

Sum and Difference Formulas

Trigonometry

Double-Angle Formulas

The Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Examples

1. Simplify to one trigonometric expression. $4\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$

2. Simplify the expression. Do not evaluate. $\cos^2(28^\circ) - \sin^2(28^\circ)$

3. Establish the identity.

$$(\sin^2 x - 1)^2 = \cos(2x) + \sin^4 x$$

4. Establish the identity.

$$\frac{1 + \cos(2t)}{\sin(2t) - \cos t} = \frac{2\cos t}{2\sin t - 1}$$

Reduction Formulas

The Formulas

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Example

1. Rewrite the expression with an exponent no higher than 1. $\cos^2 x \sin^4 x$.

Half-Angle Formulas

The Formulas

$$\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

Examples

1. Find the exact value using half-angle formulas. $\tan\left(-\frac{3\pi}{8}\right)$

Product-to-Sum Formulas

The Formulas

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos (\alpha - \beta) + \cos (\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin (\alpha + \beta) + \sin (\alpha - \beta)]$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos (\alpha - \beta) - \cos (\alpha + \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin (\alpha + \beta) - \sin (\alpha - \beta)]$$

Examples

1. Rewrite the product as a sum or difference. $10 \cos (5x) \sin (10x)$

2. Evaluate using a sum or difference of two functions. $\sin (195^\circ) \cos (15^\circ)$

1. Prove the identity.

$$\frac{\cos(2y) - \cos(4y)}{\sin(2y) + \sin(4y)} = \tan y$$

Sum-to-Product Formulas

The Formulas

$$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\sin \alpha - \sin \beta = 2 \sin \left(\frac{\alpha - \beta}{2} \right) \cos \left(\frac{\alpha + \beta}{2} \right)$$

$$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$$

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

Example

1. Rewrite the sum or difference as a product. $\sin(3x) + \sin(7x)$