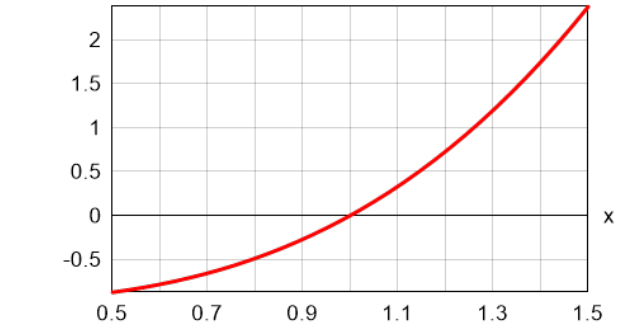
Root-finding Test Functions

Sérgio Galdino. *A family of regula falsi root-finding methods*. Proceedings of 2011 World Congress on Engineering and Technology. Shanghai, China. IEEE Press. ISBN 978-1-61284-365-0. p. 514-517

http://sergiogaldino.pbworks.com/w/file/fetch/66011429/0130-1943543

$$f_{01}(x) = x^3 - 1$$

[1.5; 2.37]

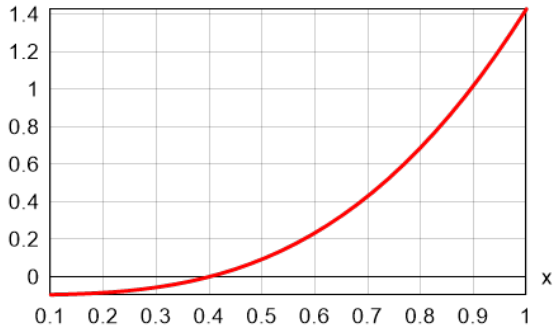


[0.5; -0.875]

$$\$\ Root\left\{f_{01}(x) = 0\ ;\ x \in [0.5\ ;\ 1.5]\right\} = 1$$

$$f_{02}(x) = x^2 \cdot \left(\frac{x^2}{3} + \sqrt{2} \cdot \sin(x)\right) - \frac{\sqrt{3}}{18}$$

[1; 1.43]

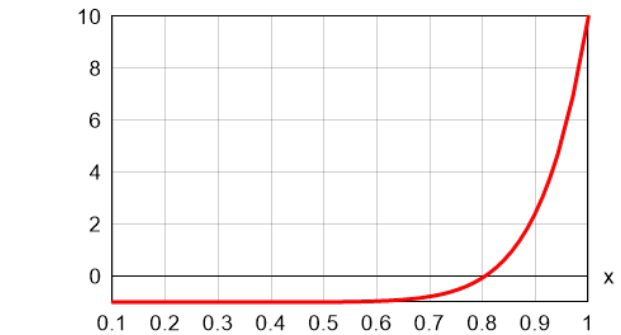


[0.1; -0.0948]

$$\$\ Root\left\{f_{02}(x) = 0\ ;\ x \in [0.1\ ;\ 1]\right\} = 0.399$$

$$f_{03}(x) = 11 \cdot x^{11} - 1$$

[1; 10]

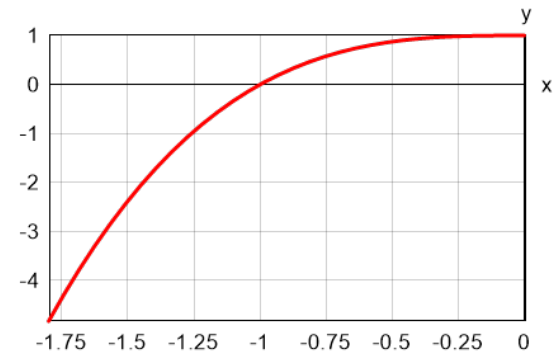


[0.1; -1]

$$\$\ Root\left\{f_{03}(x) = 0\ ;\ x \in [0.1\ ;\ 1]\right\} = 0.804$$

$$f_{04}(x) = x^3 + 1$$

[7.36E-16; 1]

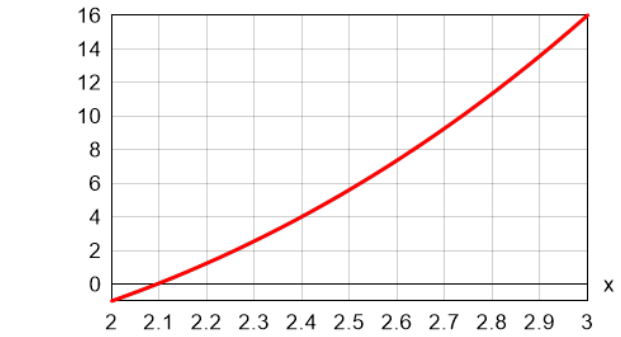


[-1.8; -4.83]

$$\$\ Root\left\{f_{04}(x) = 0\ ;\ x \in [-1.8\ ;\ 0]\right\} = -1$$

$$f_{05}(x) = x^3 - 2 \cdot x - 5$$

[3; 16]

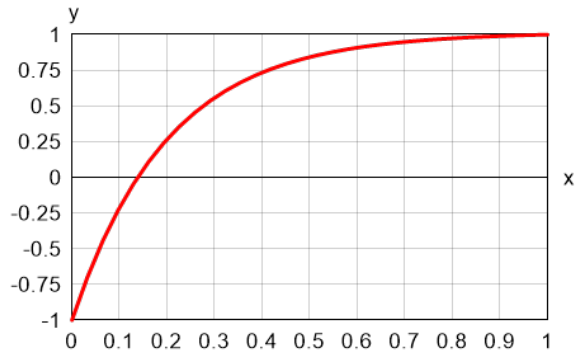


[2; -1]

$$\$\ Root\left\{f_{05}(x) = 0\ ;\ x \in [2\ ;\ 3]\right\} = 2.09$$

$$f_{06}(x) = 2 \cdot x \cdot e^{-5} + 1 - 2 \cdot e^{-5 \cdot x}$$

[1; 1]

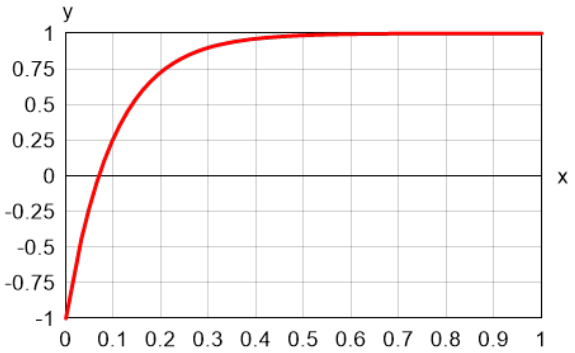


[0; -1]

$$\$\ Root\left\{f_{06}(x) = 0\ ;\ x \in [0\ ;\ 1]\right\} = 0.138$$

$$f_{07}(x) = 2 \cdot x \cdot e^{-10} + 1 - 2 \cdot e^{-10 \cdot x}$$

[1; 1]

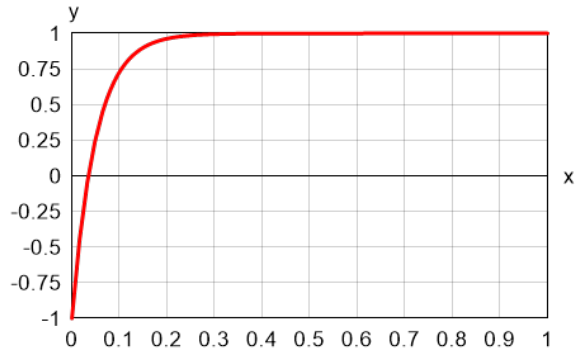


[0; -1]

$$\$\ Root\left\{f_{07}(x) = 0\ ;\ x \in [0\ ;\ 1]\right\} = 0.0693$$

$$f_{08}(x) = 2 \cdot x \cdot e^{-20} + 1 - 2 \cdot e^{-20 \cdot x}$$

[1; 1]

