\end{cases}$$



$$x_{39}=\$\text{Find}\left\{f_{39}(x);\ x\in[-11;9]\right\}=0.333$$

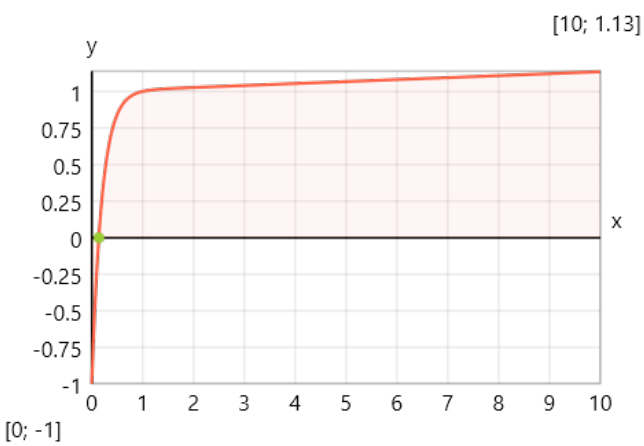
$$f_{40}(x)=\begin{cases}\text{if } x\equiv 0: & 0\\ \text{else:} & \dfrac{1}{x-2/3}\end{cases}$$



$$x_{40}=\$\text{Find}\left\{f_{40}(x);\ x\in[-11;9]\right\}=0.667$$

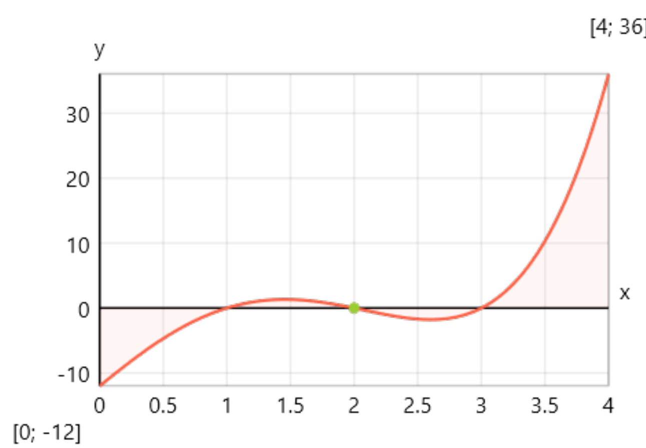
A. Swift, G. R. Lindfield, *Comparison of a continuation method with Brent's method for the numerical solution of a single nonlinear equation*, The Computer Journal, Volume 21, Issue 4, 1978, p. 359–362 https://doi.org/10.1093/comjnl/21.4.359

$$f_{41}(x)=2\cdot x\cdot e^{-5}-2\cdot e^{-5\cdot x}+1$$



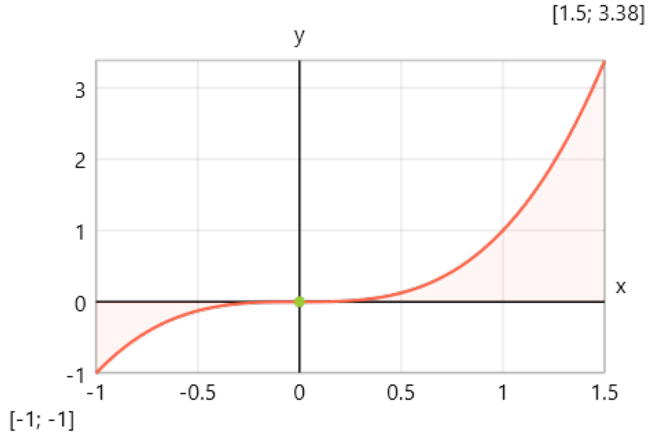
$$x_{41}=\$\text{Root}\left\{f_{41}(x)=0;\ x\in[0;\ 10]\right\}=0.138$$

$$f_{42}(x)=\left(x^2-x-6\right)\cdot\left(x^2-3\cdot x+2\right)$$



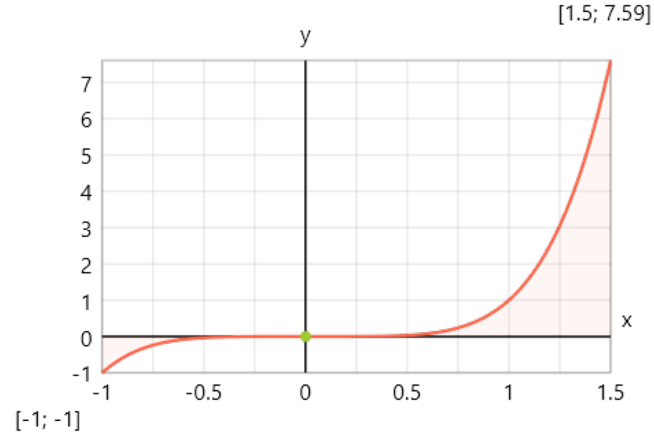
$$x_{42}=\$\text{Root}\left\{f_{42}(x)=0;\ x\in[0;\ 4]\right\}=2$$

$$f_{43}(x)=x^{3}$$



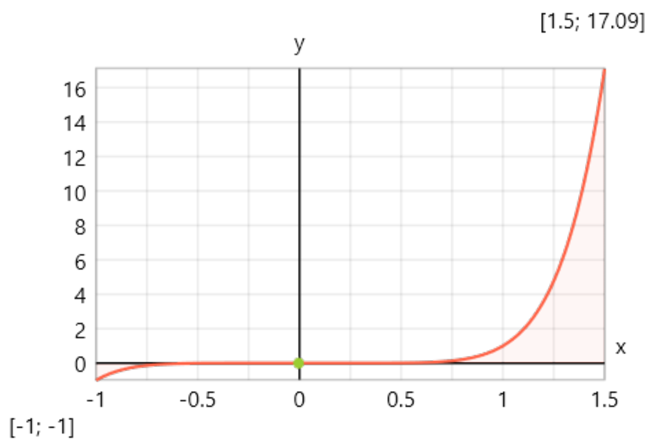
$$x_{43}=\$\mathrm{Root}\left\{f_{43}(x)=0\,;\ x\in[-1;\ 1.5]\right\}=3.81\times10^{-6}$$

$$f_{44}(x)=x^{5}$$



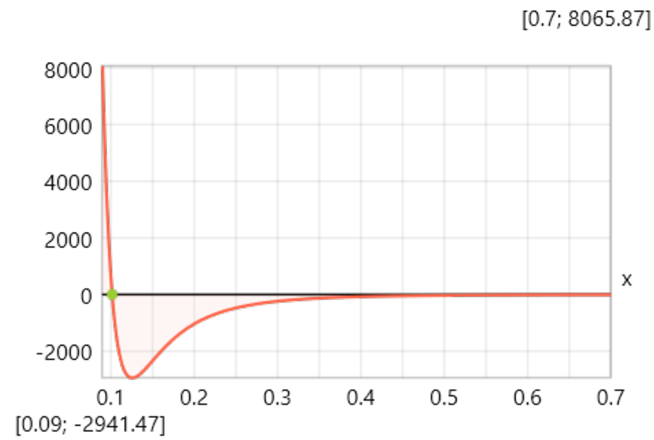
$$x_{44}=\$\mathrm{Root}\left\{f_{44}(x)=0\,;\ x\in[-1;\ 1.5]\right\}=0.000977$$

$$f_{45}(x)=x^{7}$$



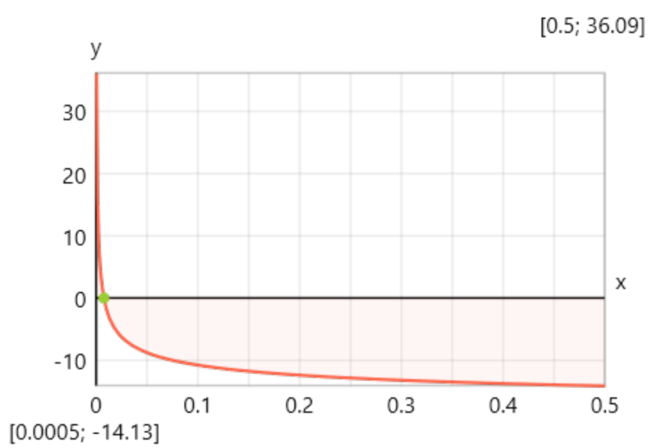
$$x_{45}=\$\mathrm{Root}\left\{f_{45}(x)=0\,;\ x\in[-1;\ 1.5]\right\}=-0.00391$$

$$f_{46}(x)=\frac{e^{-5\cdot x}-x-0.5}{x^{5}}$$



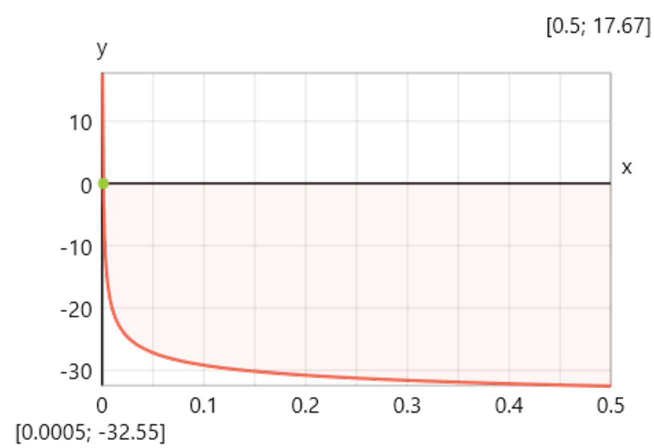
$$x_{46}=\$\mathrm{Root}\left\{f_{46}(x)=0\,;\ x\in[0.09\,;\ 0.7]\right\}=0.102$$

$$f_{47}(x)=\frac{1}{\sqrt{x}}-2\cdot\ln\left(5\cdot10^{3}\cdot\sqrt{x}\right)+0.8$$



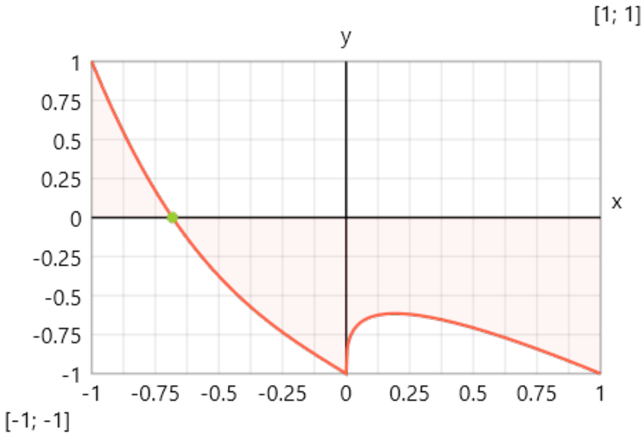
$$x_{47}=\$\mathrm{Root}\left\{f_{47}(x)=0\,;x\in[0.0005\,;0.5]\right\}=0.00773$$

$$f_{48}(x)=\frac{1}{\sqrt{x}}-2\cdot\ln\left(5\cdot10^{7}\cdot\sqrt{x}\right)+0.8$$



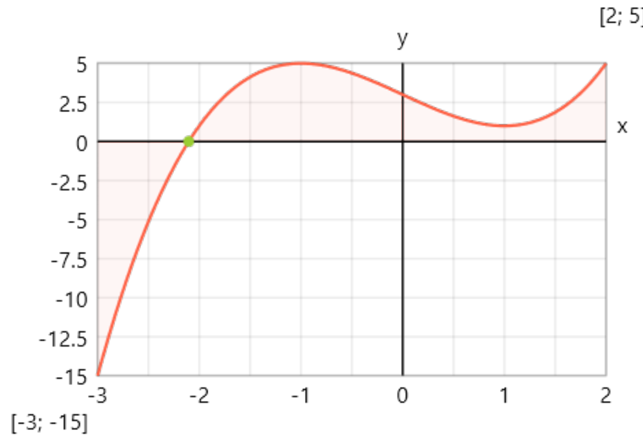
$$x_{48}=\$\mathrm{Root}\left\{f_{48}(x)=0\,;x\in[0.0005\,;0.5]\right\}=0.00128$$

$$f_{49}(x)=\begin{cases}\text{if } x\le 0: & -(x^3)-x-1\\ \text{else:} & x^{\frac{1}{3}}-x-1\end{cases}$$



$$x_{49}=\$\mathrm{Root}\{f_{49}(x)=0;\ x\in[-1;\ 1]\}=-0.682$$

$$f_{50}(x)=x^3-2\cdot x-x+3$$

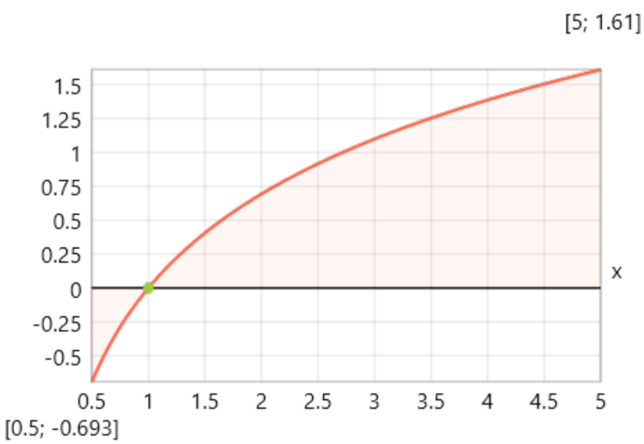


$$x_{50}=\$\mathrm{Root}\{f_{50}(x)=0;\ x\in[-3;\ 2]\}=-2.1$$

Alojz Suhadolnik, Combined bracketing methods for solving nonlinear equations, Applied Mathematics Letters, Volume 25, Issue 11, 2012, Pages 1755-1760, ISSN 0893-9659

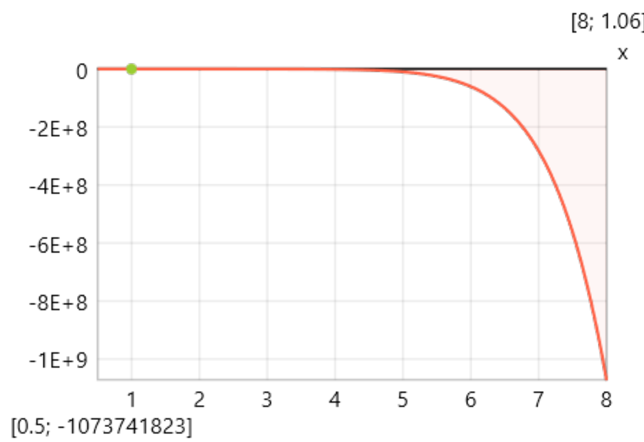
https://doi.org/10.1016/j.aml.2012.02.006

$$f_{51}(x)=\ln(x)$$



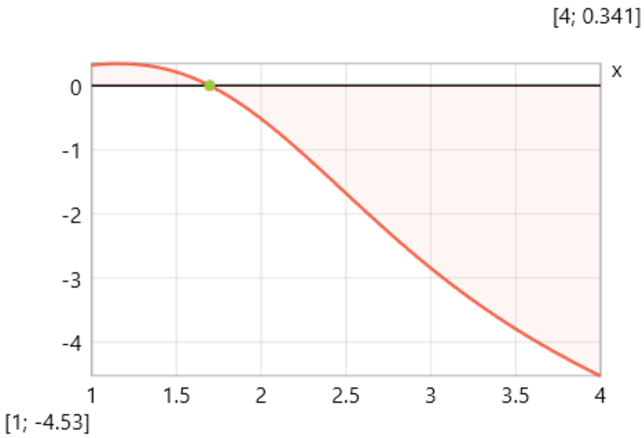
$$x_{51}=\$\mathrm{Root}\{f_{51}(x)=0;\ x\in[0.5;\ 5]\}=1$$

$$f_{52}(x)=(10-x)\cdot e^{-10\cdot x}-x^{10}+1$$



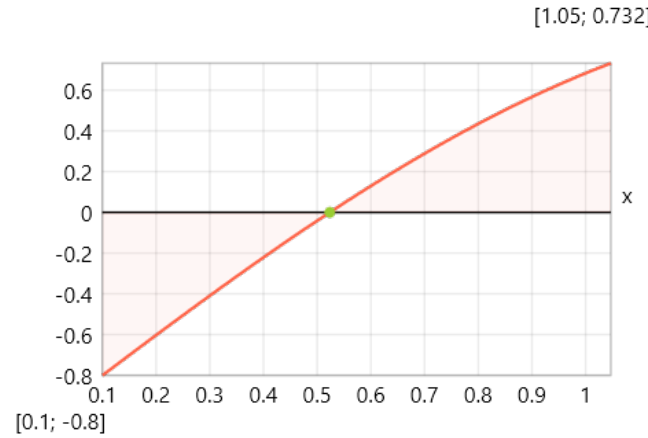
$$x_{52}=\$\mathrm{Root}\{f_{52}(x)=0;\ x\in[0.5;\ 8]\}=1$$

$$f_{53}(x) = e^{\sin(x)} - x - 1$$



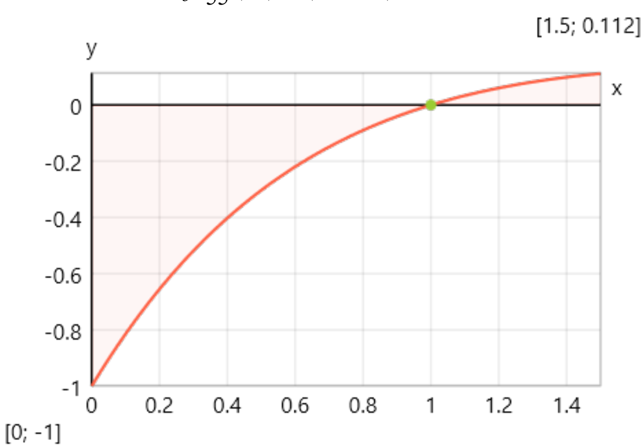
$$x_{53} = \$\mathrm{Root}\left\{f_{53}(x) = 0\,;\ x \in [1\,;\ 4]\right\} = 1.7$$

$$f_{54}(x) = 2 \cdot \sin(x) - 1$$



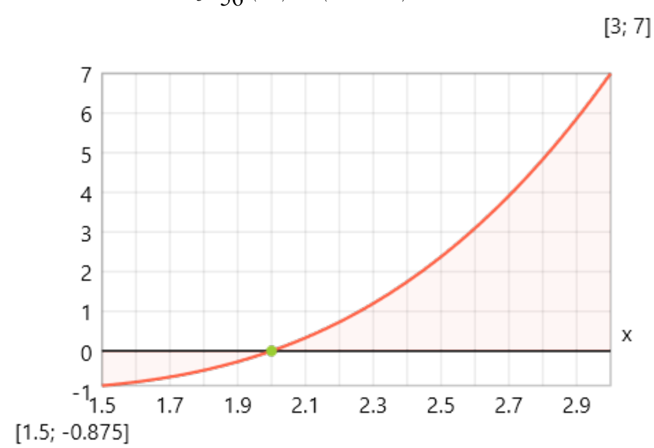
$$x_{54} = \$\mathrm{Root}\left\{f_{54}(x) = 0\,;\ x \in \left[0.1\,;\ \frac{\pi}{3}\right]\right\} = 0.524$$

$$f_{55}(x) = (x - 1) \cdot e^{-x}$$



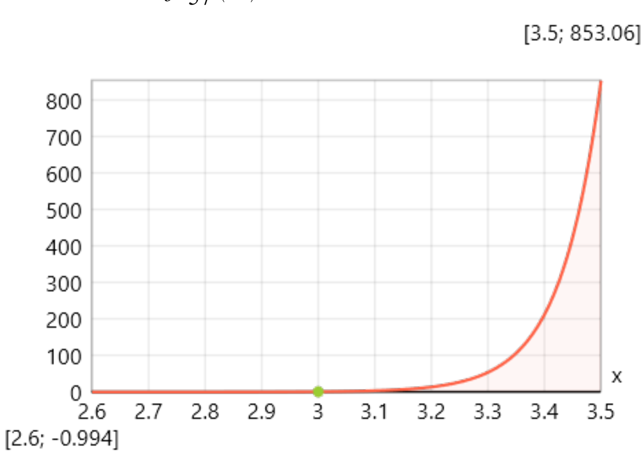
$$x_{55} = \$\mathrm{Root}\left\{f_{55}(x) = 0\,;\ x \in [0\,;\ 1.5]\right\} = 1$$

$$f_{56}(x) = (x - 1)^3 - 1$$



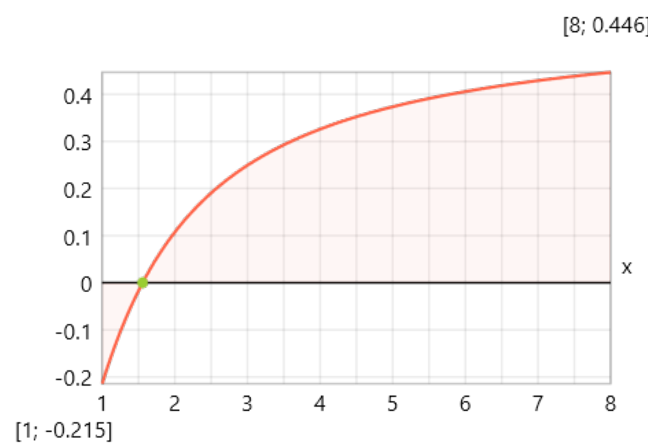
$$x_{56} = \$\mathrm{Root}\left\{f_{56}(x) = 0\,;\ x \in [1.5\,;\ 3]\right\} = 2$$

$$f_{57}(x) = e^{x^2 + 7 \cdot x - 30} - 1$$



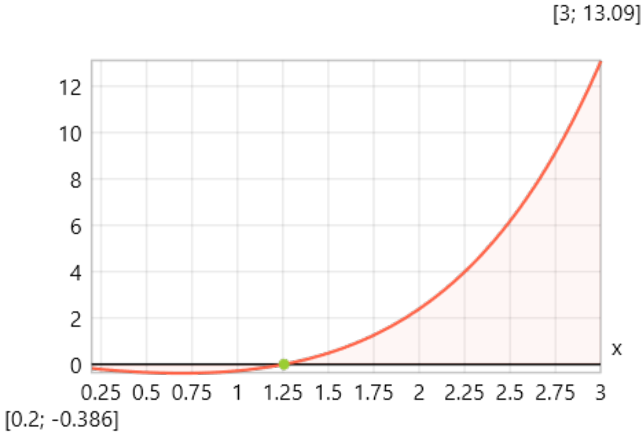
$$x_{57} = \$\mathrm{Root}\left\{f_{57}(x) = 0\,;\ x \in [2.6\,;\,3.5]\right\} = 3$$

