問2.1 (1) $\dot{y}(t) + 2y(t) = u(t) \rightarrow sy(s) + 2y(s) = u(s)$

$\rightarrow (s+2)y(s) = u(s) \rightarrow y(s) = P(s)u(s),\ P(s) = \dfrac{1}{s+2}$ //

極：-2
零点：なし

(2) $3\ddot{y}(t) + 2\dot{y}(t) + y(t) = 2\dot{u}(t) + u(t)$

$\rightarrow 3s^2 y(s) + 2sy(s) + y(s) = 2su(s) + u(s)$

$\rightarrow (3s^2 + 2s + 1)y(s) = (2s+1)u(s)$

$\rightarrow y(s) = P(s)u(s),\ P(s) = \dfrac{2s+1}{3s^2+2s+1}$ //

4-12

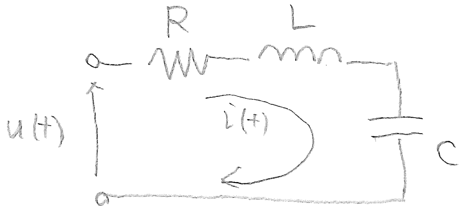
極は $3s^2 + 2s + 1 = 0$

$\rightarrow s = \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3} = \dfrac{-2 \pm \sqrt{-8}}{6} = \dfrac{-2 \pm 2\sqrt{2}j}{6}$

$= \dfrac{-1 \pm \sqrt{2}j}{3}$ //

零点は $2s + 1 = 0 \rightarrow s = -\dfrac{1}{2}$ //

問2.2



$u(t) = Ri(t) + L\dfrac{di(t)}{dt} + \dfrac{1}{C}\int_0^t i(t)dt$

$= R\dot{q}(t) + L\ddot{q}(t) + \dfrac{1}{C}q(t)$

$= R\dot{y}(t) + L\ddot{y}(t) + \dfrac{1}{C}y(t)$

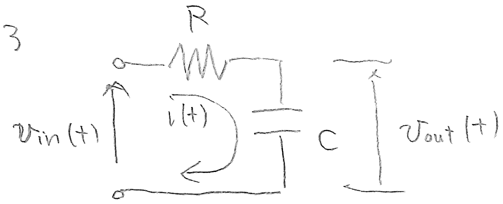
$\rightarrow u(s) = Rsy(s) + Ls^2y(s) + \dfrac{1}{C}y(s)$

$\rightarrow Cu(s) = RCsy(s) + LCs^2y(s) + y(s)$

$= (LCs^2 + RCs + 1)y(s)$

$\rightarrow y(s) = P(s)u(s),\ P(s) = \dfrac{C}{LCs^2 + RCs + 1}$ //

問2.3



(1) $u(t) = v_{in}(t),\ y(t) = i(t)$

$\rightarrow u(t) = Ry(t) + \dfrac{1}{C}\int_0^t y(t)dt$

$\rightarrow u(s) = Ry(s) + \dfrac{1}{Cs}y(s) = \dfrac{RCs+1}{Cs}y(s)$

$\rightarrow y(s) = P(s)u(s),\ P(s) = \dfrac{Cs}{RCs+1}$ //

(2) $u(t) = v_{in}(t),\ y(t) = v_{out}(t)$

$\rightarrow \begin{cases} u(t) = Ri(t) + \dfrac{1}{C}\displaystyle\int_0^t i(t)dt \\ y(t) = \dfrac{1}{C}\displaystyle\int_0^t i(t)dt \end{cases} \rightarrow \begin{cases} u(s) = Ri(s) + \dfrac{1}{Cs}i(s) \\ \quad = \dfrac{RCs+1}{Cs}i(s) \\ y(s) = \dfrac{1}{Cs}i(s) \end{cases}$

$\rightarrow P(s) = \dfrac{y(s)}{u(s)} = \dfrac{1}{RCs+1}$ //

問2.4 $u(t) - c\dot{z}(t) = M\ddot{z}(t)$

$y(t) = \dot{z}(t)$ より

$$u(t) - c\,y(t) = M\dot{y}(t) \longrightarrow M\dot{y}(t) + c\,y(t) = u(t)$$

$$\longrightarrow Ms\,y(s) + c\,y(s) = u(s) \longrightarrow (Ms + c)\,y(s) = u(s)$$

$$\longrightarrow y(s) = P(s)\,u(s),\quad P(s) = \frac{1}{Ms + c} \;/\!/$$

問2.5 $\tau(t) - \underbrace{\tau_d(t)}_{c\dot{\theta}(t)} = J\ddot{\theta}(t) \longrightarrow J\ddot{\theta}(t) + c\dot{\theta}(t) = \tau(t)$

$$\longrightarrow J\ddot{y}(t) + c\dot{y}(t) = u(t) \longrightarrow Js^2 y(s) + cs\,y(s) = u(s)$$

$$\longrightarrow (Js^2 + cs)\,y(s) = u(s) \longrightarrow y(s) = P(s)\,u(s),\quad P(s) = \frac{1}{Js^2 + cs} \;/\!/$$

問2.6 $f(t) - f_k(t) - f_{d1}(t) - f_{d2}(t) = M\ddot{z}(t)$

ここで

$$f_k(t) = k\,z(t),\quad f_{d1}(t) = c_1\dot{z}(t),\quad f_{d2}(t) = c_2\dot{z}(t)$$

なので $u(t) = f(t)$, $y(t) = z(t)$ とすると

$$u(t) - k\,y(t) - c_1\dot{y}(t) - c_2\dot{y}(t) = M\ddot{y}(t)$$

$$\longrightarrow M\ddot{y}(t) + (c_1 + c_2)\dot{y}(t) + k\,y(t) = u(t)$$

$$\longrightarrow Ms^2 y(s) + (c_1 + c_2)s\,y(s) + k\,y(s) = u(s)$$

$$\longrightarrow (Ms^2 + cs + k)\,y(s) = u(s)$$

$$\longrightarrow y(s) = P(s)\,u(s),\quad P(s) = \frac{1}{Ms^2 + cs + k},\quad c = c_1 + c_2$$

問2.7 運動エネルギー $K(t) = \frac{1}{2}M\dot{z}(t)^2 = \frac{1}{2}M\dot{q}(t)^2$

位置 —— $U(t) = 0$

散逸 —— $D(t) = \frac{1}{2}c\dot{z}(t)^2 = \frac{1}{2}c\dot{q}(t)^2$

ラグランジアン $L(t) = K(t) - U(t) = \frac{1}{2}M\dot{q}(t)^2$

$$\longrightarrow \frac{\partial L(t)}{\partial \dot{q}(t)} = \frac{1}{2}M\cdot 2\dot{q}(t) = M\dot{q}(t)$$

$$\longrightarrow \frac{d}{dt}\left(\frac{\partial L(t)}{\partial \dot{q}(t)}\right) = \frac{d}{dt}\left(M\dot{q}(t)\right) = M\ddot{q}(t)$$

$$\frac{\partial L(t)}{\partial q(t)} = 0$$

$$\frac{\partial D(t)}{\partial \dot{q}(t)} = \frac{1}{2}c\cdot 2\dot{q}(t) = c\dot{q}(t)$$

ラグランジュの運動方程式

$$\frac{d}{dt}\left(\frac{\partial L(t)}{\partial \dot{q}(t)}\right) - \frac{\partial L(t)}{\partial q(t)} + \frac{\partial D(t)}{\partial \dot{q}(t)} = \nu(t)$$

$$\longrightarrow M\ddot{q}(t) - 0 + c\dot{q}(t) = \nu(t)$$

$$\longrightarrow M\ddot{z}(t) + c\dot{z}(t) = f(t)$$

$$\longrightarrow M\ddot{y}(t) + c\dot{y}(t) = u(t) \;/\!/$$

問2.8 $P(s) = \dfrac{1}{Ms+c} = \dfrac{\frac{1}{c}}{1+\frac{M}{c}s} = \dfrac{K}{1+Ts} \rightarrow \begin{cases} T = M/c \\ K = 1/c \end{cases}$ //

問2.9 $P(s) = \dfrac{1}{LCs^2+RCs+1} = \dfrac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}} = \dfrac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$$\rightarrow \begin{cases} K\omega_n^2 = \dfrac{1}{LC} \\ \omega_n^2 = \dfrac{1}{LC} \\ 2\zeta\omega_n = \dfrac{R}{L} \end{cases} \rightarrow \begin{cases} K = 1 \\ \omega_n = \dfrac{1}{\sqrt{LC}} \\ \zeta = \dfrac{1}{2\omega_n}\dfrac{R}{L} = \dfrac{R}{2L}\sqrt{LC} = \dfrac{R}{2}\sqrt{\dfrac{C}{L}} \end{cases}$$ //

問2.10 略

問3.1 (1) $f(t) = te^{-at}$

$$\rightarrow f(s) = \int_0^\infty \tau e^{-a\tau}e^{-s\tau}d\tau = \int_0^\infty \tau e^{-(s+a)\tau}d\tau = \int_0^\infty \tau\left\{-\frac{1}{s+a}e^{-(s+a)\tau}\right\}'d\tau$$

$$= \left[\tau\left\{-\frac{1}{s+a}e^{-(s+a)\tau}\right\}\right]_0^\infty - \int_0^\infty 1\cdot\left\{-\frac{1}{s+a}e^{-(s+a)\tau}\right\}d\tau$$

$$= -\frac{1}{s+a}\left[\frac{\tau}{e^{(s+a)\tau}}\right]_0^\infty + \frac{1}{s+a}\int_0^\infty e^{-(s+a)\tau}d\tau$$

$$\begin{cases} \lim\limits_{\tau\to\infty}\dfrac{\tau}{e^{(s+a)\tau}} = 0 \quad (\mathrm{Re}[s+a]>0) \\ \lim\limits_{\tau\to 0}\dfrac{\tau}{e^{(s+a)\tau}} = \dfrac{0}{e^0} = \dfrac{0}{1} = 0 \end{cases}$$

$$= 0 + \frac{1}{s+a}\left[-\frac{1}{s+a}e^{-(s+a)\tau}\right]_0^\infty = -\frac{1}{(s+a)^2}\left[e^{-(s+a)\tau}\right]_0^\infty$$

$$= -\frac{1}{(s+a)^2}(0-1) = \frac{1}{(s+a)^2}$$ //

$\mathrm{Re}[s+a]>0$

(2) $f(t) = \sin\omega t = \dfrac{e^{j\omega t}-e^{-j\omega t}}{2j}$

$$\rightarrow f(s) = \frac{1}{2j}\left\{\mathcal{L}[e^{-(-j\omega)t}] - \mathcal{L}[e^{-j\omega t}]\right\}$$

$$= \frac{1}{2j}\left\{\frac{1}{s+(-j\omega)} - \frac{1}{s+j\omega}\right\} = \frac{1}{2j}\left(\frac{1}{s-j\omega}-\frac{1}{s+j\omega}\right)$$

$$= \frac{1}{2j}\frac{s+j\omega-(s-j\omega)}{s^2+\omega^2} = \frac{1}{2j}\frac{2j\omega}{s^2+\omega^2}$$

$$= \frac{\omega}{s^2+\omega^2}$$ //

問3.2 (1) $f(t) = 1 - e^{-5t} \rightarrow F(s) = \frac{1}{s} - \frac{1}{s+5} = \frac{s+5-s}{s(s+5)} = \frac{5}{s(s+5)}$ //

(2) $f(t) = e^{-2t} + 2e^{t} - 3$

$\rightarrow F(s) = \frac{1}{s+2} + 2 \cdot \frac{1}{s-1} - 3 \cdot \frac{1}{s} = \frac{s(s-1) + 2s(s+2) - 3(s-1)(s+2)}{s(s-1)(s+2)}$

$= \frac{s^2 - s + 2s^2 + 4s - 3s^2 - 3s + 6}{s(s-1)(s+2)}$

$= \frac{6}{s(s-1)(s+2)}$ //

(3) $f(t) = 2e^{-t} - 2\cos 2t + \sin 2t$

$\rightarrow F(s) = 2 \cdot \frac{1}{s+1} - 2 \cdot \frac{s}{s^2+2^2} + \frac{2}{s^2+2^2} = \frac{2}{s+1} + \frac{-2s+2}{s^2+4}$

$= \frac{2(s^2+4) + (s+1)(-2s+2)}{(s+1)(s^2+4)} = \frac{2(s^2+4) - 2(s^2-1)}{(s+1)(s^2+4)}$

$= \frac{10}{(s+1)(s^2+4)}$ //

$\boxed{\frac{t^2}{2!} \longleftrightarrow \frac{1}{s^3}} \Rightarrow \boxed{t^2 \longleftrightarrow \frac{2!}{s^3}}$

(4) $f(t) = 3 + 2t + 2t^2 - 3e^{-2t}$

$\rightarrow F(s) = 3 \cdot \frac{1}{s} + 2 \cdot \frac{1}{s^2} + 2 \cdot \frac{2!}{s^3} - 3 \cdot \frac{1}{s+2}$

