MPMP: How Odd is Pascal's Triangle

Solution by: Chris Sears

To solve the problem algebraically, first observe that when the number of rows is doubled, the number of odd numbers is tripled. So, if n is the number of rows, and m is the number of times the rows are doubled, then the number of odd numbers after m doublings is 3^m. Using the fact that $m = \log_2 n$ and some algebra gives the following equation.

$$\text{number of odds above and including row } n = n^{\log_2 3}$$

To count the numbers in row n and above, use the sum of consecutive whole numbers formula.

$$\text{numbers in row } n \text{ and above} = \frac{n(n+1)}{2}$$

To get the percentage of odd numbers in row n and above, divide the previous two formulas.

$$\text{fraction of odd numbers above and including row } n = \frac{2n^{(\log_2 3)-1}}{n+1}$$

Of course, this only works when n is a power of 2.

To test how well this solution works for other values of n, I wrote a R script to compare the function above to the actual values. The graph is below.