$$f_{58}(x) = \mathrm{atan}(x) - 1$$



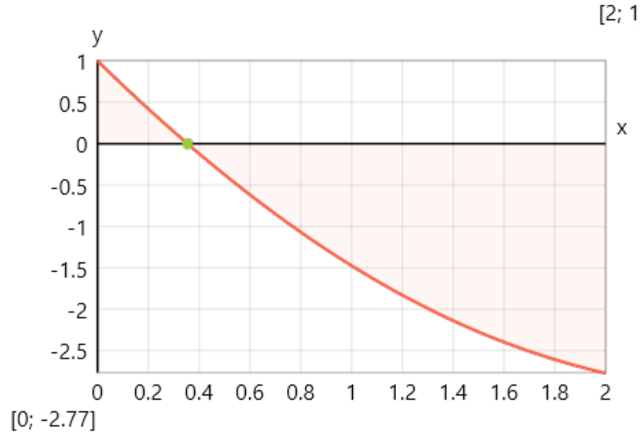
$$x_{58} = \$\mathrm{Root}\left\{f_{58}(x) = 0\,;\ x \in [1\,;8]\right\} = 1.56$$

$$f_{59}(x) = e^{x} - 2 \cdot x - 1$$



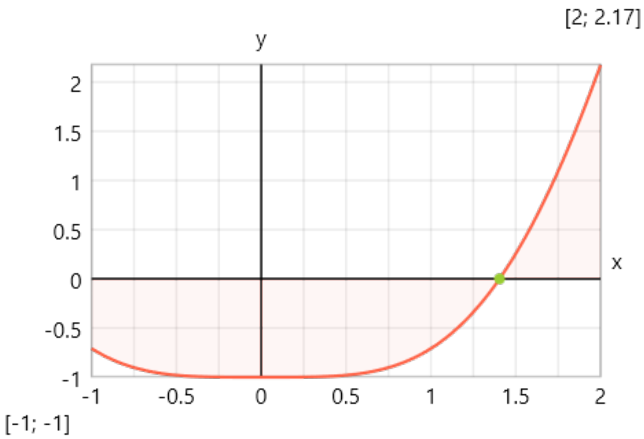
$$x_{59} = \$\text{Root}\left\{ f_{59}(x) = 0;\ x \in [0.2;\ 3] \right\} = 1.26$$

$$f_{60}(x) = e^{-x} - x - \sin(x)$$



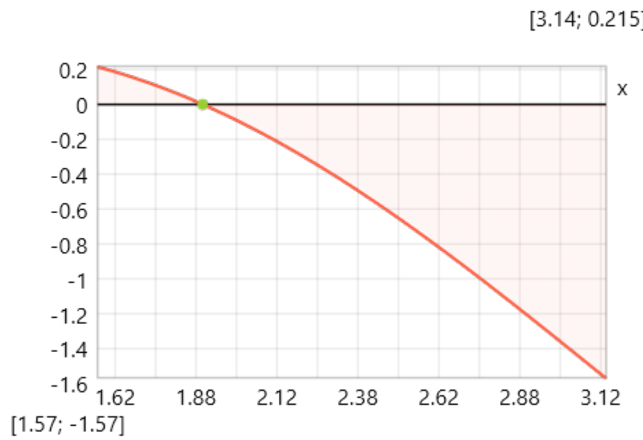
$$x_{60} = \$\text{Root}\left\{ f_{60}(x) = 0;\ x \in [0;\ 2] \right\} = 0.354$$

$$f_{61}(x) = x^{2} - \sin(x)^{2} - 1$$



$$x_{61} = \$\text{Root}\left\{ f_{61}(x) = 0;\ x \in [-1;\ 2] \right\} = 1.4$$

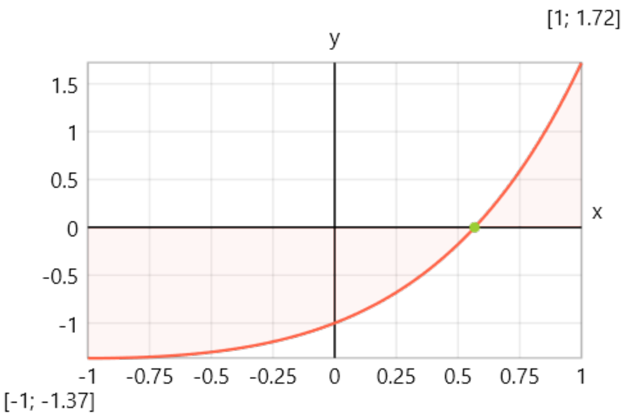
$$f_{62}(x) = \sin(x) - x/2$$



$$x_{62} = \$\text{Root}\left\{ f_{62}(x) = 0;\ x \in [\pi/2;\ \pi] \right\} = 1.9$$

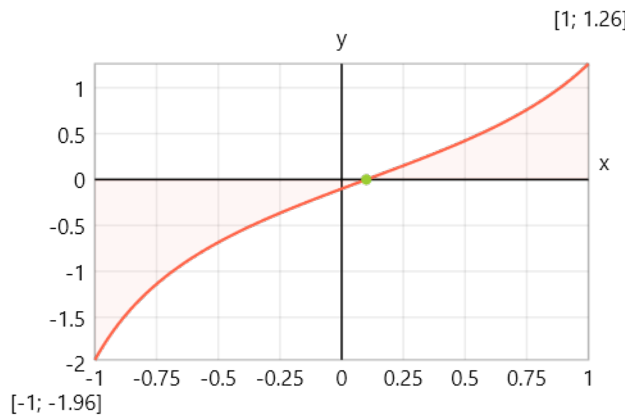
Oliveira I. F. D., Takahashi R. H. C. An Enhancement of the Bisection Method Average Performance Preserving Minmax Optimality

Lambert - $f_{63}(x) = x \cdot \exp(x) - 1$



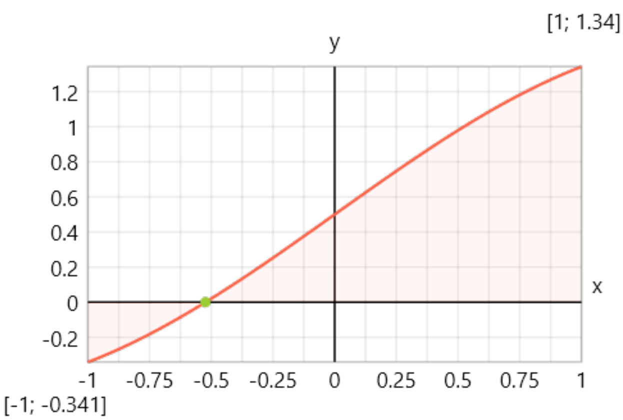
$$x_{63} = \$\mathrm{Root}\left\{f_{63}(x) = 0\,;\ x \in [-1;\ 1]\right\} = 0.567$$

Trigonometric 1 - $f_{64}(x) = \tan\left(x - \frac{1}{10}\right)$



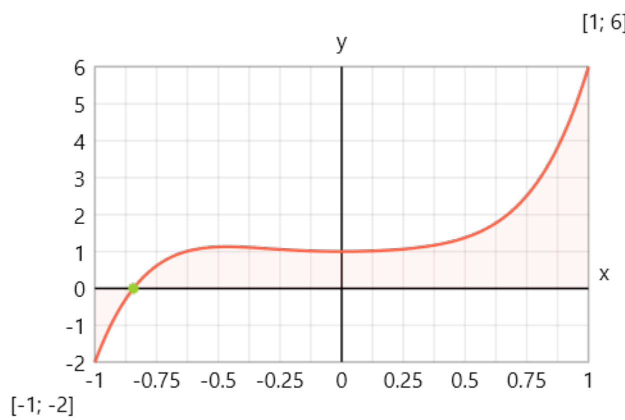
$$x_{64} = \$\mathrm{Root}\left\{f_{64}(x) = 0\,;\ x \in [-1;\ 1]\right\} = 0.1$$

Trigonometric 2 - $f_{65}(x) = \sin(x) + 0.5$



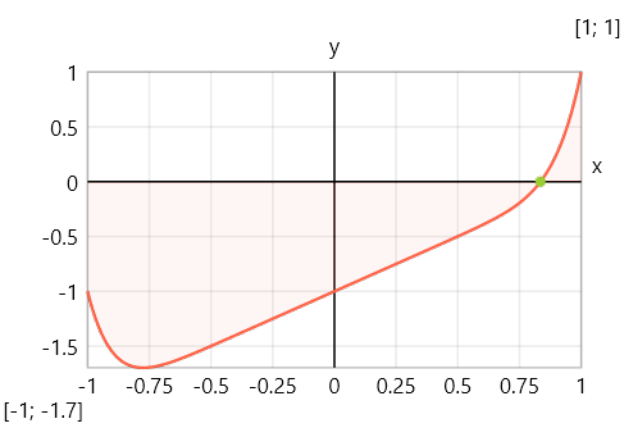
$$x_{65} = \$\mathrm{Root}\left\{f_{65}(x) = 0\,;\ x \in [-1;\ 1]\right\} = -0.524$$

Polynomial 1 - $f_{66}(x) = 4 \cdot x^5 + x \cdot x + 1$



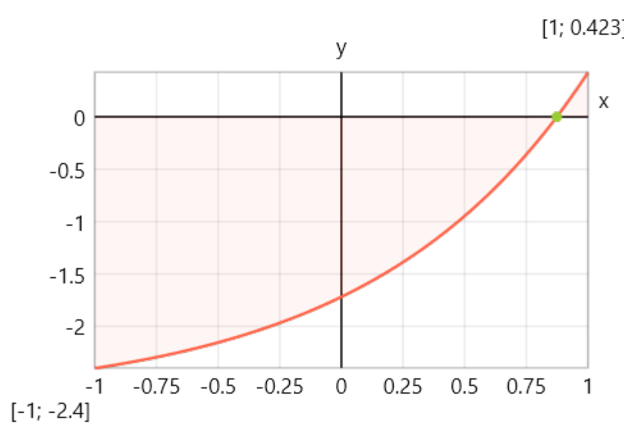
$$x_{66} = \$\mathrm{Root}\left\{f_{66}(x) = 0\,;\ x \in [-1;\ 1]\right\} = -0.844$$

Polynomial 2 - $f_{67}(x) = x + x^{10} - 1$



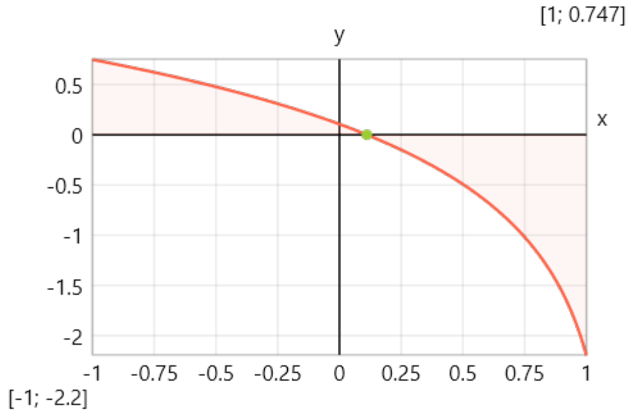
$$x_{67} = \$\mathrm{Root}\left\{f_{67}(x) = 0\,;\ x \in [-1;\ 1]\right\} = 0.835$$

Exponential - $f_{68}(x) = \pi^x - e$



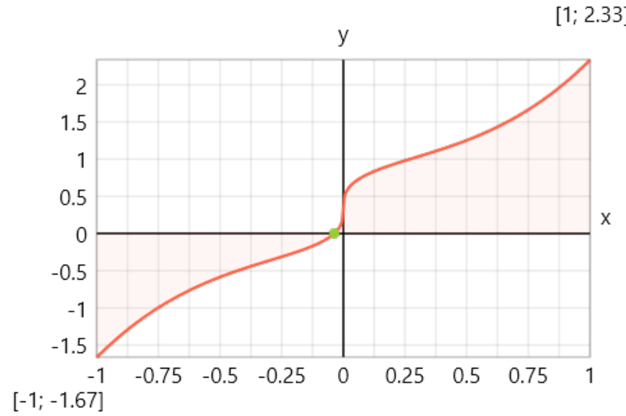
$$x_{68} = \$\mathrm{Root}\left\{f_{68}(x) = 0\,;\ x \in [-1;\ 1]\right\} = 0.874$$

Logarithmic - $f_{69}(x)=\ln\left(\left|x-\frac{10}{9}\right|\right)$



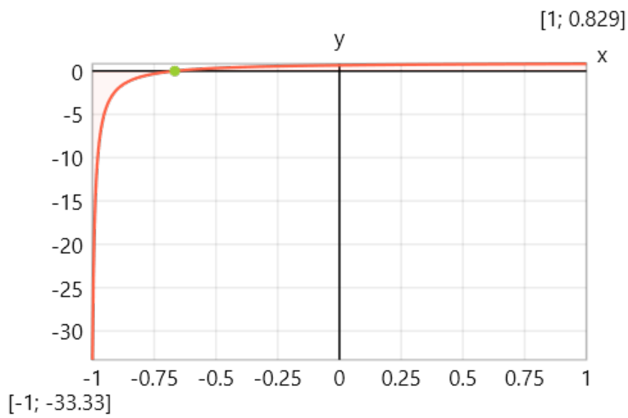
$$x_{69}=\$\mathrm{Root}\left\{f_{69}(x)=0\,;\;x\in[-1;\;1]\right\}=0.111$$

Polynomial - $f_{70}(x)=\frac{1}{3}+\mathrm{sign}(x)\cdot\sqrt[3]{|x|}+x^3$



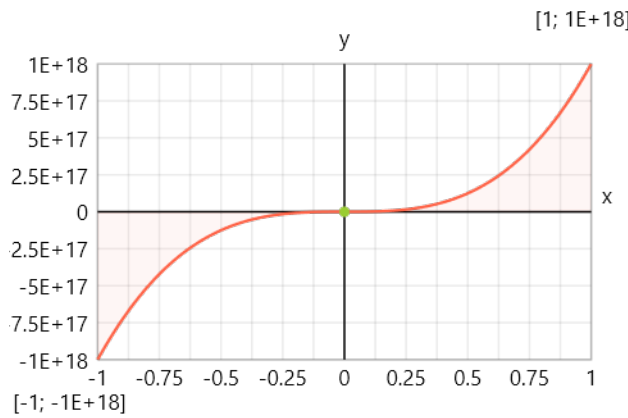
$$x_{70}=\$\mathrm{Root}\left\{f_{70}(x)=0\,;\;x\in[-1;\;1]\right\}=-\,0.037$$

Poly. Frac. - $f_{71}(x)=\dfrac{x+2/3}{x+101/100}$



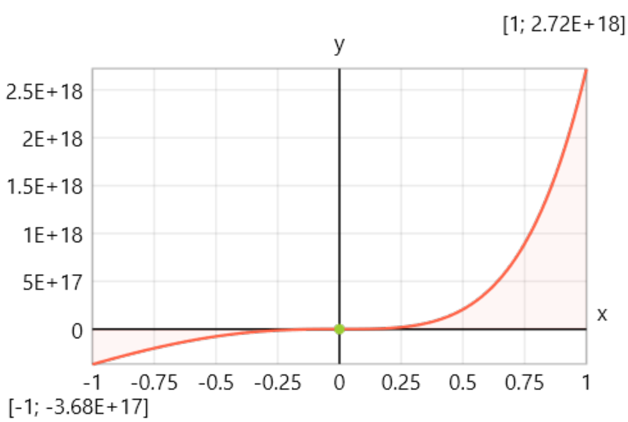
$$x_{71}=\$\mathrm{Root}\left\{f_{71}(x)=0\,;\;x\in[-1;\;1]\right\}=-\,0.667$$

Polynomial 3 - $f_{72}(x)=\left(x\cdot 10^6-1\right)^3$



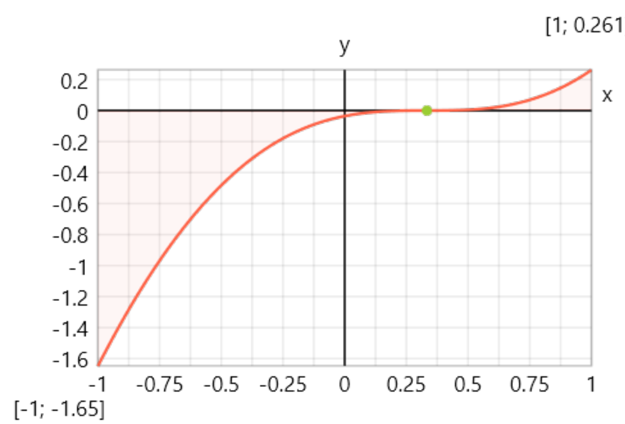
$$x_{72}=\$\mathrm{Root}\left\{f_{72}(x)=0\,;\;x\in[-1;\;1]\right\}=10^{-6}$$

Exp. Poly. - $f_{73}(x)=\exp(x)\cdot\left(x\cdot 10^6-1\right)^3$



$$x_{73}=\$\mathrm{Root}\left\{f_{73}(x)=0\,;\;x\in[-1;\;1]\right\}=10^{-6}$$

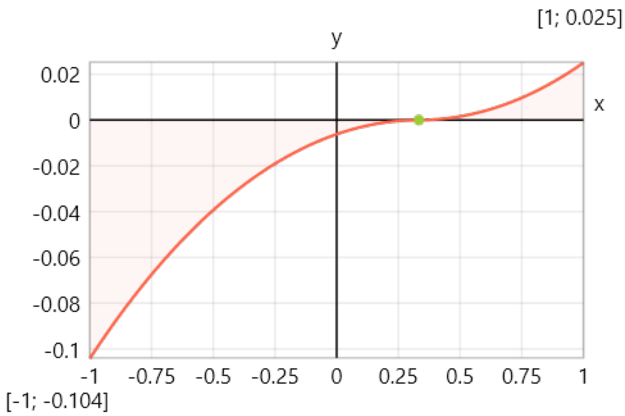
Tan. Poly. - $f_{74}(x)=\left(x-\frac{1}{3}\right)^2\cdot\left(\mathrm{atan}\left(x-\frac{1}{3}\right)\right)$



$$x_{74}=\$\mathrm{Root}\left\{f_{74}(x)=0\,;\;x\in[-1;\;1]\right\}=0.333$$

Circles

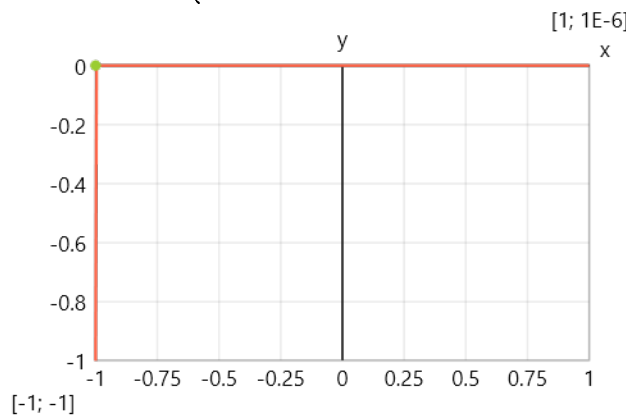
$$f_{75}(x) = \mathrm{sign}(3 \cdot x - 1) \cdot \left(1 - \sqrt{1 - \frac{(3 \cdot x - 1)^2}{81}}\right)$$



$$x_{75} = \$\mathrm{Root}\left\{f_{75}(x) = 0;\ x \in [-1;\ 1]\right\} = 0.333$$

Step Function

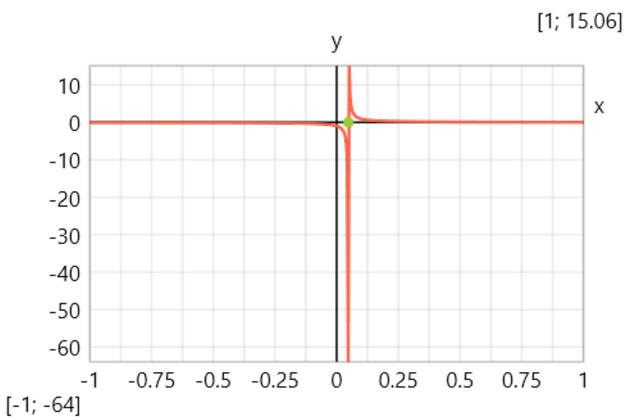
$$f_{76}(x) = \begin{cases} \text{if } x > \dfrac{1 - 10^6}{10^6}: & \dfrac{1 + 10^6}{10^6} - 1 \\ \text{else:} & 0 \end{cases}$$



$$x_{76} = \$\mathrm{Find}\left\{f_{76}(x);\ x \in [-1;\ 1]\right\} = -1$$

Geometric

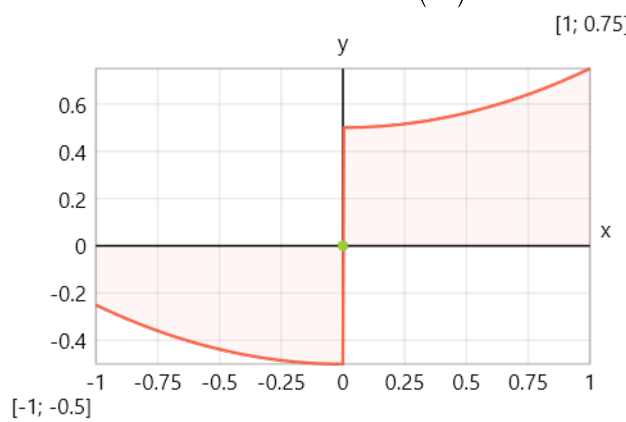
Geometric - $f_{77}(x) = \begin{cases} \text{if } x \neq \dfrac{1}{21}: & \dfrac{1}{21 \cdot x - 1} \\ \text{else:} & 0 \end{cases}$



$$x_{77} = \$\mathrm{Find}\left\{f_{77}(x);\ x \in [-1;\ 1]\right\} = 0.0476$$

Trunc. Poly.

