5. Make a decision and write a meaningful conclusion.

NOTE

To do the hypothesis test homework problems for this chapter and later chapters, make copies of the appropriate special solution sheets. See Appendix E.

9.1 Null and Alternative Hypotheses

The actual test begins by considering two **hypotheses**. They are called the **null hypothesis** and the **alternative hypothesis**. These hypotheses contain opposing viewpoints.

H_0: **The null hypothesis:** It is a statement of no difference between the variables—they are not related. This can often be considered the status quo and as a result if you cannot accept the null it requires some action.

H_a: **The alternative hypothesis:** It is a claim about the population that is contradictory to H_0 and what we conclude when we reject H_0. This is usually what the researcher is trying to prove.

Since the null and alternative hypotheses are contradictory, you must examine evidence to decide if you have enough evidence to reject the null hypothesis or not. The evidence is in the form of sample data.

After you have determined which hypothesis the sample supports, you make a **decision.** There are two options for a decision. They are "reject H_0" if the sample information favors the alternative hypothesis or "do not reject H_0" or "decline to reject H_0" if the sample information is insufficient to reject the null hypothesis.

Mathematical Symbols Used in H_0 and H_a:

H_0	H_a
equal (=)	not equal (≠) **or** greater than (>) **or** less than (<)
greater than or equal to (≥)	less than (<)
less than or equal to (≤)	more than (>)

Table 9.1

NOTE

H_0 always has a symbol with an equal in it. H_a never has a symbol with an equal in it. The choice of symbol depends on the wording of the hypothesis test. However, be aware that many researchers (including one of the co-authors in research work) use = in the null hypothesis, even with > or < as the symbol in the alternative hypothesis. This practice is acceptable because we only make the decision to reject or not reject the null hypothesis.

EXAMPLE 9.1

H_0: No more than 30% of the registered voters in Santa Clara County voted in the primary election. $p \leq .30$
H_a: More than 30% of the registered voters in Santa Clara County voted in the primary election. $p > 30$

TRY IT 9.1

A medical trial is conducted to test whether or not a new medicine reduces cholesterol by 25%. State the null and alternative hypotheses.

EXAMPLE 9.2

We want to test whether the mean GPA of students in American colleges is different from 2.0 (out of 4.0). The null and alternative hypotheses are:
H_0: $\mu = 2.0$
H_a: $\mu \neq 2.0$

TRY IT 9.2

We want to test whether the mean height of eighth graders is 66 inches. State the null and alternative hypotheses. Fill in the correct symbol (=, ≠, ≥, <, ≤, >) for the null and alternative hypotheses.

a. H_0: μ __ 66
b. H_a: μ __ 66

EXAMPLE 9.3

We want to test if college students take less than five years to graduate from college, on the average. The null and alternative hypotheses are:
H_0: $\mu \geq 5$
H_a: $\mu < 5$

TRY IT 9.3

We want to test if it takes fewer than 45 minutes to teach a lesson plan. State the null and alternative hypotheses. Fill in the correct symbol (=, ≠, ≥, <, ≤, >) for the null and alternative hypotheses.

a. H_0: μ __ 45
b. H_a: μ __ 45

EXAMPLE 9.4

In an issue of *U. S. News and World Report*, an article on school standards stated that about half of all students in France, Germany, and Israel take advanced placement exams and a third pass. The same article stated that 6.6% of U.S. students take advanced placement exams and 4.4% pass. Test if the percentage of U.S. students who take advanced placement exams is more than 6.6%. State the null and alternative hypotheses.
H_0: $p \leq 0.066$
H_a: $p > 0.066$

TRY IT 9.4

On a state driver's test, about 40% pass the test on the first try. We want to test if more than 40% pass on the first try. Fill in the correct symbol (=, ≠, ≥, <, ≤, >) for the null and alternative hypotheses.

a. H_0: p __ 0.40
b. H_a: p __ 0.40

COLLABORATIVE EXERCISE

Bring to class a newspaper, some news magazines, and some Internet articles . In groups, find articles from which your group can write null and alternative hypotheses. Discuss your hypotheses with the rest of the class.

9.2 Outcomes and the Type I and Type II Errors

When you perform a hypothesis test, there are four possible outcomes depending on the actual truth (or falseness) of the null hypothesis H_0 and the decision to reject or not. The outcomes are summarized in the following table:

ACTION	H_0 IS ACTUALLY	...
	True	False
Do not reject H_0	Correct Outcome	Type II error
Reject H_0	Type I Error	Correct Outcome

Table 9.2

The four possible outcomes in the table are:

1. The decision is **not to reject H_0** when **H_0 is true (correct decision).**
2. The decision is to **reject H_0** when **H_0 is true** (incorrect decision known as a**Type I error**).
3. The decision is **not to reject H_0** when, in fact, **H_0 is false** (incorrect decision known as a **Type II error**).
4. The decision is to **reject H_0** when **H_0 is false** (**correct decision** whose probability is called the **Power of the Test**).

Each of the errors occurs with a particular probability. The Greek letters α and β represent the probabilities.

α = probability of a Type I error = ***P*(Type I error)** = probability of rejecting the null hypothesis when the null hypothesis is true.

β = probability of a Type II error = ***P*(Type II error)** = probability of not rejecting the null hypothesis when the null hypothesis is false.

α and β should be as small as possible because they are probabilities of errors. They are rarely zero.

The Power of the Test is 1 – β. Ideally, we want a high power that is as close to one as possible. Increasing the sample size can increase the Power of the Test.

The following are examples of Type I and Type II errors.

EXAMPLE 9.5

Suppose the null hypothesis, H_0, is: Navah's rock climbing equipment is safe.

Type I error: Navah thinks that her rock climbing equipment may not be safe when, in fact, it really is safe. **Type II error**: Navah thinks that her rock climbing equipment may be safe when, in fact, it is not safe.

α = probability that Navah thinks her rock climbing equipment may not be safe when, in fact, it really is safe. **β = probability** that Navah thinks her rock climbing equipment may be safe when, in fact, it is not safe.

Notice that, in this case, the error with the greater consequence is the Type II error. (If Navah thinks her rock climbing equipment is safe, she will go ahead and use it.)

TRY IT 9.5

Suppose the null hypothesis, H_0, is: the blood cultures contain no traces of pathogen *X*. State the Type I and Type II errors.

EXAMPLE 9.6

Suppose the null hypothesis, H_0, is: The victim of an automobile accident is alive when they arrive at the emergency room of a hospital.

Type I error: The emergency crew thinks that the victim is dead when, in fact, the victim is alive. **Type II error**: The emergency crew does not know if the victim is alive when, in fact, the victim is dead.

***α* = probability** that the emergency crew thinks the victim is dead when, in fact, the victim is really alive = *P*(Type I error). ***β* = probability** that the emergency crew does not know if the victim is alive when, in fact, the victim is dead = *P*(Type II error).

The error with the greater consequence is the Type I error. (If the emergency crew thinks the victim is dead, they will not treat them.)

TRY IT 9.6

Suppose the null hypothesis, H_0, is: a patient is not sick. Which type of error has the greater consequence, Type I or Type II?

EXAMPLE 9.7

A company called Genetic Labs claims to be able to increase the likelihood that a pregnancy will result in a male being born. Statisticians want to test the claim. Suppose that the null hypothesis, H_0, is: Genetic Labs has no effect on sex outcome.

Type I error: This results when a true null hypothesis is rejected. In the context of this scenario, we would state that we believe that Genetic Labs influences the sex outcome, when in fact it has no effect. The probability of this error occurring is denoted by the Greek letter alpha, α.

Type II error: This results when we fail to reject a false null hypothesis. In context, we would state that Genetic Labs does not influence the sex outcome of a pregnancy when, in fact, it does. The probability of this error occurring is denoted by the Greek letter beta, β.