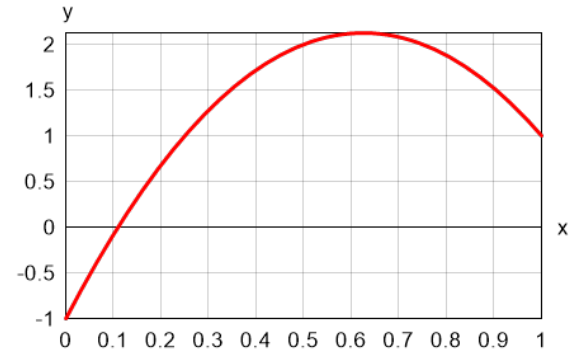


[0; -1]

$$\$\ Root\left\{f_{08}(x) = 0\ ;\ x \in [0\ ;\ 1]\right\} = 0.0347$$

$$f_{09}(x) = \left(1 + (1-5)^2\right) \cdot x^2 - (1 - 5 \cdot x)^2$$

[1; 2.12]

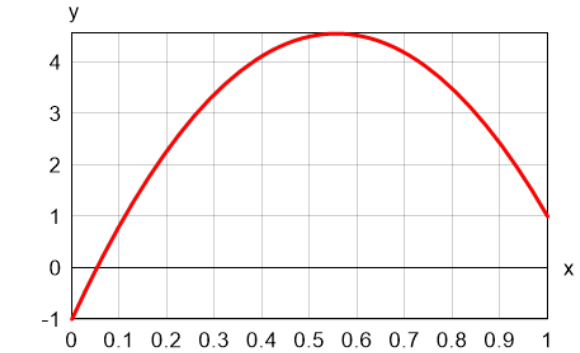


[0; -1]

$$\$\ Root\left\{f_{09}(x) = 0\ ;\ x \in [0\ ;\ 1]\right\} = 0.11$$

$$f_{10}(x) = \left(1 + (1-10)^2\right) \cdot x^2 - (1 - 10 \cdot x)^2$$

[1; 4.55]

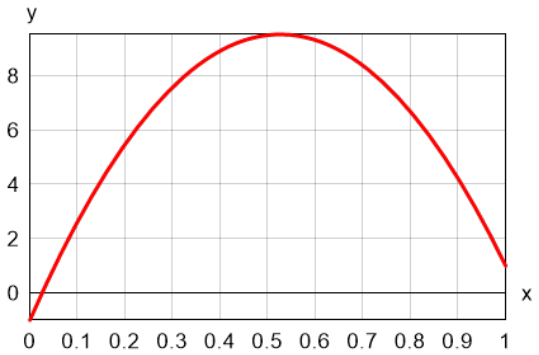


[0; -1]

$$\$\ Root\left\{f_{10}(x) = 0\ ;\ x \in [0\ ;\ 1]\right\} = 0.0525$$

$f_{11}(x)=\left(1+(1-20)^2\right)\cdot x^2-(1-20\cdot x)^2$

[1; 9.52]

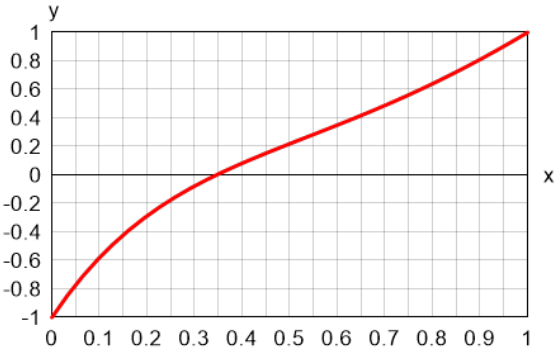


[0; -1]

$\$\ Root\left\{f_{11}(x)=0\,;\,x\in[0\,;1]\right\}=0.0256$

$f_{12}(x)=x^2-(1-x)^5$

[1; 1]

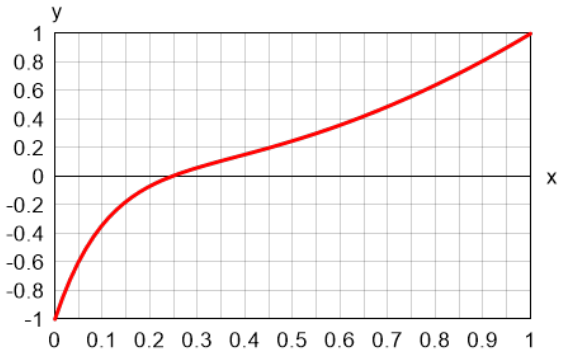


[0; -1]

$\$\ Root\left\{f_{12}(x)=0\,;\,x\in[0\,;1]\right\}=0.346$

$f_{13}(x)=x^2-(1-x)^{10}$

[1; 1]

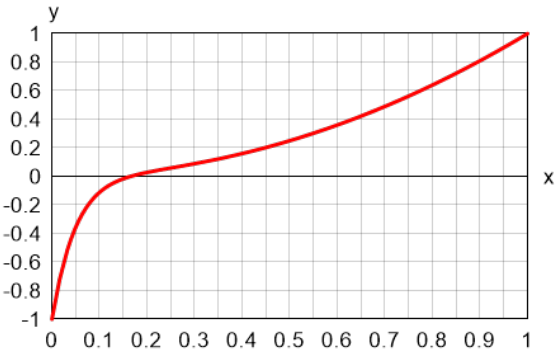


[0; -1]

$\$\ Root\left\{f_{13}(x)=0\,;\,x\in[0\,;1]\right\}=0.245$

$f_{14}(x)=x^2-(1-x)^{20}$

[1; 1]

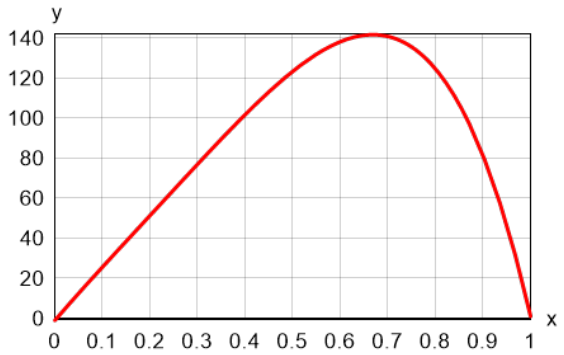


[0; -1]

$\$\ Root\left\{f_{14}(x)=0\,;\,x\in[0\,;1]\right\}=0.165$

$f_{15}(x)=\left(1+(1-5)^4\right)\cdot x-(1-5\cdot x)^4$

[1; 141.69]

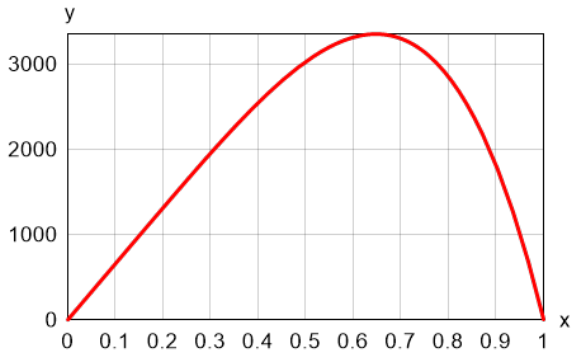


[0; -1]