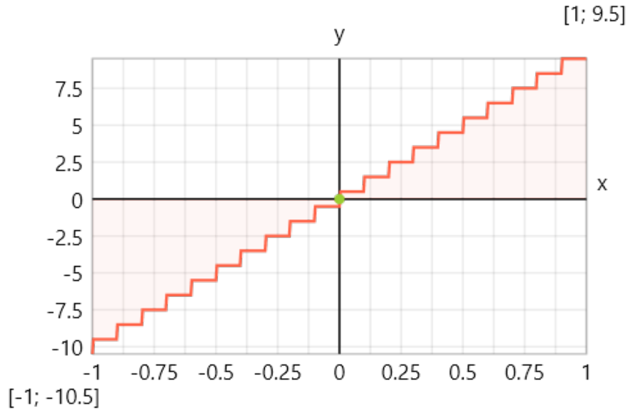
$$f_{78}(x) = \frac{x \cdot x}{4} + \mathrm{ceiling}\left(\frac{x}{2}\right) - 0.5$$



$$x_{78} = \$\mathrm{Find}\left\{f_{78}(x);\ x \in [-1;\ 1]\right\} = 7.11 \times 10^{-15}$$

Staircase

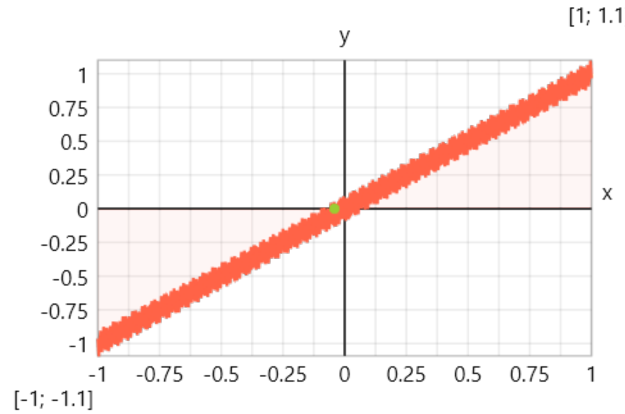
$$f_{79}(x)=\mathrm{ceiling}(10\cdot x-1)+0.5$$



$$x_{79}=\$\mathrm{Find}\left\{f_{79}(x);\ x\in[-1;\ 1]\right\}=2.91\times10^{-19}$$

Noisy Line

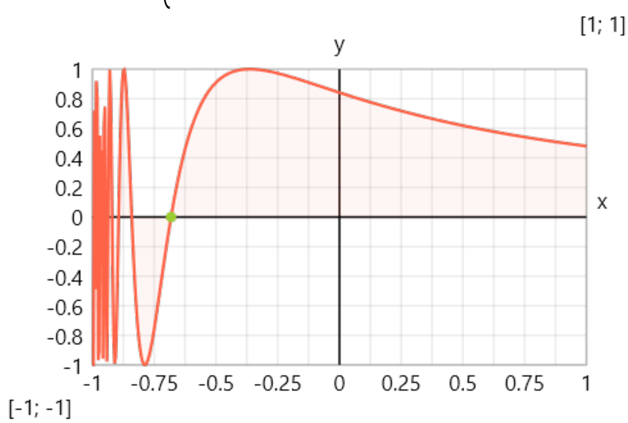
$$f_{80}(x)=x+\frac{\sin\left(x\cdot10^{6}\right)}{10}+10^{-3}$$



$$x_{80}=\$\mathrm{Root}\left\{f_{80}(x)=0;\ x\in[-1;\ 1]\right\}=-0.0415$$

Warsaw

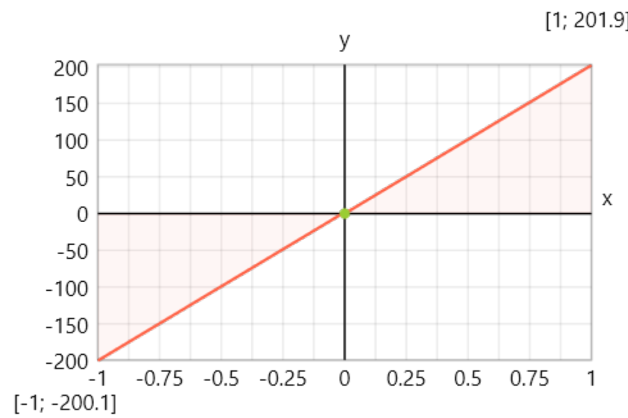
$$f_{81}(x)=\begin{cases}\text{if } x>(-1): & 1+\sin\left(\dfrac{1}{x+1}\right)-1\\ \text{else:} & 0\end{cases}$$



$$x_{81}=\$\mathrm{Root}\left\{f_{81}(x)=0;\ x\in[-1;\ 1]\right\}=-0.682$$

Sawtooth

$$f_{82}(x)=202\cdot x-2\cdot\mathrm{floor}\left(\frac{2\cdot x+10^{-2}}{2}\cdot10^{-2}\right)-0.1$$

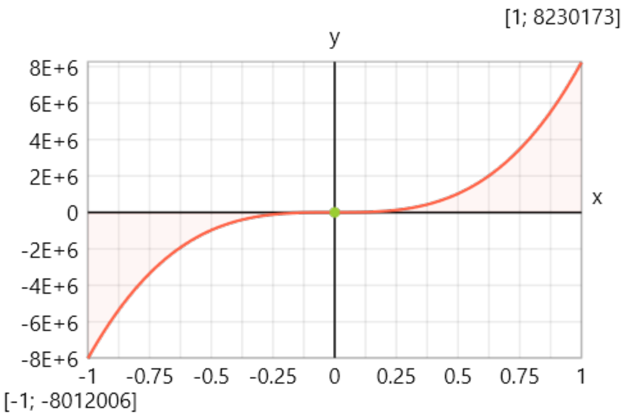


$$x_{82}=\$\mathrm{Root}\left\{f_{82}(x)=0;\ x\in[-1;\ 1]\right\}=0.000495$$

SciML Benchmarks test suite

Sawtooth Cube

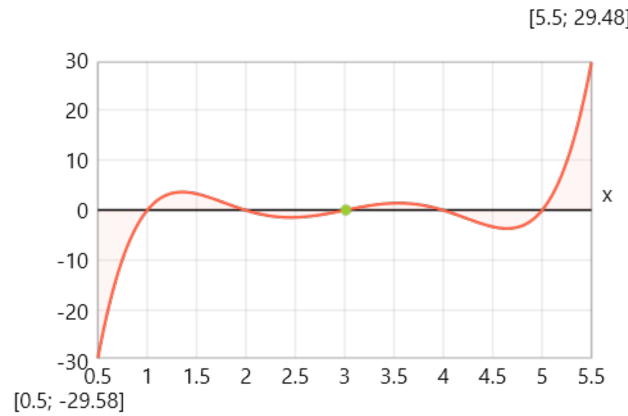
$$f_{83}(x)=\left(202\cdot x-2\cdot \mathrm{floor}\left(\frac{2\cdot x+10^{-2}}{2}\cdot 10^{-2}\right)-0.1\right)^{3}$$



$$x_{83}=\$\mathrm{Root}\left\{f_{83}(x)=0\,;\ x\in[-1;\ 1]\right\}=0.000495$$

Polynomial with multiple roots

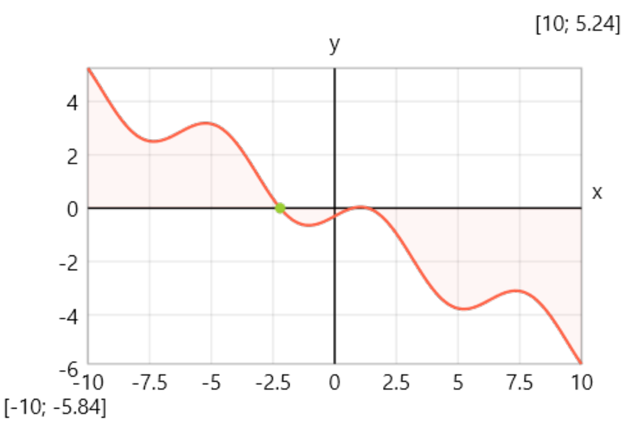
$$f_{84}(x)=(x-1)\cdot(x-2)\cdot(x-3)\cdot(x-4)\cdot(x-5)-0.05$$



$$x_{84}=\$\mathrm{Root}\left\{f_{84}(x)=0\,;\ x\in[0.5\,;\ 5.5]\right\}=3.01$$

Trigonometric with multiple roots

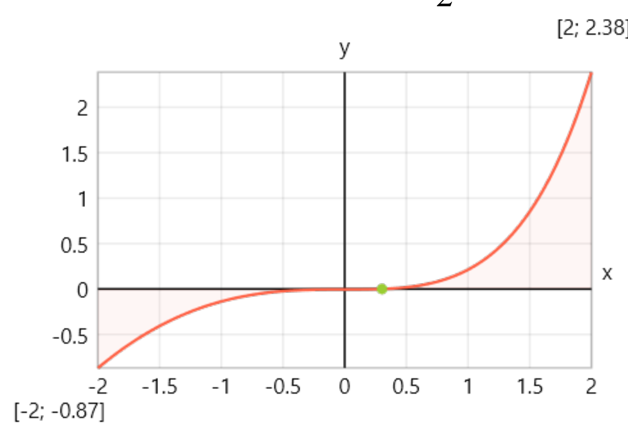
$$f_{85}(x)=\sin(x)-0.5\cdot x-0.3$$



$$x_{85}=\$\mathrm{Root}\left\{f_{85}(x)=0\,;\ x\in[-10;\ 10]\right\}=-2.21$$

Exponential function (sensitive near zero)

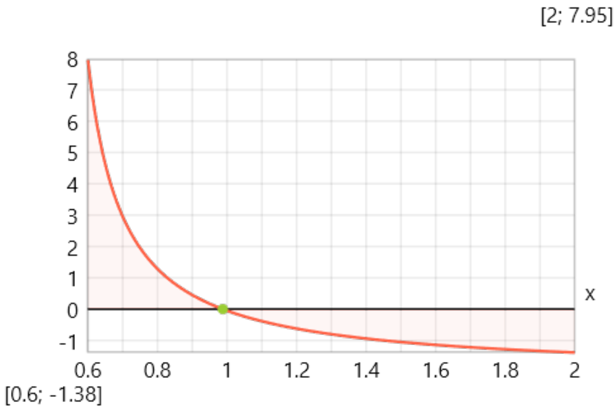
$$f_{86}(x)=\exp(x)-1-x-\frac{x\cdot x}{2}-0.005$$



$$x_{86}=\$\mathrm{Root}\left\{f_{86}(x)=0\,;\ x\in[-2;\ 2]\right\}=0.303$$

Rational function with pole

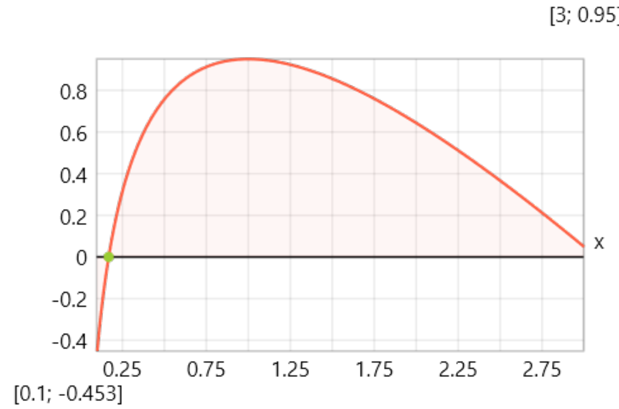
$$f_{87}(x) = \frac{1}{x-0.5} - 2 - 0.05$$



$$x_{87} = \$\text{Root}\left\{ f_{87}(x) = 0\,;\ x \in [0.6\,;\ 2] \right\} = 0.988$$

Logarithmic function

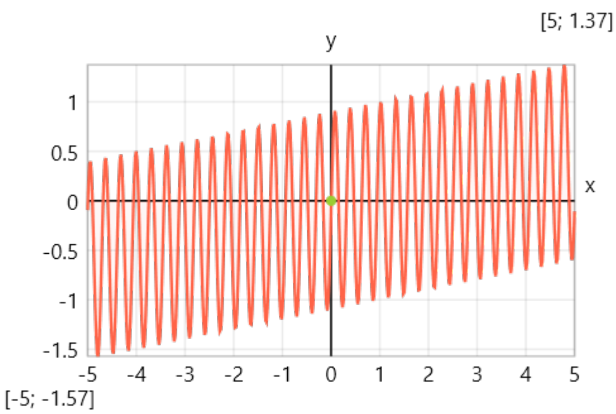
$$f_{88}(x) = \ln(x) - x + 2 - 0.05$$



$$x_{88} = \$\text{Root}\left\{ f_{88}(x) = 0\,;\ x \in [0.1\,;\ 3] \right\} = 0.168$$

High oscillation function

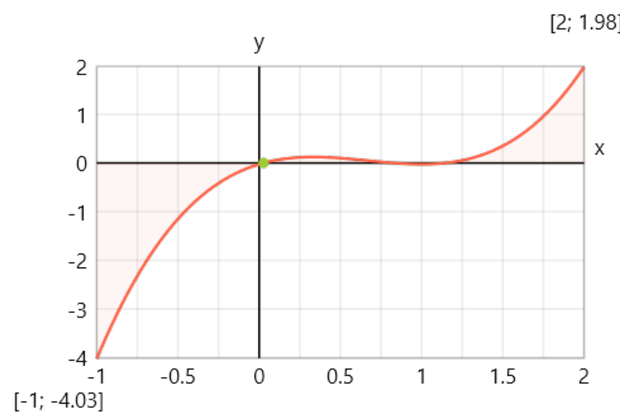
$$f_{89}(x) = \sin(20 \cdot x) + 0.1 \cdot x - 0.1$$



x_{89} = $Root{$f_{89}(x)$ @ $x \in$ [-5.0; 5.0]} - No solution

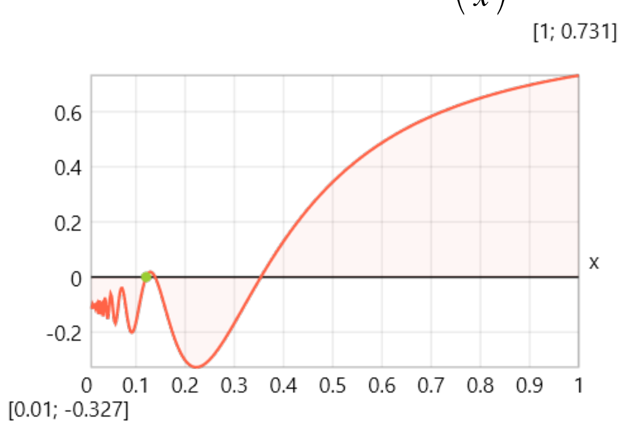
Function with very flat region

$$f_{90}(x) = x \cdot x \cdot x - 2 \cdot x \cdot x + x - 0.025$$



$$x_{90} = \$\text{Root}\left\{ f_{90}(x) = 0\,;\ x \in [-1\,;\ 2] \right\} = 0.0264$$

Bessel-like function - $f_{91}(x) = x \cdot \sin\left(\frac{1}{x}\right) - 0.1 - 0.01$



$$x_{91} = \$\text{Root}\left\{ f_{91}(x) = 0\,;\ x \in [0.01\,;\ 1] \right\} = 0.121$$

Root-finding Test Results

Results

Method	bs	fp	mfp	ill	AB	ITP	mAB	Rid	Bre
f01	1	1	1	1	1	1	1	1	1
f02	0,3994	0,3994	0,3994	0,3994	0,3994	0,3994	0,3994	0,3994	0,3994
f03	0,8041	0,8041	0,8041	0,8041	0,8041	0,8041	0,8041	0,8041	0,8041
f04	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000
f05	2,0946	2,0946	2,0946	2,0946	2,0946	2,0946	2,0946	2,0946	2,0946
f06	0,1383	0,1383	0,1383	0,1383	0,1383	0,1383	0,1383	0,1383	0,1383
f07	0,0693	0,0693	0,0693	0,0693	0,0693	0,0693	0,0693	0,0693	0,0693
f08	0,0347	0,0347	0,0347	0,0347	0,0347	0,0347	0,0347	0,0347	0,0347
f09	0,1096	0,1096	0,1096	0,1096	0,1096	0,1096	0,1096	0,1096	0,1096
f10	0,0525	0,0525	0,0525	0,0525	0,0525	0,0525	0,0525	0,0525	0,0525
f11	0,0256	0,0256	0,0256	0,0256	0,0256	0,0256	0,0256	0,0256	0,0256
f12	0,3460	0,3460	0,3460	0,3460	0,3460	0,3460	0,3460	0,3460	0,3460
f13	0,2451	0,2451	0,2451	0,2451	0,2451	0,2451	0,2451	0,2451	0,2451
f14	0,1649	0,1649	0,1649	0,1649	0,1649	0,1649	0,1649	0,1649	0,1649
f15	0,0036	0,0036	0,0036	0,0036	0,0036	0,0036	0,0036	0,0036	0,0036
f16	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002
f17	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
f18	0,5162	0,5162	0,5162	0,5162	0,5162	0,5162	0,5162	0,5162	0,5162
f19	0,5395	NaN	0,5395	0,5395	0,5395	0,5395	0,5395	0,5395	0,5395
f20	0,5527	NaN	0,5527	0,5527	0,5527	0,5527	0,5527	0,5527	0,5527
f21	0,4100	0,4100	0,4100	0,4100	0,4100	0,4100	0,4100	0,4100	0,4100
f22	0,4525	0,4525	0,4525	0,4525	0,4525	0,4525	0,4525	0,4525	0,4525
f23	0,4756	0,4756	0,4756	0,4756	0,4756	0,4756	0,4756	0,4756	0,4756
f24	3,0000	NaN	3,0000	3,0000	3,0000	3,0000	3,0000	3,0000	3,0000
f25	4,0000	NaN	4,0000	4,0000	4,0000	4,0000	4,0004	4,0000	4,0000
f26	2,4746	NaN	2,4746	2,4746	2,4746	2,4746	2,4746	2,4746	2,4746
f27	1,8892	NaN	1,8892	1,8885	1,8887	1,8892	1,8892	1,8891	1,8891
f28	7,1430	NaN	7,1430	7,1430	7,1430	7,1430	7,1430	7,1430	7,1430
f29	3,0004	NaN	3,0004	2,9996	2,9995	3,0000	3,0004	3,0005	2,9998
f30	4,2672	NaN	4,2672	4,2672	4,2672	4,2672	4,2672	4,2672	4,2672

f31	0,3183	0,3183	0,3183	0,3183	0,3183	0,3183	0,3183	0,3183	0,3183
f32	1,3258	1,3258	1,3258	1,3258	1,3258	1,3258	1,3258	1,3258	1,3258
f33	0,8655	0,8655	0,8655	0,8655	0,8655	0,8655	0,8655	0,8655	0,8655
f34	0,7391	0,7391	0,7391	0,7391	0,7391	0,7391	0,7391	0,7391	0,7391
f35	0,6567	0,6567	0,6567	0,6567	0,6567	0,6567	0,6567	0,6567	0,6567
f36	0,6667	0,6667	0,6667	0,6667	0,6667	0,6667	0,6667	0,6667	0,6667
f37	0,7778	NaN	0,7778	0,7778	0,7778	0,7778	0,7778	0,7778	0,7778
f38	0,3333	0,3333	0,3333	0,3333	0,3333	0,3333	0,3333	0,3333	0,3333
f39	0,3333	NaN	0,3333	0,3333	0,3333	0,3333	0,3333	0,3333	0,3333
f40	0,6667	-1,0000	0,6667	0,6667	0,6667	0,6667	0,6667	0,6667	0,6667
f41	0,1383	0,1383	0,1383	0,1383	0,1383	0,1383	0,1383	0,1383	0,1383
f42	1,0000	3,0000	1,0000	3,0000	3,0000	3,0000	1,0000	1,0000	3,0000
f43	0,0000	NaN	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
f44	0,0000	NaN	0,0000	0,0000	0,0000	0,0000	0,0010	0,0000	0,0000
f45	0,0000	NaN	0,0000	NaN	NaN	0,0000	-0,0039	0,0000	0,0000
f46	0,1016	NaN	0,1016	0,1016	0,1016	0,1016	0,1016	0,1016	0,1016
f47	0,0077	0,0077	0,0077	0,0077	0,0077	0,0077	0,0077	0,0077	0,0077
f48	0,0013	0,0013	0,0013	0,0013	0,0013	0,0013	0,0013	0,0013	0,0013
f49	-0,6823	-0,6823	-0,6823	-0,6823	-0,6823	-0,6823	-0,6823	-0,6823	-0,6823
f50	-2,1038	-2,1038	-2,1038	-2,1038	-2,1038	-2,1038	-2,1038	-2,1038	-2,1038
f51	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
f52	1,0000	NaN	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
f53	1,6968	1,6968	1,6968	1,6968	1,6968	1,6968	1,6968	1,6968	1,6968
f54	0,5236	0,5236	0,5236	0,5236	0,5236	0,5236	0,5236	0,5236	0,5236
f55	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
f56	2,0000	2,0000	2,0000	2,0000	2,0000	2,0000	2,0000	2,0000	2,0000
f57	3,0000	NaN	3,0000	3,0000	3,0000	3,0000	3,0000	3,0000	3,0000
f58	1,5574	1,5574	1,5574	1,5574	1,5574	1,5574	1,5574	1,5574	1,5574
f59	1,2564	1,2564	1,2564	1,2564	1,2564	1,2564	1,2564	1,2564	1,2564
f60	0,3545	0,3545	0,3545	0,3545	0,3545	0,3545	0,3545	0,3545	0,3545
f61	1,4045	1,4045	1,4045	1,4045	1,4045	1,4045	1,4045	1,4045	1,4045
f62	1,8955	1,8955	1,8955	1,8955	1,8955	1,8955	1,8955	1,8955	1,8955
f63	0,5671	0,5671	0,5671	0,5671	0,5671	0,5671	0,5671	0,5671	0,5671
f64	0,1000	0,1000	0,1000	0,1000	0,1000	0,1000	0,1000	0,1000	0,1000
f65	-0,5236	-0,5236	-0,5236	-0,5236	-0,5236	-0,5236	-0,5236	-0,5236	-0,5236

f66	-0,8439	-0,8439	-0,8439	-0,8439	-0,8439	-0,8439	-0,8439	-0,8439	-0,8439
f67	0,8351	0,8351	0,8351	0,8351	0,8351	0,8351	0,8351	0,8351	0,8351
f68	0,8736	0,8736	0,8736	0,8736	0,8736	0,8736	0,8736	0,8736	0,8736
f69	0,1111	0,1111	0,1111	0,1111	0,1111	0,1111	0,1111	0,1111	0,1111
f70	-0,0370	-0,0370	-0,0370	-0,0370	-0,0370	-0,0370	-0,0370	-0,0370	-0,0370
f71	-0,6667	NaN	-0,6667	-0,6667	-0,6667	-0,6667	-0,6667	-0,6667	-0,6667
f72	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
f73	0,0000	NaN	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
f74	0,3333	NaN	0,3333	0,3333	0,3333	0,3333	0,3333	0,3333	0,3333
f75	0,3333	NaN	0,3333	0,3333	0,3333	0,3333	0,3333	0,3333	0,3333
f76	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000
f77	0,0476	0,1429	0,0476	0,0476	0,0476	0,0476	0,0476	0,0476	0,0476
f78	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
f79	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
f80	-0,0313	-0,0205	-0,0156	-0,0145	-0,0125	-0,0226	-0,0415	-0,0447	-0,0282
f81	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000	-1,0000
f82	0,1985	0,0500	0,0500	0,0500	0,0500	0,0203	0,0500	0,0500	0,0500
f83	0,1985	NaN	0,1985	0,1490	0,1490	0,0005	0,1985	0,0005	0,2579
f84	5,0021	3,0125	3,0125	3,0125	3,0125	3,0125	3,0125	5,0021	3,0125
f85	-2,2078	-2,2078	-2,2078	-2,2078	-2,2078	-2,2078	-2,2078	-2,2078	-2,2078
f86	0,3028	NaN	0,3028	0,3028	0,3028	0,3028	0,3028	0,3028	0,3028
f87	0,9878	0,9878	0,9878	0,9878	0,9878	0,9878	0,9878	0,9878	0,9878
f88	0,1684	0,1684	0,1684	0,1684	0,1684	0,1684	0,1684	0,1684	0,1684
f89	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
f90	0,0264	1,1476	0,0264	1,1476	1,1476	0,0264	0,0264	0,0264	1,1476
f91	0,3539	0,3539	0,1208	0,3539	0,3539	0,3539	0,1208	0,1208	0,3539