$= \frac{3s^2 + 2s + 4}{s^3} - \frac{3}{s+2} = \frac{(3s^2+2s+4)(s+2) - 3s^3}{s^3(s+2)}$

$= \frac{3s^3 + 2s^2 + 4s + 6s^2 + 4s + 8 - 3s^3}{s^3(s+2)}$

$= \frac{8s^2 + 8s + 8}{s^3(s+2)} = \frac{8(s^2+s+1)}{s^3(s+2)}$ //

問3.3 (1) $F(s) = \frac{1}{s} - \frac{1}{s+5} \rightarrow f(t) = 1 - e^{-5t}$ //

(2) $F(s) = \frac{3}{s-1} - \frac{2}{s+1} \rightarrow f(t) = 3e^{t} - 2e^{-2t}$ //

(3) $F(s) = \frac{s+1}{s^2+25} = \frac{s}{s^2+5^2} + \frac{1}{5} \frac{5}{s^2+5^2} \rightarrow f(t) = \cos 5t + \frac{1}{5}\sin 5t$ //

(4) $F(s) = \frac{2s+5}{s^2+2s+5} = \frac{2s+5}{(s+1)^2+2^2}$

$= 2 \cdot \frac{s+1}{(s+1)^2+2^2} + \frac{3}{2} \frac{2}{(s+1)^2+2^2}$

$\rightarrow f(t) = 2e^{-t}\cos 2t + \frac{3}{2} e^{-t} \sin 2t$

$= e^{-t}(2\cos 2t + \frac{3}{2}\sin 2t)$ //

問3.4 (1) $P(s) = \frac{3}{s+2}$

$$\rightarrow y(s) = P(s)u(s) = \frac{3}{s(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+2}$$

$$\begin{cases} k_1 = s\,y(s)\big|_{s=0} = \left.\frac{3}{s+2}\right|_{s=0} = \frac{3}{2} \,/\!/ \\ k_2 = (s+2)\,y(s)\big|_{s=-2} = \left.\frac{3}{s}\right|_{s=-2} = -\frac{3}{2} \,/\!/ \end{cases}$$

$$\therefore y(s) = \frac{3}{2}\frac{1}{s} - \frac{3}{2}\frac{1}{s+2}$$

$$\rightarrow y(t) = \frac{3}{2} - \frac{3}{2}e^{-2t} = \frac{3}{2}(1-e^{-2t}) \,/\!/$$

(2) $P(s) = \frac{s+4}{(s+1)(s+2)}$

$$\rightarrow y(s) = P(s)u(s) = \frac{s+4}{s(s+1)(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+2}$$

$$\begin{cases} k_1 = s\,y(s)\big|_{s=0} = \left.\frac{s+4}{(s+1)(s+2)}\right|_{s=0} = \frac{4}{1\cdot 2} = 2 \,/\!/ \\ k_2 = (s+1)\,y(s)\big|_{s=-1} = \left.\frac{s+4}{s(s+2)}\right|_{s=-1} = \frac{3}{-1\cdot 1} = -3 \,/\!/ \\ k_3 = (s+2)\,y(s)\big|_{s=-2} = \left.\frac{s+4}{s(s+1)}\right|_{s=-2} = \frac{2}{-2\cdot(-1)} = 1 \,/\!/ \end{cases}$$

$$\therefore y(s) = \frac{2}{s} - \frac{3}{s+1} + \frac{1}{s+2}$$

$$\rightarrow y(t) = 2 - 3e^{-t} + e^{-2t} \,/\!/$$

(3) $P(s) = \frac{2}{(s+1)^2}$

$$\rightarrow y(s) = P(s)u(s) = \frac{2}{s(s+1)^2} = \frac{k_1}{s} + \frac{k_{22}}{(s+1)^2} + \frac{k_{21}}{s+1}$$

$$\begin{cases} k_1 = s\,y(s)\big|_{s=0} = \left.\frac{2}{(s+1)^2}\right|_{s=0} = \frac{2}{1} = 2 \,/\!/ \\ k_{22} = (s+1)^2 y(s)\big|_{s=-1} = \left.\frac{2}{s}\right|_{s=-1} = \frac{2}{-1} = -2 \,/\!/ \\ k_{21} = \frac{d}{ds}(s+1)^2 y(s)\Big|_{s=-1} = \frac{d}{ds}\left(\frac{2}{s}\right)\Big|_{s=-1} = -\left.\frac{2}{s^2}\right|_{s=-1} = -\frac{2}{1} = -2 \,/\!/ \end{cases}$$

$$\therefore y(s) = \frac{2}{s} - \frac{2}{(s+1)^2} - \frac{2}{s+1}$$

$$\rightarrow y(t) = 2 - 2te^{-t} - 2e^{-t} = 2(1 - te^{-t} - e^{-t})$$

(4) $P(s) = \dfrac{5}{s^2+2s+5}$

$\rightarrow\ y(s) = P(s)u(s) = \dfrac{5}{s(s^2+2s+5)} = \dfrac{k_1}{s} + \dfrac{h_1 s + h_2}{s^2+2s+5}$

$k_1 = s\,y(s)\big|_{s=0} = \dfrac{5}{s^2+2s+5}\Big|_{s=0} = \dfrac{5}{5} = 1$ //

$\rightarrow\ y(s) = \dfrac{1}{s} + \dfrac{h_1 s + h_2}{s^2+2s+5} = \dfrac{s^2+2s+5+h_1 s^2 + h_2 s}{s(s^2+2s+5)}$

$= \dfrac{(1+h_1)s^2 + (2+h_2)s + 5}{s(s^2+2s+5)}$

$\rightarrow\ \begin{cases} 1+h_1 = 0 \\ 2+h_2 = 0 \end{cases} \rightarrow \begin{cases} h_1 = -1 \\ h_2 = -2 \end{cases}$

$\rightarrow\ y(s) = \dfrac{1}{s} - \dfrac{s+2}{s^2+2s+5} = \dfrac{1}{s} - \dfrac{s+2}{(s+1)^2+2^2}$

$= \dfrac{1}{s} - \left\{ \dfrac{s+1}{(s+1)^2+2^2} + \dfrac{1}{2}\dfrac{2}{(s+1)^2+2^2} \right\}$

$\rightarrow\ y(t) = 1 - \left(e^{-t}\cos 2t + \dfrac{1}{2}e^{-t}\sin 2t\right) = 1 - e^{-t}\left(\cos 2t + \dfrac{1}{2}\sin 2t\right)$ //

問3.5 (1) 極 $s = -1, -2 \rightarrow$ 安定

(2) 極 $s = 1, -2 \rightarrow$ 正の極 $s=1$ を含むので不安定

(3) $s^2 - 2s + 2 = (s-1)^2 - 1^2 + 2 = (s-1)^2 + 1^2 = 0$

$\rightarrow (s-1)^2 = -1^2 \rightarrow s-1 = \pm j \rightarrow$ 極 : $s = 1 \pm j$

$\rightarrow$ 実部 1 が正なので不安定.

(4) $s^2 + 2s + 2 = (s+1)^2 - 1^2 + 2 = (s+1)^2 + 1^2 = 0$

$\rightarrow (s+1)^2 = -1^2 \rightarrow s+1 = \pm j \rightarrow s = -1 \pm j$

$\rightarrow$ 極 : $s = -1, -1 \pm j \rightarrow$ 安定.

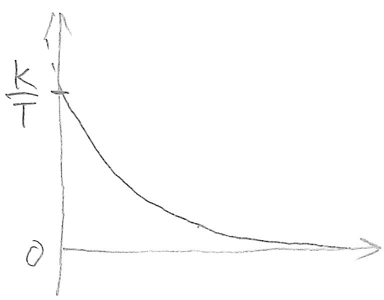
問3.6 (1) $y_\infty = P(0) = \dfrac{1}{2}$

(2) $y_\infty = P(0) = \dfrac{2}{1^{10}} = 2$

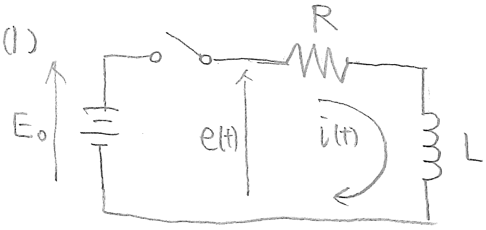
問3.7 $y(s) = P(s)\cdot 1 = P(s) = \dfrac{K}{1+Ts} = \dfrac{\frac{K}{T}}{s + \frac{1}{T}}$

$\rightarrow y(t) = \dfrac{K}{T} e^{-\frac{1}{T}t}$ //

$\begin{cases} y(0) = \dfrac{K}{T}\cdot e^0 = \dfrac{K}{T} \\ y_\infty = 0 \end{cases}$



問3.8 (1)



$$e(t) = Ri(t) + L\frac{di(t)}{dt}$$

$$\rightarrow u(t) = Ry(t) + L\dot{y}(t)$$

$$\rightarrow (Ls+R)y(s) = u(s)$$

$$\rightarrow y(s) = P(s)u(s),\quad P(s) = \frac{1}{Ls+R} = \frac{\frac{1}{R}}{1+\frac{L}{R}s} = \frac{K}{1+Ts}$$

$$\rightarrow T = \frac{L}{R},\ K = \frac{1}{R}$$

(2) $u(t) = e(t) = \begin{cases} 0 & (t<0) \\ E_0 & (t \geqq 0) \end{cases} \rightarrow u(s) = \frac{E_0}{s}$

$$\rightarrow y(s) = P(s)u(s) = \frac{1}{Ls+R}\cdot\frac{E_0}{s} = \frac{\frac{E_0}{L}}{s\left(s+\frac{R}{L}\right)}$$

$$= \frac{k_1}{s} + \frac{k_2}{s+\frac{R}{L}}$$

$$\begin{cases} k_1 = sy(s)\big|_{s=0} = \left.\frac{E_0/L}{s+R/L}\right|_{s=0} = \frac{E_0/L}{R/L} = \frac{E_0}{R} \\ k_2 = \left(s+\frac{R}{L}\right)y(s)\big|_{s=-\frac{R}{L}} = \left.\frac{E_0/L}{s}\right|_{s=-\frac{R}{L}} = \frac{E_0/L}{-R/L} = -\frac{E_0}{R} \end{cases}$$

$$\rightarrow y(s) = \frac{E_0/R}{s} - \frac{E_0/R}{s+R/L} = \frac{E_0}{R}\left(\frac{1}{s} - \frac{1}{s+R/L}\right)$$

$$\rightarrow y(t) = \frac{E_0}{R}\left(1 - e^{-\frac{R}{L}t}\right) \quad /\!/$$

$$i_\infty = \lim_{t\to\infty} y(t) = \frac{E_0}{R}$$

(3) 「$R \rightarrow$ 大」とすると「$T = \frac{L}{R} \rightarrow$ 小」なので反応が速くなる

「$L \rightarrow$ 大」とすると「$T = \frac{L}{R} \rightarrow$ 大」なので反応が遅くなる

問3.9 $\begin{cases} i_\infty = \frac{E_0}{R} = \frac{1}{R} = 0.02 \\ T = \frac{L}{R} = 0.004 \end{cases} \rightarrow \begin{cases} R = \frac{1}{0.02} = \frac{100}{2} = 50\,[\Omega] \ /\!/ \\ L = 0.004 \times R = 0.004 \times 50 = 0.2\,[\mathrm{H}] \ /\!/ \end{cases}$

問3.10 $P(s) = \dfrac{C}{LCs^2 + RCs + 1} = \dfrac{\frac{1}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \dfrac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$\rightarrow \begin{cases} K\omega_n^2 = \dfrac{1}{L} \\ \omega_n^2 = \dfrac{1}{LC} \\ 2\zeta\omega_n = \dfrac{R}{L} \end{cases} \rightarrow \begin{cases} K = C \\ \omega_n = \dfrac{1}{\sqrt{LC}} \\ \zeta = \dfrac{1}{2\omega_n}\dfrac{R}{L} = \dfrac{R}{2L}\sqrt{LC} = \dfrac{R}{2}\sqrt{\dfrac{C}{L}} \ /\!/ \end{cases}$$

オーバーシュートを生じないのは

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} \geq 1 \rightarrow R \geq 2\sqrt{\frac{L}{C}} \ /\!/$$

問3.11 (1) $P(s) = \dfrac{1}{Ms^2 + cs + k} = \dfrac{\frac{1}{M}}{s^2 + \frac{c}{M}s + \frac{k}{M}} = \dfrac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$\rightarrow \begin{cases} K\omega_n^2 = \dfrac{1}{M} \\ \omega_n^2 = \dfrac{k}{M} \\ 2\zeta\omega_n = \dfrac{c}{M} \end{cases} \rightarrow \begin{cases} K = \dfrac{1}{k} \\ \omega_n = \sqrt{\dfrac{k}{M}} \\ \zeta = \dfrac{1}{2\omega_n}\dfrac{c}{M} = \dfrac{c}{2M}\sqrt{\dfrac{M}{k}} = \dfrac{c}{2\sqrt{kM}} \end{cases}$$