$\$\ Root\left\{f_{15}(x)=0\,;\,x\in[0\,;1]\right\}=0.00362$

$f_{16}(x)=\left(1+(1-10)^4\right)\cdot x-(1-10\cdot x)^4$

[1; 3350.27]

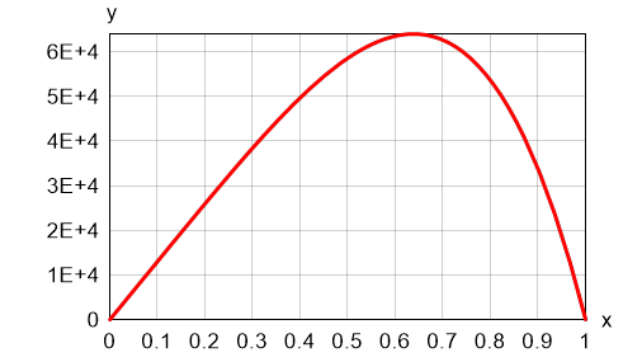


[0; -1]

$\$\ Root\left\{f_{16}(x)=0\,;\,x\in[0\,;1]\right\}=0.000151$

$$f_{17}(x)=\left(1+(1-20)^4\right)\cdot x-(1-20\cdot x)^4$$

[1; 64018.7]

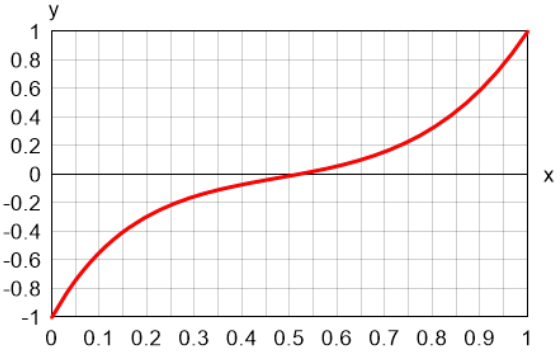


[0; -1]

$$\$\ Root\left\{f_{17}(x)=0\,;x\in[0\,;1]\right\}=7.67\times10^{-06}$$

$$f_{18}(x)=e^{-5\cdot x}\cdot(x-1)+x^5$$

[1; 1]

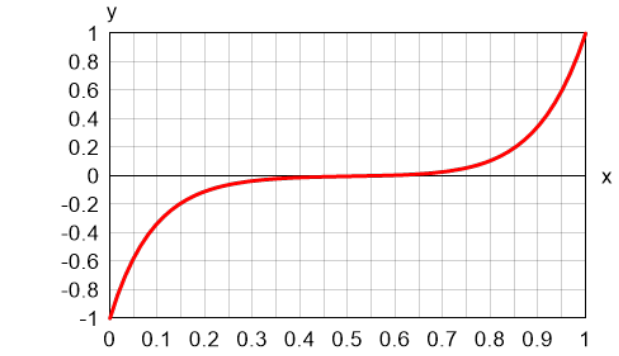


[0; -1]

$$\$\ Root\left\{f_{18}(x)=0\,;x\in[0\,;1]\right\}=0.516$$

$$f_{19}(x)=e^{-10\cdot x}\cdot(x-1)+x^{10}$$

[1; 1]

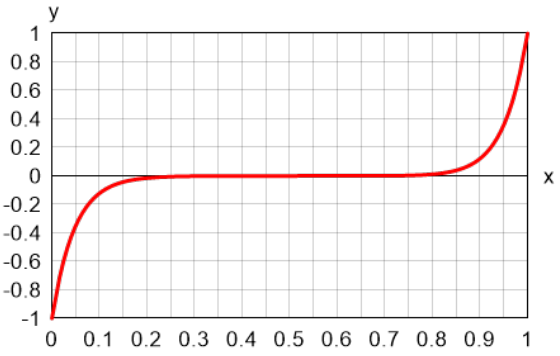


[0; -1]

$$\$\ Root\left\{f_{19}(x)=0\,;x\in[0\,;1]\right\}=0.54$$

$$f_{20}(x)=e^{-20\cdot x}\cdot(x-1)+x^{20}$$

[1; 1]

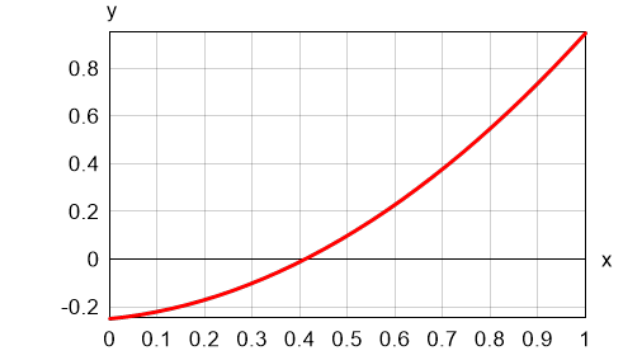


[0; -1]

$$\$\ Root\left\{f_{20}(x)=0\,;x\in[0\,;1]\right\}=0.553$$

$$f_{21}(x)=x^2+\sin\left(\frac{x}{5}\right)-\frac{1}{4}$$

[1; 0.949]

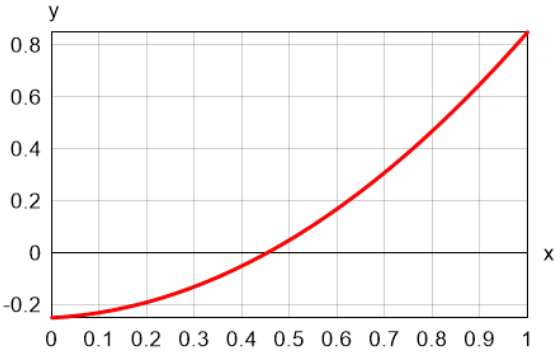


[0; -0.25]

$$\$\ Root\left\{f_{21}(x)=0\,;x\in[0\,;1]\right\}=0.41$$

$$f_{22}(x)=x^2+\sin\left(\frac{x}{10}\right)-\frac{1}{4}$$

[1; 0.85]

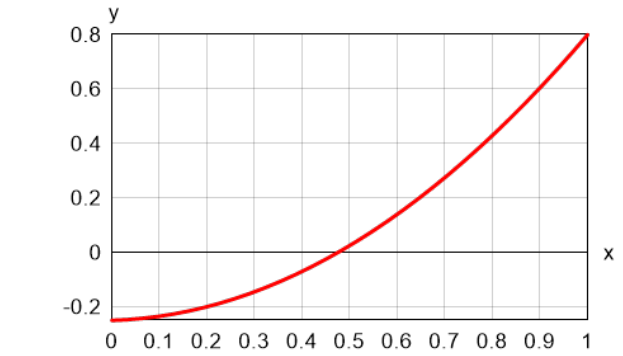


[0; -0.25]

$$\$\ Root\left\{f_{22}(x)=0\,;x\in[0\,;1]\right\}=0.453$$

$$f_{23}(x)=x^2+\sin\left(\frac{x}{20}\right)-\frac{1}{4}$$

[1; 0.8]