Iteration count

Method	bs	fp	mfp	ill	AB	ITP	mAB	Rid	Bre
f01	1	33	1	9	7	9	1	2	9
f02	48	85	14	11	11	36	10	14	12
f03	48	107	17	15	22	50	11	12	11
f04	48	48	13	11	10	10	8	10	9
f05	48	31	15	8	7	50	7	12	7

f06	48	29	14	8	9	11	9	10	9
f07	48	29	15	11	10	13	10	12	11
f08	48	30	16	12	12	15	11	10	11
f09	48	16	14	11	8	47	9	14	9
f10	48	13	14	9	7	10	7	14	9
f11	48	11	13	9	7	8	7	14	8
f12	48	39	14	9	8	10	9	12	8
f13	48	80	17	11	9	48	9	14	9
f14	48	167	17	11	11	49	10	14	12
f15	48	7	8	8	6	9	6	10	7
f16	48	5	6	6	5	8	6	10	6
f17	48	5	5	6	5	8	5	10	6
f18	48	79	15	9	7	36	8	12	8
f19	48	200	15	13	8	10	9	14	8
f20	48	200	17	21	9	11	10	14	12
f21	48	33	13	9	8	8	9	10	9
f22	48	32	14	10	8	8	7	10	9
f23	48	31	13	11	8	9	7	10	9
f24	48	200	47	90	107	49	18	78	132
f25	48	200	47	174	189	48	10	60	111
f26	48	200	47	90	107	49	16	76	132
f27	12	200	12	33	36	21	12	22	30
f28	48	200	47	89	106	49	14	72	132
f29	10	200	10	57	44	1	10	16	29
f30	48	200	17	51	56	49	12	14	13
f31	48	179	18	13	4	49	11	14	11
f32	48	90	16	13	10	50	10	14	12
f33	48	190	13	14	14	10	10	16	13
f34	48	10	10	9	7	9	8	14	9
f35	48	13	21	13	12	35	16	20	10
f36	48	39	47	42	48	42	48	46	38
f37	48	200	21	23	25	15	15	22	30
f38	48	47	47	54	54	48	48	52	46
f39	48	200	47	156	156	49	48	74	72

f40	48	7	47	90	90	49	48	58	48
f41	48	30	18	12	11	15	12	12	12
f42	48	21	14	11	8	13	11	16	9
f43	48	200	47	89	106	49	17	78	115
f44	48	200	47	172	191	49	9	54	113
f45	48	200	47	200	200	49	7	50	123
f46	48	200	18	23	21	49	12	12	14
f47	48	154	18	15	12	50	13	16	15
f48	48	43	20	14	12	19	17	16	15
f49	48	24	13	11	8	11	9	10	10
f50	48	39	17	13	11	11	10	12	11
f51	48	26	18	10	8	10	9	10	9
f52	48	200	19	36	19	50	13	16	16
f53	48	30	34	10	10	11	10	16	12
f54	48	14	13	7	6	10	7	10	7
f55	48	68	13	11	8	9	8	2	10
f56	48	56	13	11	8	9	8	12	10
f57	48	200	20	19	29	50	10	12	11
f58	48	24	16	10	7	11	9	10	9
f59	48	142	20	14	13	49	9	12	12
f60	48	13	12	8	6	9	6	12	7
f61	48	31	14	12	12	12	9	16	10
f62	48	31	14	9	7	8	7	12	8
f63	48	29	15	10	8	10	8	8	9
f64	48	37	10	7	6	10	7	10	8
f65	48	10	16	7	7	9	7	14	7
f66	48	31	16	12	10	39	10	14	9
f67	48	48	16	11	9	49	11	10	11
f68	48	14	10	8	6	9	7	8	7
f69	48	60	12	10	8	8	8	10	9
f70	48	12	71	12	10	49	9	14	10
f71	48	200	15	14	5	49	8	14	12
f72	48	2	47	2	2	50	33	4	127
f73	48	200	47	90	108	50	33	4	131

f74	48	200	47	88	107	49	16	78	133
f75	25	200	25	26	35	27	22	36	65
f76	48	47	47	35	35	48	48	48	46
f77	48	13	47	94	94	49	48	64	49
f78	48	47	47	53	52	48	48	38	47
f79	48	45	44	48	48	45	13	34	43
f80	48	23	18	21	21	26	23	22	20
f81	48	43	46	13	13	49	18	38	47
f82	48	2	3	2	2	12	2	4	3
f83	48	200	47	68	64	49	24	64	116
f84	48	63	64	8	6	9	6	16	7
f85	48	12	13	12	10	14	9	18	11
f86	48	200	16	18	18	49	11	18	15
f87	48	134	15	13	4	15	5	14	11
f88	48	27	14	12	10	15	11	12	12
f89	0	0	0	0	0	0	0	0	0
f90	48	167	18	12	12	50	11	16	14
f91	48	12	24	12	10	47	10	12	12
SUM	**4176**	**7709**	**2089**	**2684**	**2700**	**2581**	**1227**	**1990**	**2625**
AVE	**46**	**85**	**23**	**29**	**30**	**28**	**13**	**22**	**29**
MAX	**48**	**200**	**71**	**200**	**200**	**50**	**48**	**78**	**133**
Method	**bs**	**fp**	**mfp**	**ill**	**AB**	**ITP**	**mAB**	**Rid**	**Bre**

Precision = 10^{-14}

Legend:

bs – Bisection method

fp – False position

mfp – Modified false position

ill – Illinois method

AB – Anderson-Bjork

ITP – Interpolate, truncate, project

mAB – Modified Anderson-Bjork (new)

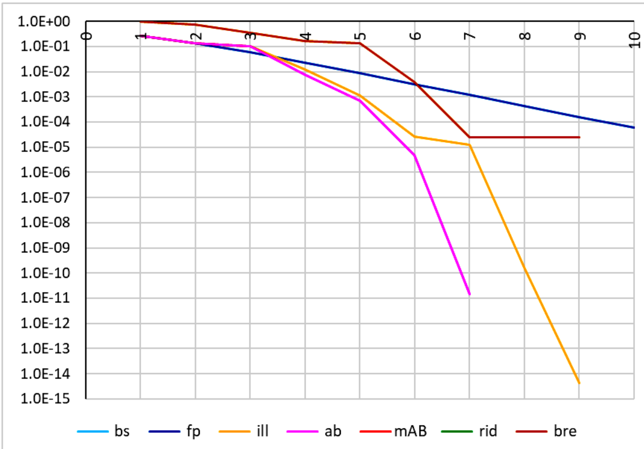
Rid – Ridders

Bre – Brent

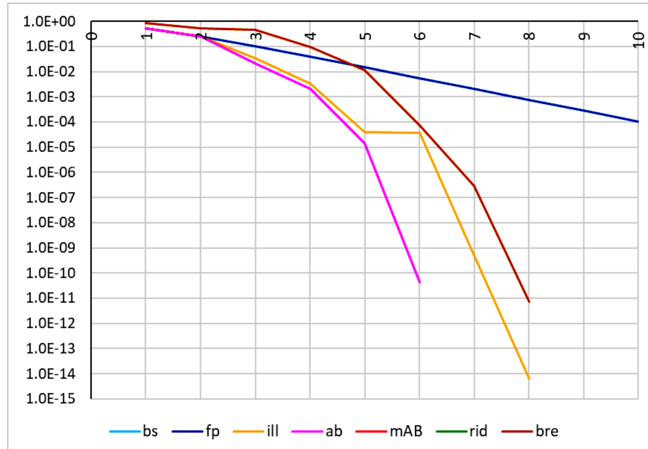
Convergence plots

$$f_{01}(x) = x^3 - 1$$

TO THE VALUES OF THE ARGUMENT

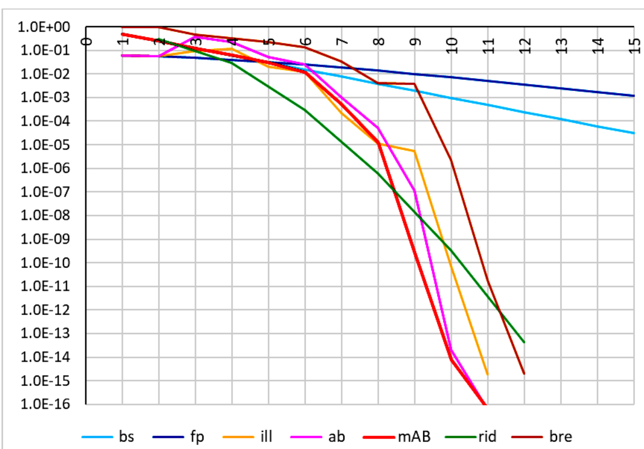


TO THE VALUES OF THE FUNCTION



$$f_{02}(x) = x^2 \cdot \left(\frac{x^2}{3} + \sqrt{2} \cdot \sin(x) \right) - \frac{\sqrt{3}}{18}$$

TO THE VALUES OF THE ARGUMENT



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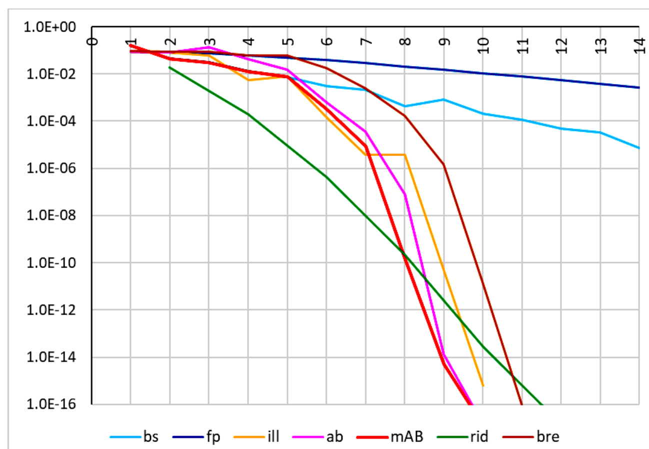
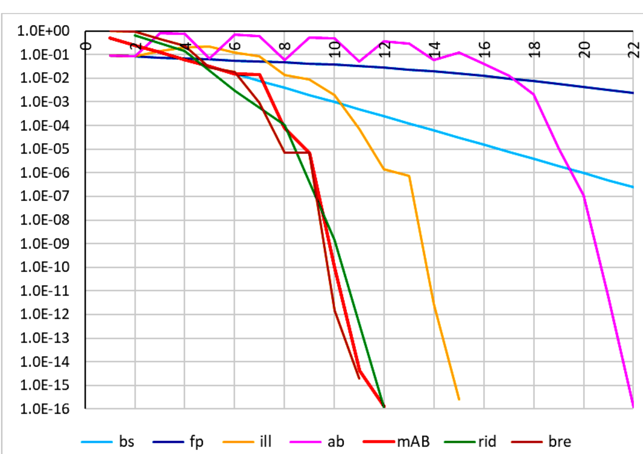
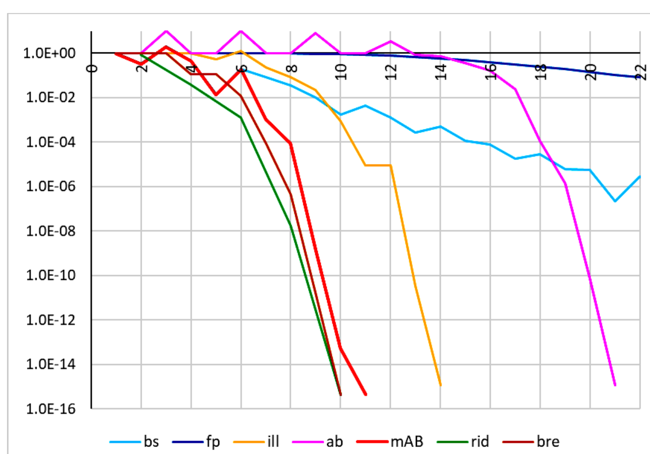


$$f_{03}(x) = 11\,x^{11} - 1$$

TO THE VALUES OF THE ARGUMENT

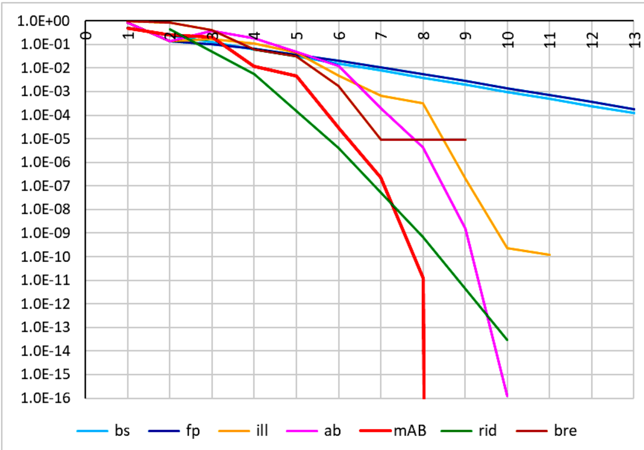


TO THE VALUES OF THE FUNCTION

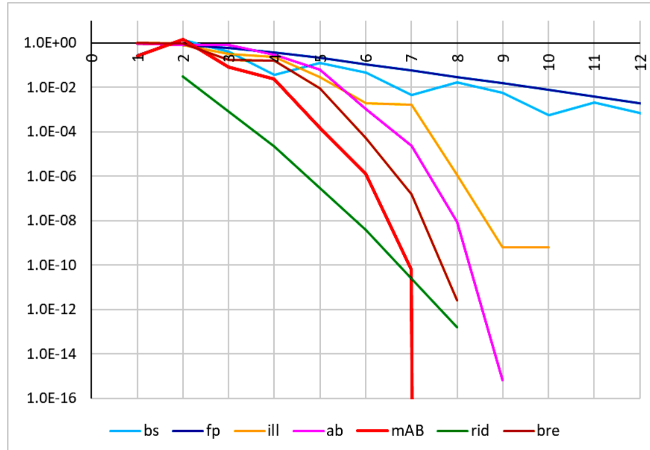


$$f_{04}(x) = x^3 + 1$$

TO THE VALUES OF THE ARGUMENT

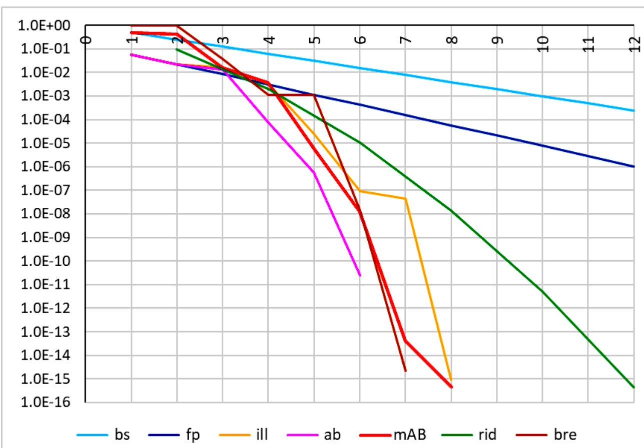


TO THE VALUES OF THE FUNCTION

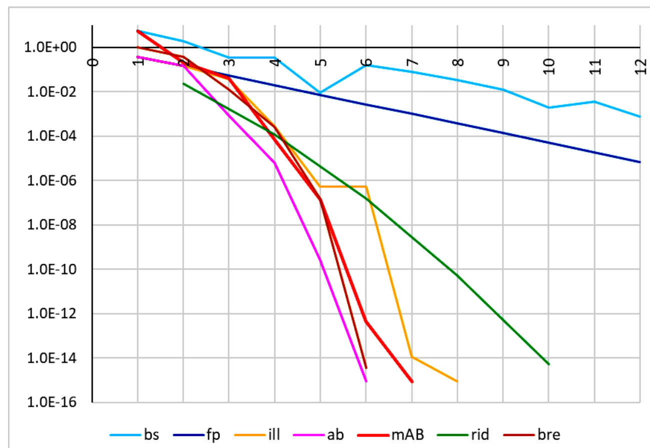


$$f_{05}(x) = x^3 - 2x - 5$$

TO THE VALUES OF THE ARGUMENT

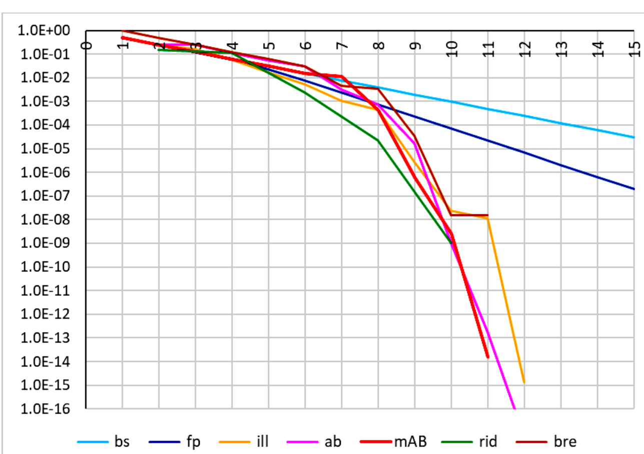


TO THE VALUES OF THE FUNCTION



$$f_{06}(x) = 2x \cdot e^{-20} + 1 - 2e^{-20x}$$

TO THE VALUES OF THE ARGUMENT



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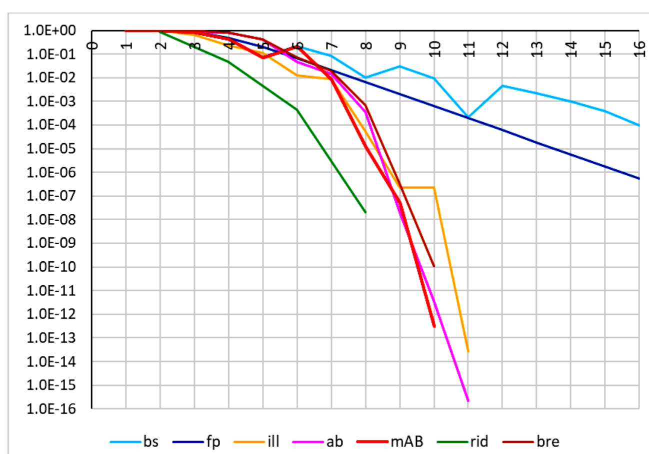
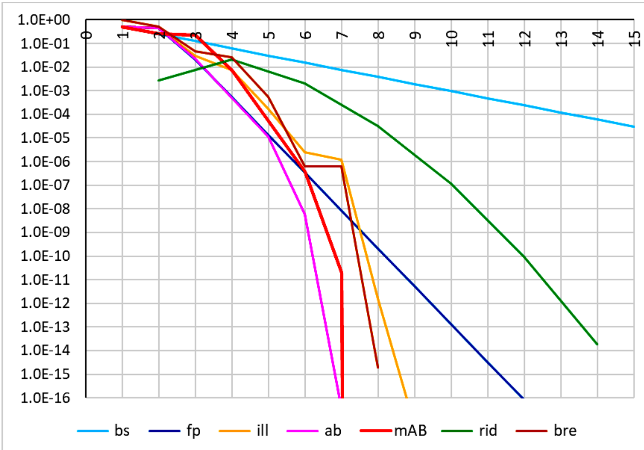


$$f_{07}(x) = (1 + (1 - 20)^2) \cdot x^2 - (1 - 20\,x)^2$$

TO THE VALUES OF THE ARGUMENT



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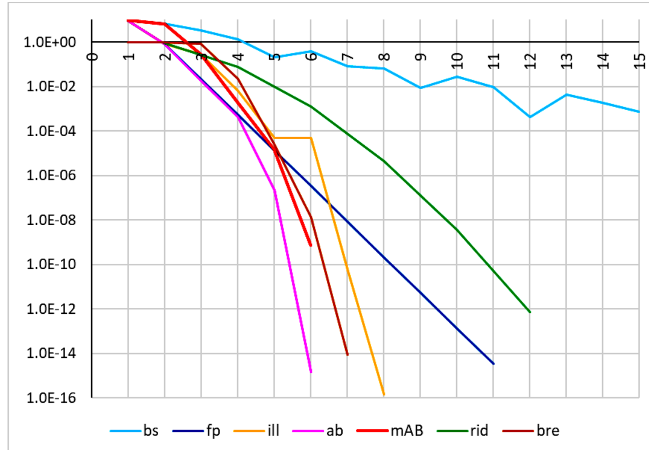
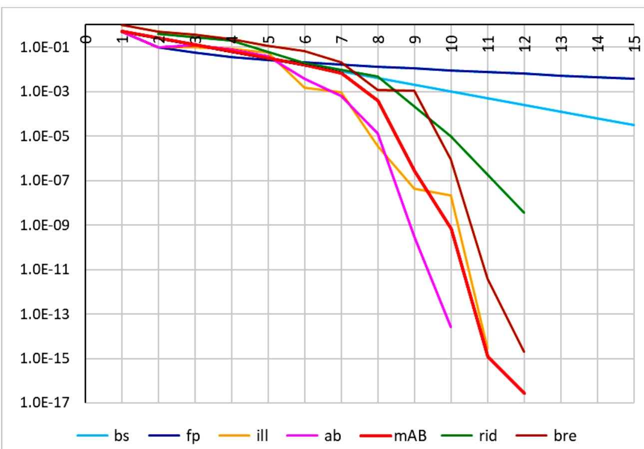