(2) $\begin{cases} y_\infty = K \\ A_{max} = K e^{-\zeta\omega_n T_p} \\ T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} \end{cases}$

$\rightarrow K = y_\infty = 0.04 \ /\!/$

$A_{max} = y_\infty e^{-\zeta\omega_n T_p}$

$\rightarrow e^{-\zeta\omega_n T_p} = \dfrac{A_{max}}{y_\infty}$

$\rightarrow -\zeta\omega_n T_p = \log_e \dfrac{A_{max}}{y_\infty}$

$\rightarrow \xi = \zeta\omega_n = -\dfrac{1}{T_p}\log_e\dfrac{A_{max}}{y_\infty} = 2.7726 \ /\!/$

$\rightarrow \omega_n\sqrt{1-\zeta^2} = \dfrac{\pi}{T_p} \rightarrow \omega_n^2(1-\zeta^2) = \left(\dfrac{\pi}{T_p}\right)^2 \rightarrow \omega_n^2 = (\zeta\omega_n)^2 + \left(\dfrac{\pi}{T_p}\right)^2$

$= \xi^2 + \left(\dfrac{\pi}{T_p}\right)^2$

$\rightarrow \omega_n = \sqrt{\xi^2 + \left(\dfrac{\pi}{T_p}\right)^2} = 6.8677 \ /\!/$

$\zeta = \dfrac{\xi}{\omega_n} = 0.40371 \ /\!/$

(3) $k = \dfrac{1}{K} = \dfrac{1}{0.04} = 25 \ /\!/$

$M = \dfrac{k}{\omega_n^2} = 0.53005 \ /\!/$

$c = 2\zeta\omega_n M = 2.9392 \ /\!/$

問4.1 $e = r - y = r - P(u+d) = r - P(Ce+d)$

$\rightarrow (1+PC)e = r - Pd \rightarrow e = \frac{1}{1+PC} r - \frac{P}{1+PC} d$

$\rightarrow \begin{cases} G_{er} = \frac{1}{1+PC} \\ G_{ed} = -\frac{P}{1+PC} \end{cases}$ //

$u = Ce = C(r-y) = C\{r - P(u+d)\}$

$\rightarrow (1+PC)u = Cr - PCd \rightarrow u = \frac{C}{1+PC} r - \frac{PC}{1+PC} d$

$\rightarrow \begin{cases} G_{ur} = \frac{C}{1+PC} \\ G_{ud} = -\frac{PC}{1+PC} \end{cases}$ /

問4.2 (1) $P(s) = \frac{1}{s-1} \rightarrow \begin{cases} N_p(s) = 1 \\ D_p(s) = s-1 \end{cases}$

$C(s) = \frac{2s+1}{s} \rightarrow \begin{cases} N_c(s) = 2s+1 \\ D_c(s) = s \end{cases}$

$\Delta(s) = D_p(s)D_c(s) + N_p(s)N_c(s) = (s-1)\cdot s + 1\cdot(2s+1)$
$= s^2 + s + 1 = 0$

$\rightarrow (s+\frac{1}{2})^2 - (\frac{1}{2})^2 + 1 = (s+\frac{1}{2})^2 + \frac{3}{4} = (s+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 = 0$

$\rightarrow s = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} j$ → 実部が負なので内部安定である

(2) $P(s) = \frac{1}{(s-1)(s+2)} \rightarrow \begin{cases} N_p(s) = 1 \\ D_p(s) = (s-1)(s+2) \end{cases}$

$C(s) = 1 = \frac{1}{1} \rightarrow \begin{cases} N_c(s) = 1 \\ D_c(s) = 1 \end{cases}$

$\Delta(s) = D_p(s)D_c(s) + N_p(s)N_c(s) = (s-1)(s+2)\cdot 1 + 1\cdot 1$
$= s^2 + s - 1 = 0$

$\rightarrow (s+\frac{1}{2})^2 - (\frac{1}{2})^2 - 1 = (s+\frac{1}{2})^2 - \frac{5}{4} = (s+\frac{1}{2})^2 - (\frac{\sqrt{5}}{2})^2 = 0$

$\rightarrow s = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$ → $-\frac{1}{2} + \frac{\sqrt{5}}{2} > 0$ なので内部安定ではない

問4.3 (1) $\Delta(s) = s^3 + 4s^2 + 14s + 20 \longrightarrow \Delta(s) = (s+2)(s^2+2s+10)$

条件I
$$\begin{cases} a_3 = 1 > 0 \\ a_2 = 4 > 0 \\ a_1 = 14 > 0 \\ a_0 = 20 > 0 \end{cases}$$

$$\begin{array}{r} 14 \\ \underline{\times\ 4} \\ 56 \end{array}$$

条件II
$$H = \begin{bmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{bmatrix} = \begin{bmatrix} 4 & 20 & 0 \\ 1 & 14 & 0 \\ 0 & 4 & 20 \end{bmatrix}$$

$$\longrightarrow H_2 = \begin{vmatrix} 4 & 20 \\ 1 & 14 \end{vmatrix} = 4 \cdot 14 - 20 \cdot 1 = 36 > 0$$

条件I, IIを満足するので、内部安定

(2) $\Delta(s) = s^4 + 2s^3 + 5s^2 + 34s + 30 \longrightarrow \Delta(s) = (s+1)(s+3)(s^2-2s+10)$

条件I
$$\begin{cases} a_4 = 1 > 0 \\ a_3 = 2 > 0 \\ a_2 = 5 > 0 \\ a_1 = 34 > 0 \\ a_0 = 30 > 0 \end{cases}$$

条件II
$$H = \begin{bmatrix} a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_0 & 0 \\ 0 & a_3 & a_1 & 0 \\ 0 & a_4 & a_2 & a_0 \end{bmatrix} = \begin{bmatrix} 2 & 34 & 0 & 0 \\ 1 & 5 & 30 & 0 \\ 0 & 2 & 34 & 0 \\ 0 & 1 & 5 & 30 \end{bmatrix}$$

$$\longrightarrow H_3 = \begin{vmatrix} 2 & 34 & 0 \\ 1 & 5 & 30 \\ 0 & 2 & 34 \end{vmatrix} = 2 \cdot 5 \cdot 34 - 2 \cdot 2 \cdot 30 - 1 \cdot 34 \cdot 34$$
$$= 340 - 120 - 1156$$
$$= -936 < 0$$

条件Iを満足するが条件IIを満足しないので、内部安定ではない

問4.4 (1) $P(s) = \dfrac{5}{s^2+2s+2} \longrightarrow \begin{cases} N_P(s) = 5 \\ D_P(s) = s^2+2s+2 \end{cases}$

$C(s) = k_P = \dfrac{k_P}{1} \longrightarrow \begin{cases} N_C(s) = k_P \\ D_C(s) = 1 \end{cases}$

$$\Delta(s) = D_P(s)D_C(s) + N_P(s)N_C(s) = (s^2+2s+2) \cdot 1 + 5 \cdot k_P$$
$$= s^2 + 2s + 2 + 5k_P$$

条件I
$$\begin{cases} a_2 = 1 > 0 \\ a_1 = 2 > 0 \\ a_0 = 2 + 5k_P > 0 \rightarrow k_P > -\dfrac{2}{5} \end{cases}$$

条件II 不要

条件I, IIより $k_P > -\dfrac{2}{5}$

(2) $P(s) = \dfrac{5}{s^2+2s+2} \rightarrow \begin{cases} N_p(s) = 5 \\ D_p(s) = s^2+2s+2 \end{cases}$

$C(s) = \dfrac{k_P s + k_I}{s} \rightarrow \begin{cases} N_c(s) = k_P s + k_I \\ D_c(s) = s \end{cases}$

$$\Delta(s) = D_p(s)D_c(s) + N_p(s)N_c(s) = (s^2+2s+2)\cdot s + 5\cdot(k_P s + k_I)$$
$$= s^3 + 2s^2 + (2+5k_P)s + 5k_I$$

条件I $\begin{cases} a_3 = 1 > 0 \\ a_2 = 2 > 0 \\ a_1 = 2+5k_P > 0 \rightarrow k_P > -\frac{2}{5} \\ a_0 = 5k_I > 0 \rightarrow k_I > 0 \end{cases}$

条件II

$$H = \begin{bmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{bmatrix} = \begin{bmatrix} 2 & 5k_I & 0 \\ 1 & 2+5k_P & 0 \\ 0 & 2 & 5k_I \end{bmatrix}$$

$$\rightarrow H_2 = \begin{vmatrix} 2 & 5k_I \\ 1 & 2+5k_P \end{vmatrix} = 2(2+5k_P) - 5k_I\cdot 1$$
$$= 4 + 10k_P - 5k_I > 0$$
$$\rightarrow k_I < \frac{4}{5} + 2k_P$$

条件I, II より

$$k_P > -\frac{2}{5}, \quad 0 < k_I < \frac{4}{5} + 2k_P \quad //$$

問4.5 (1) $\Delta(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0$

条件I $a_3 = 1 > 0$, $a_2 = 4 > 0$, $a_1 = 14 > 0$, $a_0 = 20 > 0$

条件II

ラウス表

s^3	$a_3 = 1$	$a_1 = 14$
s^2	$a_2 = 4$	$a_0 = 20$
s^1	$\dfrac{14\cdot4 - 1\cdot20}{4} = 9$	0
s^0	$\dfrac{20\cdot9 - 4\cdot0}{9} = 20$	*

ラウス数列 {1, 4, 9, 20} の符号がすべて正

条件I, II より内部安定

(2) $\Delta(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$

条件I
$$\begin{cases} a_4 = 1 > 0 \\ a_3 = 2 > 0 \\ a_2 = 5 > 0 \\ a_1 = 34 > 0 \\ a_0 = 30 > 0 \end{cases}$$

条件II ラウス表

$$\begin{array}{c|lll}
s^4 & a_4 = 1 & a_2 = 5 & a_0 = 30 \\
s^3 & a_3 = 2 & a_1 = 34 & 0 \\
s^2 & \dfrac{5\cdot 2 - 1\cdot 34}{2} = -12 & \dfrac{30\cdot 34 - 5\cdot 0}{34} = 30 & 0 \\
s^1 & \dfrac{34\cdot(-12) - 2\cdot 30}{-12} = 39 & 0 & \\
s^0 & \dfrac{30\cdot 39 - (-12)\cdot 0}{39} = 30 & * &
\end{array}$$

ラウス数列 $\{1, 2, -12, 39, 30\}$ の符号は $\{+, +, -, +, +\}$ のように2回変化なので不適

- 条件I は満足するが条件II は満足しない
- ラウス数列の符号が2回変化するので、不安定極の数は 2個

問4.6 (1) $P(s) = \dfrac{1}{s(s+1)(s+2)(s+3)}$, $\tilde{C}(s) = 1$ $\left(C(s) = k\tilde{C}(s) \right)$

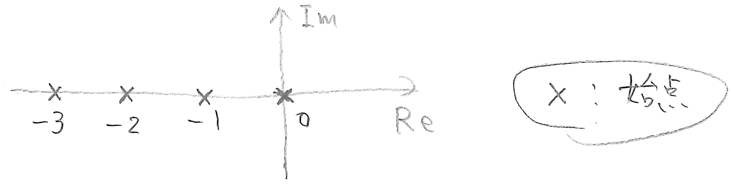
$\longrightarrow P(s)\tilde{C}(s) = \dfrac{1}{s(s+1)(s+2)(s+3)}$

性質1 $P(s)\tilde{C}(s)$ の極 $p_1 = 0$, $p_2 = -1$, $p_3 = -2$, $p_4 = -3$ (個数 $n = 4$)

零点なし (個数 $m = 0$)

$\longrightarrow$ 始点は $(0, 0)$, $(-1, 0)$, $(-2, 0)$, $(-3, 0)$

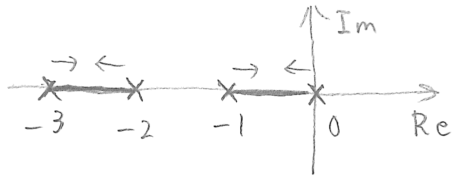
終点は $m = 0$ なので $n - m = 4$ 個の無限遠点



性質2 実軸上で右側に奇数個の極もしくは零点が存在するのは

$-1 \sim 0$, $-3 \sim -2$

の区間であり，この部分は根軌跡の一部



性質3 $n - m = 4$ 個の無限遠点に向かう $n - m = 4$ 本の漸近線の始点は

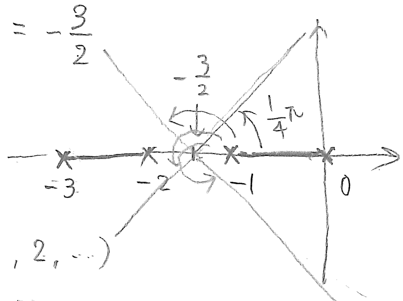
$\dfrac{1}{n-m}\left(\sum p_i - \sum z_i \right) = \dfrac{1}{4}(0 - 1 - 2 - 3) = -\dfrac{3}{2}$