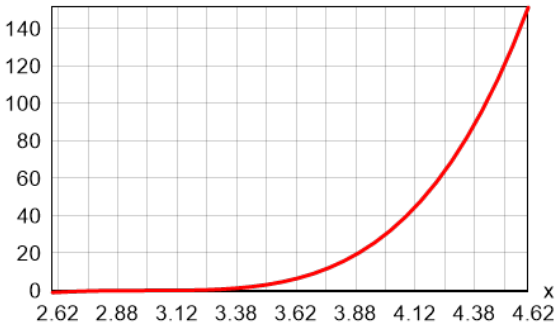


[0; -0.25]

$$\$\ Root\left\{f_{23}(x)=0\,;\,x\in[0\,;\,1]\right\}=0.476$$

$$f_{24}(x)=(x+2)\cdot(x+1)\cdot(x-3)^3$$

[4.6; 151.39]

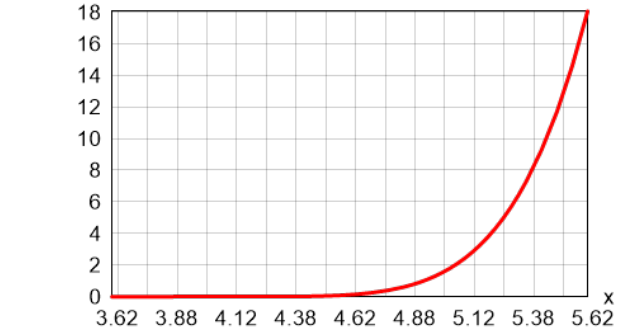


[2.6; -1.06]

$$\$\ Root\left\{f_{24}(x)=0\,;\,x\in[2.6\,;\,4.6]\right\}=3$$

$$f_{25}(x)=(x-4)^5\cdot\ln(x)$$

[5.6; 18.06]

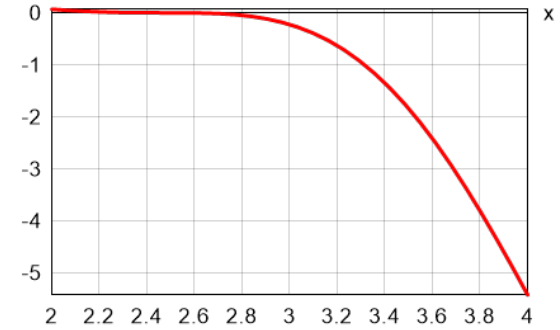


[3.6; -0.0131]

$$\$\ Root\left\{f_{25}(x)=0\,;\,x\in[3.6\,;\,5.6]\right\}=4$$

$$f_{26}(x)=\left(\sin(x)-\frac{x}{4}\right)^3$$

[4; 0.0686]



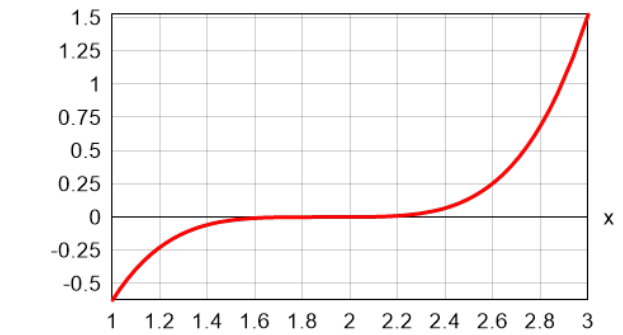
[2; -5.42]

$$\$\ Root\left\{f_{26}(x)=0\,;\,x\in[2\,;\,4]\right\}=2.47$$

$$f_{27}(x)=(81-p(x)\cdot(108-p(x)\cdot(54-p(x)\cdot(12-p(x)$$

$$p(x)=x+1.1\,1$$

[3; 1.52]

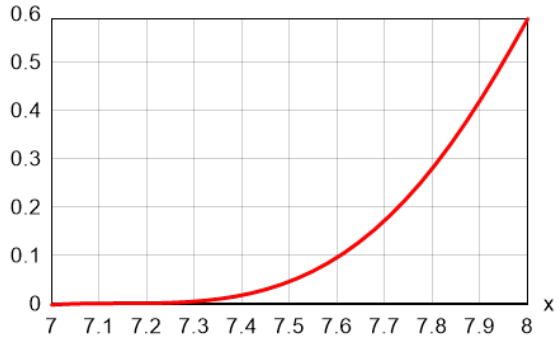


[1; -0.624]

$$\$\ Root\left\{f_{27}(x)=0\,;\,x\in[1\,;3]\right\}=1.89$$

$$f_{28}(x)=\sin\left((x-7.14)^3\right)$$

[8; 0.589]

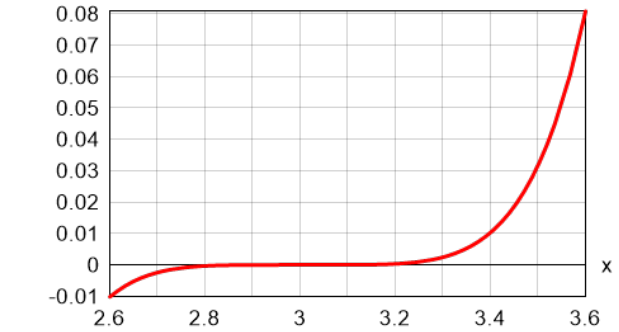


[7; -0.00292]

$$\$\ Root\left\{f_{28}(x)=0\,;\,x\in[7\,;8]\right\}=7.14$$

$$f_{29}(x)=e^{(x-3)^5}-1$$

[3.6; 0.0809]

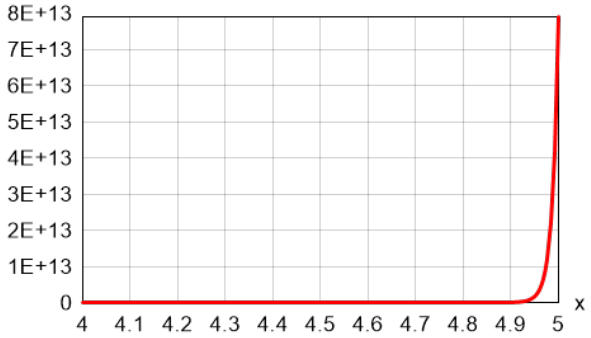


[2.6; -0.0102]

$$\$\ Root\left\{f_{29}(x)=0\ ;\ x\in[2.6\ ;\ 3.6]\right\}=3$$

$$f_{30}(x)=e^{(x-3)^5}-e^{x-1}$$

[5; 78962960182620]



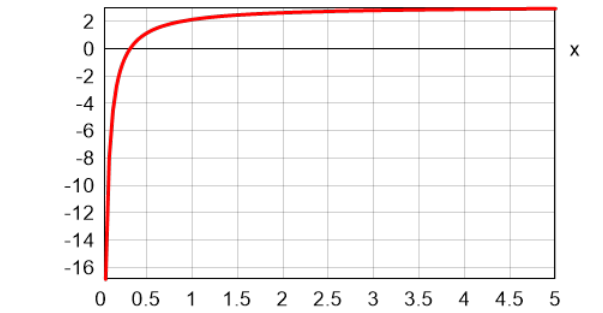
[4; -17.52]

$$\$\ Root\left\{f_{30}(x)=0\ ;\ x\in[4\ ;\ 5]\right\}=4.27$$

My functions

$$f_{31}(x)=\pi-\frac{1}{x}$$

[5; 2.94]