$$f_{08}(x) = x^2 - (1 - x)^{20}$$

TO THE VALUES OF THE ARGUMENT



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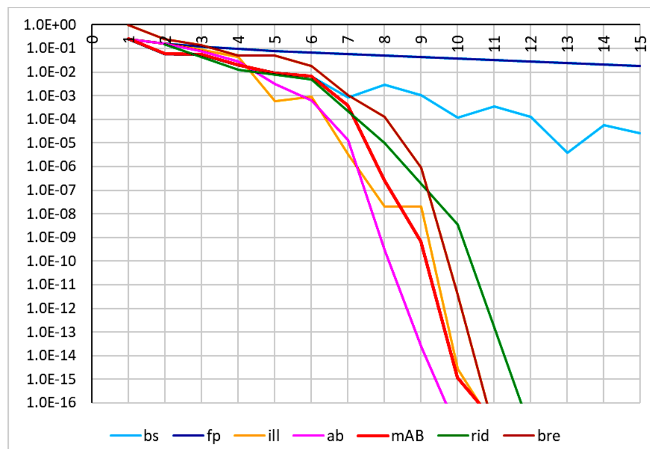
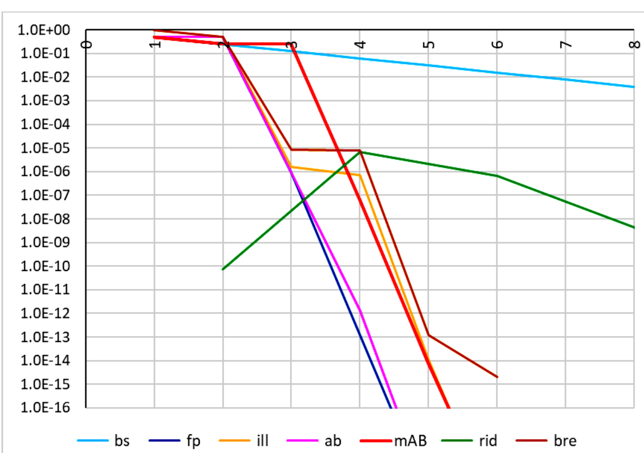


$$f_{09}(x) = (1 + (1 - 20)^4) \cdot x - (1 - 20\,x)^4$$

TO THE VALUES OF THE ARGUMENT



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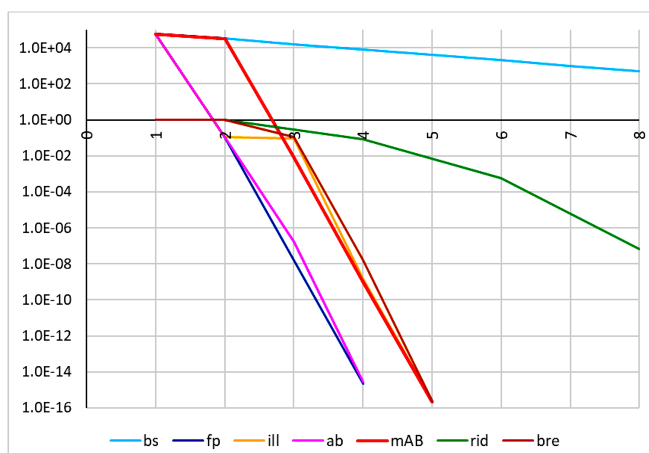
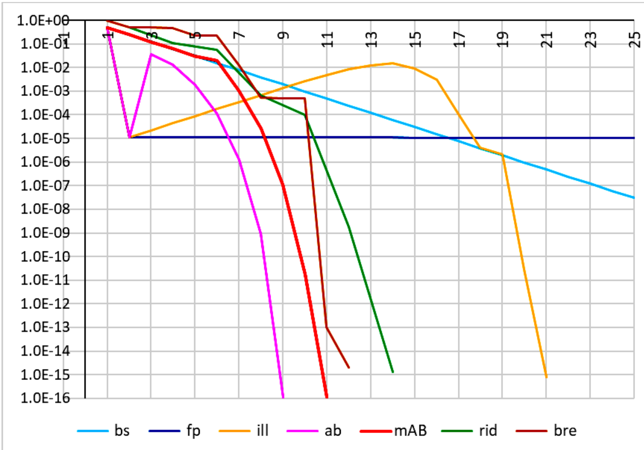


$$f_{10}(x)=e^{-20x}(x-1)+x^{20}$$

TO THE VALUES OF THE ARGUMENT



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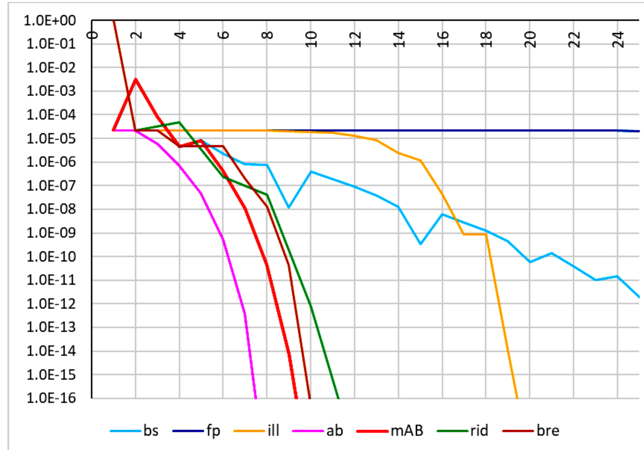
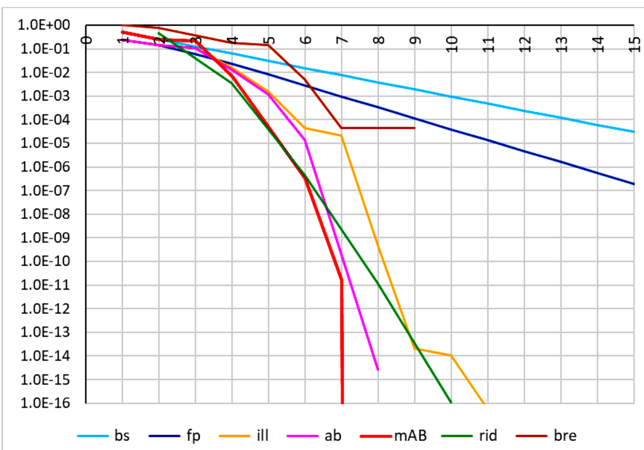


$$f_{23}(x)=x^2+\sin\left(\frac{x}{20}\right)-\frac{1}{4}$$

TO THE VALUES OF THE ARGUMENT



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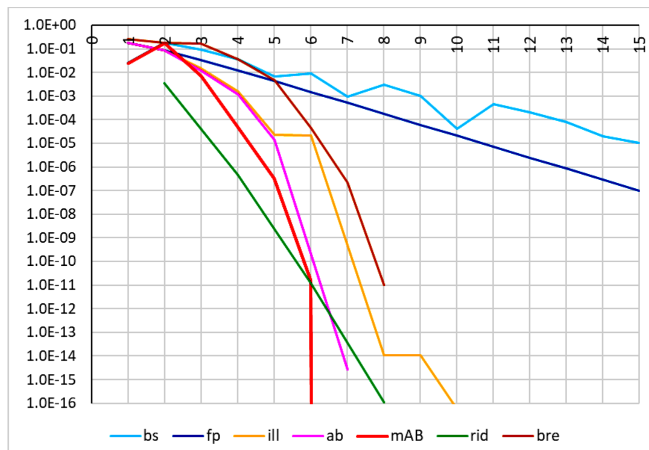
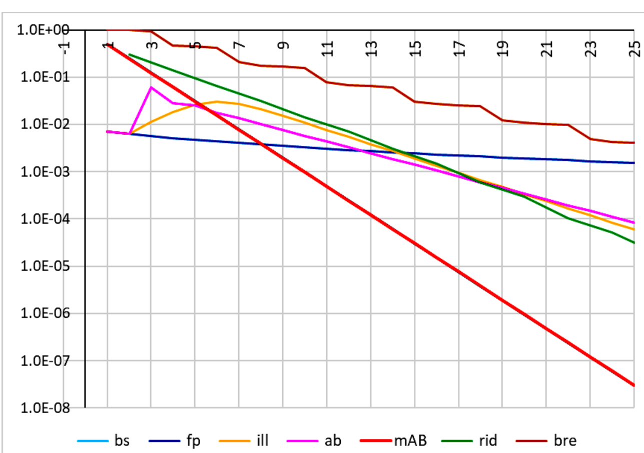


$$f_{12}(x)=(x+2)\cdot(x+1)\cdot(x-3)^3$$

TO THE VALUES OF THE ARGUMENT



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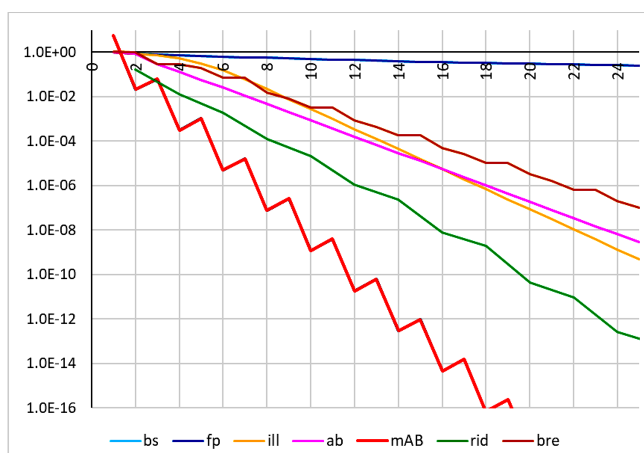
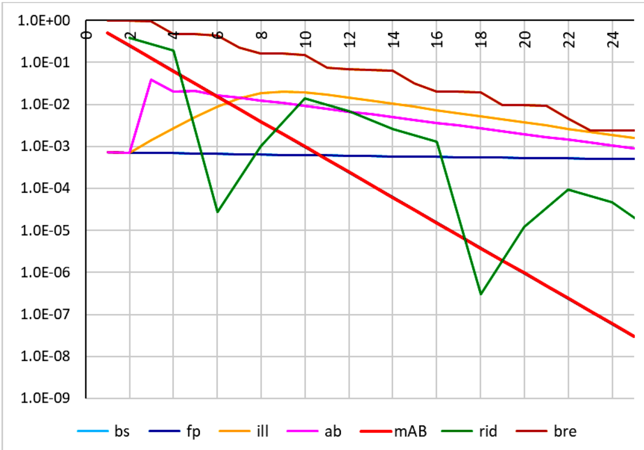


$$f_{13}(x)=(x-4)^5\cdot\ln(x)$$

TO THE VALUES OF THE ARGUMENT



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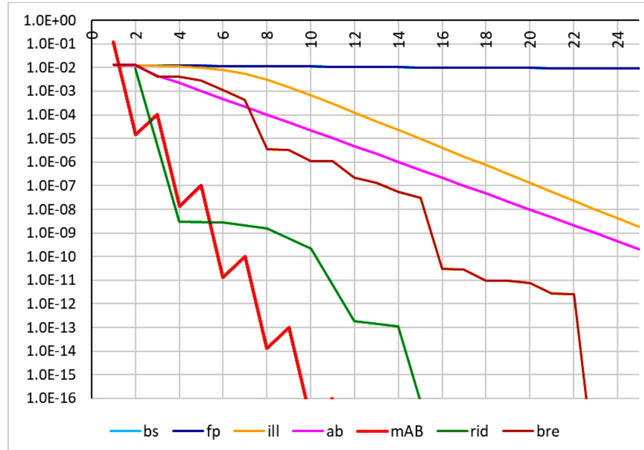
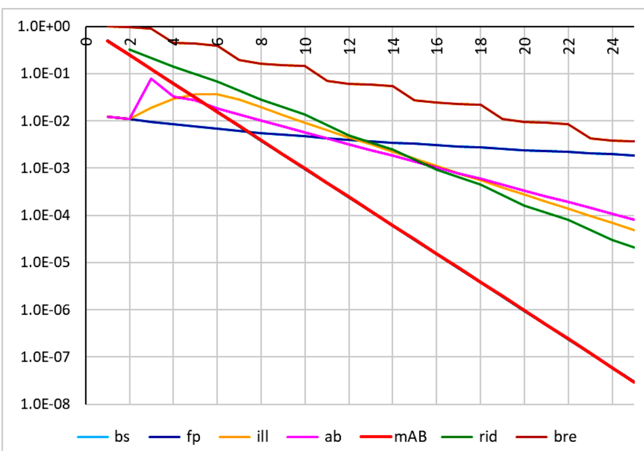


$$f_{14}(x)=\left(\sin(x)-\frac{x}{4}\right)^3$$

TO THE VALUES OF THE ARGUMENT



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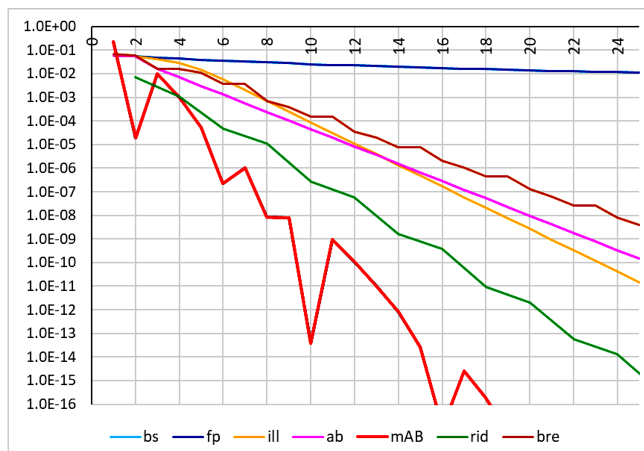
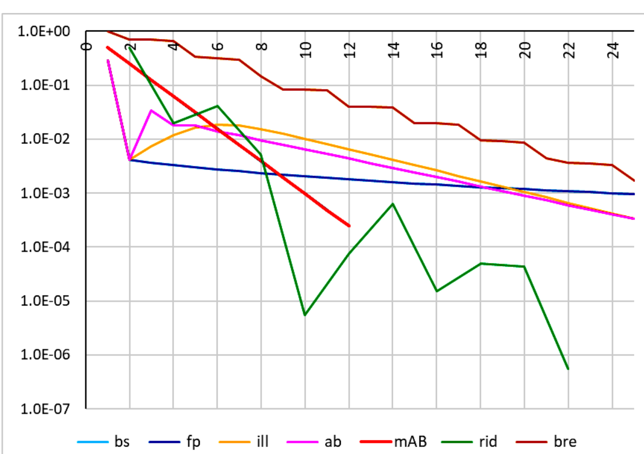


$$f_{15}(x)=\Big(81-p(x)\cdot\big(108-p(x)\cdot\big(54-p(x)\cdot(12-p(x))\big)\big)\Big)\cdot sign\big(p(x)-3\big),\ \textit{където}\ p(x)=x+1.11$$

$$\textit{където}\ p(x)=x+1.11$$

TO THE VALUES OF THE ARGUMENT



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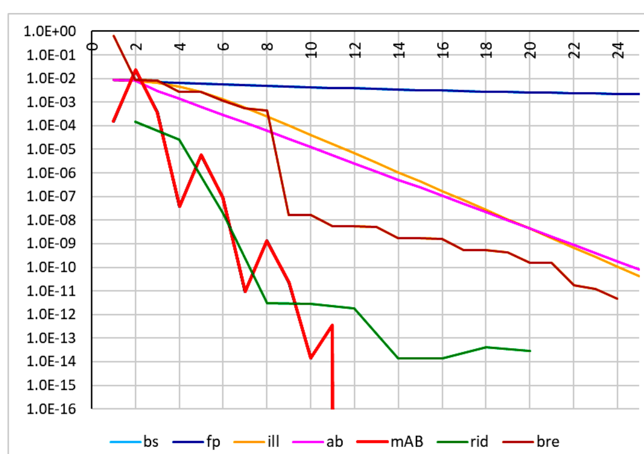
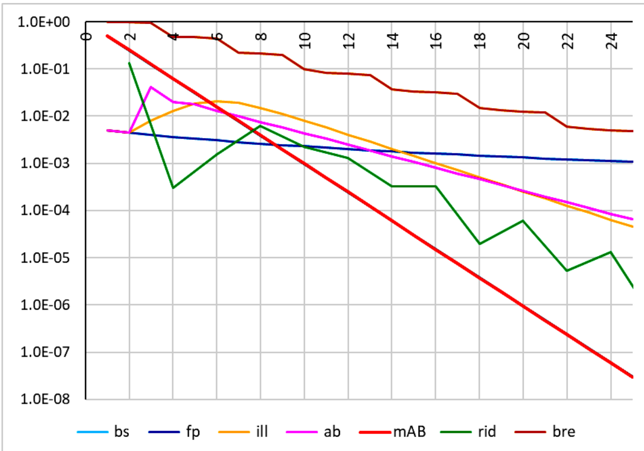


$$f_{16}(x) = \sin\left((x - 7.14)^3\right)$$

TO THE VALUES OF THE ARGUMENT



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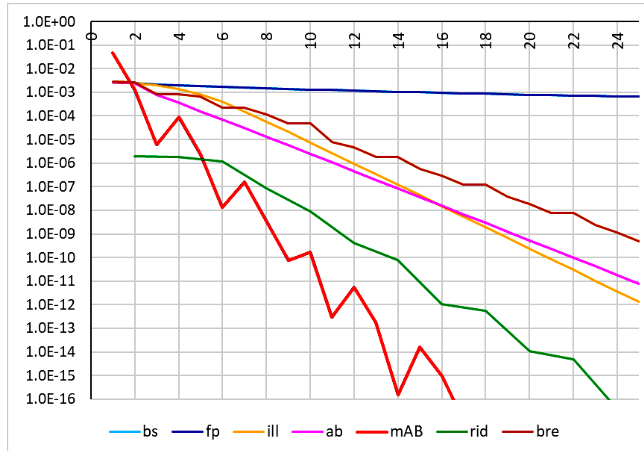
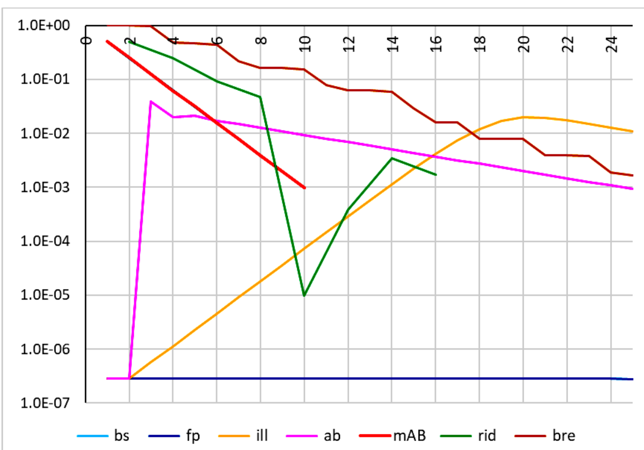


$$f_{17}(x) = e^{(x-3)^5} - 1$$

TO THE VALUES OF THE ARGUMENT



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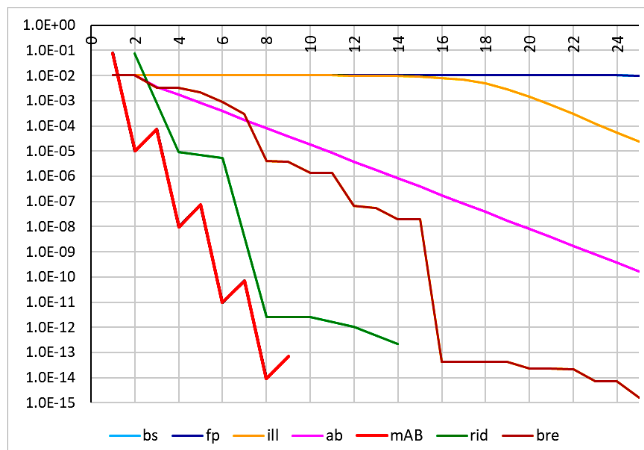
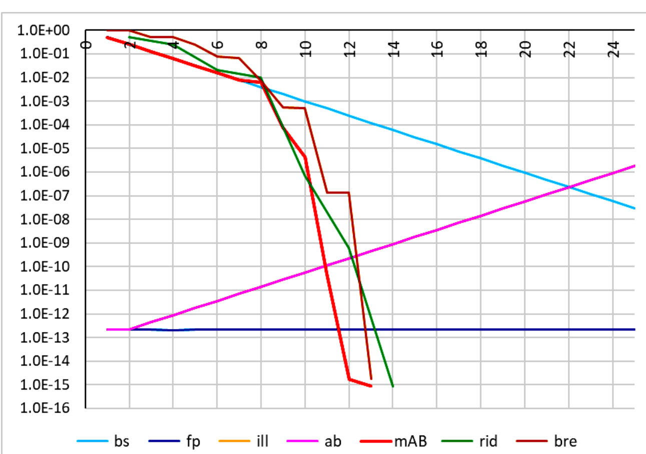