The error of greater consequence would be the Type I error since people would use the Genetic Labs product in hopes of increasing the chances of having a male.

TRY IT 9.7

"Red tide" is a bloom of poison-producing algae–a few different species of a class of plankton called dinoflagellates. When the weather and water conditions cause these blooms, shellfish such as clams living in the area develop dangerous levels of a paralysis-inducing toxin. In Massachusetts, the Division of Marine Fisheries (DMF) monitors levels of the toxin in shellfish by regular sampling of shellfish along the coastline. If the mean level of toxin in clams exceeds 800 μg (micrograms) of toxin per kg of clam meat in any area, clam harvesting is banned there until the bloom is over and levels of toxin in clams subside. Describe both a Type I and a Type II error in this context, and state which error has the greater consequence.

EXAMPLE 9.8

A certain experimental drug claims a cure rate of at least 75% for males with prostate cancer. Describe both the Type I and Type II errors in context. Which error is the more serious?

Type I: A cancer patient believes the cure rate for the drug is less than 75% when it actually is at least 75%.

Type II: A cancer patient believes the experimental drug has at least a 75% cure rate when it has a cure rate that is less than 75%.

In this scenario, the Type II error contains the more severe consequence. If a patient believes the drug works at least 75% of the time, this most likely will influence the patient's (and doctor's) choice about whether to use the drug as a treatment option.

TRY IT 9.8

Determine both Type I and Type II errors for the following scenario:

Assume a null hypothesis, H_0, that states the percentage of adults with jobs is at least 88%.

Identify the Type I and Type II errors from these four statements.

a. Not to reject the null hypothesis that the percentage of adults who have jobs is at least 88% when that percentage is actually less than 88%
b. Not to reject the null hypothesis that the percentage of adults who have jobs is at least 88% when the percentage is actually at least 88%.
c. Reject the null hypothesis that the percentage of adults who have jobs is at least 88% when the percentage is actually at least 88%.
d. Reject the null hypothesis that the percentage of adults who have jobs is at least 88% when that percentage is actually less than 88%.

9.3 Probability Distribution Needed for Hypothesis Testing

Earlier in the course, we discussed sampling distributions. Particular distributions are associated with various types of hypothesis testing.

The following table summarizes various hypothesis tests and corresponding probability distributions that will be used to conduct the test (based on the assumptions shown below):

Type of Hypothesis Test	Population Parameter	Estimated value (point estimate)	Probability Distribution Used
Hypothesis test for the mean, when the population standard deviation is known	Population mean μ	Sample mean $\overline{x}$	Normal distribution, $\overline{X} \sim N\left(\mu_X, \frac{\sigma_X}{\sqrt{n}}\right)$
Hypothesis test for the mean, when the population standard deviation is unknown and the distribution of the sample mean is approximately normal	Population mean μ	Sample mean $\overline{x}$	Student's t-distribution, t_{df}
Hypothesis test for proportions	Population proportion p	Sample proportion p'	Normal distribution, $P' \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right)$

Table 9.3

Assumptions

When you perform a hypothesis test of a single population mean μ using a normal distribution (often called a z-test), you take a **simple random sample** from the population. The population you are testing is **normally distributed**, or your sample size is sufficiently large. You know the value of the population **standard deviation**, which, in reality, is rarely known.

When you perform a hypothesis test of a single population mean μ using a **Student's t-distribution** (often called a *t*-test), there are fundamental assumptions that need to be met in order for the test to work properly. Your data should be a simple random sample that comes from a population that is approximately normally distributed. You use the sample standard deviation to approximate the population standard deviation. (Note that if the sample size is sufficiently large, a *t*-test will work even if the population is not approximately normally distributed).

When you perform a hypothesis test of a single population proportion *p*, you take a simple random sample from the population. You must meet the conditions for a **binomial distribution**: there are a certain number *n* of independent trials, the outcomes of any trial are success or failure, and each trial has the same probability of a success *p*. The shape of the binomial distribution needs to be similar to the shape of the normal distribution. To ensure this, the quantities *np* and *nq* must both be greater than five ($np > 5$ and $nq > 5$). Then the binomial distribution of a sample (estimated) proportion can be approximated by the normal distribution with $\mu = p$ and $\sigma = \sqrt{\frac{pq}{n}}$. Remember that $q = 1 - pq$.

9.4 Rare Events, the Sample, Decision and Conclusion

Establishing the type of distribution, sample size, and known or unknown standard deviation can help you figure out how to go about a hypothesis test. However, there are several other factors you should consider when working out a hypothesis test.

Rare Events

Suppose you make an assumption about a property of the population (this assumption is the **null hypothesis**). Then you gather sample data randomly. If the sample has properties that would be very **unlikely** to occur if the assumption is true, then you would conclude that your assumption about the population is probably incorrect. (Remember that your assumption is just an **assumption**—it is not a fact and it may or may not be true. But your sample data are real and the data are showing you a fact that seems to contradict your assumption.)

For example, Didi and Ali are at a birthday party of a very wealthy friend. They hurry to be first in line to grab a prize from a tall basket that they cannot see inside because they will be blindfolded. There are 200 plastic bubbles in the basket and Didi and Ali have been told that there is only one with a $100 bill. Didi is the first person to reach into the basket and pull out a bubble. Her bubble contains a $100 bill. The probability of this happening is $\frac{1}{200}$ = 0.005. Because this is so unlikely, Ali is hoping that what the two of them were told is wrong and there are more $100 bills in the basket. A "rare event" has occurred (Didi getting the $100 bill) so Ali doubts the assumption about only one $100 bill being in the basket.

Using the Sample to Test the Null Hypothesis

Use the sample data to calculate the actual probability of getting the test result, called the ***p*-value**. The *p*-value is the **probability that, if the null hypothesis is true, the results from another randomly selected sample will be as extreme or more extreme as the results obtained from the given sample.**

A large *p*-value calculated from the data indicates that we should not reject the **null hypothesis**. The smaller the *p*-value, the more unlikely the outcome, and the stronger the evidence is against the null hypothesis. We would reject the null hypothesis if the evidence is strongly against it.

Draw a graph that shows the *p*-value. The hypothesis test is easier to perform if you use a graph because you see the problem more clearly.

EXAMPLE 9.9

Suppose a baker claims that his bread height is more than 15 cm, on average. Several of his customers do not believe him. To persuade his customers that he is right, the baker decides to do a hypothesis test. He bakes 10 loaves of bread. The mean height of the sample loaves is 17 cm. The baker knows from baking hundreds of loaves of bread that the **standard deviation** for the height is 0.5 cm. and the distribution of heights is normal.

The null hypothesis could be H_0: $\mu \leq 15$ The alternate hypothesis is H_a: $\mu > 15$

The words **"is more than"** translates as a ">" so "μ > 15" goes into the alternate hypothesis. The null hypothesis must contradict the alternate hypothesis.

By the **Central Limit Theorem**, the distribution of sample means, for samples of 10 loaves, will be normal with mean μ = 15 and standard deviation $\frac{\sigma}{\sqrt{n}} = \frac{0.5}{\sqrt{10}} = 0.16.$

Suppose the null hypothesis is true (the mean height of the loaves is no more than 15 cm). Then is the mean height (17 cm) calculated from the sample unexpectedly large? The hypothesis test works by asking the question how **unlikely** the sample mean would be if the null hypothesis were true. The graph shows how far out the sample mean is on the normal curve. The *p*-value is the probability that, if we were to take other samples, any other sample mean would fall at least as far out as 17 cm.

The *p*-value, then, is the probability that a sample mean is the same or greater than 17 cm. when the population mean is, in fact, 15 cm. We can calculate this probability using the normal distribution for means.

