Law of Sines

Trigonometry
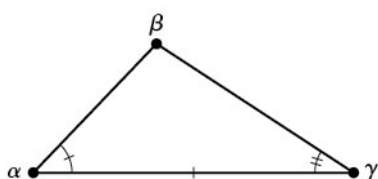
Introduction

Definition - Oblique Triangles

Any triangle that is not a right triangle is an **oblique triangle**.
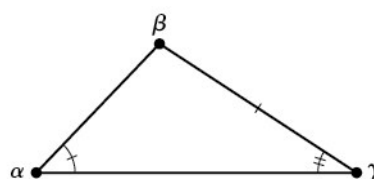
Definition - Solving a Triangle

Solving a triangle means finding the measures of all angles and sides given incomplete information.
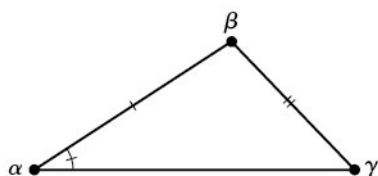
Types of Triangles for the Law of Sines

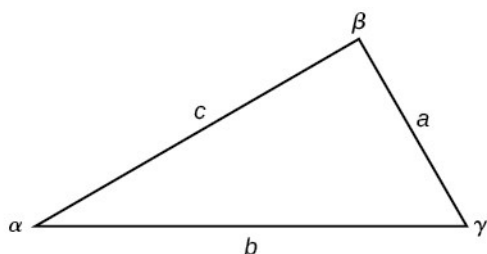**ASA (angle-side-angle)**

AAS (angle-angle-side)

SSA (side-side-angle)

Law of Sines

Fact - Law of Sines

For triangles labeled as the triangle to the right, with angles α, β, and γ, and opposite corresponding sides a, b, and c, respectively, the following proportions are true.

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Examples

For the following exercises, assume the angles and sides are as in the triangle above. Solve each triangle, if possible. Round each answer to the nearest tenth.

1. $\alpha = 43°, \gamma = 69°, a = 20$
2. $\alpha = 37°, \beta = 49°, c = 5$

You Try It

For the following exercises, assume the angles are sides are as in the triangle above. Solve each triangle, if possible. Round each answer to the nearest tenth.
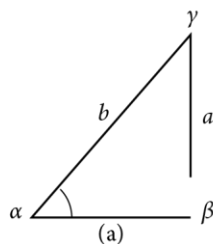
1. $a = 4, \alpha = 60°, \beta = 100°$
2. $\alpha = 132°, \gamma = 23°, b = 10$

The Ambiguous Case
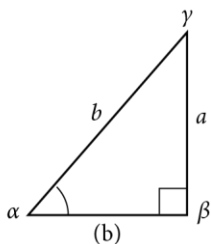
Possible Cases for SSA triangles

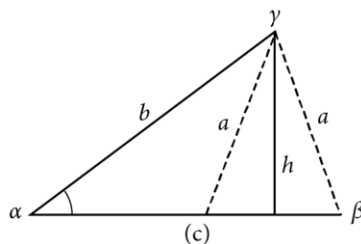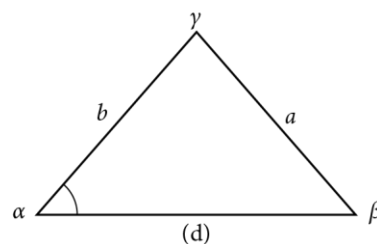No triangle, $a < h$ Right triangle, $a = h$ Two triangles, $a > h$, $a < b$ One triangle, $a \geq b$

Comparison of SSA Triangles

Examples

Determine whether there is no triangle, one triangle, or two triangles. Then solve each triangle, if possible. Round each answer to the nearest tenth.

1. $\gamma = 113°, b = 10, c = 32$

2. $a = 7, c = 9, \alpha = 43.$

3. $\beta = 119°, b = 8.2, a = 11.3.$