$$f_{18}(x) = e^{(x-3)^5} - e^{x-1}$$

TO THE VALUES OF THE ARGUMENT



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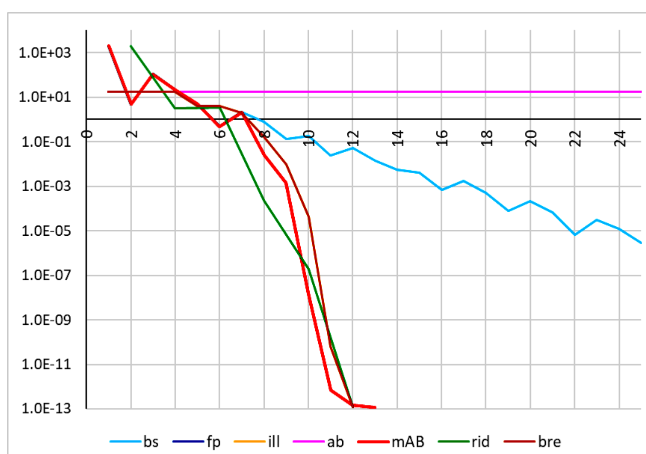
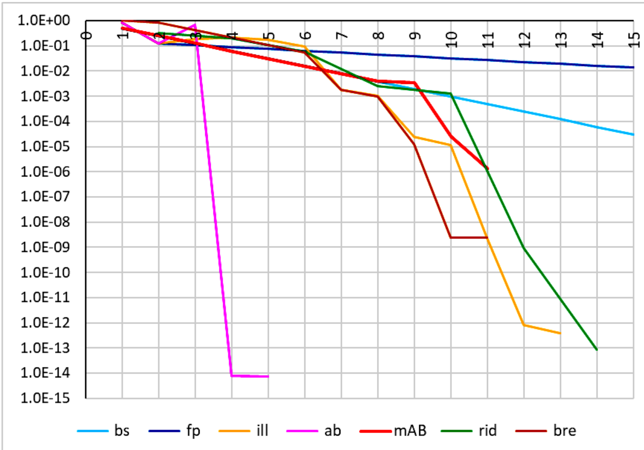


$$f_{19}(x) = \pi - \frac{1}{x}$$

TO THE VALUES OF THE ARGUMENT



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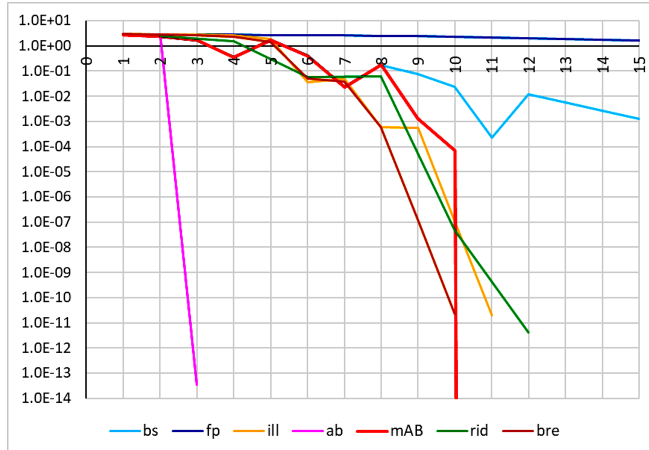
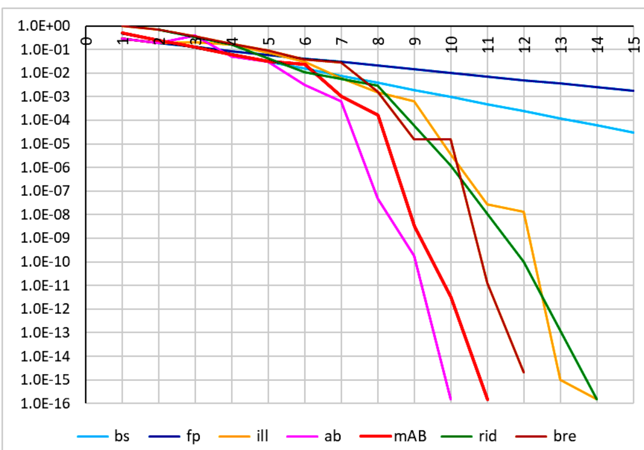


$$f_{20}(x) = 4 - \tan(x)$$

TO THE VALUES OF THE ARGUMENT



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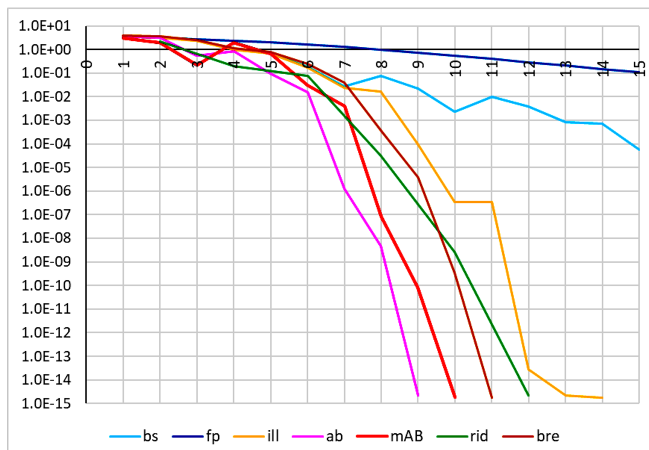
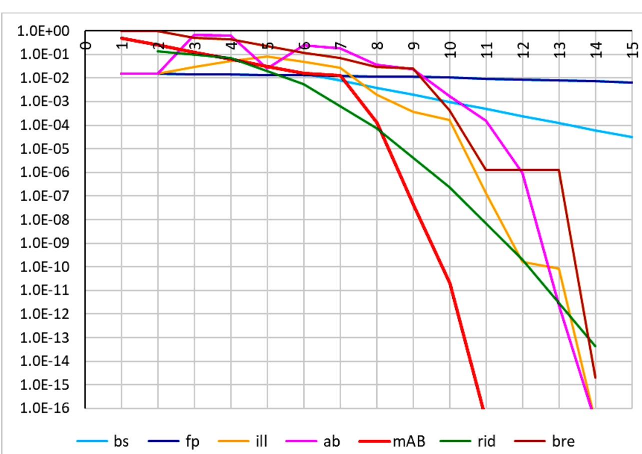


$$f_{21}(x) = \cos(x) - x^3$$

TO THE VALUES OF THE ARGUMENT



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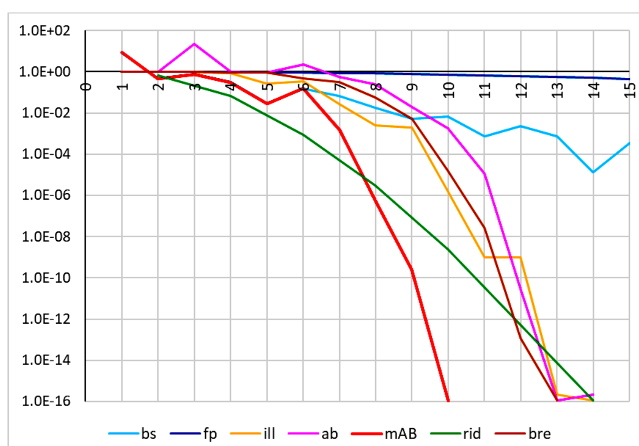
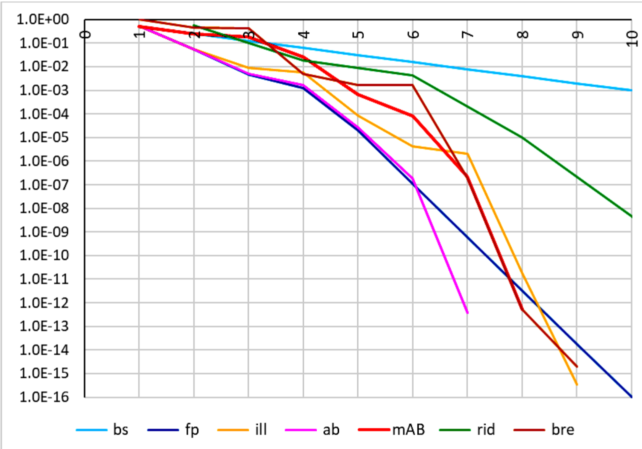


$$f_{22}(x)=\cos(x)-x$$

TO THE VALUES OF THE ARGUMENT



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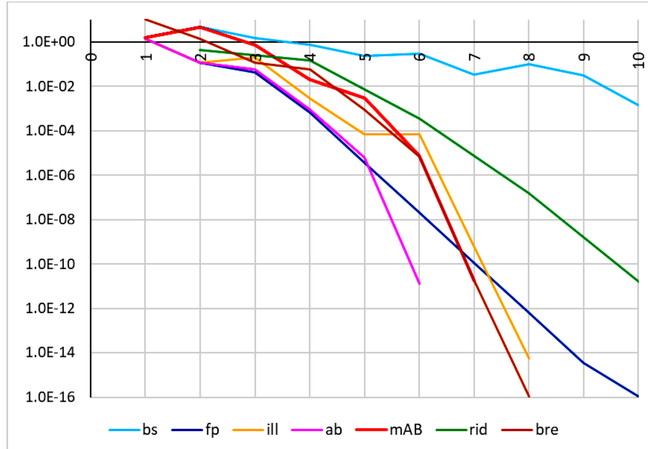
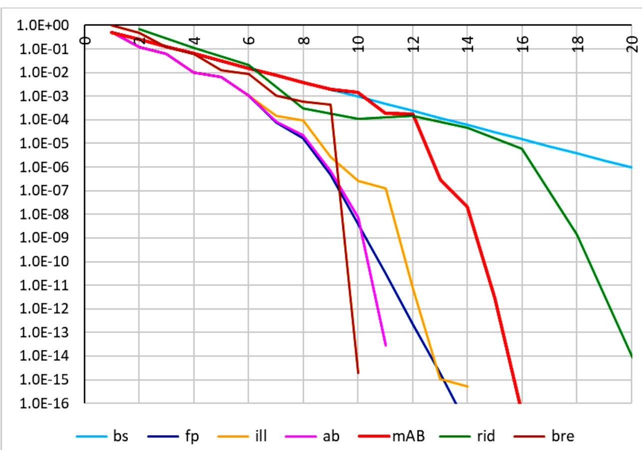


$$f_{23}(x)=\sqrt{\left|x-\frac{2}{3}\right|}\cdot if\left(x\leq\frac{2}{3};1;-1\right)-0.1$$

TO THE VALUES OF THE ARGUMENT



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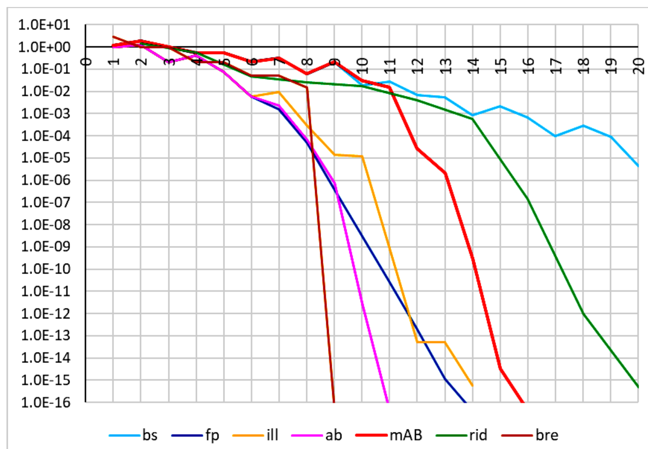
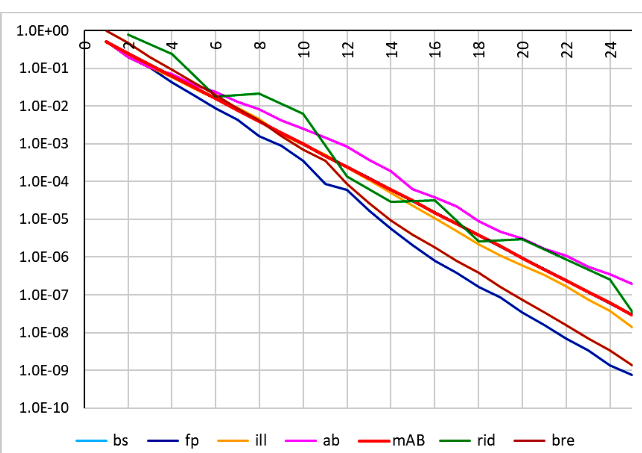
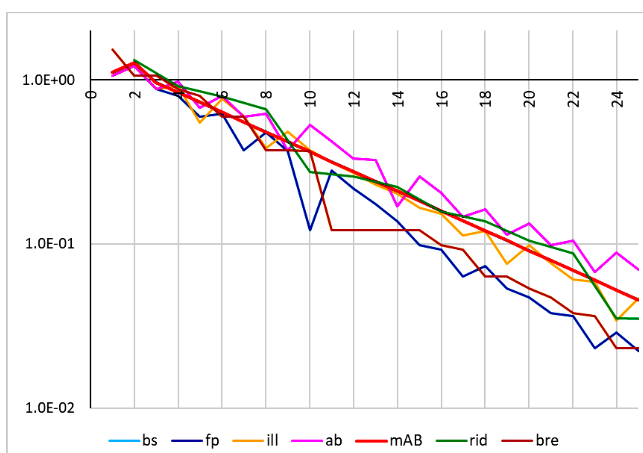


$$f_{24}(x)=\left|x-\frac{2}{3}\right|^{0.2}\cdot if\left(x\leq\frac{2}{3};1;-1\right)$$

TO THE VALUES OF THE ARGUMENT

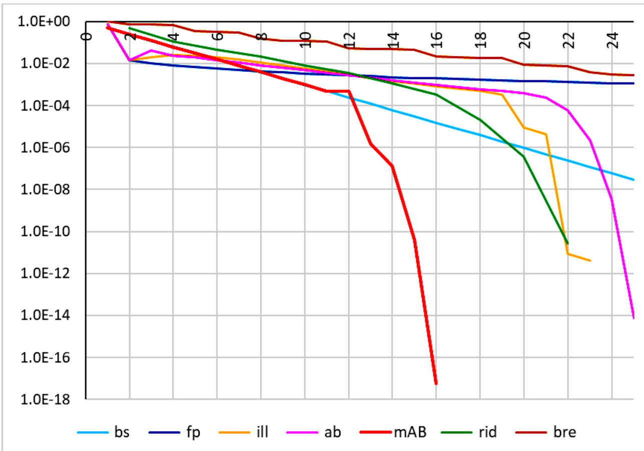


TO THE VALUES OF THE FUNCTION



$$f_{25}(x)=\left(x-\frac{7}{9}\right)^3+\left(x-\frac{7}{9}\right)\cdot 10^{-3}$$

TO THE VALUES OF THE ARGUMENT



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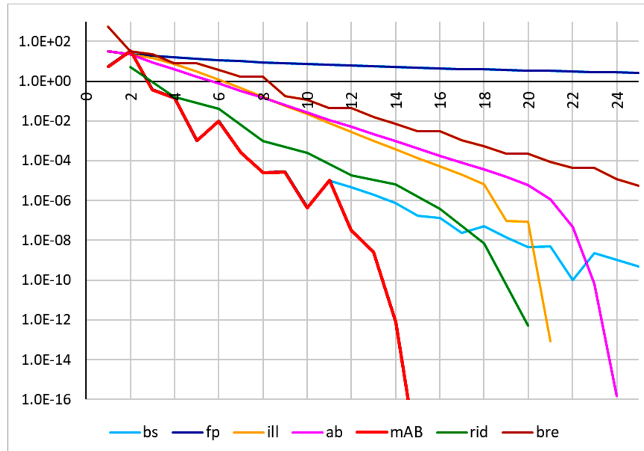
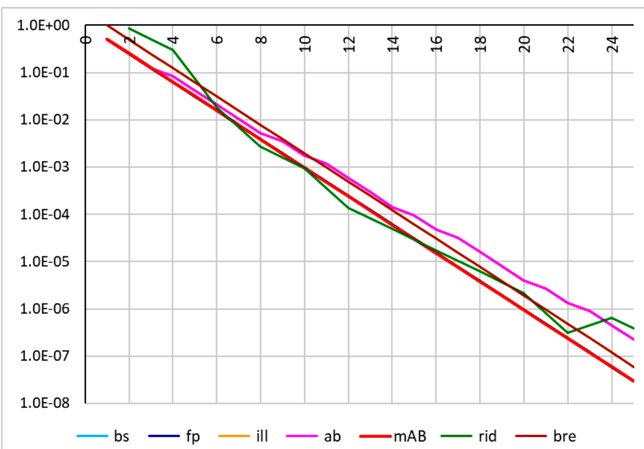


$$f_{26}(x)=\begin{cases}-0.5\,,\ if\ x\le 1/3\\ +0.5\,,\ otherwise\end{cases}$$

TO THE VALUES OF THE ARGUMENT



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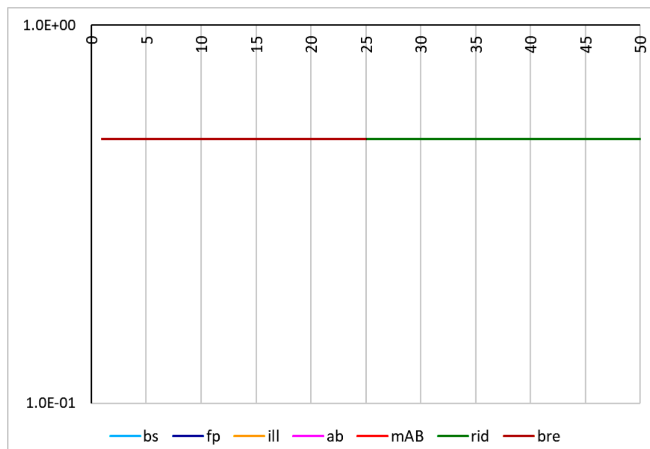
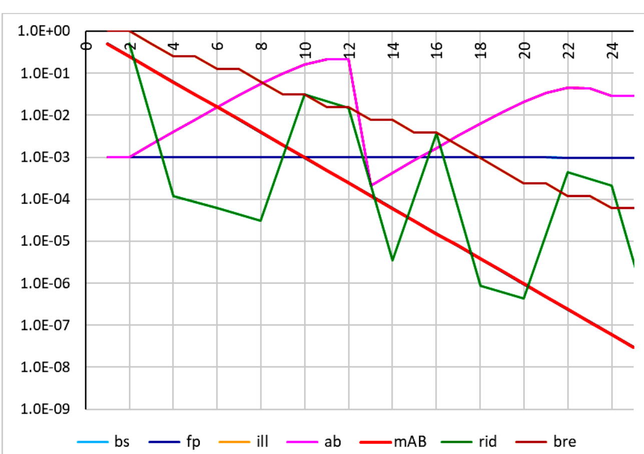


$$f_{27}(x)=\begin{cases}-10^{-3}\,,\ if\ x\le 1/3\\ 1-10^{-3}\,,\ otherwise\end{cases}$$

TO THE VALUES OF THE ARGUMENT



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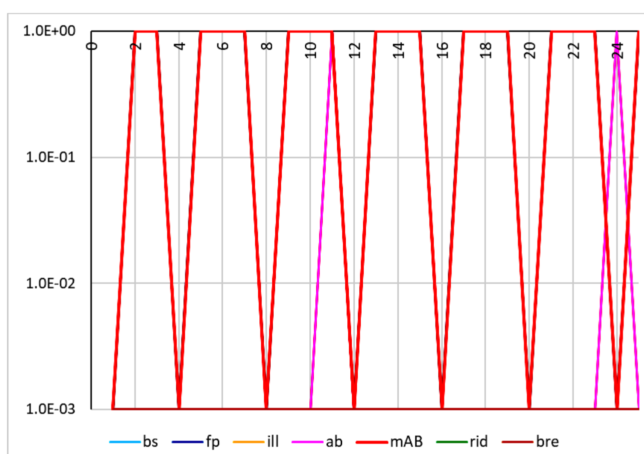
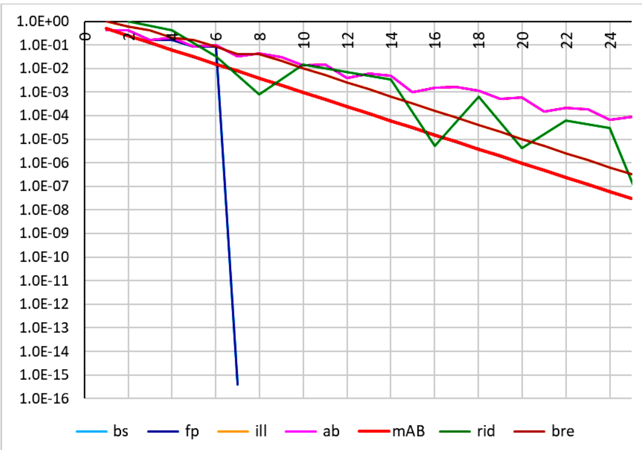


$$f_{28}(x)=\begin{cases} 0\,,\ if\ x=0 \\ \dfrac{1}{x-2/3}\,,\ otherwise \end{cases}$$