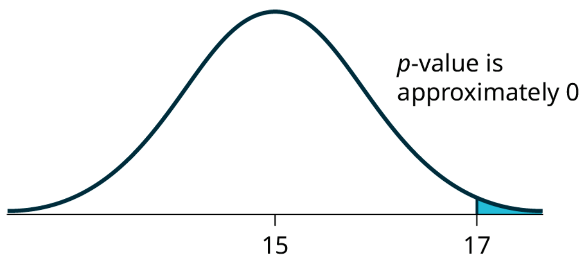

Figure 9.2

p-value= $P(\overline{x} > 17)$ which is approximately zero.

A *p*-value of approximately zero tells us that it is highly unlikely that a loaf of bread rises no more than 15 cm, on average. That is, almost 0% of all loaves of bread would be at least as high as 17 cm. **purely by CHANCE** had the population mean height really been 15 cm. Because the outcome of 17 cm. is so **unlikely (meaning it is happening NOT by chance alone)**, we conclude that the evidence is strongly against the null hypothesis (the mean height is at most 15 cm.). There is sufficient evidence that the true mean height for the population of the baker's loaves of bread is greater than 15 cm.

TRY IT 9.9

A normal distribution has a standard deviation of 1. We want to verify a claim that the mean is greater than 12. A sample of 36 is taken with a sample mean of 12.5.

H_0: $\mu \le 12$
H_a: $\mu > 12$
The *p*-value is 0.0013
Draw a graph that shows the *p*-value.

Decision and Conclusion

A systematic way to make a decision of whether to reject or not reject the **null hypothesis** is to compare the *p*-value and a **preset or preconceived α (also called a "significance level")**. A preset α is the probability of a **Type I error** (rejecting the null hypothesis when the null hypothesis is true). It may or may not be given to you at the beginning of the problem.

When you make a **decision** to reject or not reject H_0, do as follows:

- If α > *p*-value, reject H_0. The results of the sample data are significant. There is sufficient evidence to conclude that H_0 is an incorrect belief and that the **alternative hypothesis**, H_a, may be correct.
- If $\alpha \le$ *p*-value, do not reject H_0. The results of the sample data are not significant.There is not sufficient evidence to conclude that the alternative hypothesis,H_a, may be correct.
- When you "do not reject H_0", it does not mean that you should believe that H_0 is true. It simply means that the sample data have **failed** to provide sufficient evidence to cast serious doubt about the truthfulness of H_0.

Conclusion: After you make your decision, write a thoughtful **conclusion** about the hypotheses in terms of the given problem.

EXAMPLE 9.10

When using the *p*-value to evaluate a hypothesis test, it is sometimes useful to use the following memory device

If the *p*-value is low, the null must go.

If the *p*-value is high, the null must fly.

This memory aid relates a *p*-value less than the established alpha (the *p* is low) as rejecting the null hypothesis and, likewise, relates a *p*-value higher than the established alpha (the *p* is high) as not rejecting the null hypothesis.

Problem

Fill in the blanks.

Reject the null hypothesis when ___________________________________.

The results of the sample data __________________________________.

Do not reject the null when hypothesis when _______________________________________.

The results of the sample data ____.

Solution

Reject the null hypothesis when **the *p*-value is less than the established alpha value**. The results of the sample data **support the alternative hypothesis**.

Do not reject the null hypothesis when **the *p*-value is greater than the established alpha value**. The results of the sample data **do not support the alternative hypothesis**.

TRY IT 9.10

It's a Boy Genetics Labs claim their procedures improve the chances of a boy being born. The results for a test of a single population proportion are as follows:

H_0: $p = 0.50$, H_a: $p > 0.50$

$\alpha = 0.01$

p-value = 0.025

Interpret the results and state a conclusion in simple, non-technical terms.

9.5 Additional Information and Full Hypothesis Test Examples

- In a **hypothesis test** problem, you may see words such as "the **level of significance** is 1%." The "1%" is the preconceived or preset α.
- The statistician setting up the hypothesis test selects the value of α to use **before** collecting the sample data.
- **If no level of significance is given, a common standard to use is α = 0.05.**
- When you calculate the *p*-value and draw the picture, the *p*-value is the area in the left tail, the right tail, or split evenly between the two tails. For this reason, we call the hypothesis test left, right, or two tailed.
- The **alternative hypothesis**, H_a, tells you if the test is left, right, or two-tailed. It is the **key** to conducting the appropriate test.
- H_a **never** has a symbol that contains an equal sign.
- **Thinking about the meaning of the *p*-value**: A data analyst (and anyone else) should have more confidence that he made the correct decision to reject the null hypothesis with a smaller *p*-value (for example, 0.001 as opposed to 0.04) even if using the 0.05 level for alpha. Similarly, for a large *p*-value such as 0.4, as opposed to a *p*-value of 0.056 (alpha = 0.05 is less than either number), a data analyst should have more confidence that she made the correct decision in not rejecting the null hypothesis. This makes the data analyst use judgment rather than mindlessly applying rules.

The following examples illustrate a left-, right-, and two-tailed test.

EXAMPLE 9.11

H_0: $\mu = 5$, H_a: $\mu < 5$

Test of a single population mean. H_a tells you the test is left-tailed. The picture of the *p*-value is as follows:

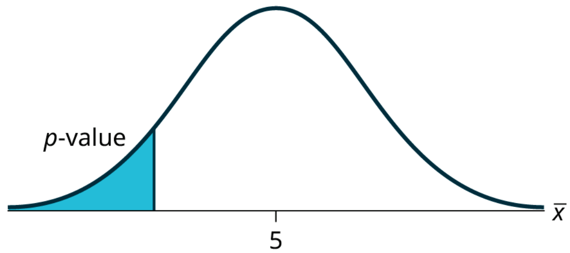

Figure 9.3

TRY IT 9.11

H_0: $\mu = 10$, H_a: $\mu < 10$

Assume the *p*-value is 0.0935. What type of test is this? Draw the picture of the *p*-value.

EXAMPLE 9.12

H_0: $p \leq 0.2$ H_a: $p > 0.2$

This is a test of a single population proportion. H_a tells you the test is **right-tailed**. The picture of the *p*-value is as follows:

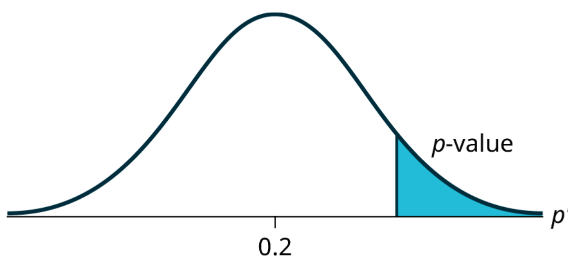

Figure 9.4

TRY IT 9.12

H_0: $\mu \leq 1$, H_a: $\mu > 1$

Assume the *p*-value is 0.1243. What type of test is this? Draw the picture of the *p*-value.

EXAMPLE 9.13

H_0: $p = 50$ H_a: $p \neq 50$

This is a test of a single population mean. H_a tells you the test is **two-tailed**. The picture of the *p*-value is as follows.

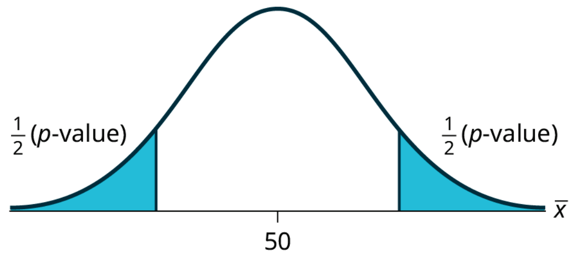

Figure 9.5

TRY IT 9.13

H_0: $p = 0.5$, H_a: $p \neq 0.5$

Assume the *p*-value is 0.2564. What type of test is this? Draw the picture of the *p*-value.