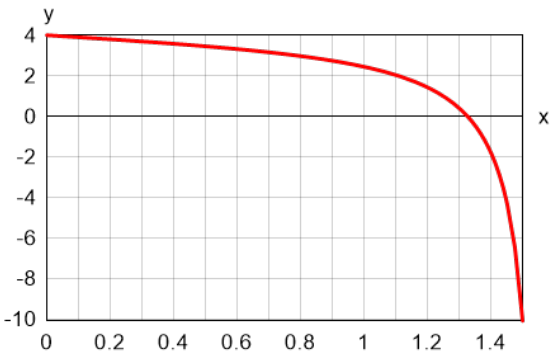


[0.05; -16.86]

$$\$\ Root\left\{f_{31}(x)=0\ ;\ x\in[0.05\ ;\ 5]\right\}=0.318$$

$$f_{32}(x)=4-\tan(x)$$

[1.5; 4]



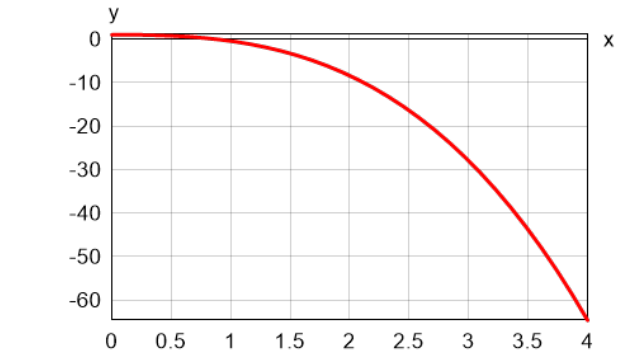
[0; -10.1]

$$\$\ Root\left\{f_{32}(x)=0\ ;\ x\in[0\ ;\ 1.5]\right\}=1.33$$

Steven A. Stage. *Comments on An Improvement to the Brent's Method*. International Journal of Experimental Algorithms (IJEA), Volume (4) : Issue (1) : 2013

$$f_{33}(x)=\cos(x)-x^3$$

[4; 1]

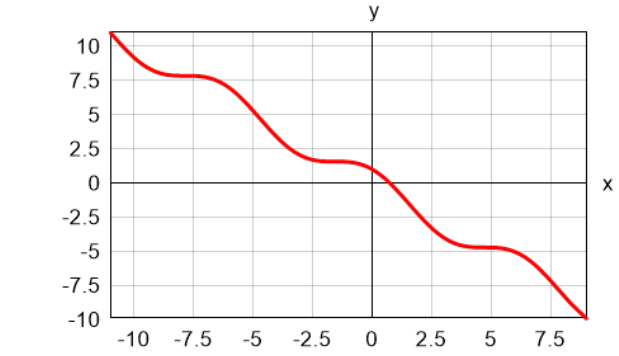


[0; -64.65]

$$\$\ Root\left\{f_{33}(x)=0\,;x\in[0\,;4]\right\}=0.865$$

$$f_{34}(x)=\cos(x)-x$$

[9; 11]

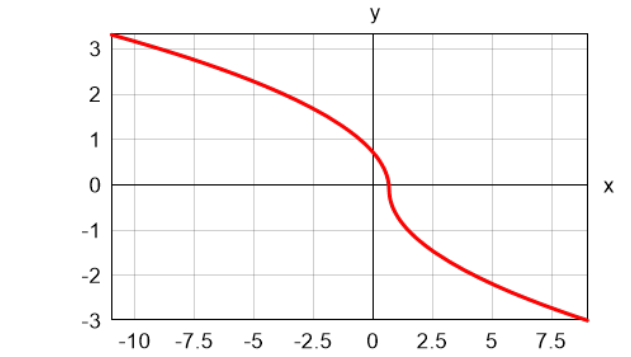


[-11; -9.91]

$$\$\ Root\left\{f_{34}(x)=0\,;x\in[-11\,;9]\right\}=0.739$$

$$f_{35}(x)=\sqrt{\left|\left(x-\frac{2}{3}\right)\right|}\cdot if\left(x\le\frac{2}{3}\,;1\,;-1\right)-0.1$$

[9; 3.32]

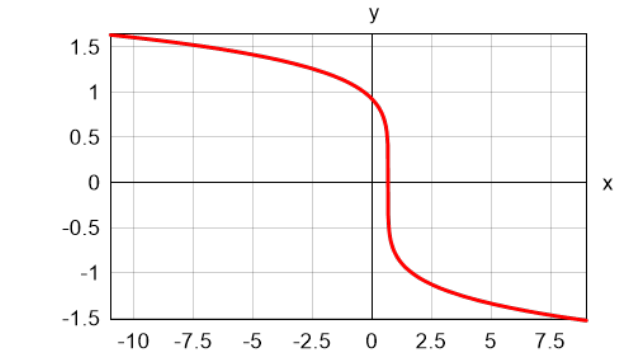


[-11; -2.99]

$$\$\ Root\left\{f_{35}(x)=0\,;x\in[-11\,;9]\right\}=0.657$$

$$f_{36}(x)=\left(\left|\left(x-\frac{2}{3}\right)\right|\right)^{0.2}\cdot if\left(x\le\frac{2}{3}\,;1\,;-1\right)$$

[9; 1.63]

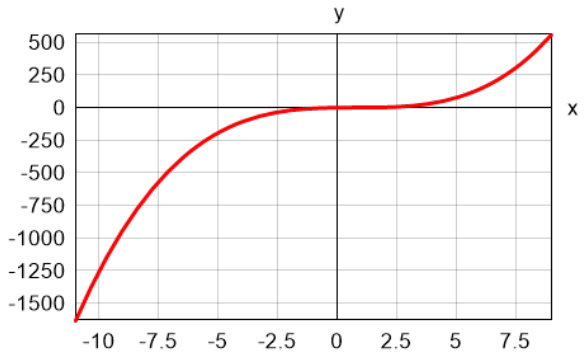


[-11; -1.53]

No solution for: $Root{f_36(x) = 0; x ∈ [-11; 9]}

$$f_{37}(x) := \left(x - \frac{7}{9}\right)^3 + \left(x - \frac{7}{9}\right) \cdot 10^{-3}$$

[9; 555.87]

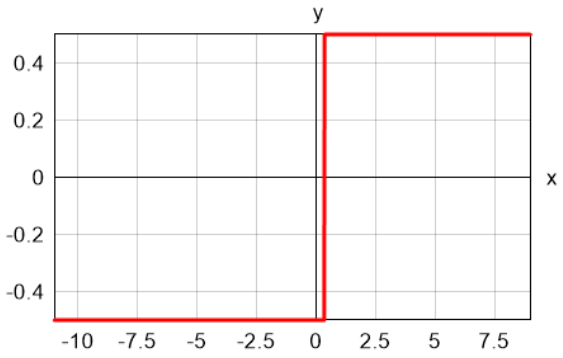


[-11; -1633.78]

$\$ Root\left\{f_{37}(x) = 0 ; x \in [-11 ; 9]\right\} = 0.778$

$$f_{38}(x) := if\left(x \le \frac{1}{3} ; -0.5 ; 0.5\right)$$

[9; 0.5]

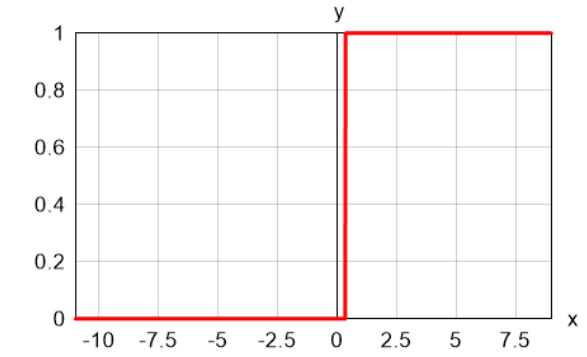


[-11; -0.5]