$\longrightarrow \left(-\dfrac{3}{2}, 0 \right)$

であり，勾配は

$\phi = \dfrac{(2l+1)\pi}{n-m} = \dfrac{2l+1}{4}\pi$ $(l = 0, 1, 2, \cdots)$

$= \dfrac{1}{4}\pi$, $\dfrac{3}{4}\pi$, $\dfrac{5}{4}\pi$, $\dfrac{7}{4}\pi$, $\cdots$



性質4 $\dfrac{d}{ds}\left(\dfrac{1}{P(s)\tilde{C}(s)} \right) = \dfrac{d}{ds}\{ s(s+1)(s+2)(s+3) \} = \dfrac{d}{ds}(s^4 + 6s^3 + 11s^2 + 6s)$

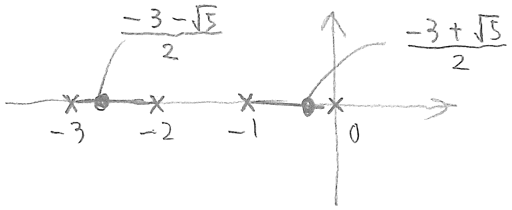
$= 4s^3 + 18s^2 + 22s + 6 = 2(2s^3 + 9s^2 + 11s + 3)$

$= 2(2s+3)(s^2 + 3s + 1) = 0$

$\longrightarrow s = -\dfrac{3}{2}$, $\dfrac{-3 \pm \sqrt{5}}{2}$

$\longrightarrow$ 根軌跡上にあるのは $\left(\dfrac{-3 \pm \sqrt{5}}{2}, 0 \right)$ であり，この点が合流もしくは分岐の点

-0.3820, -2.618



性質5　$\Delta(s) = s(s+1)(s+2)(s+3)\cdot 1 + 1\cdot k$

$= s^4 + 6s^3 + 11s^2 + 6s + k$

条件I
$$\begin{cases} a_4 = 1 > 0 \\ a_3 = 6 > 0 \\ a_2 = 11 > 0 \\ a_1 = 6 > 0 \\ a_0 = k > 0 \end{cases}$$

条件II
$$H = \begin{bmatrix} a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_0 & 0 \\ 0 & a_3 & a_1 & 0 \\ 0 & a_4 & a_2 & a_0 \end{bmatrix}$$

$$\rightarrow H_3 = \begin{vmatrix} a_3 & a_1 & 0 \\ a_4 & a_2 & a_0 \\ 0 & a_3 & a_1 \end{vmatrix} = \begin{vmatrix} 6 & 6 & 0 \\ 1 & 11 & k \\ 0 & 6 & 6 \end{vmatrix}$$

$$= 6\cdot 11\cdot 6 - 6\cdot 6\cdot 1 - 6\cdot 6\cdot k$$

$$= 36(10-k) > 0$$

$\rightarrow k < 10$

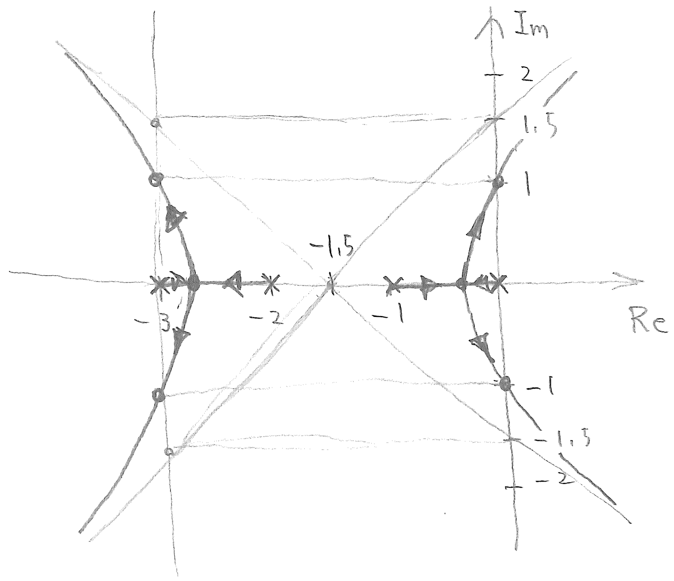
条件I, IIより安定条件は $0 < k < 10$

$k = 10$ のとき

$$\Delta(s) = s^4 + 6s^3 + 11s^2 + 6s + 10$$

$$= (s^2+1)(s^2+6s+10) = 0$$

$\rightarrow s = \pm j,\ -3 \pm j$



(2) $P(s) = \dfrac{s+2}{s(s+1)}$, $\tilde{C}(s) = 1$ $(C(s) = k\tilde{C}(s))$

$\longrightarrow P(s)\tilde{C}(s) = \dfrac{s+2}{s(s+1)}$

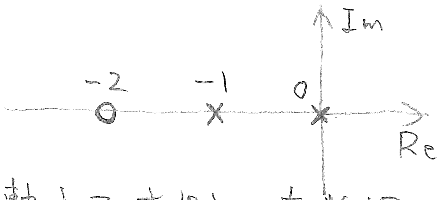
性質1　$P(s)\tilde{C}(s)$ の極　$p_1 = 0$, $p_2 = -1$　$(n=2)$

零点　$z_1 = -2$　$(m=1)$

→ 始点は $(0,0)$, $(-1,0)$

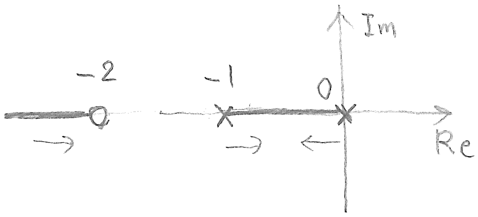
終点は　$m=1$ 個の $(-2,0)$

$n-m = 2-1 = 1$ 個の無限遠点



性質2　実軸上で右側に奇数個の極もしくは零点が存在するのは

$-1 \sim 0$, $-\infty \sim -2$



性質3　$n-m=1$ 個の無限遠点に向かう $n-m=1$ 本の漸近線の始点は

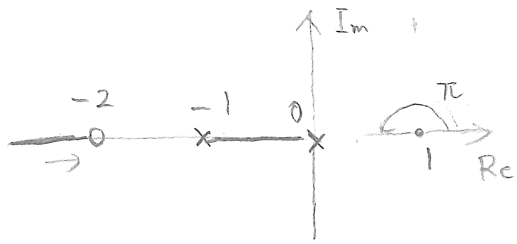
$$\frac{1}{n-m}\left(\sum p_i - \sum z_i\right) = \frac{1}{1}\{(0-1)-(-2)\} = 1$$

$\longrightarrow (1,0)$

であり勾配は

$$\phi = \frac{(2l+1)\pi}{n-m} = \frac{2l+1}{1}\pi \quad (l = 0, 1, 2, \cdots)$$

$$= \pi, 3\pi, \cdots$$



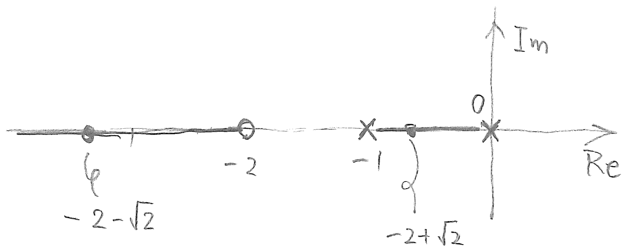
性質4 $\displaystyle \frac{d}{ds}\left(\frac{1}{P(s)\hat{C}(s)}\right) = \frac{d}{ds}\left\{\frac{s(s+1)}{s+2}\right\} = \frac{(2s+1)(s+2)-s(s+1)\cdot 1}{(s+2)^2}$

$$= \frac{s^2+4s+2}{(s+2)^2} = 0$$

$\longrightarrow$ $s^2+4s+2=0$ $\rightarrow$ $s = -2 \pm \sqrt{2^2-2}$

$= -2 \pm \sqrt{2} = -3.4142,\ -0.5858$

$\longrightarrow$ $(-2\pm\sqrt{2},\ 0)$ はともに根軌跡上にあり，この点が分岐点もしくは合流点



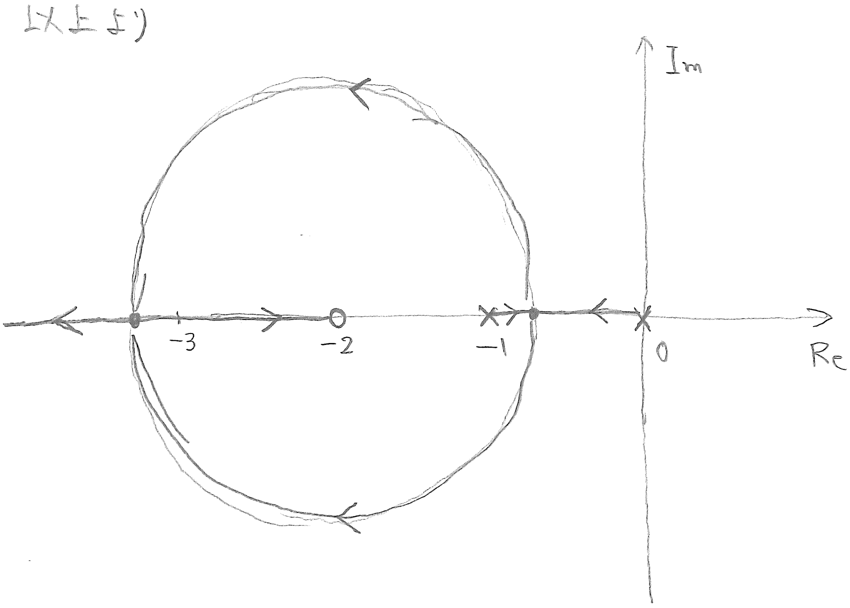
性質5 $\Delta(s) = s(s+1)\cdot 1 + (s+2)\cdot k$

$= s^2 + (1+k)s + 2k$

条件I $\begin{cases} a_2 = 1 > 0 \\ a_1 = 1+k > 0 & \rightarrow k > -1 \\ a_0 = 2k > 0 & \rightarrow k > 0 \end{cases}$

$\longrightarrow$ $k > 0$ なら安定.

以上より

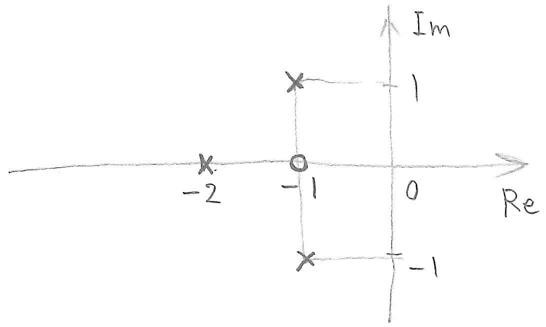


(3) $P(s) = \dfrac{s+1}{(s+2)(s^2+2s+2)}$, $\tilde{C}(s) = 1$ $(C(s) = k\tilde{C}(s))$

$\longrightarrow P(s)\tilde{C}(s) = \dfrac{s+1}{(s+2)(s^2+2s+2)}$

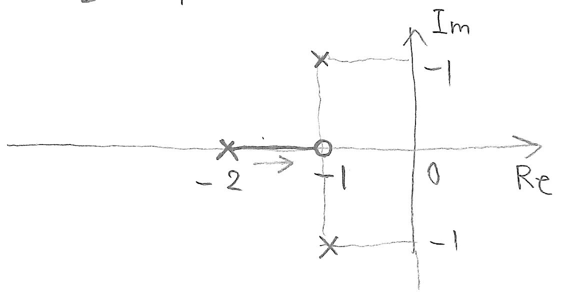
性質1　$P(s)\tilde{C}(s)$ の極　$p_1 = -2$, $p_2 = -1+j$, $p_3 = -1-j$ $(n=3)$

零点　$z = -1$ $(m=1)$



性質2　実軸上で右側に奇数個の極もしくは零点が存在するのは

$-2 \sim -1$



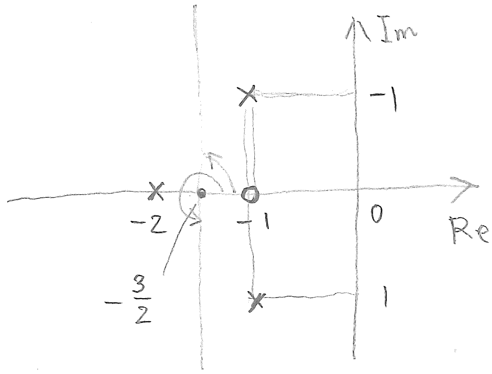
性質3　$n-m = 3-1 = 2$ 個の無限遠点に向かう $n-m=2$ 本の漸近線の始点は

$$\frac{1}{n-m}\left(\Sigma p_i - \Sigma z_i\right) = \frac{1}{2}\{(-2-1+j-1-j)-(-1)\} = -\frac{3}{2}$$