Full Hypothesis Test Examples

EXAMPLE 9.14

Problem

Jeffrey, as an eight-year old, **established a mean time of 16.43 seconds** for swimming the 25-yard freestyle, with a **standard deviation of 0.8 seconds**. His dad, Frank, thought that Jeffrey could swim the 25-yard freestyle faster using goggles. Frank bought Jeffrey a new pair of expensive goggles and timed Jeffrey for **15 25-yard freestyle swims**. For the 15 swims, **Jeffrey's mean time was 16 seconds. Frank thought that the goggles helped Jeffrey to swim faster than the 16.43 seconds.** Conduct a hypothesis test using a preset $\alpha = 0.05$. Assume that the swim times for the 25-yard freestyle are normal.

Solution

Set up the Hypothesis Test:

Since the problem is about a mean, this is a **test of a single population mean**.

H_0: $\mu = 16.43$ $\quad$ H_a: $\mu < 16.43$

For Jeffrey to swim faster, his time will be less than 16.43 seconds. The "<" tells you this is left-tailed.

Determine the distribution needed:

Random variable: $\overline{X}$ = the mean time to swim the 25-yard freestyle.

Distribution for the test: $\overline{X}$ is normal (population **standard deviation** is known: $\sigma = 0.8$)

$\overline{X} \sim N\left(\mu, \frac{\sigma_X}{\sqrt{n}}\right)$ Therefore, $\overline{X} \sim N\left(16.43, \frac{0.8}{\sqrt{15}}\right)$

$\mu = 16.43$ comes from H_0 and not the data. $\sigma = 0.8$, and $n = 15$.

Calculate the *p*-value using the normal distribution for a mean:

p-value = $P(\bar{x} < 16) = 0.0187$ where the sample mean in the problem is given as 16.

p-value = 0.0187 (This is called the **actual level of significance**.) The *p*-value is the area to the left of the sample mean is given as 16.

Graph:

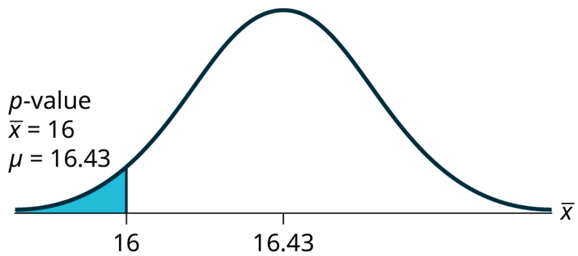

Figure 9.6

μ = 16.43 comes from H_0. Our assumption is μ = 16.43.

Interpretation of the *p*-value: If H_0 is true, there is a 0.0187 probability (1.87%)that Jeffrey's mean time to swim the 25-yard freestyle is 16 seconds or less. Because a 1.87% chance is small, the mean time of 16 seconds or less is unlikely to have happened randomly. It is a rare event.

Compare α and the *p*-value:

α = 0.05 *p*-value = 0.0187 α > *p*-value

Make a decision: Since α >*p*-value, reject H_0.

This indicates that you reject the null hypothesis that the mean time to swim the 25-yard freestyle is at least 16.43 seconds.

Conclusion: At the 5% significance level, there is sufficient evidence that Jeffrey's mean time to swim the 25-yard freestyle is less than 16.43 seconds. Thus, based on the sample data, we conclude that Jeffrey swims faster using the new goggles.

The Type I and Type II errors for this problem are as follows:
The Type I error is to conclude that Jeffrey swims the 25-yard freestyle, on average, in less than 16.43 seconds when, in fact, he actually swims the 25-yard freestyle, on average, in at least 16.43 seconds. (Reject the null hypothesis when the null hypothesis is true.)

The Type II error is that there is not evidence to conclude that Jeffrey swims the 25-yard freestyle, on average, in less than 16.43 seconds when, in fact, he actually does swim the 25-yard free-style, on average, in less than 16.43 seconds. (Do not reject the null hypothesis when the null hypothesis is false.)

USING THE TI-83, 83+, 84, 84+ CALCULATOR

Press STAT and arrow over to TESTS. Press `1:Z-Test`. Arrow over to `Stats` and press ENTER. Arrow down and enter 16.43 for μ_0 (null hypothesis), .8 for σ, 16 for the sample mean, and 15 for *n*. Arrow down to μ : (alternate hypothesis) and arrow over to < μ_0. Press ENTER. Arrow down to `Calculate` and press ENTER. The calculator not only calculates the *p*-value (*p* = 0.0187) but it also calculates the test statistic (*z*-score) for the sample mean. μ < 16.43 is the alternative hypothesis. Do this set of instructions again except arrow to `Draw`(instead of `Calculate`). Press ENTER. A shaded graph appears with *z* = -2.08 (test statistic) and *p* = 0.0187 (*p*-value). Make sure when you use `Draw` that no other equations are highlighted in *Y* = and the plots are turned off.

TRY IT 9.14

The mean throwing distance of a football for Marco, a high school quarterback, is 40 yards, with a standard deviation of two yards. The team coach tells Marco to adjust his grip to get more distance. The coach records the distances for 20 throws. For the 20 throws, Marco's mean distance was 45 yards. The coach thought the different grip helped Marco throw farther than 40 yards. Conduct a hypothesis test using a preset α = 0.05. Assume the throw distances for footballs are normal.

First, determine what type of test this is, set up the hypothesis test, find the *p*-value, sketch the graph, and state your conclusion.

USING THE TI-83, 83+, 84, 84+ CALCULATOR

Press STAT and arrow over to TESTS. Press `1:Z-Test`. Arrow over to Stats and press ENTER. Arrow down and enter 40 for μ0 (null hypothesis), 2 for σ, 45 for the sample mean, and 20 for *n*. Arrow down to μ: (alternative hypothesis) and set it either as <, ≠, or >. Press ENTER. Arrow down to Calculate and press ENTER. The calculator not only calculates the *p*-value but it also calculates the test statistic (*z*-score) for the sample mean. Select <, ≠, or > for the alternative hypothesis. Do this set of instructions again except arrow to Draw (instead of Calculate). Press ENTER. A shaded graph appears with test statistic and *p*-value. Make sure when you use Draw that no other equations are highlighted in *Y* = and the plots are turned off.

Historical Note (Example 9.14)

The traditional way to compare the two probabilities, α and the *p*-value, is to compare the critical value (*z*-score from α) to the test statistic (*z*-score from data). The calculated test statistic for the *p*-value is –2.08. (From the Central Limit Theorem, the test statistic formula is $z = \frac{\overline{x} - \mu_X}{\left(\frac{\sigma_X}{\sqrt{n}}\right)}$. For this problem, $\overline{x}$ = 16, μ_X = 16.43 from the null hypothes is, σ_X = 0.8, and *n* = 15.) You can find the critical value for α = 0.05 in the normal table (see **15.Tables** in the Table of Contents). The *z*-score for an area to the left equal to 0.05 is midway between –1.65 and –1.64 (0.05 is midway between 0.0505 and 0.0495). The *z*-score is –1.645. Since –1.645 > –2.08 (which demonstrates that α > *p*-value), reject H_0. Traditionally, the decision to reject or not reject was done in this way. Today, comparing the two probabilities α and the *p*-value is very common. For this problem, the *p*-value, 0.0187 is considerably smaller than α, 0.05. You can be confident about your decision to reject. The graph shows α, the *p*-value, and the test statistic and the critical value.

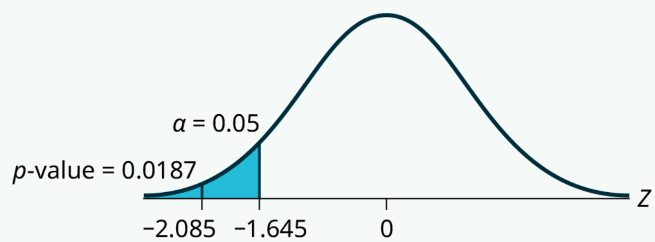

Figure 9.7

EXAMPLE 9.15

Problem

A college football coach records the mean weight that the players can bench press as **275 pounds**, with a **standard deviation of 55 pounds**. Three of the players thought that the mean weight was **more than** that amount. They asked **30** of their teammates for their estimated maximum lift on the bench press exercise. The data ranged from 205 pounds to 385 pounds. The actual different weights were (frequencies are in parentheses) 205(3); 215(3); 225(1); 241(2); 252(2); 265(2); 275(2); 313(2); 316(5); 338(2); 341(1); 345(2); 368(2); 385(1).