No solution for: $Root{f_38(x) = 0; x ∈ [-11; 9]}

$$f_{39}(x) := if\left(x \le \frac{1}{3} ; -\left(10^{-3}\right) ; 1 - 10^{-3}\right)$$

[9; 0.999]

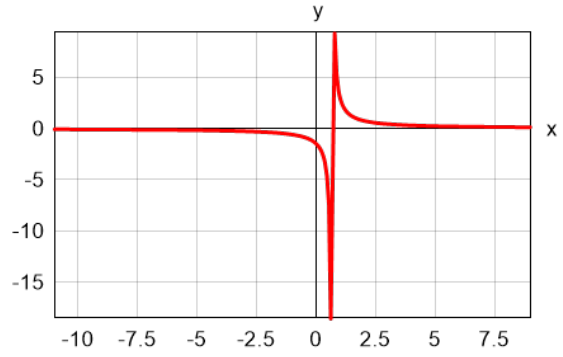


[-11; -0.001]

No solution for: $Root{f_39(x) = 0; x ∈ [-11; 9]}

$$f_{40}(x) := if\left[x \equiv 0 ; 0 ; \frac{1}{x - \frac{2}{3}}\right]$$

[9; 9.3]



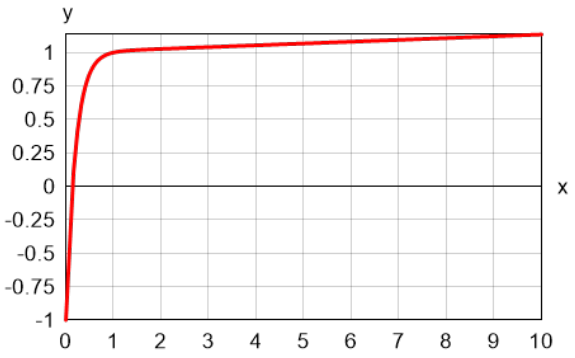
[-11; -18.6]

No solution for: $Root{f_40(x) = 0; x ∈ [-11; 9]}

A. Swift, G. R. Lindfield, *Comparison of a continuation method with Brent's method for the numerical solution of a single nonlinear equation*, The Computer Journal, Volume 21, Issue 4, 1978, Pages 359–362 https://doi.org/10.1093/comjnl/21.4.359

$$f_{41}(x)=2\cdot x\cdot e^{-5}-2\cdot e^{-5\cdot x}+1$$

[10; 1.13]

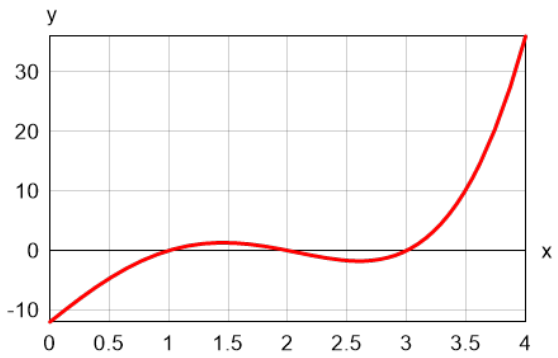


[0; -1]

$$\$\ Root\left\{f_{41}(x)=0\,;\,x\in[0\,;\,10]\right\}=0.138$$

$$f_{42}(x)=\left(x^2-x-6\right)\cdot\left(x^2-3\cdot x+2\right)$$

[4; 36]

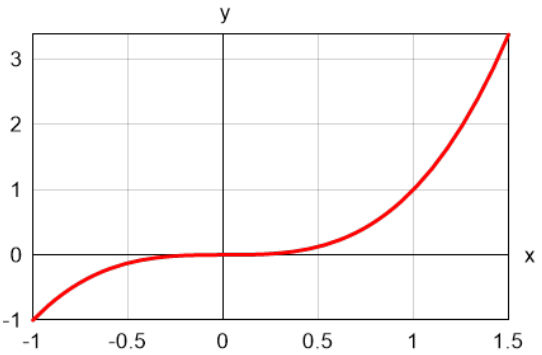


[0; -12]

$$\$\ Root\left\{f_{42}(x)=0\,;\,x\in[0\,;\,4]\right\}=2$$

$$f_{43}(x)=x^3$$

[1.5; 3.37]

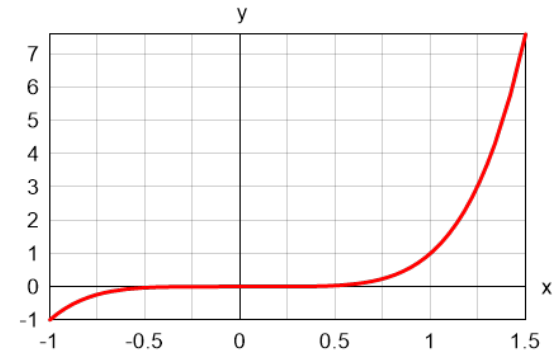


[-1; -1]

$$\$\ Root\left\{f_{43}(x)=0\,;\,x\in[-1\,;\,1.5]\right\}=6.1\times10^{-05}$$

$$f_{44}(x)=x^5$$

[1.5; 7.59]

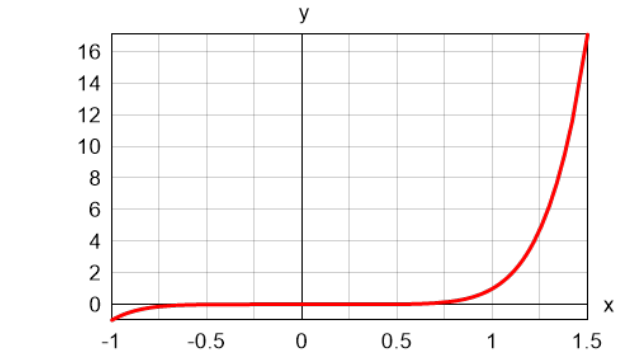


[-1; -1]

$$\$\ Root\left\{f_{44}(x)=0\,;\,x\in[-1\,;\,1.5]\right\}=0.000977$$

$$f_{45}(x)=x^7$$

[1.5; 17.09]

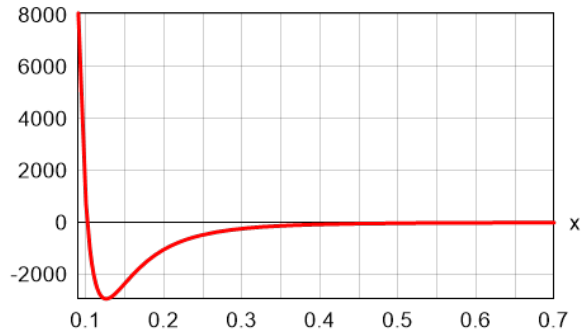


[-1; -1]

$$\$\ Root\left\{f_{45}(x)=0\,;\,x\in[-1\,;\,1.5]\right\}=0.0156$$

$$f_{46}(x)=\frac{e^{-5\cdot x}-x-0.5}{x^5}$$

[0.7; 8065.87]