$\longrightarrow \left(-\frac{3}{2}, 0\right)$

であり，勾配は

$$\phi = \frac{(2l+1)\pi}{n-m} = \frac{2l+1}{2}\pi \quad (l = 0, 1, 2, \cdots)$$
$$= \frac{1}{2}\pi, \frac{3}{2}\pi, \cdots$$



性質4 $\frac{d}{ds}\left(\frac{1}{P(s)\hat{C}(s)}\right) = \frac{d}{ds}\left\{\frac{(s+2)(s^2+2s+2)}{s+1}\right\} = \frac{2s^3+7s^2+8s+2}{(s+1)^2} = 0$

$\longrightarrow s = -0.3427,\ -1.5786 \pm 0.6526j$

$\longrightarrow$ 根軌跡上にないので、分岐点はない

性質5 $\Delta(s) = (s+2)(s^2+2s+2)\cdot 1 + (s+1)\cdot k$

$= s^3+4s^2+(k+6)s+k+4$

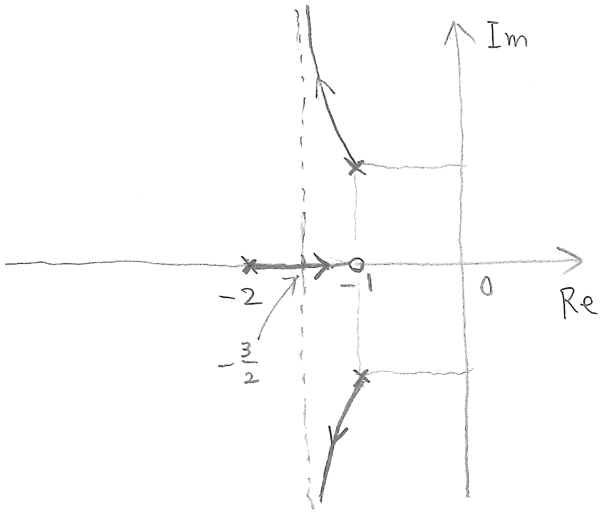
条件I
$$\begin{cases} a_3 = 1 > 0 \\ a_2 = 4 > 0 \\ a_1 = k+6 > 0 & \longrightarrow k > -6 \\ a_0 = k+4 > 0 & \longrightarrow k > -4 \end{cases}$$

条件II
$$H = \begin{bmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{bmatrix}$$

$$\longrightarrow H_2 = \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = \begin{vmatrix} 4 & k+4 \\ 1 & k+6 \end{vmatrix} = 4(k+6)-(k+4)$$

$$= 3k+20 > 0 \quad \rightarrow k > -\frac{20}{3}$$

条件I, IIより $k>0$ のときは安定



問4.7 (1) $G_{er}(s) = \frac{1}{1+P(s)C(s)} = \frac{1}{1+\frac{5}{s^2+2s+2}k_P} = \frac{s^2+2s+2}{s^2+2s+2+5k_P}$

$\longrightarrow e_\infty = G_{er}(0) = \frac{2}{2+5k_P}$

$\longrightarrow k_P = 2$ のとき $e_\infty = \frac{2}{2+5\cdot 2} = \frac{1}{6}$ //

$k_P = 5$ のとき $e_\infty = \frac{2}{2+5\cdot 5} = \frac{2}{27}$ //

(問4.4 (1)' より $k_P > -\frac{2}{5}$ のとき安定)

(2) $C(s) = \dfrac{k_P s + k_I}{s}$

$$\rightarrow G_{er}(s) = \frac{1}{1+P(s)C(s)} = \frac{1}{1+\dfrac{5}{s^2+2s+2}\dfrac{k_P s+k_I}{s}}$$

$$= \frac{s(s^2+2s+2)}{s^3+2s^2+(2+5k_P)s+5k_I}$$

問4.4(2)より

$k_P > -\dfrac{2}{5}$ かつ $0 < k_I < \dfrac{4}{5} + 2k_P$

のとき安定であり,

$k_P = 2,\ k_I = 1.25 = \dfrac{5}{4}$

はこの条件を満足する. したがって

$$e_\infty = G_{er}(0) = 0 \ /\!/$$

問4.8 (1) $$G_{yd}(s) = \frac{P(s)}{1+P(s)C(s)} = \frac{\dfrac{5}{s^2+2s+2}}{1+\dfrac{5}{s^2+2s+2}\cdot k_P} = \frac{5}{s^2+2s+2+5k_P}$$

$$\rightarrow y_\infty = G_{yd}(0) = \frac{5}{2+5k_P}$$

$$\rightarrow k_P = 2 \text{ のとき } y_\infty = \frac{5}{2+5\cdot 2} = \frac{5}{12} \ /\!/$$

$$k_P = 5 \text{ のとき } y_\infty = \frac{5}{2+5\cdot 5} = \frac{5}{27} \ /\!/$$

(2) $$G_{yd}(s) = \frac{P(s)}{1+P(s)C(s)} = \frac{\dfrac{5}{s^2+2s+2}}{1+\dfrac{5}{s^2+2s+2}\dfrac{k_P s+k_I}{s}}$$

$$= \frac{5s}{s^3+2s^2+(2+5k_P)s+5k_I}$$

$$\rightarrow y_\infty = G_{yd}(0) = 0 \ /\!/$$

問5.1 $P(s) = \dfrac{b_0}{s^2+a_1 s+a_0}$, $C_1(s) = k_P$

$$\rightarrow G_{yr}(s) = \frac{P(s)C_1(s)}{1+P(s)C_1(s)} = \frac{\dfrac{b_0}{s^2+a_1 s+a_0}k_P}{1+\dfrac{b_0}{s^2+a_1 s+a_0}k_P}$$

$$= \frac{b_0 k_P}{s^2+a_1 s+a_0+b_0 k_P} /\!/ = \frac{K_1\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} /\!/$$

$$G_{yd}(s) = \frac{P(s)}{1+P(s)C_1(s)} = \frac{\dfrac{b_0}{s^2+a_1 s+a_0}}{1+\dfrac{b_0}{s^2+a_1 s+a_0}k_P}$$

$$= \frac{b_0}{s^2+a_1 s+a_0+b_0 k_P} /\!/ = \frac{K_2\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} /\!/$$

$$\rightarrow \begin{cases} K_1\omega_n^2 = b_0 k_P \\ K_2\omega_n^2 = b_0 \\ 2\zeta\omega_n = a_1 \\ \omega_n^2 = a_0+b_0 k_P \end{cases} \rightarrow \begin{cases} K_1 = \dfrac{b_0 k_P}{a_0+b_0 k_P} \\ K_2 = \dfrac{b_0}{a_0+b_0 k_P} \\ \omega_n = \sqrt{a_0+b_0 k_P} \\ \zeta = \dfrac{a_1}{2\omega_n} = \dfrac{a_1}{2\sqrt{a_0+b_0 k_P}} \end{cases} /\!/$$

$\zeta = \zeta_m$ とする k_P は

$$\zeta_m = \frac{a_1}{2\sqrt{a_0+b_0 k_P}} \rightarrow 4(a_0+b_0 k_P) = \frac{a_1^2}{\zeta_m}$$

$$\rightarrow k_P = \frac{1}{b_0}\left(\frac{a_1^2}{4\zeta_m} - a_0\right) /\!/$$

問5.2 $P(s) = \dfrac{5}{s^2+2s+2}$, $C_1(s) = k_P + k_D s$, $C_2(s) = k_P$

$$\rightarrow G_{yr}(s) = \frac{PC_2(s)}{1+PC_1(s)} = \frac{\dfrac{b_0}{s^2+a_1 s+a_0}k_P}{1+\dfrac{b_0}{s^2+a_1 s+a_0}(k_P+k_D s)}$$

$$= \frac{b_0 k_P}{s^2+(a_1+b_0 k_D)s+a_0+b_0 k_P} /\!/ = \frac{K_1\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} /\!/$$

$$G_{yd}(s) = \frac{P(s)}{1+PC_1(s)} = \frac{\dfrac{b_0}{s^2+a_1 s+a_0}}{1+\dfrac{b_0}{s^2+a_1 s+a_0}(k_P+k_D s)}$$

$$= \frac{b_0}{s^2+(a_1+b_0 k_D)s+a_0+b_0 k_P} /\!/ = \frac{K_2\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} /\!/$$

$$\rightarrow \begin{cases} K_1\omega_n^2 = b_0 k_P \\ K_2\omega_n^2 = b_0 \\ 2\zeta\omega_n = a_1+b_0 k_D \\ \omega_n^2 = a_0+b_0 k_P \end{cases} \rightarrow \begin{cases} K_1 = \dfrac{b_0 k_P}{a_0+b_0 k_P} \\ K_2 = \dfrac{b_0}{a_0+b_0 k_P} \\ \omega_n = \sqrt{a_0+b_0 k_P} \\ \zeta = \dfrac{a_1+b_0 k_D}{2\omega_n} = \dfrac{a_1+b_0 k_D}{2\sqrt{a_0+b_0 k_P}} \end{cases} /\!/$$

問5.3 $k_I = \dfrac{\omega_m a_0}{2\zeta_m b_0}$, $k_P = \dfrac{\omega_m^2}{b_0}$, $k_D = \dfrac{2\zeta_m \omega_m}{b_0} - \dfrac{a_1}{b_0} + \dfrac{a_0}{2\zeta_m \omega_m b_0}$

のとき

$$N_{yr}(s) = b_0(k_P s + k_I) = b_0\left(\frac{\omega_m^2}{b_0} s + \frac{\omega_m a_0}{2\zeta_m b_0}\right)$$
$$= \omega_m^2\left(s + \frac{a_0}{2\zeta_m \omega_m}\right)$$

$$D_{yr}(s) = s^3 + (a_1 + b_0 k_D)s^2 + (a_0 + b_0 k_P)s + b_0 k_I$$
$$= s^3 + \left(\cancel{a_1} + 2\zeta_m\omega_m - \cancel{a_1} + \frac{a_0}{2\zeta_m\omega_m}\right)s^2 + (a_0 + \omega_m^2)s + \frac{\omega_m a_0}{2\zeta_m}$$
$$= (s^2 + 2\zeta_m\omega_m s + \omega_m^2)\left(s + \frac{a_0}{2\zeta_m\omega_m}\right)$$

なので、

$$G_{yr}(s) = \frac{N_{yr}(s)}{D_{yr}(s)} = \frac{\omega_m^2}{s^2 + 2\zeta_m\omega_m s + \omega_m^2} \quad //$$

問5.4 $P(s) = \dfrac{1}{Ms^2 + cs} = \dfrac{\frac{1}{M}}{s^2 + \frac{c}{M}s + 0}$

$$= \frac{b_0}{s^2 + a_1 s + a_0} \ , \ a_1 = \frac{c}{M} \ , \ a_0 = 0 \ , \ b_0 = \frac{1}{M} \ /\!/$$

このとき (5.37)式は

$$k_I = \frac{\omega_m^3}{b_0} = \frac{\omega_m^3}{1/M} = M\omega_m^3 \quad //$$
$$k_P = \frac{\alpha_1\omega_m^2 - a_0}{b_0} = \frac{\alpha_1\omega_m^2}{1/M} = M\alpha_1\omega_m^2 \quad //$$
$$k_D = \frac{\alpha_2\omega_m - a_1}{b_0} = \frac{\alpha_2\omega_m - c/M}{1/M} = M\alpha_2\omega_m - c \quad //$$

問6.1 $u(t) = \sin t \rightarrow u(s) = \dfrac{1}{s^2+1^2} = \dfrac{1}{s^2+1} = \dfrac{1}{(s-j)(s+j)}$

$\rightarrow y(s) = P(s)\,u(s) = \dfrac{1}{s-1}\cdot\dfrac{1}{s^2+1}$

$= \dfrac{1}{(s-1)(s-j)(s+j)} = \dfrac{h}{s-1} + \dfrac{k_1}{s-j} + \dfrac{k_2}{s+j}$

$$\begin{cases} h = (s-1)\,y(s)\big|_{s=1} = \dfrac{1}{s^2+1}\Big|_{s=1} = \dfrac{1}{2} \\ k_1 = (s-j)\,y(s)\big|_{s=j} = \dfrac{1}{(j-1)\,2j} = \dfrac{1}{2(-1-j)} = -\dfrac{1}{2}\dfrac{1}{1+j} = -\dfrac{1}{2}\dfrac{1-j}{2} \\ \quad = -\dfrac{1}{4}(1-j) \\ k_2 = \bar{k}_1 = -\dfrac{1}{4}(1+j) \end{cases}$$