Conduct a hypothesis test using a 2.5% level of significance to determine if the bench press mean is **more than 275 pounds**.

Solution

Set up the Hypothesis Test:

Since the problem is about a mean weight, this is a **test of a single population mean**.

H_0: $\mu = 275$
H_a: $\mu > 275$
This is a right-tailed test.

Calculating the distribution needed:

Random variable: $\overline{X}$ = the mean weight, in pounds, lifted by the football players.

Distribution for the test: It is normal because σ is known.

$\overline{X} \sim N\left(275, \frac{55}{\sqrt{30}}\right)$

$\overline{x} = 286.2$ pounds (from the data).

$\sigma = 55$ pounds **(Always use σ if you know it.)** We assume $\mu = 275$ pounds unless our data shows us otherwise.

Calculate the *p*-value using the normal distribution for a mean and using the sample mean as input (see Appendix G NOTEs for the TI-83, 83+, 84, 84+ Calculators for using the data as input):

$p\text{-value} = P(\overline{x} > 286.2) = 0.1323.$

Interpretation of the *p*-value: If H_0 is true, then there is a 0.1331 probability (13.23%) that the football players can lift a mean weight of 286.2 pounds or more. Because a 13.23% chance is large enough, a mean weight lift of 286.2 pounds or more is not a rare event.

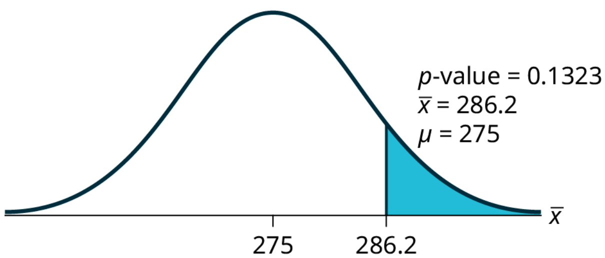

Figure 9.8

Compare α and the *p*-value:

$\alpha = 0.025$ *p*-value = 0.1323

Make a decision: Since α <*p*-value, do not reject H_0.

Conclusion: At the 2.5% level of significance, from the sample data, there is not sufficient evidence to conclude that the true mean weight lifted is more than 275 pounds.

USING THE TI-83, 83+, 84, 84+ CALCULATOR

Put the data and frequencies into lists. Press STAT and arrow over to TESTS. Press `1:Z-Test`. Arrow over to `Data` and press ENTER. Arrow down and enter 275 for μ_0, 55 for σ, the name of the list where you put the data, and the name of the list where you put the frequencies. Arrow down to μ: and arrow over to > μ_0. Press ENTER. Arrow down to `Calculate` and press ENTER. The calculator not only calculates the *p*-value ($p = 0.1331$, a little different from the previous calculation - in it we used the sample mean rounded to one decimal place instead of the data) but it also calculates the test statistic (*z*-score) for the sample mean, the sample mean, and the sample standard deviation. $\mu > 275$ is the alternative hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press ENTER. A shaded graph appears with $z = 1.112$ (test statistic) and $p = 0.1331$ (*p*-value). Make sure when you use `Draw` that no other equations are highlighted in *Y* = and the plots are turned off.

TRY IT 9.15

A company records the mean time of employees working in a day. The mean comes out to be 475 minutes, with a standard deviation of 45 minutes. A manager recorded times of 20 employees. The times of working were (frequencies are in parentheses) 460(3); 465(2); 470(3); 475(1); 480(6); 485(3); 490(2).

Conduct a hypothesis test using a 2.5% level of significance to determine if the mean time is *more than 475*.

EXAMPLE 9.16

Problem

Statistics students believe that the mean score on the first statistics test is 65. A statistics instructor thinks the mean score is higher than 65. He samples ten statistics students and obtains the scores 65; 65; 70; 67; 66; 63; 63; 68; 72; 71. He performs a hypothesis test using a 5% level of significance. The data are assumed to be from a normal distribution.

Solution

Set up the hypothesis test:

A 5% level of significance means that $\alpha = 0.05$. This is a test of a **single population mean**.

H_0: $\mu = 65$ H_a: $\mu > 65$

Since the instructor thinks the average score is higher, use a ">". The ">" means the test is right-tailed.

Determine the distribution needed:

Random variable: $\overline{X}$ = average score on the first statistics test.

Distribution for the test: If you read the problem carefully, you will notice that there is **no population standard deviation given**. You are only given $n = 10$ sample data values. Notice also that the data come from a normal distribution. This means that the distribution for the test is a student's *t*.

Use t_{df}. Therefore, the distribution for the test is t_9 where $n = 10$ and $df = 10 - 1 = 9$.

Calculate the *p*-value using the Student's *t*-distribution:

p-value = $P(\overline{x} > 67) = 0.0396$ where the sample mean and sample standard deviation are calculated as 67 and 3.1972 from the data.

Interpretation of the *p*-value: If the null hypothesis is true, then there is a 0.0396 probability (3.96%) that the sample mean is 67 or more.

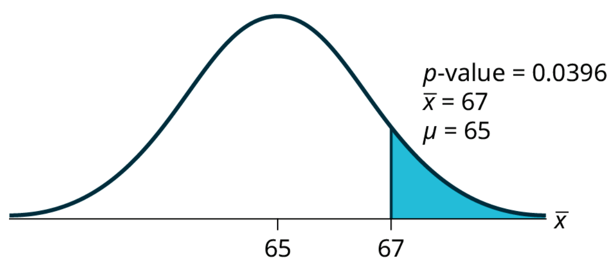

Figure 9.9

Compare α and the *p*-value:

Since $\alpha = 0.05$ and *p*-value = 0.0396. $\alpha >$ *p*-value.

Make a decision: Since $\alpha >$ *p*-value, reject H_0.

This means you reject $\mu = 65$. In other words, you believe the average test score is greater than 65.

Conclusion: At a 5% level of significance, the sample data show sufficient evidence that the mean (average) test score is greater than 65, just as the math instructor thinks.

USING THE TI-83, 83+, 84, 84+ CALCULATOR

Put the data into a list. Press STAT and arrow over to TESTS. Press `2:T-Test`. Arrow over to `Data` and press ENTER. Arrow down and enter 65 for μ_0, the name of the list where you put the data, and 1 for `Freq:`. Arrow down to μ: and arrow over to > μ_0. Press ENTER. Arrow down to `Calculate` and press ENTER. The calculator not only calculates the *p*-value (p = 0.0396) but it also calculates the test statistic (*t*-score) for the sample mean, the sample mean, and the sample standard deviation. $\mu > 65$ is the alternative hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press ENTER. A shaded graph appears with t = 1.9781 (test statistic) and p = 0.0396 (*p*-value). Make sure when you use `Draw` that no other equations are highlighted in Y = and the plots are turned off.

TRY IT 9.16

It is believed that a stock price for a particular company will grow at a rate of $5 per week with a standard deviation of $1. An investor believes the stock won't grow as quickly. The changes in stock price is recorded for ten weeks and are as follows: $4, $3, $2, $3, $1, $7, $2, $1, $1, $2. Perform a hypothesis test using a 5% level of significance. State the null and alternative hypotheses, find the *p*-value, state your conclusion, and identify the Type I and Type II errors.

EXAMPLE 9.17

Problem

Joon believes that 50% of first-time brides in the United States are younger than their grooms. She performs a hypothesis test to determine if the percentage is **the same or different from 50%**. Joon samples **100 first-time brides** and **53** reply that they are younger than their grooms. For the hypothesis test, she uses a 1% level of significance.