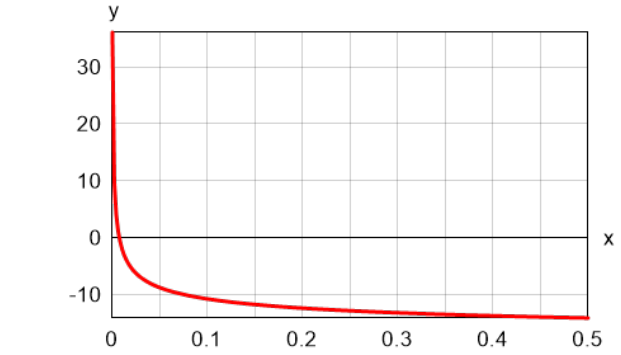


[0.09; -2941.49]

$$\$\ Root\left\{f_{46}(x)=0\,;\,x\in[0.09\,;\,0.7]\right\}=0.102$$

$$f_{47}(x)=\frac{1}{\sqrt{x}}-2\cdot\ln\left(5\cdot10^3\cdot\sqrt{x}\right)+0.8$$

[0.5; 36.09]

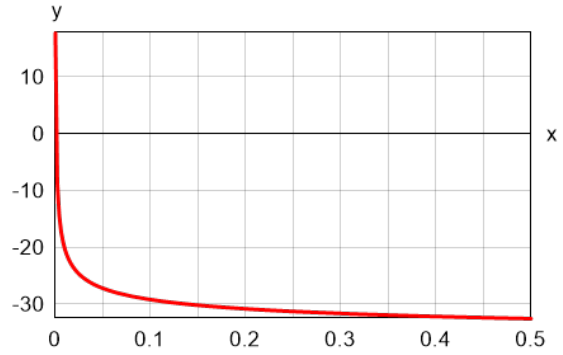


[0.0005; -14.13]

$$\$\ Root\left\{f_{47}(x)=0\,;\,x\in[0.0005\,;\,0.5]\right\}=0.00773$$

$$f_{48}(x)=\frac{1}{\sqrt{x}}-2\cdot\ln\left(5\cdot10^7\cdot\sqrt{x}\right)+0.8$$

[0.5; 17.67]

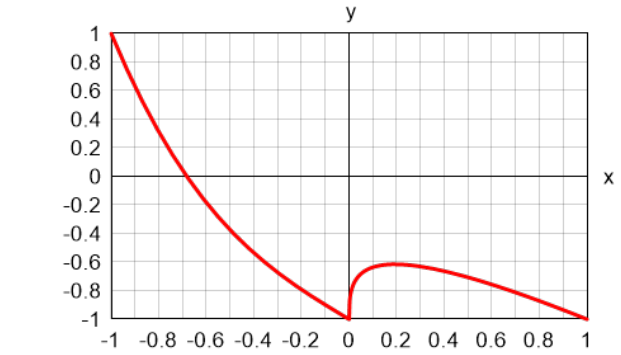


[0.0005; -32.55]

$$\$\ Root\left\{f_{48}(x)=0\,;\,x\in[0.0005\,;\,0.5]\right\}=0.00128$$

$$f_{49}(x)=if\left(x\le0\,;-(x^3)-x-1\,;\,x^{\frac{1}{3}}-x-1\right)$$

[1; 1]

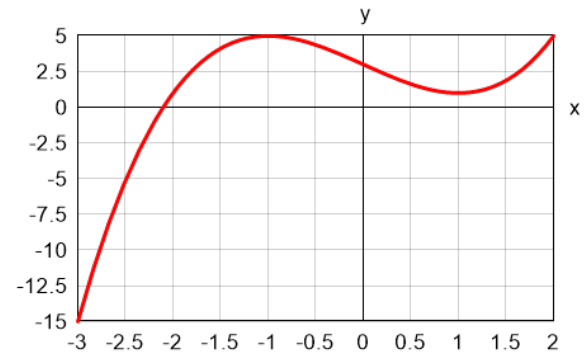


[-1; -1]

$$\$\ Root\left\{f_{49}(x)=0\,;\,x\in[-1\,;\,1]\right\}=-0.682$$

$$f_{50}(x)=x^3-2\cdot x-x+3$$

[2; 5]



[-3; -15]

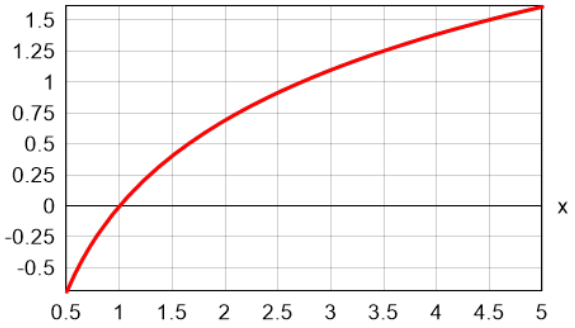
$$\$\ Root\left\{f_{50}(x)=0\,;\,x\in[-3\,;\,2]\right\}=-2.1$$

Alojz Suhadolnik, Combined bracketing methods for solving nonlinear equations, Applied Mathematics Letters, Volume 25, Issue 11, 2012, Pages 1755-1760, ISSN 0893-9659

https://doi.org/10.1016/j.aml.2012.02.006

$f_{51}(x)=\ln(x)$

[5; 1.61]

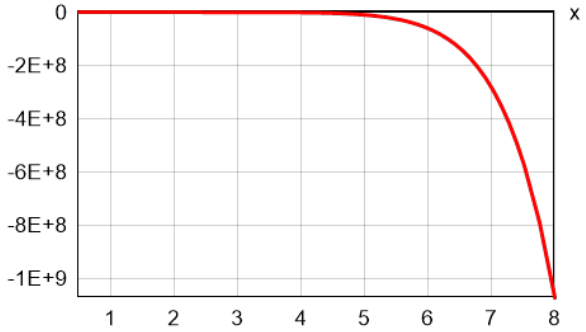


[0.5; -0.693]

$\$\ Root\{f_{51}(x)=0\,;x\in[0.5\,;5]\}=1$

$f_{52}(x)=(10-x)\cdot e^{-10\cdot x}-x^{10}+1$

[8; 1.06]

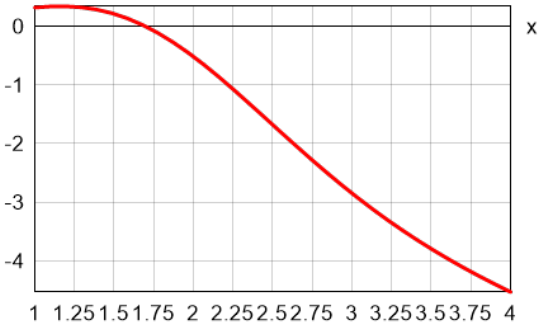


[0.5; -1073741823]

$\$\ Root\{f_{52}(x)=0\,;x\in[0.5\,;8]\}=1$

$f_{53}(x)=e^{\sin(x)}-x-1$

[4; 0.34]

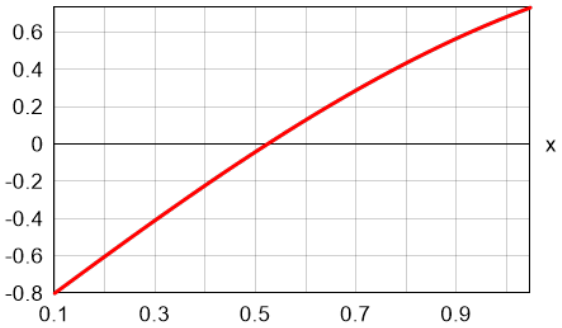


[1; -4.53]

