# Sum and Difference Formulas

### Trigonometry

## The Formulas

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

## Examples

Find the exact value of the following.

1. $\cos\left(\frac{7\pi}{12}\right)$

2. $\sin\left(\frac{5\pi}{12}\right)$

3. $\tan\left(\frac{19\pi}{12}\right)$

4. Rewrite in terms of $\sin x$ and $\cos x$.

$$\sin\left(x-\frac{3\pi}{4}\right)$$

Given $\sin(a) = \frac{2}{3}$ and $\cos(b) = -\frac{1}{4}$, with $a$ and $b$ both in the interval $[\frac{\pi}{2}, \pi)$, find $\sin(a+b)$

Prove the identity.

1. $\tan\left(x+\frac{\pi}{4}\right) = \frac{\tan x+1}{1-\tan x}$

2. $\cos(a+b)+\cos(a-b) = 2\cos(a)\cos(b)$

3. $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} = \frac{1-\tan(\alpha)\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$