Solution

Set up the hypothesis test:

The 1% level of significance means that α = 0.01. This is a **test of a single population proportion**.

H_0: $p = 0.50$ H_a: $p \neq 0.50$

The words **"is the same or different from"** tell you this is a two-tailed test.

Calculate the distribution needed:

Random variable: P' = the percent of of first-time brides who are younger than their grooms.

Distribution for the test: The problem contains no mention of a mean. The information is given in terms of percentages. Use the distribution for P', the estimated proportion.

$P' \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right)$ Therefore, $P' \sim N\left(0.5, \sqrt{\frac{0.5 \cdot 0.5}{100}}\right)$

where p = 0.50, q = 1-p = 0.50, and n = 100

Calculate the *p*-value using the normal distribution for proportions:

p-value = $P(p' < 0.47 \text{ or } p' > 0.53) = 0.5485$

where x = 53, $p' = \frac{x}{n} = \frac{53}{100}$ = 0.53.

Interpretation of the *p*-value: If the null hypothesis is true, there is 0.5485 probability (54.85%) that the sample (estimated) proportion p' is 0.53 or more OR 0.47 or less (see the graph in Figure 9.10).

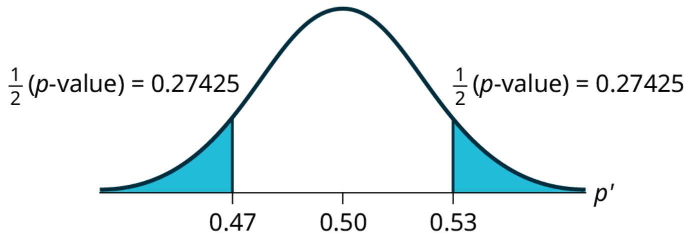

Figure 9.10

$\mu = p = 0.50$ comes from H_0, the null hypothesis.

$p' = 0.53$. Since the curve is symmetrical and the test is two-tailed, the p' for the left tail is equal to $0.50 - 0.03 = 0.47$ where $\mu = p = 0.50$. (0.03 is the difference between 0.53 and 0.50.)

Compare α and the p-value:

Since $\alpha = 0.01$ and p-value = 0.5485. $\alpha < p$-value.

Make a decision: Since $\alpha < p$-value, you cannot reject H_0.

Conclusion: At the 1% level of significance, the sample data do not show sufficient evidence that the percentage of first-time brides who are younger than their grooms is different from 50%.

USING THE TI-83, 83+, 84, 84+ CALCULATOR

Press STAT and arrow over to TESTS. Press `5:1-PropZTest`. Enter .5 for p_0, 53 for x and 100 for n. Arrow down to `Prop` and arrow to `not equals` p_0. Press ENTER. Arrow down to `Calculate` and press ENTER. The calculator calculates the p-value ($p = 0.5485$) and the test statistic (z-score). `Prop not equals` .5 is the alternate hypothesis. Do this set of instructions again except arrow to `Draw` (instead of `Calculate`). Press ENTER. A shaded graph appears with $z = 0.6$ (test statistic) and $p = 0.5485$ (p-value). Make sure when you use `Draw` that no other equations are highlighted in $Y =$ and the plots are turned off.

The Type I and Type II errors are as follows:

The Type I error is to conclude that the proportion of first-time brides who are younger than their grooms is different from 50% when, in fact, the proportion is actually 50%. (Reject the null hypothesis when the null hypothesis is true).

The Type II error is there is not enough evidence to conclude that the proportion of first time brides who are younger than their grooms differs from 50% when, in fact, the proportion does differ from 50%. (Do not reject the null hypothesis when the null hypothesis is false.)

TRY IT 9.17

A teacher believes that 85% of students in the class will want to go on a field trip to the local zoo. The teacher performs a hypothesis test to determine if the percentage is the same or different from 85%. The teacher samples 50 students and 39 reply that they would want to go to the zoo. For the hypothesis test, use a 1% level of significance.

First, determine what type of test this is, set up the hypothesis test, find the p-value, sketch the graph, and state your conclusion.

EXAMPLE 9.18

Problem

Suppose a consumer group suspects that the proportion of households that have three cell phones is 30%. A cell phone company has reason to believe that the proportion is not 30%. Before they start a big advertising campaign, they conduct a hypothesis test. Their marketing people survey 150 households with the result that 43 of the households have three cell phones.

a. The value that helps determine the *p*-value is p'. Calculate p'.

b. What is a **success** for this problem?

c. What is the level of significance?

d. Draw the graph for this problem. Draw the horizontal axis. Label and shade appropriately.
Calculate the *p*-value.

e. Make a decision. _____________(Reject/Do not reject) H_0 because____________.

Solution

Set up the Hypothesis Test:

H_0: $p = 0.30$ H_a: $p \neq 0.30$

Determine the distribution needed:

The **random variable** is P' = proportion of households that have three cell phones.

The **distribution** for the hypothesis test is $P' \sim N\left(0.30, \sqrt{\frac{(0.30)\cdot(0.70)}{150}}\right)$

a. $p' = \frac{x}{n}$ where x is the number of successes and n is the total number in the sample.

$x = 43$, $n = 150$

$p' = \frac{43}{150}$

b. A success is having three cell phones in a household.

c. The level of significance is the preset α. Since α is not given, assume that $\alpha = 0.05$.

d. *p*-value = 0.7216

e. Assuming that $\alpha = 0.05$, $\alpha <$ *p*-value. The decision is do not reject H_0 because there is not sufficient evidence to conclude that the proportion of households that have three cell phones is not 30%.

TRY IT 9.18

Marketers believe that 92% of adults in the United States own a cell phone. A cell phone manufacturer believes that number is actually lower. 200 American adults are surveyed, of which, 174 report having cell phones. Use a 5% level of significance. State the null and alternative hypothesis, find the *p*-value, state your conclusion, and identify the Type I and Type II errors.

The next example is a poem written by a statistics student named Nicole Hart. The solution to the problem follows the poem. Notice that the hypothesis test is for a single population proportion. This means that the null and alternate hypotheses use the parameter p. The distribution for the test is normal. The estimated proportion p' is the proportion of fleas killed to the total fleas found on Fido. This is sample information. The problem gives a preconceived $\alpha = 0.01$, for comparison, and a 95% confidence interval computation. The poem is clever and humorous, so please enjoy it!

EXAMPLE 9.19

Problem

My dog has so many fleas,
They do not come off with ease.
As for shampoo, I have tried many types

Even one called Bubble Hype,
Which only killed 25% of the fleas,
Unfortunately I was not pleased.

I've used all kinds of soap,
Until I had given up hope
Until one day I saw
An ad that put me in awe.

A shampoo used for dogs
Called GOOD ENOUGH to Clean a Hog
Guaranteed to kill more fleas.

I gave Fido a bath
And after doing the math
His number of fleas
Started dropping by 3's!

Before his shampoo
I counted 42.
At the end of his bath,
I redid the math
And the new shampoo had killed 17 fleas.
So now I was pleased.

Now it is time for you to have some fun
With the level of significance being .01,
You must help me figure out
Use the new shampoo or go without?

Solution

Set up the hypothesis test:

H_0: $p \leq 0.25$ H_a: $p > 0.25$

Determine the distribution needed:

In words, CLEARLY state what your random variable $\overline{X}$ or P' represents.

P' = The proportion of fleas that are killed by the new shampoo

State the distribution to use for the test.

Normal: $N\left(0.25, \sqrt{\frac{(0.25)(1-0.25)}{42}}\right)$

Test Statistic: $z = 2.3163$

Calculate the p-value using the normal distribution for proportions:

p-value = 0.0103

In one to two complete sentences, explain what the p-value means for this problem.