$y(t) = h e^{t} + k_1 e^{jt} + k_2 e^{-jt}$

$= \dfrac{1}{2}e^{t} - \dfrac{1}{4}(1-j)(\cos t + j\sin t) - \dfrac{1}{4}(1+j)(\cos t - j\sin t)$

$= \dfrac{1}{2}e^{t} - \dfrac{1}{2}\cos t - \dfrac{1}{2}\sin t = \dfrac{1}{2}\{e^{t} - (\cos t + \sin t)\}$ //

$= \dfrac{1}{2}\{e^{t} - \sqrt{2}\sin(t + \dfrac{\pi}{4})\}$ //

$t \to \infty$ で $y(t) \to \infty$ となる（$y(t)$は発散）

問6.1 $u(t) = A\sin\omega t \rightarrow u(s) = \dfrac{A\omega}{s^2+\omega^2}$

を加えると

$$y(s) = P(s)u(s) = \frac{1}{s-1}\frac{A\omega}{s^2+\omega^2} = \frac{h}{s-1} + \frac{k_2}{s-j\omega} + \frac{k_2}{s+j\omega}$$

$$\rightarrow y(t) = he^t + k_1 e^{j\omega t} + k_2 e^{-j\omega t}$$

$t\rightarrow\infty$ で $e^t\rightarrow\infty$ なので、$y(t)$ は発散するので (6.6)式で近似できない。

問6.2 (1) $P(s) = \dfrac{1}{s+5} \rightarrow P(j\omega) = \dfrac{1}{5+j\omega}$

$$\begin{cases} G_g(\omega) = |P(j\omega)| = \dfrac{1}{\sqrt{25+\omega^2}} \\ G_p(\omega) = \angle P(j\omega) = \tan^{-1}0 - \tan^{-1}\dfrac{\omega}{5} = -\tan^{-1}\dfrac{\omega}{5} \end{cases}$$

(2) $P(s) = \dfrac{2}{s^2+2s+2} \rightarrow P(j\omega) = \dfrac{2}{2-\omega^2+j\cdot 2\omega}$

$$\begin{cases} G_g(\omega) = |P(j\omega)| = \dfrac{2}{\sqrt{(2-\omega^2)^2+(2\omega)^2}} = \dfrac{2}{\sqrt{4-4\omega^2+\omega^4+4\omega^2}} \\ \qquad = \dfrac{2}{\sqrt{4+\omega^4}} \\ G_p(\omega) = \angle P(j\omega) = \tan^{-1}0 - \tan^{-1}\dfrac{2\omega}{2-\omega^2} = -\tan^{-1}\dfrac{2\omega}{2-\omega^2} \end{cases}$$

(3) $P(s) = \dfrac{s(s+1)}{2(s+3)(s+4)(s+5)}$

$$\begin{cases} N_1(s) = \frac{1}{2} \\ N_2(s) = s \\ N_3(s) = s+1 \\ D_1(s) = s+3 \\ D_2(s) = s+4 \\ D_3(s) = s+5 \end{cases} \rightarrow \begin{cases} N_1(j\omega) = \frac{1}{2} + j\cdot 0 \\ N_2(j\omega) = 0 + j\omega \\ N_3(j\omega) = 1 + j\omega \\ D_1(j\omega) = 3 + j\omega \\ D_2(j\omega) = 4 + j\omega \\ D_3(j\omega) = 5 + j\omega \end{cases}$$

$$\rightarrow \begin{cases} |N_1(j\omega)| = \frac{1}{2} \\ |N_2(j\omega)| = \omega \\ |N_3(j\omega)| = \sqrt{1+\omega^2} \\ |D_1(j\omega)| = \sqrt{9+\omega^2} \\ |D_2(j\omega)| = \sqrt{16+\omega^2} \\ |D_3(j\omega)| = \sqrt{25+\omega^2} \end{cases} \rightarrow G_g(\omega) = |P(j\omega)|$$

$$= \frac{\omega\sqrt{1+\omega^2}}{2\sqrt{(9+\omega^2)(16+\omega^2)(25+\omega^2)}}$$

$$= \frac{\omega}{2}\sqrt{\frac{1+\omega^2}{(9+\omega^2)(16+\omega^2)(25+\omega^2)}}$$

$$\begin{cases} \angle N_1(j\omega) = 0 \\ \angle N_2(j\omega) = \tan^{-1}\frac{\omega}{0} = \tan^{-1}\infty = 90^\circ \\ \angle N_3(j\omega) = \tan^{-1}\frac{\omega}{1} = \tan^{-1}\omega \\ \angle D_1(j\omega) = \tan^{-1}\frac{\omega}{3} \\ \angle D_2(j\omega) = \tan^{-1}\frac{\omega}{4} \\ \angle D_3(j\omega) = \tan^{-1}\frac{\omega}{5} \end{cases} \rightarrow G_p(\omega) = 90^\circ + \tan^{-1}\omega - \left(\tan^{-1}\frac{\omega}{3} + \tan^{-1}\frac{\omega}{4} + \tan^{-1}\frac{\omega}{5}\right) \quad /\!/$$

(4) $P(s) = \dfrac{1}{(s+1)^{10}} = \dfrac{1}{D(s)^{10}}$

$$D(s) = s+1 \rightarrow D(j\omega) = 1 + j\omega \rightarrow \begin{cases} |D(j\omega)| = \sqrt{1+\omega^2} \\ \angle D(j\omega) = \tan^{-1}\omega \end{cases}$$

$$\rightarrow G_g(\omega) = |P(j\omega)| = \frac{1}{|D(j\omega)|^{10}} = \frac{1}{(1+\omega^2)^5} \quad /\!/$$

$$G_p(\omega) = \angle P(j\omega) = 0 - 10\angle D(j\omega) = -10\tan^{-1}\omega \quad /$$

問6.3 $P(s) = \dfrac{s+2}{s^2+2s+2} \rightarrow P(j\omega) = \dfrac{2+j\omega}{2-\omega^2+j\cdot 2\omega}$

$$\rightarrow \begin{cases} G_g(\omega) = |P(j\omega)| = \dfrac{\sqrt{4+\omega^2}}{\sqrt{(2-\omega^2)^2+(2\omega)^2}} = \sqrt{\dfrac{4+\omega^2}{4+\omega^4}} \\ G_p(\omega) = \angle P(j\omega) = \tan^{-1}\dfrac{\omega}{2} - \tan^{-1}\dfrac{2\omega}{2-\omega^2} \end{cases}$$

$u(t) = \sin t$ を加えると

$$y(t) \fallingdotseq G_g(\omega)|_{\omega=1}\sin(t + G_p(\omega)|_{\omega=1}) = G_g(1)\sin(t+G_p(1))$$
$$= 1\cdot\sin(t+\phi)$$
$$= \sin(t+\phi) \quad /\!/ , \quad \phi = G_p(1) \fallingdotseq \tan^{-1}\frac{1}{2} - \tan^{-1}\frac{2}{1}$$
$$= \tan^{-1}\frac{1}{2} - \tan^{-1}2 = -\tan^{-1}\frac{3}{4} \quad /\!/$$

$$\boxed{\begin{aligned} &\phi = \phi_1 - \phi_2 \\ &\begin{cases} \phi_1 = \tan^{-1}\frac{1}{2} \\ \phi_2 = \tan^{-1}2 \end{cases} \rightarrow \begin{cases} \tan\phi_1 = \frac{1}{2} \\ \tan\phi_2 = 2 \end{cases} \text{より} \\ &\tan\phi = \tan(\phi_1-\phi_2) = \frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\tan\phi_2} = \frac{\frac{1}{2}-2}{1+\frac{1}{2}\cdot 2} \\ &= \frac{-\frac{3}{2}}{2} = -\frac{3}{4} \rightarrow \phi = \tan^{-1}\left(-\frac{3}{4}\right) = -\tan^{-1}\frac{3}{4} \end{aligned}}$$

問6.4 (1) $P(s) = e^{-Ls} \quad (L>0)$

$\rightarrow P(j\omega) = e^{-j\omega L} = \cos\omega L - j\sin\omega L$

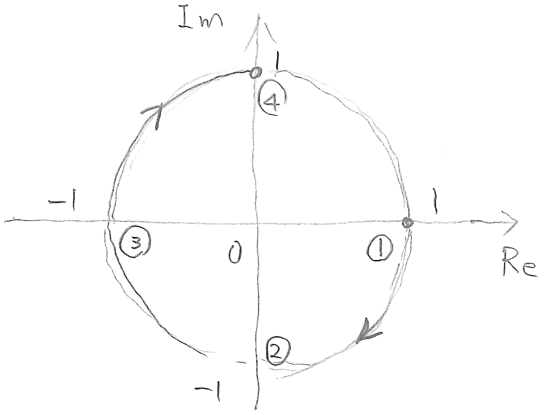
$$\rightarrow \begin{cases} |P(j\omega)| = \sqrt{\cos^2\omega L + \sin^2\omega L} = 1 \;/\!/ \\ \angle P(j\omega) = -\tan^{-1}\dfrac{\sin\omega L}{\cos\omega L} = -\tan^{-1}(\tan\omega L) = -\omega L \;/\!/ \end{cases}$$

① $(1,0)$ で交わるのは $\omega L = 2n\pi \longrightarrow \omega = 0, \frac{2\pi}{L}, \frac{4\pi}{L}, \cdots$

② $(0,-1)$ —— $\omega L = \frac{\pi}{2} + 2n\pi \longrightarrow \omega = \frac{\pi}{2L}, \frac{5\pi}{2L}, \frac{9\pi}{2L}, \cdots$

③ $(-1,0)$ —— $\omega L = \pi + 2n\pi \longrightarrow \omega = \frac{\pi}{L}, \frac{3\pi}{L}, \frac{5\pi}{L}, \cdots$

④ $(0,1)$ —— $\omega L = \frac{3}{2}\pi + 2n\pi \longrightarrow \omega = \frac{3\pi}{2L}, \frac{7\pi}{2L}, \frac{11\pi}{2L}, \cdots$



(2) $P(s) = \dfrac{e^{-Ls}}{1+Ts} \quad (L=1,\ T=0.2)$

$\rightarrow P(j\omega) = \dfrac{e^{-j\omega L}}{1+j\omega T} = \dfrac{\cos\omega L - j\sin\omega L}{1+j\omega T}$

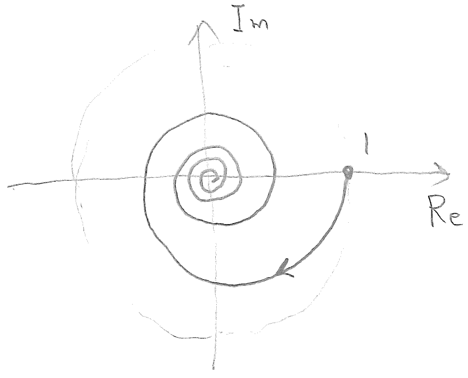
$$\rightarrow \begin{cases} |P(j\omega)| = \dfrac{|\cos\omega L - j\sin\omega L|}{|1+j\omega T|} = \dfrac{1}{\sqrt{1+(\omega T)^2}} \\ \angle P(j\omega) = -\tan^{-1}\dfrac{\sin\omega L}{\cos\omega L} - \tan^{-1}\dfrac{\omega T}{1} = -(\omega L + \tan^{-1}\omega T) \end{cases}$$

$\omega = 0$ のとき

$|P(j\omega)| = 1$, $\angle P(j\omega) = 0$ なので $(1,0)$

$\omega \to \infty$ のとき

$|P(j\omega)| = 0$ なので $(0,0)$



問6.5 $\dfrac{B(\omega_1)}{A} = 10$, $\dfrac{B(\omega_2)}{A} = 1$, $\dfrac{B(\omega_3)}{A} = \dfrac{1}{10} = 0.1$, $\dfrac{B(\omega_4)}{A} = \dfrac{1}{100} = 0.01$