TO THE VALUES OF THE ARGUMENT



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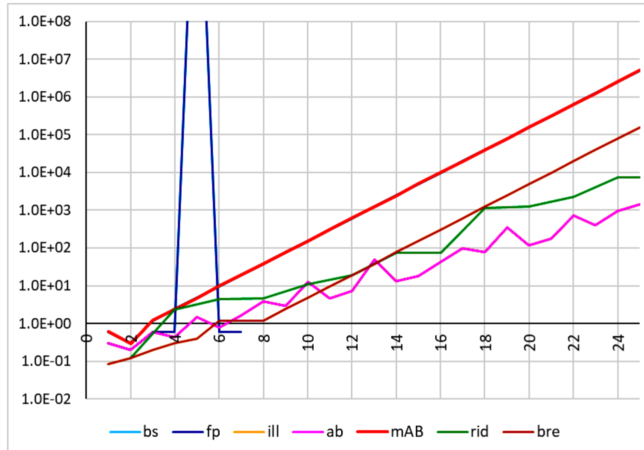
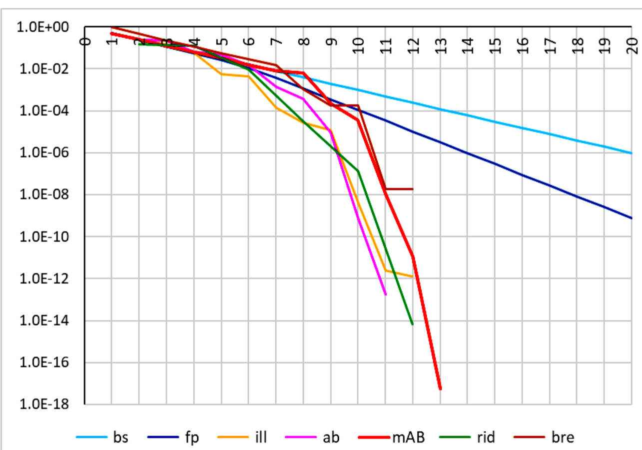


$$f_{29}(x)=2\,x\cdot e^{-5}-2\,e^{-5\,x}+1$$

TO THE VALUES OF THE ARGUMENT



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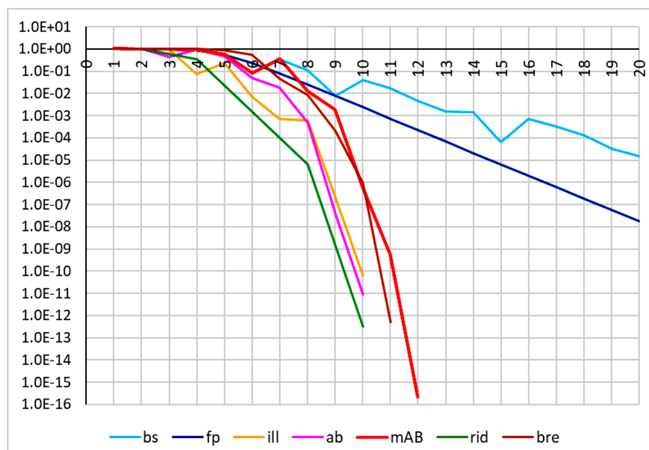
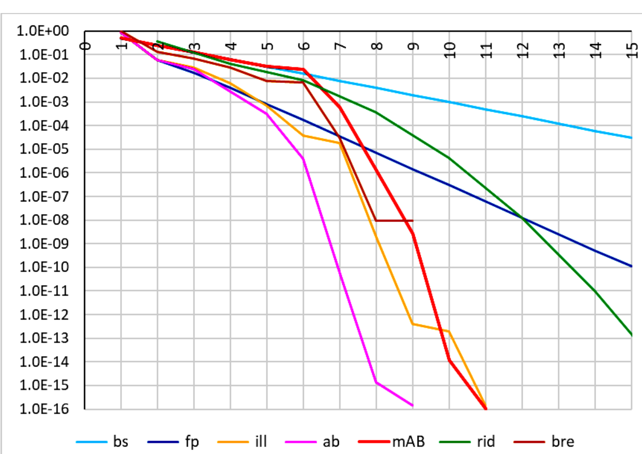


$$f_{30}(x)=(x^2-x-6)\cdot(x^2-3\,x+2)$$

TO THE VALUES OF THE ARGUMENT



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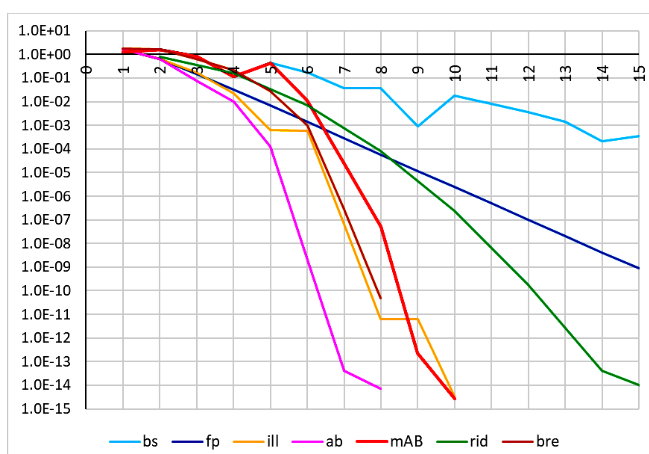
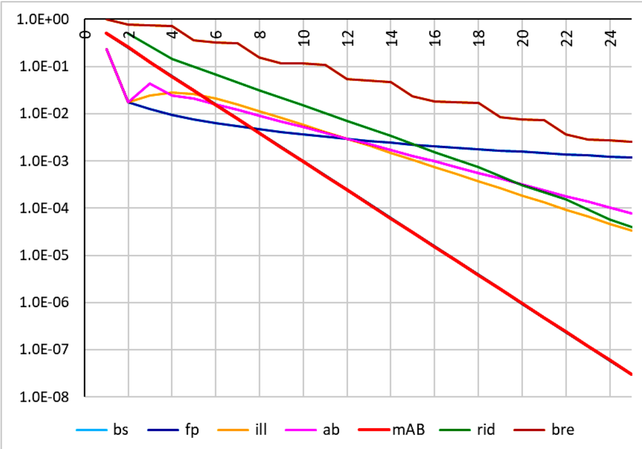


$$f_{31}(x)=x^3$$

TO THE VALUES OF THE ARGUMENT



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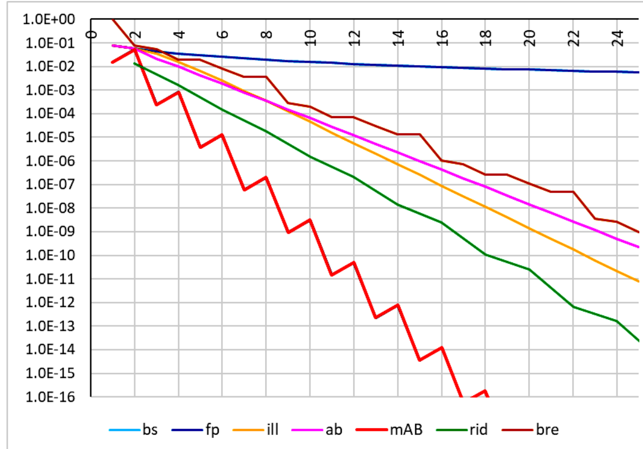
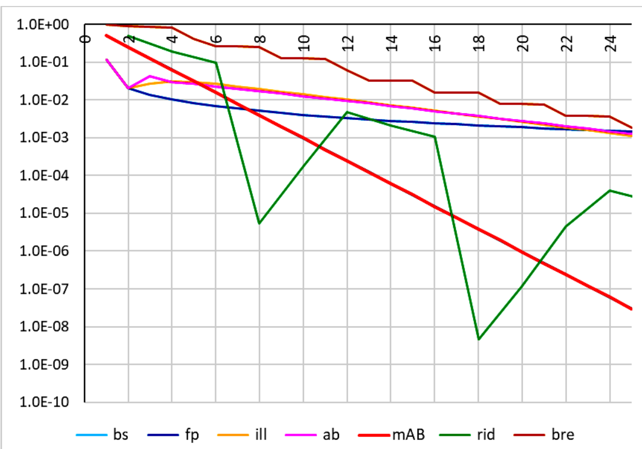


$$f_{32}(x)=x^5$$

TO THE VALUES OF THE ARGUMENT



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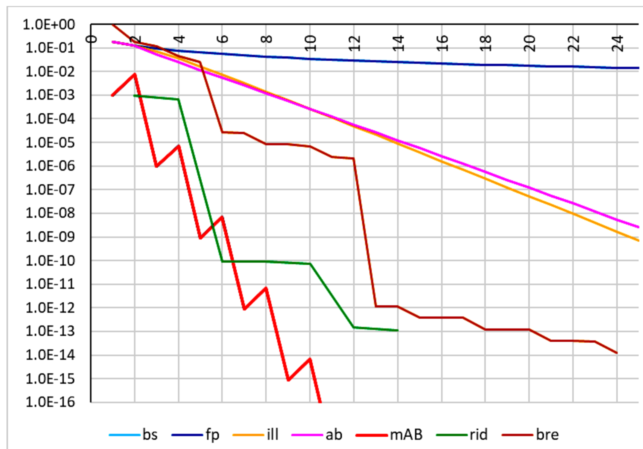
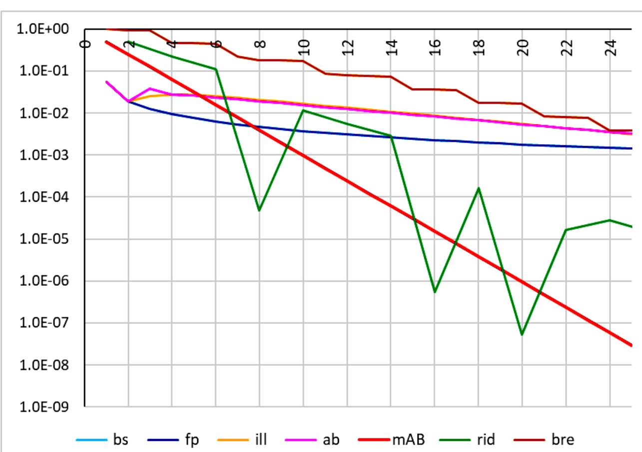


$$f_{33}(x)=x^7$$

TO THE VALUES OF THE ARGUMENT



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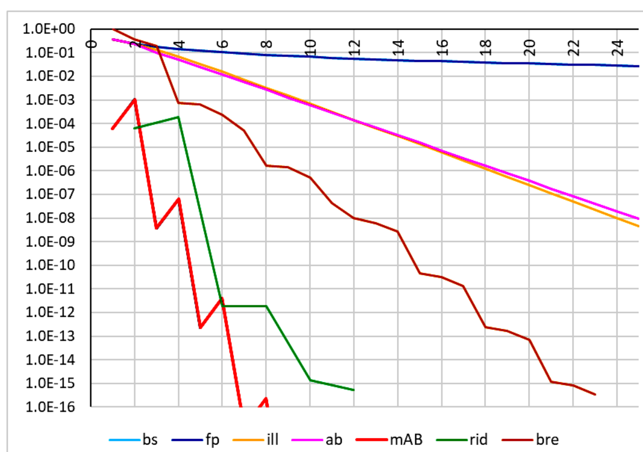
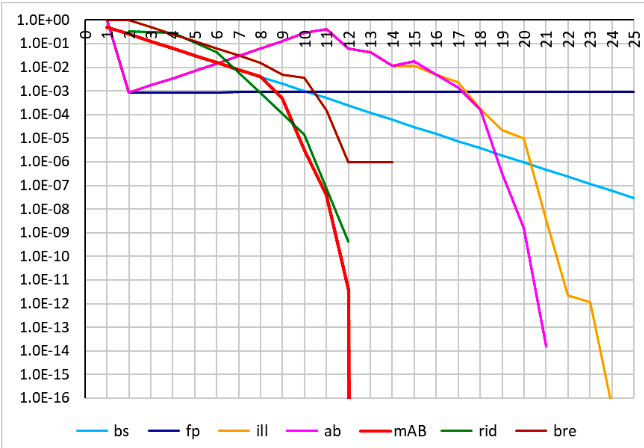


$$f_{34}(x)=\frac{e^{-5x}-x-0.5}{x^5}$$

TO THE VALUES OF THE ARGUMENT



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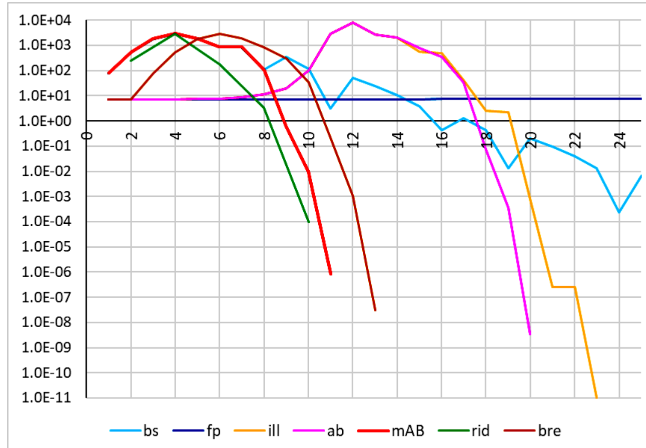
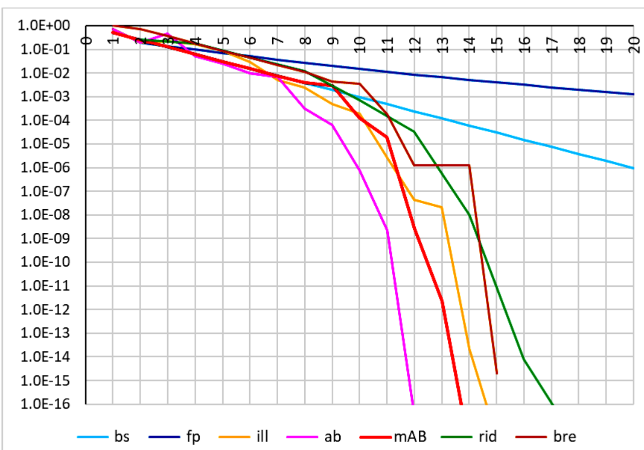


$$f_{35}(x)=\frac{1}{\sqrt{x}}-2\ln\left(5\cdot 10^3\sqrt{x}\right)+0.8$$

TO THE VALUES OF THE ARGUMENT



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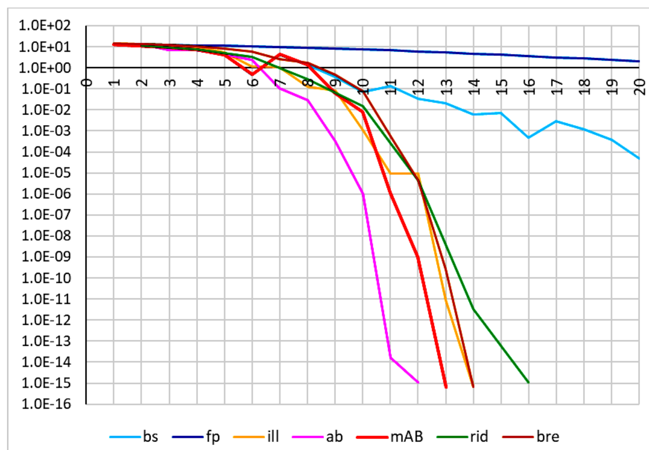
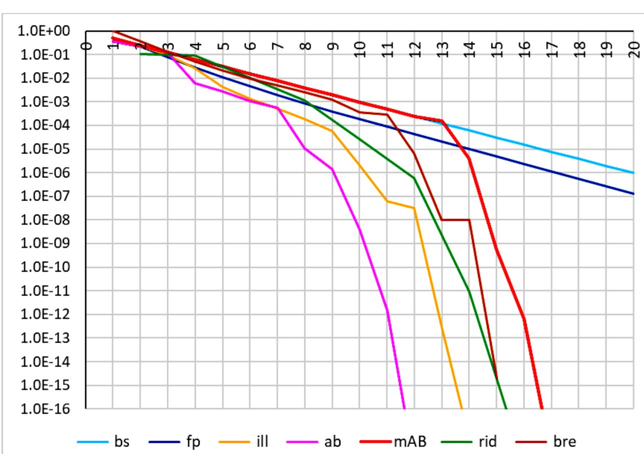
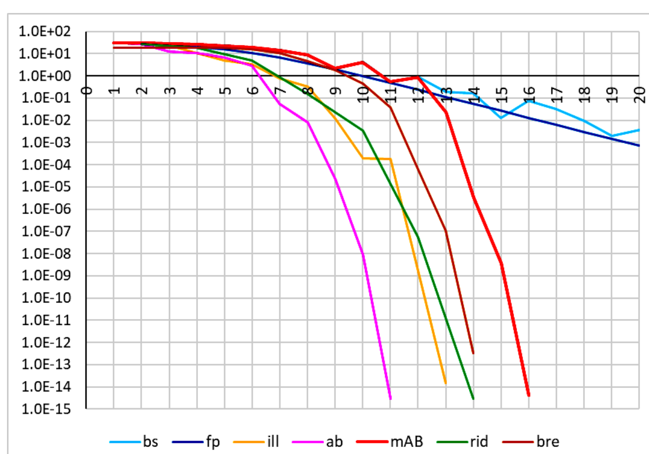


$$f_{36}(x)=\frac{1}{\sqrt{x}}-2\ln\left(5\cdot 10^7\sqrt{x}\right)+0.8$$

TO THE VALUES OF THE ARGUMENT

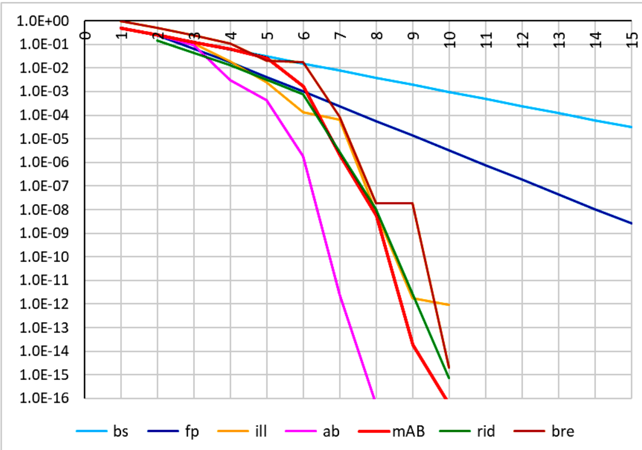


TO THE VALUES OF THE FUNCTION



$$f_{37}(x)=\begin{cases}(-x)^3-x-1\,,\ if\ x\le 0\\ x^{1/3}-x-1\,,\ otherwise\end{cases}$$

TO THE VALUES OF THE ARGUMENT



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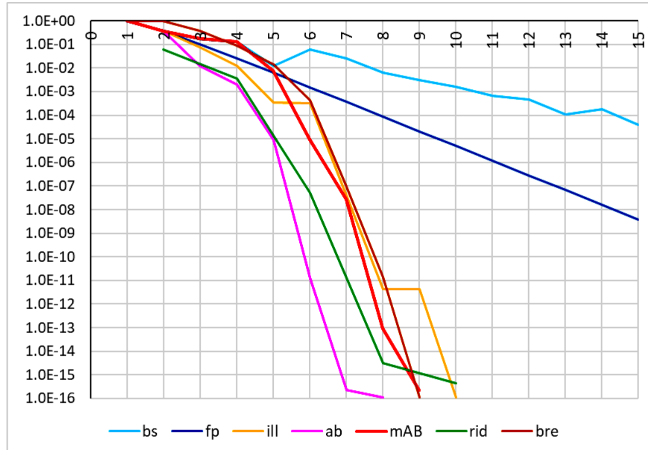
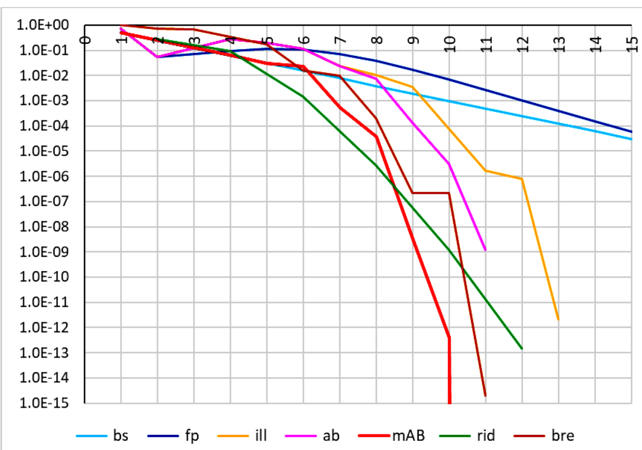


$$f_{38}(x)=x^3-2x-x+3$$

TO THE VALUES OF THE ARGUMENT



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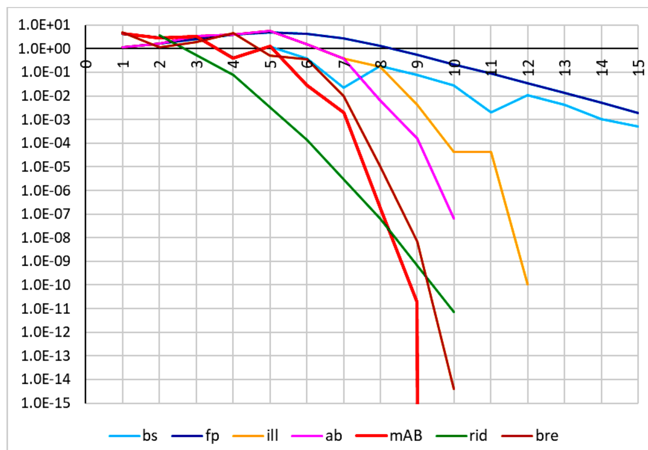
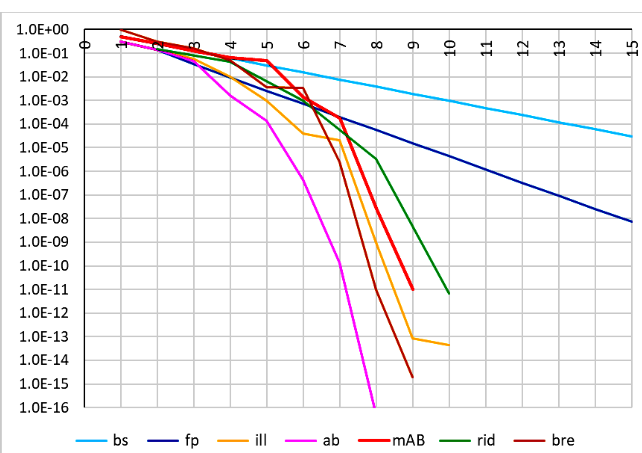


$$f_{39}(x)=\ln(x)$$

TO THE VALUES OF THE ARGUMENT



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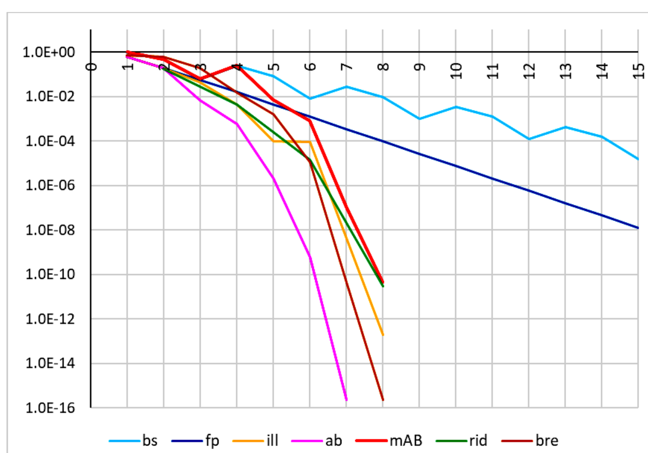
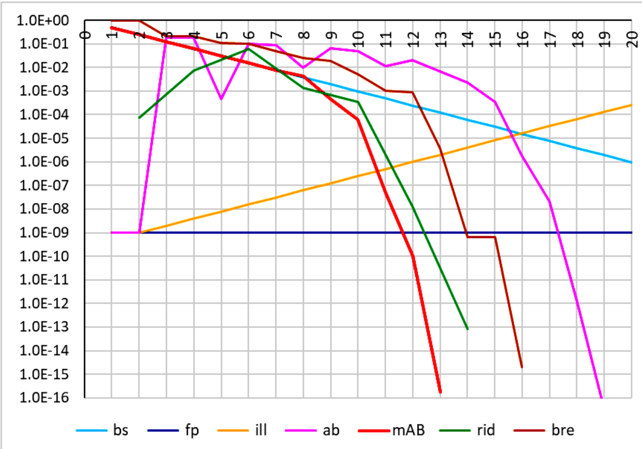


