

most Frequent Value : given an array of size  $n$ , return the value which occurs the most.

Problem size:  $n$ , the number of rows or columns.

To make analysis easier, we can divide the algorithm into three parts, thus making  $T(n) = T_1(n) + T_2(n) + T_3(n)$

$T_1(n)$  : Initializing variable count

when we initialize an array it will always require  $n$  operations.

In Java an array will be initialized with the value zero, but in languages like C/C++ we would need another loop to initialize element values.

$$\therefore T_1(n) = O(n)$$

$$T_1(n) = \Omega(n)$$

$T_2(n)$

The second section is our nested for loops. When analyzing algorithms we want to focus on instructions that are important for the algorithm.

This algorithm we'll focus on

```
int val = arr[row][col];    ①  
count++;                  ②
```

We can decompose ① into 2 operations  $arr[row][col]$  and assigning a value to  $val$ .  $arr[row][col]$  counts as 1 operation since we access the address in memory by multiplying the size of  $int$  by the offset ( $row \times col$ ). We

currently have ~~3~~ operations. We can add an additional  $n$  operations by incrementing  $col$  in each inner loop.

Adding our declaration of col brings our inner loop to  $4n+1$ , which the outer loop will always do  $n$  times.

$$n(4n+1) = 4n^2 + n$$

$$\therefore T_2(n) = O(n^2)$$

$$T_2(n) = \Omega(n^2)$$

For our last section we need to look at two scenarios.

I. Worst Case: when count is in ascending order.

First we initialize max  $O(n) = 1$

Next, our for loop performs a linear search across our array. The body of the for loop will perform 4 operation each loop (when its the worst case scenario)

1.  $\text{counts}[\text{val}]$  (we'll call this  $\alpha$ )
2.  $\alpha > \text{max}$
3.  $\text{counts}[\text{val}]$  (we'll call this  $\beta$ )
4.  $\text{max} = \beta$

Since these will fire each loop in the worst case scenario

$$T(n) = O(4n+1) = O(n)$$

## II. Best case scenario

The zeroth element is the largest

This would change our last formula

to  $T(n) = \Omega(n+5)$  since the if statement is only fired once.

In both cases  $T_3 = O(n)$

$$T_3 = \Omega(n)$$

Now we can sum these equations together

$$\begin{aligned}T(n) &= T_1(n) + T_2(n) + T_3(n) \\&= O(n) + O(n^2) + O(n) \\&= \max(O(n), O(n^2)) + O(n) \\&= O(n^2) + O(n) \\&= \max(O(n^2), O(n)) \\&= O(n^2)\end{aligned}$$

$$\begin{aligned}T(n) &= \Omega(n) + \Omega(n^2) + \Omega(n) \\&= \max(\Omega(n), \Omega(n^2)) + \Omega(n) \\&= \Omega(n^2) + \Omega(n) \\&= \max(\Omega(n^2), \Omega(n)) \\&= \Omega(n^2)\end{aligned}$$

Since  $O(n^2) = \Omega(n^2)$  then  
we can say  $\Theta(n^2)$

Here we were able to create the  
equations for our algorithm. We simplified  
steps in order to determine  $O(\cdot)$  and  $\Omega(\cdot)$   
but we could have easily found  
constants  $c_1$ ,  $c_2$  and  $n_0$