問6.6 $P(j\omega) = \dfrac{1}{1+j\omega T} = \dfrac{1}{1+(\omega T)^2} + j\left\{-\dfrac{\omega T}{1+(\omega T)^2}\right\} = \alpha + j\beta$

$$\rightarrow \left(\alpha - \frac{1}{2}\right)^2 + \beta^2 = \left\{\frac{1}{1+(\omega T)^2} - \frac{1}{2}\right\}^2 + \left\{-\frac{\omega T}{1+(\omega T)^2}\right\}^2$$

$$\downarrow \ \bar{\omega} = \omega T$$

$$= \left(\frac{1}{1+\bar{\omega}^2} - \frac{1}{2}\right)^2 + \left(-\frac{\bar{\omega}}{1+\bar{\omega}^2}\right)^2$$

$$= \frac{1}{(1+\bar{\omega}^2)^2} - \frac{1}{1+\bar{\omega}^2} + \frac{1}{4} + \frac{\bar{\omega}^2}{(1+\bar{\omega}^2)^2}$$

$$= \frac{1-(1+\bar{\omega}^2)+\bar{\omega}^2}{(1+\bar{\omega}^2)^2} + \frac{1}{4}$$

$$= \left(\frac{1}{2}\right)^2$$

なので中心 $\left(\frac{1}{2}, 0\right)$，半径 $\frac{1}{2}$ の円となる．

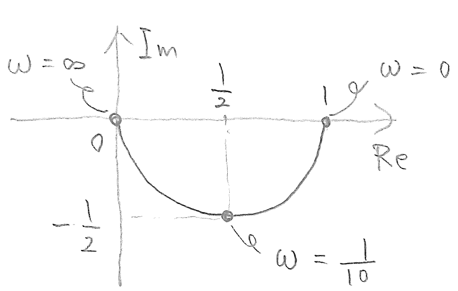
問6.7 $P(j\omega) = \dfrac{1}{1+j\cdot 10\omega}$

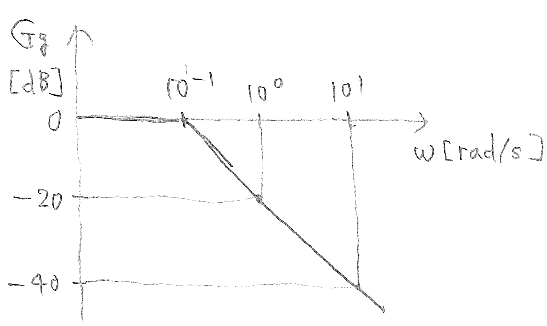
$$\begin{cases} G_g(\omega) = |P(j\omega)| = \dfrac{1}{\sqrt{1+(10\omega)^2}} \\ G_p(\omega) = \angle P(j\omega) = -\tan^{-1} 10\omega \end{cases}$$

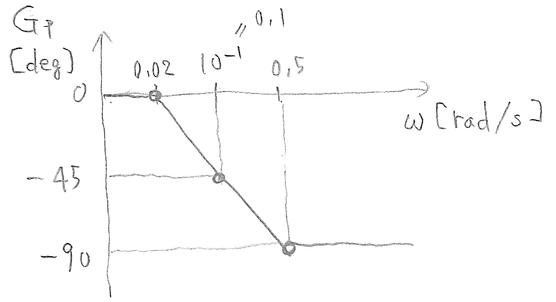
(i) $\omega \ll \frac{1}{10}$ のとき $(10\omega \ll 1)$　$G_g(\omega) \fallingdotseq 1$，$G_p(\omega) \fallingdotseq 0$

(ii) $\omega = \frac{1}{10}$ のとき $(10\omega = 1)$　$G_g(\omega) = \frac{1}{\sqrt{2}}$，$G_p(\omega) = -45°$

(iii) $\omega \gg \frac{1}{10}$ のとき $(10\omega \gg 1)$　$G_g(\omega) \fallingdotseq \frac{1}{10\omega}$，$G_p(\omega) \fallingdotseq -90°$







問6.8 (1) $P(s) = \dfrac{1}{LCs^2 + RCs + 1} = \dfrac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$

$$= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\rightarrow \begin{cases} \omega_n^2 = \dfrac{1}{LC} \\ 2\zeta\omega_n = \dfrac{R}{L} \end{cases} \rightarrow \begin{cases} \omega_n = \dfrac{1}{\sqrt{LC}} \\ \zeta = \dfrac{1}{2\omega_n}\dfrac{R}{L} = \dfrac{R}{2L}\sqrt{LC} = \dfrac{R}{2}\sqrt{\dfrac{C}{L}} \end{cases}$$

$4 \cdot 10^4$

共振が生じないのは，$L = 200\,[\mathrm{mH}]$，$C = 10\,[\mu\mathrm{F}]$ のとき．

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} \geq \frac{1}{\sqrt{2}} \rightarrow R \geq \frac{2}{\sqrt{2}}\sqrt{\frac{L}{C}} = \sqrt{\frac{2L}{C}} = \sqrt{\frac{2 \times 200 \cdot 10^{-3}}{10 \cdot 10^{-6}}}$$

$$= 2 \cdot 10^2 = 200\,[\Omega] \;/\!/$$

(2) $R = 100\,[\Omega]$，$L = 200\,[\mathrm{mH}]$，$C = 10\,[\mu\mathrm{F}]$ のとき．

$$\begin{cases} \omega_n = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{200 \cdot 10^{-3} \times 10 \cdot 10^{-6}}} = \dfrac{1}{\sqrt{2 \cdot 10^2 \cdot 10^{-3} \cdot 10^{-5}}} = \dfrac{10^3}{\sqrt{2}} = 500\sqrt{2} \\ \zeta = \dfrac{R}{2}\sqrt{\dfrac{C}{L}} = \dfrac{100}{2}\sqrt{\dfrac{10 \cdot 10^{-6}}{200 \cdot 10^{-3}}} = \dfrac{10^2}{2}\sqrt{\dfrac{10^{-5}}{2 \cdot 10^{-1}}} = \dfrac{1}{2\sqrt{2}} \end{cases}$$

なので

$$\omega_p = \omega_n\sqrt{1 - 2\zeta^2} = \frac{10^3}{\sqrt{2}}\sqrt{1 - 2 \cdot \frac{1}{4 \cdot 2}} = \frac{10^3}{\sqrt{2}}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2\sqrt{2}} \times 10^3\,[\mathrm{rad/s}]$$

$$M_p = \frac{1}{2\zeta\sqrt{1 - \zeta^2}} = \frac{1}{2 \cdot \frac{1}{2\sqrt{2}}\sqrt{1 - \frac{1}{4 \cdot 2}}} = \frac{1}{\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{7}}{2\sqrt{2}}} = \frac{4}{\sqrt{7}}$$

問7.1 $P(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$, $C(s) = k_P > 0$

$$\rightarrow P(j\omega) = \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j\cdot 2\zeta\omega_n\omega} = \frac{1}{1-\eta^2 + j\cdot 2\zeta\eta} \ , \ \eta = \frac{\omega}{\omega_n}$$

$$\rightarrow L(j\omega) = P(j\omega)C(j\omega) = \frac{k_P}{\alpha + j\beta}$$

$$= \frac{k_P(\alpha - j\beta)}{\alpha^2 + \beta^2} \ , \ \begin{cases} \alpha = 1-\eta^2 \\ \beta = 2\zeta\eta \end{cases}$$

- 虚軸と交わる $0 < \omega < \infty$ は

$$\mathrm{Re}[L(j\omega)] = 0 \rightarrow \alpha = 1-\eta^2 = 0 \rightarrow \eta = 1 \rightarrow \omega = \omega_n \ /\!/$$

このとき

$$L(j\omega)|_{\eta=1} = \frac{k_P(0 - j\cdot 2\zeta)}{0^2 + (2\zeta)^2} = -j\frac{k_P}{2\zeta} \rightarrow \left(0, -\frac{k_P}{2\zeta}\right)$$

- 実軸と交わる $0 < \omega < \infty$ は

$$\mathrm{Im}[L(j\omega)] = 0 \rightarrow \beta = 2\zeta\eta = \frac{2\zeta\omega}{\omega_n} > 0 \ /\!/$$

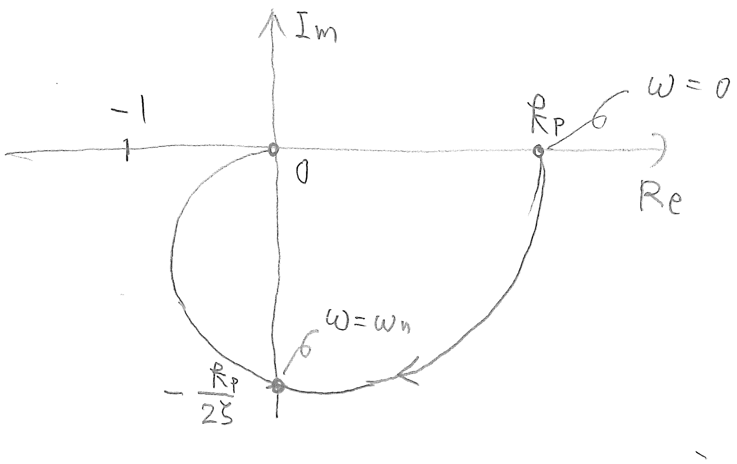
より存在しない.

- 始点は $L(j\omega)|_{\omega=0} = L(j\omega)|_{\eta=0} = \dfrac{k_P(1 - j\cdot 0)}{1} = k_P$ //

$\rightarrow (k_P, 0)$

- 終点は $L(j\omega)|_{\omega\to\infty} = L(j\omega)|_{\eta\to\infty} = 0$

$\rightarrow (0, 0)$



$L(j\omega)$ は $(-1, 0)$ を常に左に見るので安定.

問7.2 (1) $P(s) = \dfrac{1}{(s+1)^2(s+2)}$, $C(s) = k_P > 0$

ナイキスト

$$L(s) = P(s)C(s) = \frac{k_P}{(s+1)^2(s+2)} = \frac{k_P}{(s^2+2s+1)(s+2)}$$

$$= \frac{k_P}{s^3+4s^2+5s+2}$$

$$L(j\omega) = \frac{k_P}{2-4\omega^2+j(5\omega-\omega^3)} = \frac{k_P}{\alpha+j\beta} \quad \begin{cases} \alpha = 2(1-2\omega^2) \\ \beta = \omega(5-\omega^2) \end{cases}$$

$$= \frac{k_P(\alpha-j\beta)}{\alpha^2+\beta^2}$$

- $\omega = 0$ のとき

$$\begin{cases} \alpha = 2 \\ \beta = 0 \end{cases} \rightarrow L(j\omega) = \frac{k_P(2-j\cdot 0)}{4+0} = \frac{1}{2}k_P + j\cdot 0 \;/\!/$$

$\rightarrow$ 始点 $\left(\frac{1}{2}k_P, 0\right)$

- $\omega \to \infty$ のとき

$L(j\omega)$の分母 $\gg$ 分子 なので $L(j\omega) = 0 = 0 + j\cdot 0 \rightarrow$ 終点 $(0, 0)$

- 虚軸との交点は

$$\alpha = 2(1-2\omega^2) = 0 \rightarrow \omega = \frac{1}{\sqrt{2}} \rightarrow \beta = \frac{1}{\sqrt{2}}\left(5-\frac{1}{2}\right) = \frac{9}{2\sqrt{2}}$$

$$\rightarrow L(j\omega)\big|_{\omega=\frac{1}{\sqrt{2}}} = \frac{k_P}{j\cdot\frac{9}{2\sqrt{2}}} = 0 - j\cdot\frac{2\sqrt{2}}{9}k_P$$

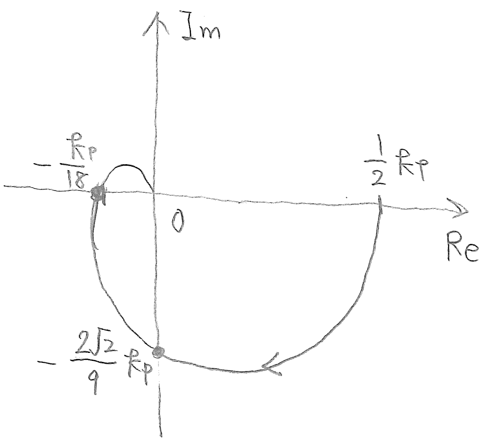
$$\rightarrow \left(0, -\frac{2\sqrt{2}}{9}k_P\right)$$

- 実軸との交点は

$$\beta = \omega(5-\omega^2) = 0 \rightarrow \omega = \omega_{pc} = \sqrt{5} \rightarrow \alpha = 2(1-2\cdot 5) = -18$$

$$\rightarrow L(j\omega)\big|_{\omega=\sqrt{5}} = \frac{k_P}{-18+j\cdot 0} = -\frac{k_P}{18} + j\cdot 0$$

$$\rightarrow \left(-\frac{k_P}{18}, 0\right)$$



安定となるのは

$$-1 < -\frac{k_P}{18} \rightarrow k_P < 18$$

$k_P > 0$ なので

$$0 < k_P < 18 \;/\!/$$

フルビッツ

$$\Delta(s) = (s+1)^2(s+2) + k_p = s^3 + 4s^2 + 5s + 2 + k_p$$
$$= a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

条件I
$$\begin{cases} a_3 = 1 > 0 \\ a_2 = 4 > 0 \\ a_1 = 5 > 0 \\ a_0 = 2 + k_p > 0 \quad \rightarrow k_p > -2 \end{cases}$$