If the null hypothesis is true (the proportion is 0.25), then there is a 0.0103 probability that the sample (estimated) proportion is 0.4048 $\left(\frac{17}{42}\right)$ or more.

Use the previous information to sketch a picture of this situation. CLEARLY, label and scale the horizontal axis and shade the region(s) corresponding to the p-value.

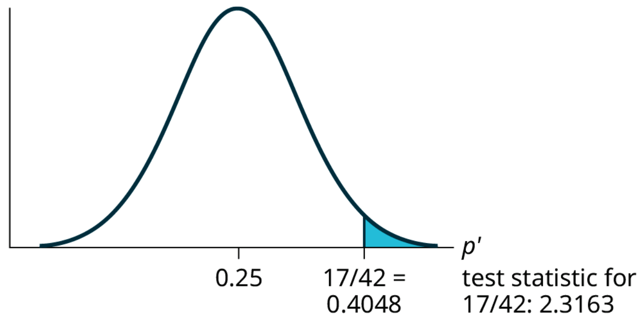

Figure 9.11

Compare α and the *p*-value:

Indicate the correct decision ("reject" or "do not reject" the null hypothesis), the reason for it, and write an appropriate conclusion, using complete sentences.

alpha	decision	reason for decision
0.01	Do not reject H_0	α < *p*-value

Table 9.4

Conclusion: At the 1% level of significance, the sample data do not show sufficient evidence that the percentage of fleas that are killed by the new shampoo is more than 25%.

Construct a 95% confidence interval for the true mean or proportion. Include a sketch of the graph of the situation. Label the point estimate and the lower and upper bounds of the confidence interval.

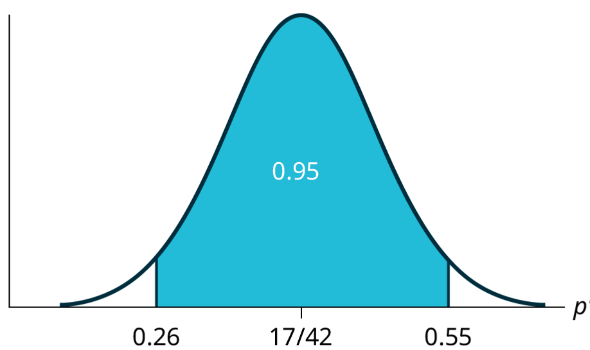

Figure 9.12

Confidence Interval: (0.26,0.55) We are 95% confident that the true population proportion *p* of fleas that are killed by the new shampoo is between 26% and 55%.

NOTE

This test result is not very definitive since the *p*-value is very close to alpha. In reality, one would probably do more tests by giving the dog another bath after the fleas have had a chance to return.

TRY IT 9.19

A car soap gets rid of 30% of stains on the car. After adding a new compound to the soap, the soap is used on a car and found to wash 20 stains out of the 50 stains on the car. With the level of significance being 0.01, find out if adding the new compound to soap is beneficial.

EXAMPLE 9.20

Problem

The National Institute of Standards and Technology provides exact data on conductivity properties of materials. Following are conductivity measurements for 11 randomly selected pieces of a particular type of glass.

1.11; 1.07; 1.11; 1.07; 1.12; 1.08; .98; .98; 1.02; .95; .95
Is there convincing evidence that the average conductivity of this type of glass is greater than one? Use a significance level of 0.05. Assume the population is normal.

Solution

Let's follow a four-step process to answer this statistical question.

1. **State the Question**: We need to determine if, at a 0.05 significance level, the average conductivity of the selected glass is greater than one. Our hypotheses will be
   a. H_0: $\mu \leq 1$
   b. H_a: $\mu > 1$
2. **Plan**: We are testing a sample mean without a known population standard deviation. Therefore, we need to use a Student's t-distribution. Assume the underlying population is normal.
3. Based on the sample of 11 data values shown above, sample mean, sample standard deviation, and test statistic are calculated as follows:

$$\begin{aligned}\overline{x} &= 1.04\\ s &= 0.0659\\ t &= 2.014\end{aligned}$$

   To calculate the *p*-value, note that this is a right-tailed test. Then, find the area under the *t*-distribution to the right of the test statistic 2.014 (using 10 degrees of freedom). This area in the right tail is 0.036, and thus the *p*-value = 0.036.
4. **State the Conclusions**: Since the *p*-value (p = 0.036) is less than our alpha value, we will reject the null hypothesis. It is reasonable to state that the data supports the claim that the average conductivity level is greater than one.

TRY IT 9.20

The boiling point of a specific liquid is measured for 15 samples, and the boiling points are obtained as follows:

205; 206; 206; 202; 199; 194; 197; 198; 198; 201; 201; 202; 207; 211; 205

Is there convincing evidence that the average boiling point is greater than 200? Use a significance level of 0.1. Assume the population is normal.

EXAMPLE 9.21

Problem

In a study of 420,019 cell phone users, 172 of the subjects developed brain cancer. Test the claim that cell phone users developed brain cancer at a greater rate than that for non-cell phone users (the rate of brain cancer for non-cell phone users is 0.0340%). Since this is a critical issue, use a 0.005 significance level. Explain why the significance level should be so low in terms of a Type I error.

Solution

We will follow the four-step process.

1. We need to conduct a hypothesis test on the claimed cancer rate. Our hypotheses will be

a. H_0: $p \leq 0.00034$
b. H_a: $p > 0.00034$

If we commit a Type I error, we are essentially accepting a false claim. Since the claim describes cancer-causing environments, we want to minimize the chances of incorrectly identifying causes of cancer.

2. We will be testing a sample proportion with $x = 172$ and $n = 420{,}019$. The sample is sufficiently large because we have $np = 420{,}019(0.00034) = 142.8$, $nq = 420{,}019(0.99966) = 419{,}876.2$, two independent outcomes, and a fixed probability of success $p = 0.00034$. Thus we will be able to generalize our results to the population.
3. The associated TI results are

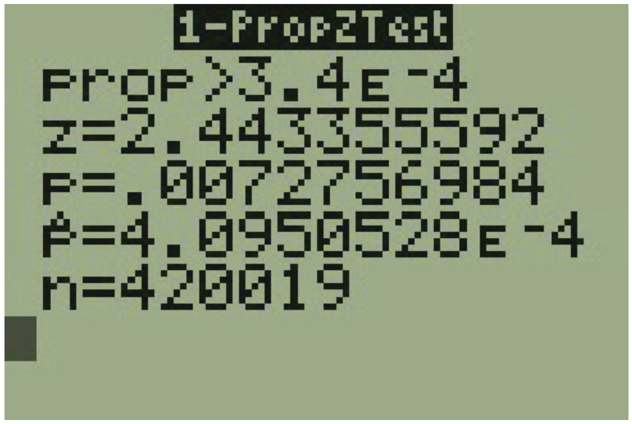

Figure 9.13

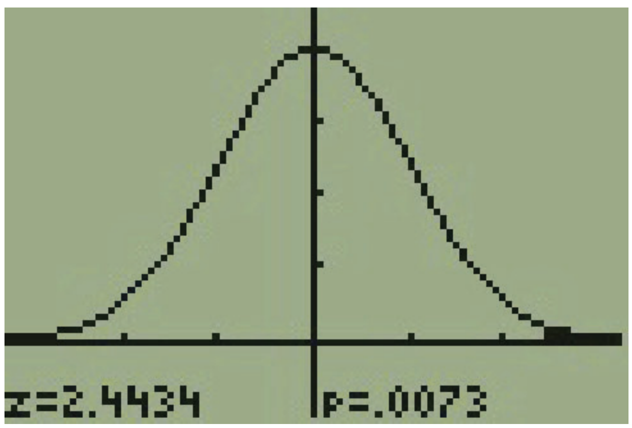

Figure 9.14

4. Since the p-value = 0.0073 is greater than our alpha value = 0.005, we cannot reject the null. Therefore, we conclude that there is not enough evidence to support the claim of higher brain cancer rates for the cell phone users.