$\$\ Root\{f_{53}(x)=0\,;x\in[1\,;4]\}=1.7$

$f_{54}(x)=2\cdot\sin(x)-1$

[1.05; 0.732]

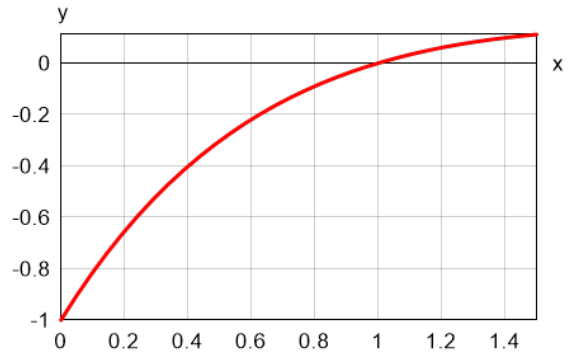


[0.1; -0.8]

$\$\ Root\left\{f_{54}(x)=0\,;x\in\left[0.1\,;\frac{\pi}{3}\right]\right\}=0.524$

$f_{55}(x)=(x-1)\cdot e^{-x}$

[1.5; 0.112]

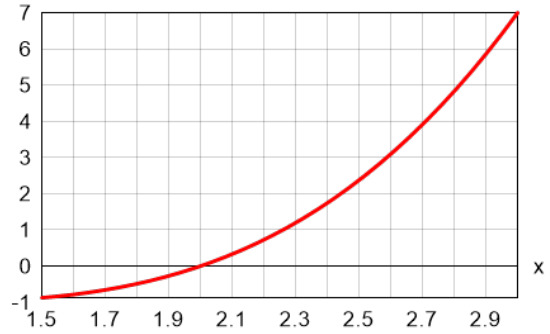


[0; -1]

$\$\ Root\left\{f_{55}(x)=0\ ;x\in[0\ ;1.5]\right\}=1$

$f_{56}(x)=(x-1)^3-1$

[3; 7]

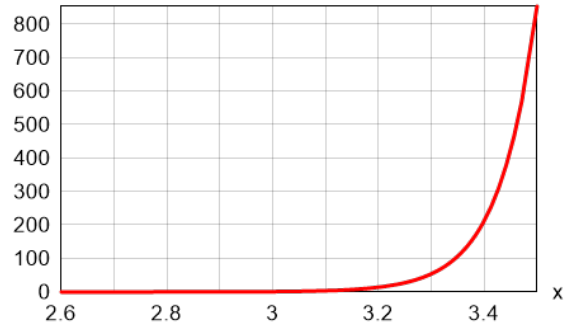


[1.5; -0.875]

$\$\ Root\left\{f_{56}(x)=0\ ;x\in[1.5\ ;3]\right\}=2$

$f_{57}(x)=e^{x^2+7\cdot x-30}-1$

[3.5; 853.06]

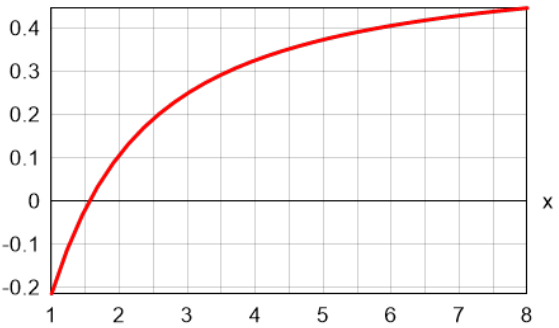


[2.6; -0.994]

$\$\ Root\left\{f_{57}(x)=0\ ;x\in[2.6\ ;3.5]\right\}=3$

$f_{58}(x)=atan(x)-1$

[8; 0.446]

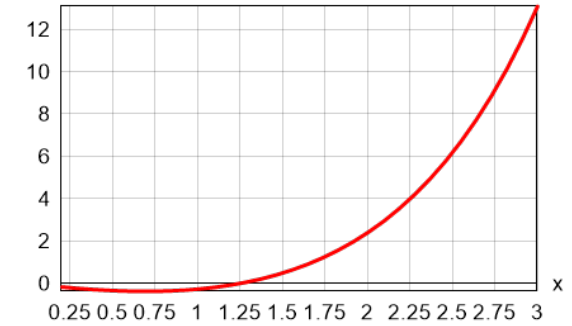


[1; -0.215]

$\$\ Root\left\{f_{58}(x)=0\ ;x\in[1\ ;8]\right\}=1.56$

$f_{59}(x)=e^{x}-2\cdot x-1$

[3; 13.09]

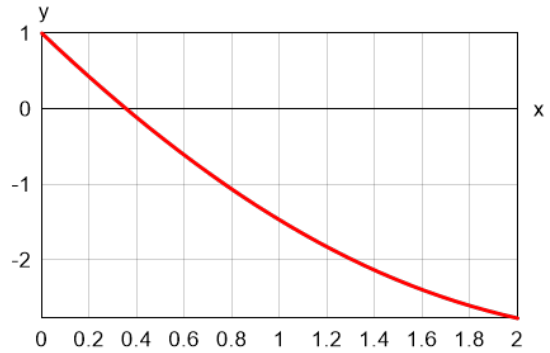


[0.2; -0.385]

$\$\ Root\left\{f_{59}(x)=0\ ;x\in[0.2\ ;3]\right\}=1.26$

$f_{60}(x)=e^{-x}-x-\sin(x)$

[2; 1]

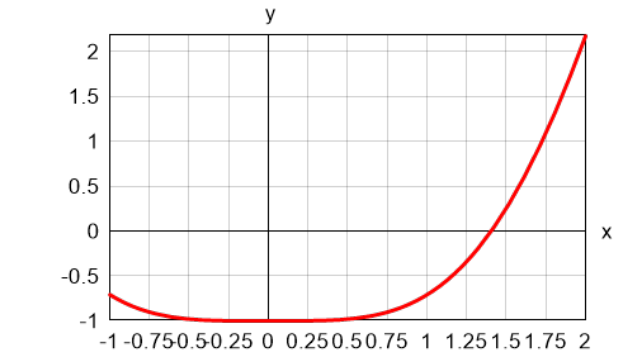


[0; -2.77]

$\$\ Root\left\{f_{60}(x)=0\ ;x\in[0\ ;2]\right\}=0.354$

$$f_{61}(x)=x^2-\sin(x)^2-1$$

[2; 2.17]

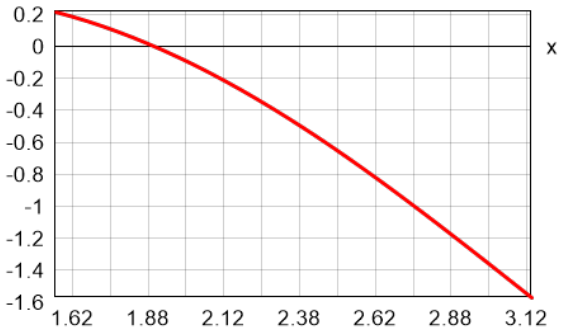


[-1; -1]

$$\$\ Root\left\{f_{61}(x)=0\,;\,x\in[-1\,;\,2]\right\}=1.4$$

$$f_{62}(x)=\sin(x)-\frac{x}{2}$$

[3.14; 0.215]



[1.57; -1.57]

$$\$\ Root\left\{f_{62}(x)=0\,;\,x\in\left[\frac{\pi}{2}\,;\,\pi\right]\right\}=1.9$$