$$f_{40}(x)=(10-x)\cdot e^{-10x}-x^{10}+1$$

TO THE VALUES OF THE ARGUMENT



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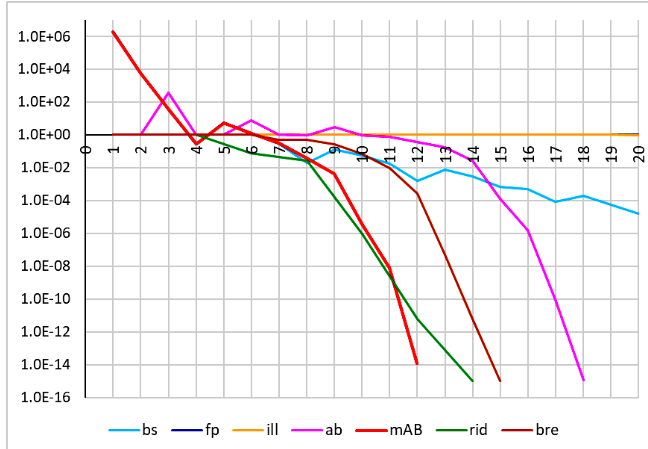
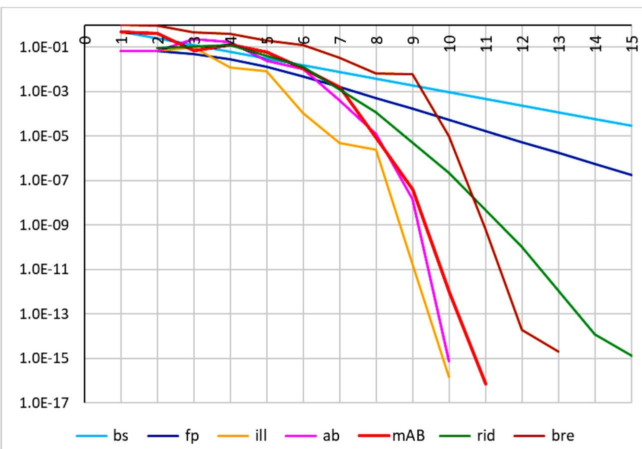


$$f_{41}(x)=e^{\sin(x)}-x-1$$

TO THE VALUES OF THE ARGUMENT



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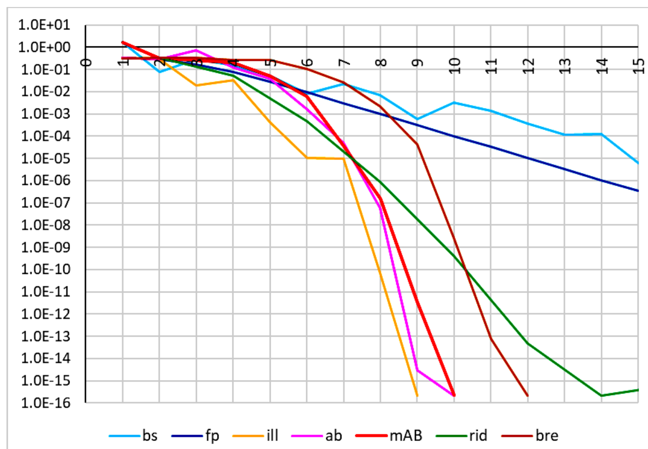
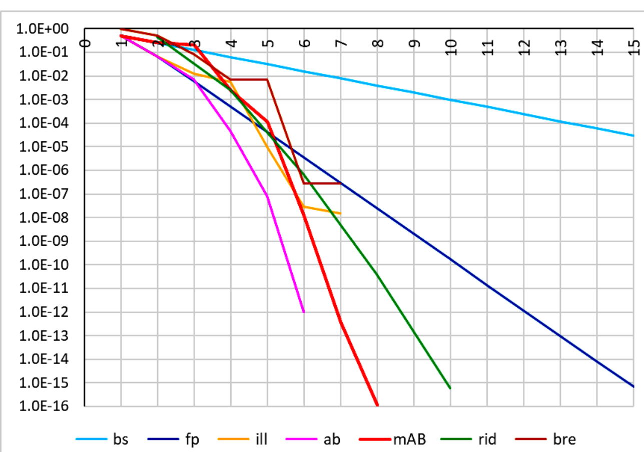


$$f_{42}(x)=2\sin(x)-1$$

TO THE VALUES OF THE ARGUMENT



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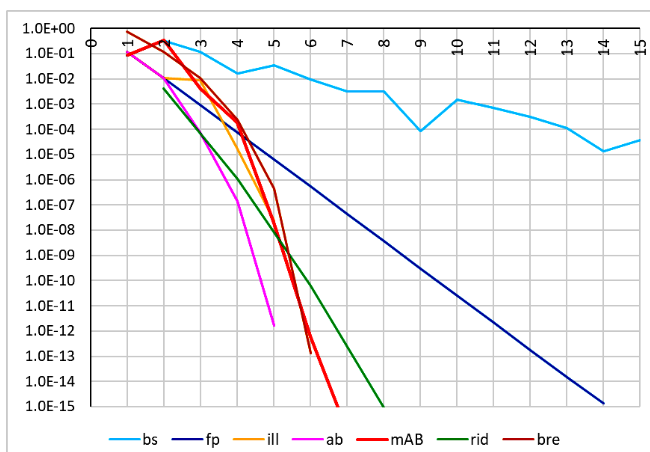
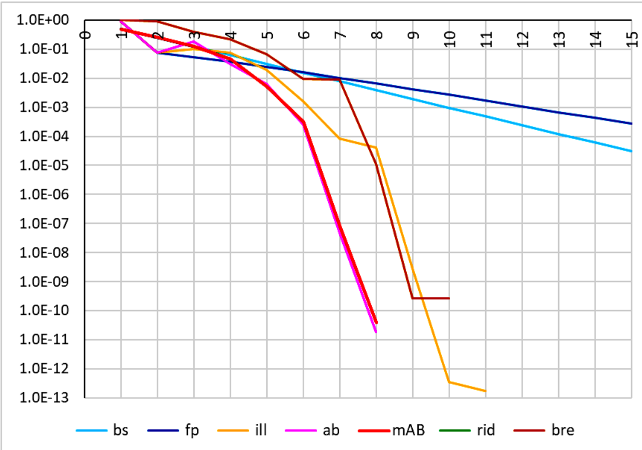


$$f_{43}(x)=(x-1)\cdot e^{-x}$$

TO THE VALUES OF THE ARGUMENT



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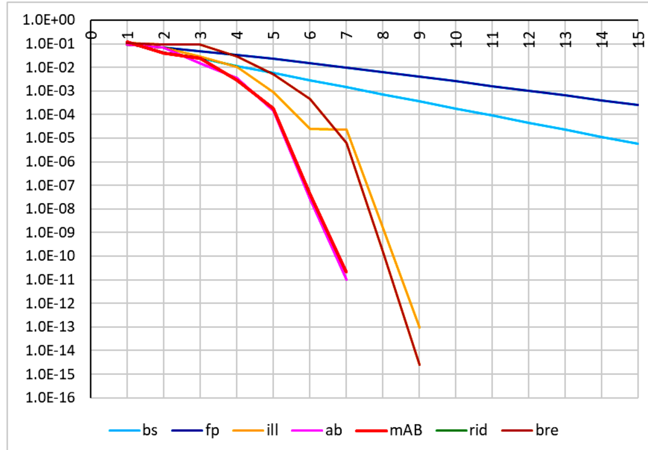
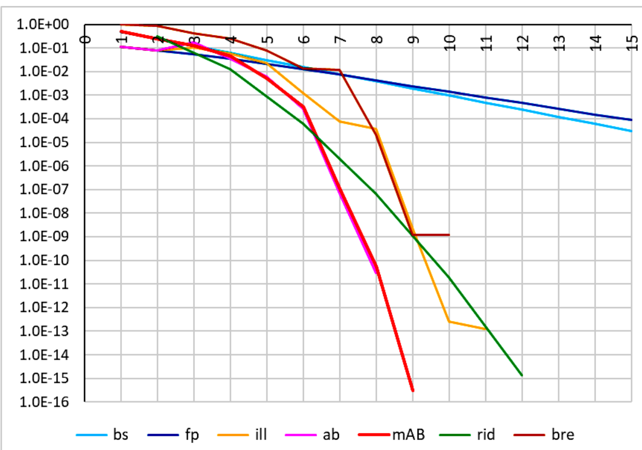


$$f_{44}(x)=(x-1)^3-1$$

TO THE VALUES OF THE ARGUMENT



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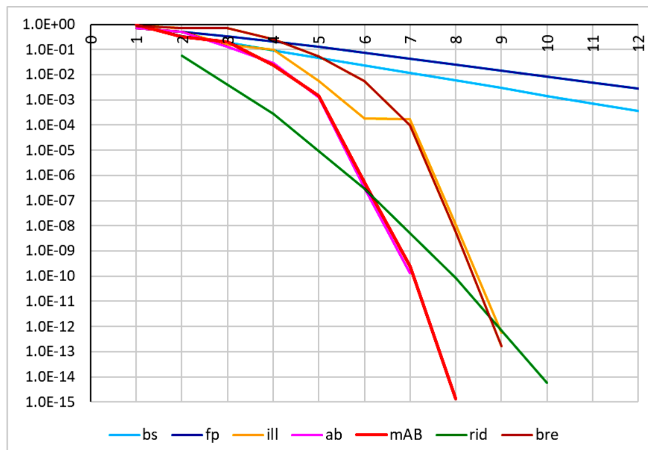
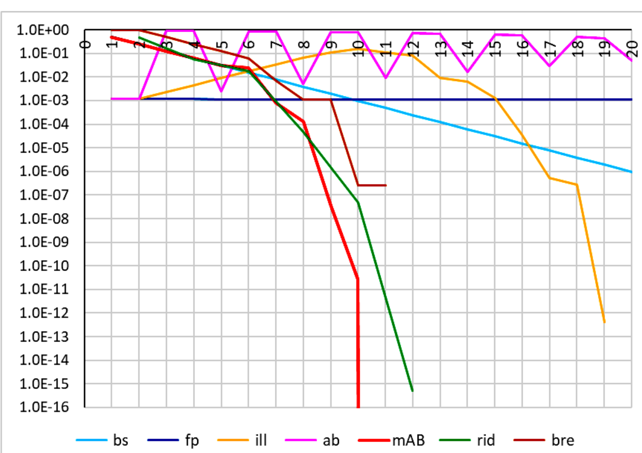


$$f_{45}(x)=e^{x^2+7x-30}-1$$

TO THE VALUES OF THE ARGUMENT



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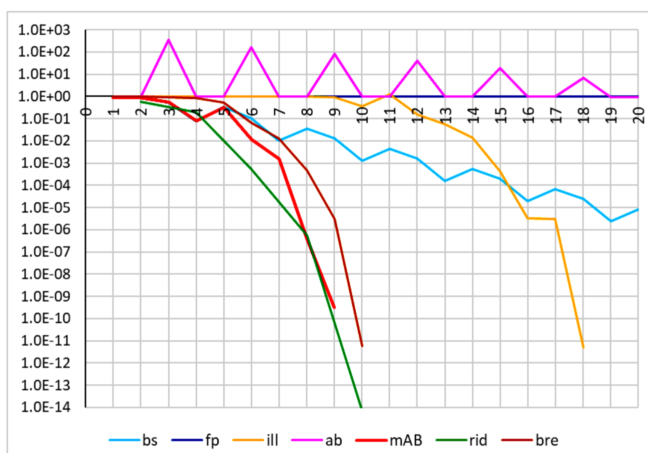
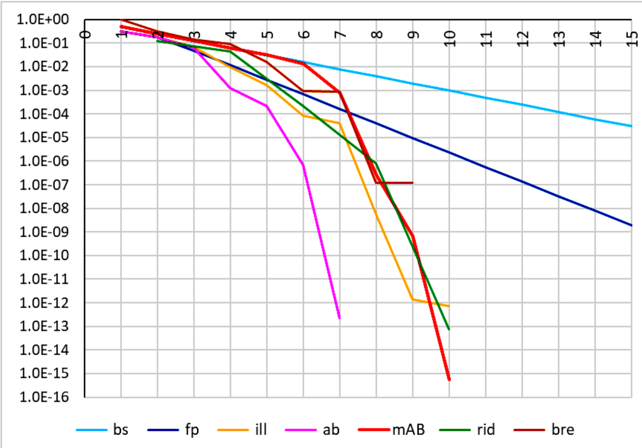


$$f_{46}(x) = \arctan(x) - 1$$

TO THE VALUES OF THE ARGUMENT



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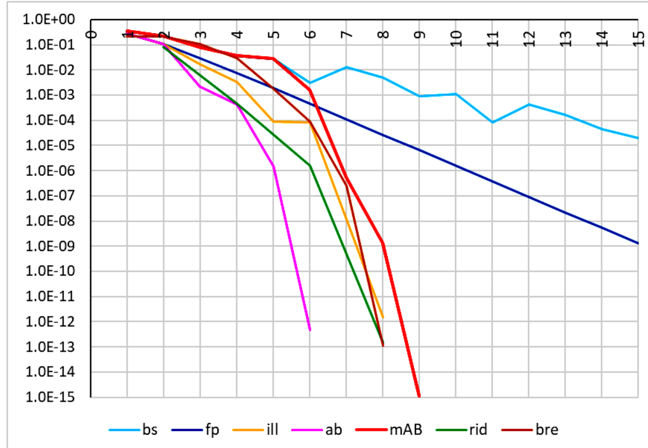
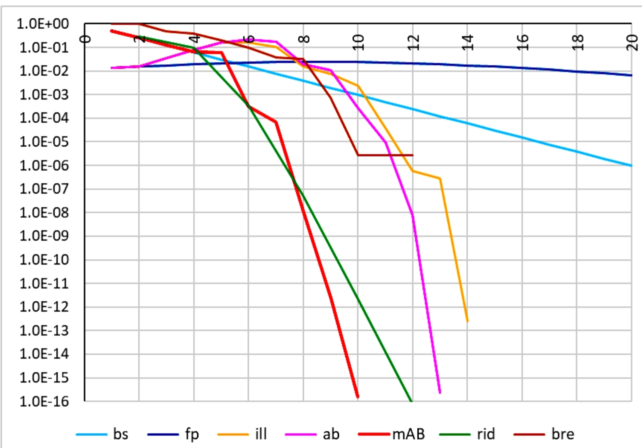


$$f_{47}(x) = e^{x} - 2x - 1$$

TO THE VALUES OF THE ARGUMENT



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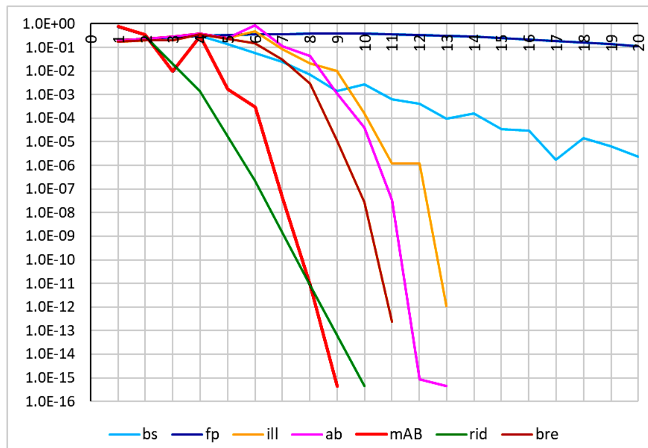

$$f_{48}(x) = e^{-x} - x - \sin(x)$$

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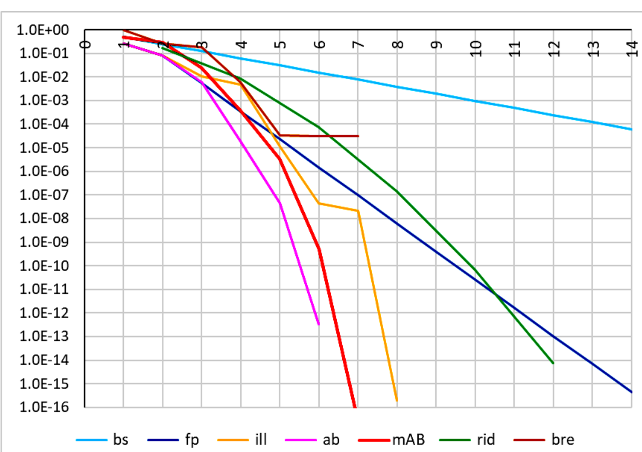

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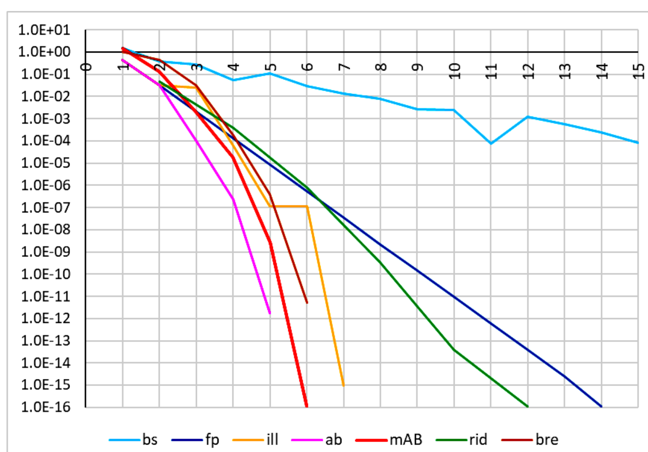
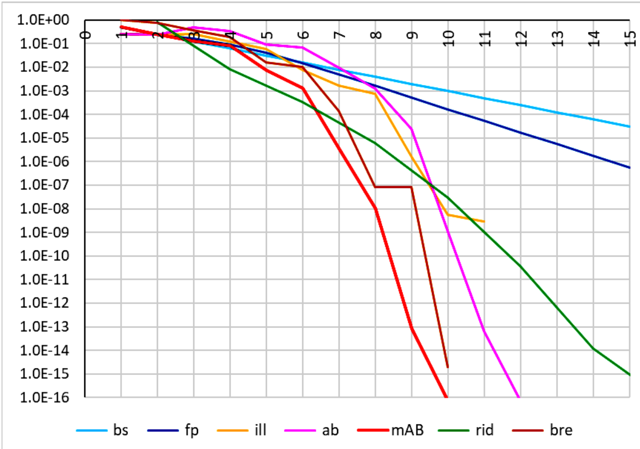


$$f_{49}(x) = x^2 - \sin^2(x) - 1$$

TO THE VALUES OF THE ARGUMENT



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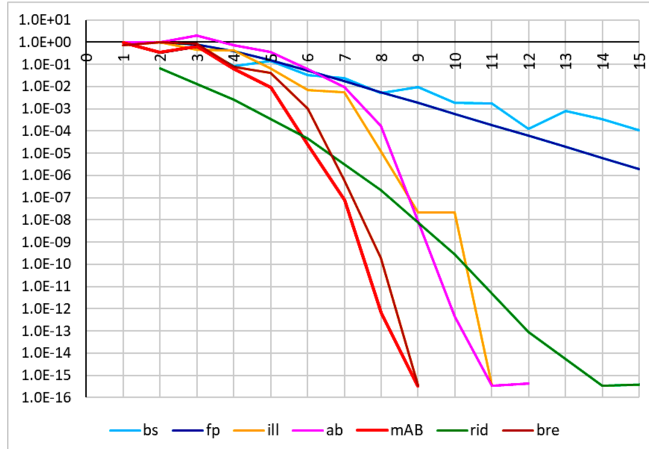
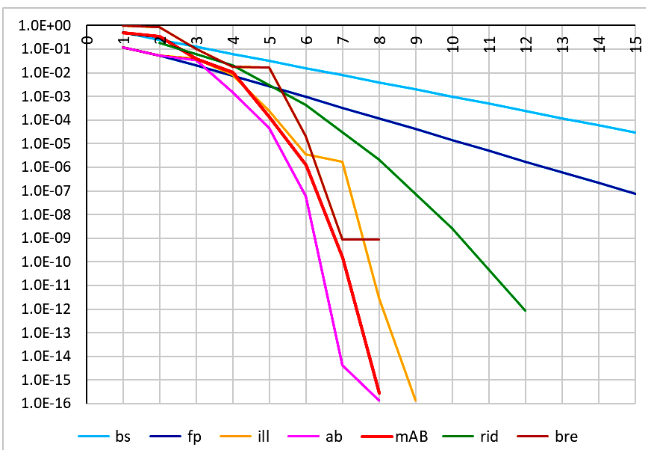


$$f_{50}(x) = \sin(x) - \frac{x}{2}$$

TO THE VALUES OF THE ARGUMENT



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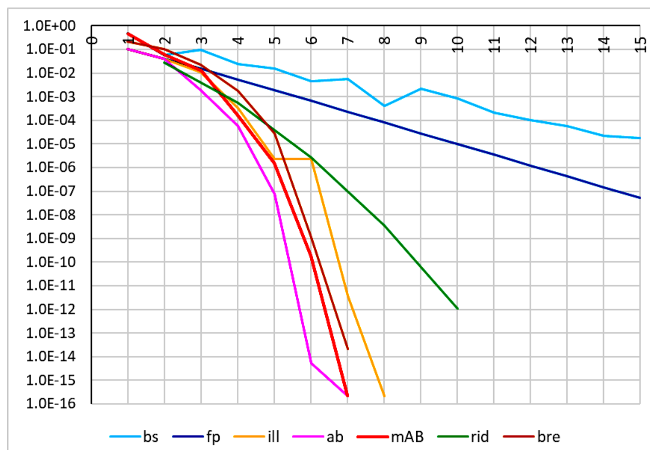

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