TRY IT 9.21

In a study of 390,000 moisturizer users, 138 of the subjects developed skin diseases. Test the claim that moisturizer users developed skin diseases at a greater rate than that for non-moisturizer users (the rate of skin diseases for non-moisturizer users is 0.041%). Since this is a critical issue, use a 0.005 significance level. Explain why the significance level should be so low in terms of a Type I error.

EXAMPLE 9.22

Problem

Statistical data indicates that in a certain country there are approximately 268,608,618 residents aged 12 and older. For a certain period of time, statistical data also indicates that the percentage of residents with blood type AB negative (AB-) is 207,754 individuals. This translates into a percentage of 0.078% with this rather rare blood type. In a certain province of the country, there were 11 people with blood type AB- out of the population of 37,937. Conduct an appropriate hypothesis test to determine if there is a statistically significant difference between the percentage of residents in the entire country with blood type AB- versus the percentage in the local province. Use a significance level of 0.01.

Solution

We will follow the four-step plan.

1. We need to test whether the proportion of residents with AB- blood type in the local province is statistically different as compared to the proportion in the entire country.
2. Since we are presented with proportions, we will use a one-proportion *z*-test. The hypotheses for the test will be:

$$H_0 : p = 0.00078$$
$$H_a : p \neq 0.00078$$

3. Note the sample proportion is $p' = \frac{11}{37,937} = 0.00029$
   The test statistic is calculated as z = -3.4189. To calculate the *p*-value, note that this is a two-tailed test. Find the area under the normal distribution to the left of the test statistic and then double this area. The area to the left of the test statistic is 0.000314, and this area doubled results in the *p*-value of 0.00063.
4. Since the *p*-value, p = 0.00063, is less than the alpha level of 0.01, the sample data indicates that we should reject the null hypothesis. In conclusion, the sample data support the claim that the proportion of individuals with blood type AB- in the local province is different from the proportion of individuals in the entire country.

TRY IT 9.22

According to the U.S. Census, there are approximately 201,456,463 residents 20 and older. Statistics from the Criminal National Network indicate that, on average, 104,354 murders occur each year for people aged 20 and older. This translates into a percentage of murder of 0.052%. In Ohio, there were reported 127 murders for a population of 427,648. Conduct an appropriate hypothesis test to determine if there is a statistically significant difference between the local murder percentage and the national murder percentage. Use a significance level of 0.01.

9.6 Hypothesis Testing of a Single Mean and Single Proportion

Stats Lab

Hypothesis Testing of a Single Mean and Single Proportion

Class Time:

Names:

Student Learning Outcomes

- The student will select the appropriate distributions to use in each case.
- The student will conduct hypothesis tests and interpret the results.

Television Survey

In a recent survey, it was stated that Americans watch television on average four hours per day. Assume that σ = 2. Using your class as the sample, conduct a hypothesis test to determine if the average for students at your school is lower.

1. H_0: ____________
2. H_a: ____________

3. In words, define the random variable. __________ = ______________________
4. The distribution to use for the test is _______________________.
5. Determine the test statistic using your data.
6. Draw a graph and label it appropriately. Shade the actual level of significance.
    a. Graph:

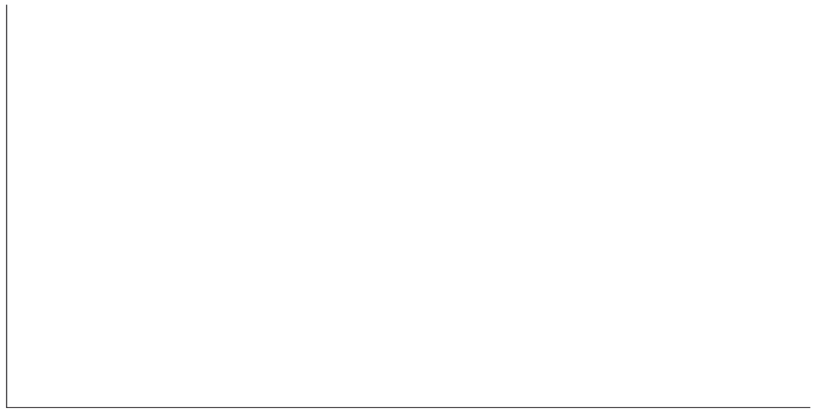

Figure 9.15

    b. Determine the *p*-value.

7. Do you or do you not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

Language Survey

About 42.3% of Californians and 19.6% of all Americans over age five speak a language other than English at home. Using your class as the sample, conduct a hypothesis test to determine if the percent of the students at your school who speak a language other than English at home is different from 42.3%.

1. H_0: ___________
2. H_a: ___________
3. In words, define the random variable. __________ = _______________
4. The distribution to use for the test is ________________
5. Determine the test statistic using your data.
6. Draw a graph and label it appropriately. Shade the actual level of significance.
    a. Graph:

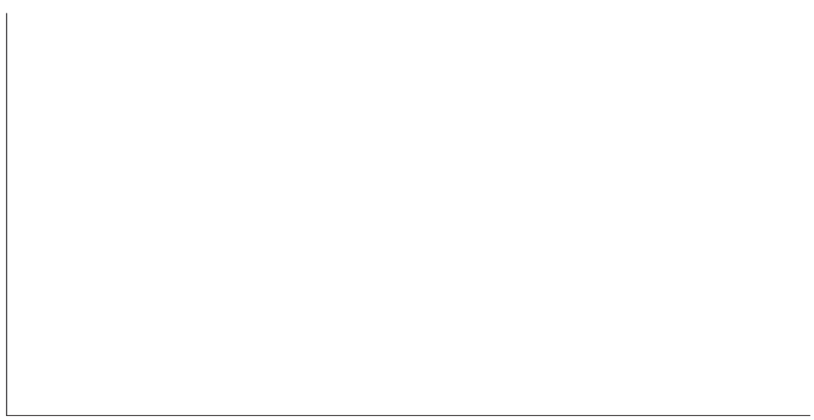

Figure 9.16

    b. Determine the *p*-value.

7. Do you or do you not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.

Jeans Survey

Suppose that young adults own an average of three pairs of jeans. Survey eight people from your class to determine if the average is higher than three. Assume the population is normal.

1. H_0: ____________
2. H_a: ____________
3. In words, define the random variable. __________ = ______________________
4. The distribution to use for the test is _______________________.
5. Determine the test statistic using your data.
6. Draw a graph and label it appropriately. Shade the actual level of significance.
    a. Graph:

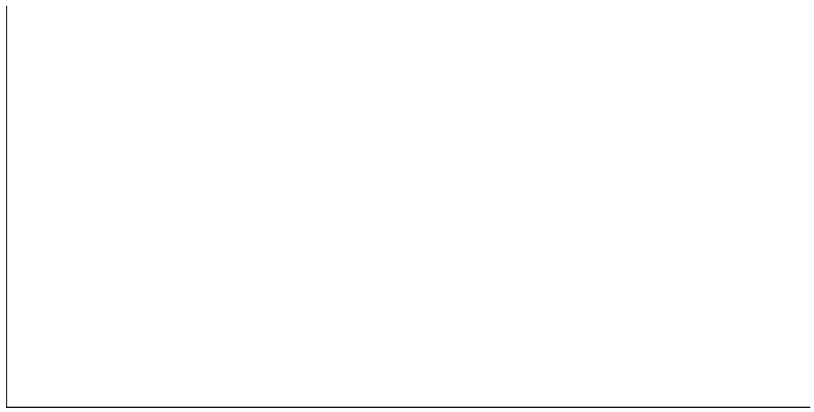

Figure 9.17

    b. Determine the *p*-value.
7. Do you or do you not reject the null hypothesis? Why?
8. Write a clear conclusion using a complete sentence.