条件II
$$H = \begin{bmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{bmatrix}$$

$$\rightarrow H_2 = \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = \begin{vmatrix} 4 & 2+k_p \\ 1 & 5 \end{vmatrix} = 20 - (2 + k_p)$$
$$= 18 - k_p > 0$$
$$\rightarrow k_p < 18$$

$k_p > 0$ かつ 条件I, II より $0 < k_p < 18$ //

(2) $P(s) = \dfrac{1}{s(s+1)(s+2)}$, $C(s) = k_p > 0$

ナイキスト

$$L(s) = P(s)C(s) = \frac{k_p}{s(s+1)(s+2)} = \frac{k_p}{s^3+3s^2+2s}$$

$$L(j\omega) = \frac{k_p}{-3\omega^2 + j(2\omega-\omega^3)} = \frac{k_p}{\alpha + j\beta} \quad \begin{cases} \alpha = -3\omega^2 \\ \beta = \omega(2-\omega^2) \end{cases}$$

$$= \frac{k_p(\alpha - j\beta)}{\alpha^2+\beta^2}$$

ここで

$$\mathrm{Re}[L(j\omega)] = \frac{k_p\alpha}{\alpha^2+\beta^2} = -\frac{3k_p\omega^2}{9\omega^4+\omega^2(2-\omega^2)^2} = -\frac{3k_p}{9\omega^2+(2-\omega^2)^2}$$

$$= -\frac{3k_p}{\omega^4+5\omega^2+4}$$

$$\mathrm{Im}[L(j\omega)] = -\frac{k_p\beta}{\alpha^2+\beta^2} = -\frac{k_p\omega(2-\omega^2)}{9\omega^4+\omega^2(2-\omega^2)^2} = -\frac{k_p(2-\omega^2)}{9\omega^3+\omega(2-\omega^2)^2}$$

$$= -\frac{k_p(2-\omega^2)}{\omega(\omega^4+5\omega^2+4)}$$

より

$$\begin{cases} \lim_{\omega\to 0} \mathrm{Re}[L(j\omega)] = -\frac{3}{4}k_p \\ \lim_{\omega\to +0} \mathrm{Im}[L(j\omega)] = -\infty \end{cases}$$ (プラス側から ω を 0 に近づける)

→ 始点 $\left(-\frac{3}{4}k_p, -\infty\right)$

一方 $\omega \to \infty$ とすると

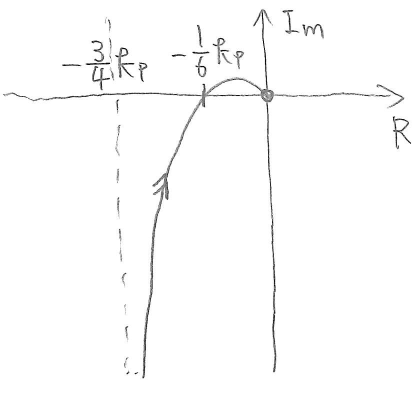
$L(j\omega) \to 0$ ⇒ 終点 $(0, 0)$

- 虚軸と交わる $0<\omega<\infty$ は. 存在しない ($\mathrm{Re}[L(j\omega)] < 0$)
- 実軸と交わる $0<\omega<\infty$ は

$\mathrm{Im}[L(j\omega)] = 0 \to 2-\omega^2 = 0 \to \omega = \omega_{pc} = \sqrt{2}$

$\to \begin{cases} \alpha = -3\cdot 2 = -6 \\ \beta = 0 \end{cases} \to L(j\omega) = \dfrac{k_p}{-6+j\cdot 0} = -\dfrac{k_p}{6} + j\cdot 0$

$\to \left(-\dfrac{k_p}{6}, 0\right)$



安定となるのは

$-1 < -\dfrac{k_p}{6} \to k_p < 6$

$k_p > 0$ なので

$0 < k_p < 6$ //

フルビッツ

$$\Delta(s) = s(s+1)(s+2) + k_p = s^3 + 3s^2 + 2s + k_p$$
$$= a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

条件I
$$\begin{cases} a_3 = 1 > 0 \\ a_2 = 3 > 0 \\ a_1 = 2 > 0 \\ a_0 = k_p > 0 \end{cases}$$

条件II
$$H = \begin{bmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{bmatrix}$$

$$\longrightarrow H_2 = \begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = \begin{vmatrix} 3 & k_p \\ 1 & 2 \end{vmatrix} = 6 - k_p > 0 \rightarrow k_p < 6$$

$k_p > 0$ かつ 条件I, II より $0 < k_p < 6$ //

問7.3 (1) $P(s) = \dfrac{1}{(s+2)^4}$, $C(s) = k_p > 0$

$$\rightarrow L(s) = P(s)C(s) = \frac{k_p}{(s+2)^4} \rightarrow L(j\omega) = \frac{k_p}{(2+j\omega)^4}$$

$$\rightarrow \begin{cases} |L(j\omega)| = \dfrac{k_p}{(4+\omega^2)^2} \\ \angle L(j\omega) = -\tan^{-1}\dfrac{\omega}{2} \times 4 = -4\tan^{-1}\dfrac{\omega}{2} \end{cases}$$

$\underline{\omega_{pc},\ G_m}$

$$\angle L(j\omega_{pc}) = -4\tan^{-1}\frac{\omega_{pc}}{2} = -180^\circ \rightarrow \tan^{-1}\frac{\omega_{pc}}{2} = 45^\circ$$

$$\rightarrow \frac{\omega_{pc}}{2} = \tan 45^\circ = 1 \rightarrow \omega_{pc} = 2\ [\mathrm{rad/s}]_{/\!/}$$

$$\rightarrow |L(j\omega_{pc})| = \frac{k_p}{(4+2^2)^2} = \frac{k_p}{64}$$

$$\rightarrow G_m = -20\log_{10}|L(j\omega_{pc})| = -20\log_{10}\frac{k_p}{64}\ [\mathrm{dB}]_{/\!/}$$

$\underline{\omega_{gc},\ P_m}$

$$|L(j\omega_{gc})| = \frac{k_p}{(4+\omega_{gc}^2)^2} = 1 \rightarrow (4+\omega_{gc}^2)^2 = k_p$$

$$\rightarrow 4+\omega_{gc}^2 = \sqrt{k_p} > 0 \rightarrow \omega_{gc} = \sqrt{\sqrt{k_p}-4}\ [\mathrm{rad/s}]\ (k_p > 16)_{/\!/}$$

$$\rightarrow P_m = 180^\circ + \angle L(j\omega_{gc}) = 180^\circ - 4\tan^{-1}\frac{\omega_{gc}}{2}$$

$$= 180^\circ - 4\tan^{-1}\frac{\sqrt{\sqrt{k_p}-4}}{2}\ [\mathrm{deg}]_{/\!/}$$

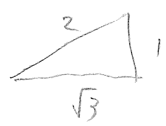
(2) $P_m = 180^\circ - 4\tan^{-1}\dfrac{\sqrt{\sqrt{k_p}-4}}{2} = 60^\circ$

$$\rightarrow 4\tan^{-1}\frac{\sqrt{\sqrt{k_p}-4}}{2} = 120^\circ \rightarrow \tan^{-1}\frac{\sqrt{\sqrt{k_p}-4}}{2} = 30^\circ$$

$$\rightarrow \frac{\sqrt{\sqrt{k_p}-4}}{2} = \tan 30^\circ = \frac{1}{\sqrt{3}} \rightarrow \sqrt{\sqrt{k_p}-4} = \frac{2}{\sqrt{3}}$$

$$\rightarrow \sqrt{k_p} - 4 = \frac{4}{3} \rightarrow \sqrt{k_p} = \frac{4}{3} + 4 = \frac{16}{3}$$

$$\rightarrow k_p = \frac{256}{9} \fallingdotseq 28.44_{/\!/}$$



問8.1 (1) $M\ddot{z} = c\dot{z} + f$

ここで

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}, \quad u = f, \quad y = z$$

とすると

$$\dot{x}_1 = \dot{z} = x_2$$
$$= 0 \cdot x_1 + 1 \cdot x_2 + 0 \cdot u$$
$$\dot{x}_2 = \ddot{z} = -\frac{c}{M}\dot{z} + \frac{1}{M}f$$
$$= 0 \cdot x_1 - \frac{c}{M}x_2 + \frac{1}{M}u$$
$$y = z = x_1$$
$$= 1 \cdot x_1 + 0 \cdot x_2 + 0 \cdot u$$

なので

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{c}{M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot u \end{cases} \rightarrow \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$\rightarrow \quad A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{c}{M} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0 \qquad //$$

一方，伝達関数表現に変換すると．

$$y(s) = P(s)u(s)$$

$$P(s) = C(sI - A)^{-1}B + D$$
$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \left(s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -\frac{c}{M} \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} + 0$$
$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ 0 & s + \frac{c}{M} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}$$
$$= \frac{1}{s\left(s + \frac{c}{M}\right)} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s + \frac{c}{M} & 1 \\ 0 & s \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}$$
$$= \frac{\frac{1}{M}}{s\left(s + \frac{c}{M}\right)} = \frac{1}{s(Ms + c)} \qquad //$$

(2) 回路方程式は

$$\begin{cases} u = Ri + L\dfrac{di}{dt} + \dfrac{1}{C}\displaystyle\int_0^t i\,dt = R\dot{q} + L\ddot{q} + \dfrac{1}{C}q \\ y = \dfrac{1}{C}\displaystyle\int_0^t i\,dt = \dfrac{1}{C}q \end{cases}$$

($i = \dot{q}$, q : 電荷)

ここで

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

とすると.

$$\begin{aligned} \dot{x}_1 &= \dot{q} = x_2 \\ &= 0 \cdot x_1 + 1 \cdot x_2 + 0 \cdot u \\ \dot{x}_2 &= \ddot{q} = \frac{1}{L}\left(u - R\dot{q} - \frac{1}{C}q\right) \\ &= -\frac{1}{LC}q - \frac{R}{L}\dot{q} + \frac{1}{L}u \\ &= -\frac{1}{LC}x_1 - \frac{R}{L}x_2 + \frac{1}{L}u \\ y &= \frac{1}{C}q \\ &= \frac{1}{C}x_1 + 0 \cdot x_2 + 0 \cdot u \end{aligned}$$

なので,

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{LC} & -\dfrac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} u \\ y = \begin{bmatrix} \dfrac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot u \end{cases} \rightarrow \begin{cases} \dot{x} = Ax + Bu \\ y = \mathbb{C}x + Du \end{cases}$$

$$\rightarrow A = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{LC} & -\dfrac{R}{L} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix}$$

$$\mathbb{C} = \begin{bmatrix} \dfrac{1}{C} & 0 \end{bmatrix}, \quad D = 0 \quad //$$

伝達関数表現に変換すると

$$y(s) = P(s)u(s)$$

$$
\begin{aligned}
P(s) &= \mathbb{C}(sI - A)^{-1}B + D \\
&= \begin{bmatrix} \frac{1}{C} & 0 \end{bmatrix} \left(s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} + 0 \\
&= \begin{bmatrix} \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ \frac{1}{LC} & s + \frac{R}{L} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \\
&= \frac{1}{s\left(s + \frac{R}{L}\right) + \frac{1}{LC}} \begin{bmatrix} \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} s + \frac{R}{L} & 1 \\ -\frac{1}{LC} & s \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \\
&= \frac{\frac{1}{LC}}{s\left(s + \frac{R}{L}\right) + \frac{1}{LC}} \\
&= \frac{1}{LCs^2 + RCs + 1} \;/\!/
\end{aligned}
$$

問8.2 $A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$

$$\rightarrow (sI-A)^{-1} = \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}^{-1} = \frac{1}{s^2+5s+6}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}$$

$$= \frac{1}{(s+2)(s+3)}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix} = \frac{1}{s+2}K_1 + \frac{1}{s+3}K_2$$

$$K_1 = (s+2)(sI-A)^{-1}\Big|_{s=-2} = \frac{1}{s+3}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}\Bigg|_{s=-2}$$

$$= \frac{1}{1}\begin{bmatrix} 3 & 1 \\ -6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -6 & -2 \end{bmatrix} /\!/$$

$$K_2 = (s+3)(sI-A)^{-1}\Big|_{s=-3} = \frac{1}{s+2}\begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}\Bigg|_{s=-3}$$

$$= \frac{1}{-1}\begin{bmatrix} 2 & 1 \\ -6 & -3 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 6 & 3 \end{bmatrix} /\!/$$

$$\rightarrow e^{At} = \mathcal{L}^{-1}\left[(sI-A)^{-1}\right] = e^{-2t}K_1 + e^{-3t}K_2$$

$$= \begin{bmatrix} 3e^{-2t} - 2e^{-3t} & e^{-2t} - e^{-3t} \\ -6e^{-2t} + 6e^{-3t} & -2e^{-2t} + 3e^{-3t} \end{bmatrix} /\!/$$