

Solving Trigonometric Equations (Part 1)

Trigonometry

Introduction

How To - Solve Linear Trigonometric Equations

To solve an equation of the form $\sin x = a$, $\cos x = a$, or $\tan x = a$ on the interval $0 \leq x < 2\pi$:

1. Use an inverse function to find one solution to the equation. Call this solution x_0 .
2. Find a second solution.
 - For sine, the second solution is $x_1 = \pi - x_0$.
 - For cosine, the second solution is $x_1 = 2\pi - x_0$.
 - For tangent, the second solution is $x_1 = x_0 + \pi$.
3. If you have a negative solution, add 2π to that solution. This adds one period to the sine case. It adds two periods when you have a tangent function. (We added one period to the solution with a tangent function back in step 2.)

How To - Find the General Solution to a Trigonometric Equation

1. Find the solutions to the equation on the interval $0 \leq x < 2\pi$. Only use one solution in the case of tangent.
2. Add multiples of the period, $2n\pi$ or $n\pi$ to the solutions from step 1.

Examples

Solve the following equations on the interval $0 \leq x < 2\pi$. Then find the general solutions.

1. $\sin \theta = -\frac{\sqrt{2}}{2}$

2. $\cos \theta = -\frac{\sqrt{2}}{2}$

3. $\tan \theta = -\sqrt{3}$

General Hints

1. Look for a pattern that suggests an algebraic property, such as the difference of squares or a factoring opportunity.
2. Substitute the trigonometric expression with a single variable, such as x or u .
3. Solve the equation the same way an algebraic equation would be solved.
4. Substitute the trigonometric expression back in for the variable in the resulting expressions.
5. Solve for the angle.

Examples

Solve the following equations on the interval $0 \leq x < 2\pi$.

1. $4\sin^2 x - 2 = 0$.

2. $\sec x \sin x - 2 \sin x = 0$.

3. $2 \sin x \cos x - \sin x + 2 \cos x - 1 = 0$

Equations of Quadratic Type

Examples

Solve the following equations on the interval $0 \leq x < 2\pi$.

1. $\tan^2 x - \sqrt{3} \tan x = 0$

2. $\sin^2 x + \sin x - 2 = 0$

3. $3 \cos^2 x - 2 \cos x - 2 = 0$