ACT Practice – Pass 1

ACT PREP

3/7/2024

Overview

Number and Quantity

Rates, Ratios, Proportions, and Percents

Algebra

Tables, Graphs, Statistics, and Probability

Functions

Geometry

Number and Quantity

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Exponent Rules

- **Important Idea**: Exponents count factors. All exponent rules are really counting rules.

- See page 175(ish)

- Multiplying two exponentials with same base, add exponents.
$$a^M \cdot a^N = a^{M+N}$$

- Dividing two exponential with same base, subtract exponents.
$$\frac{a^M}{a^N} = a^{M-N}$$

- Raising a power to a power, multiply exponents.
$$(a^M)^N = a^{M \cdot N}$$

- Multiplying inside a base, "distribute" the exponent.
$$(a \cdot b)^M = a^M \cdot b^M$$

- Zero exponent is 1. (Except 0^0)
$$a^0 = 1$$

- Negative exponents mean reciprocal.
$$a^{-M} = \frac{1}{a^M} \text{ and } \frac{1}{a^{-M}} = a^M$$

Scientific Notation

- A number is in scientific notation if it's a number between 1 and 10 (including 1 but excluding 10) times 10 raised to a power.

- If a number is not in scientific notation, move the decimal until it is behind the first digit. Count the number of places you moved the decimal.
  - If the original number is less than one, the exponent will be negative
  - If the original number is greater than 10, the exponent will be positive.

- You can add or subtract numbers in scientific notation if the exponent is the same.

- You can multiply or divide numbers in scientific notation using the rules of exponents.

Radical Rules

- A radical is a root.

- The rules for radicals are virtually the same as the rules for exponents. (That's not coincidence.)

- Dividing inside a radical, pull out the division.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

- Multiplying inside a radical, pull out the multiplication.

$$\sqrt{a \cdot b} = \sqrt{a} + \sqrt{b}$$

- Radicals can be rewritten as fractional exponents.

$$\sqrt{a} = a^{\frac{1}{2}}, \sqrt[3]{a} = a^{\frac{1}{3}}, \text{etc.}$$

- Fractional exponents can be rewriting as radicals.

$$a^{\frac{M}{N}} = \sqrt[N]{a^M} = (\sqrt[N]{a})^M$$

- Cube roots of negative numbers are negative.

Absolute Value

- The absolute value of a number is its distance from 0 on the number line.

- To find the absolute value of a number, ignore the sign.

Rates, Ratios, Proportions, and Percents

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Rates, Ratios, and Proportions

- Rates often have the word "per" in the units.

- A ratio is a comparison between numbers with the same units.

- Simplify ratios like fractions. Cancel the units.

- A proportion is two ratios, or two rates, set equal.

- Solve a proportion by cross-multiplying.

Percents

- To calculate percent, use

$$\text{percent} = \frac{\text{part}}{\text{whole}} \cdot 100\%.$$

- To calculate percent change, use

$$\text{percent change} = \frac{\text{amount of change}}{\text{original amount}} \cdot 100\%$$

Algebra

Polynomials

- A polynomial is a sum of numbers and numbers times variables raises to whole number exponents.

- Add exponents by combining like terms.
  - Like terms have the same variables raised to the same exponents.
  - Combine like terms by adding coefficients, leave the variables and exponents.

- Simplify polynomials by factoring out common factors.
  - Find a factor that divides evenly into all terms.
  - Factor out that factor by doing the distributive property in reverse.

Solving Equations

1. Simplify fractions by multiplying by the LCD.
2. Simplify parentheses by distributing.
3. Simplify like terms by combining like terms.
4. Simplify variable terms across the equal sign.
5. Simplify constant terms across the equal sign.
6. Simplify the variable by dividing by the coefficient.

Linear Graphs

- Slope formula

$$\frac{y_2 - y_1}{x_2 - x_1}$$

- To find the equation of a line from two points:

1. Calculate the slope.

2. Write the equation in $y = mx + b$ form.

3. Solve for the y-intercept.

Solve Inequalities

- Same as solving a linear equation, except:
  - When multiplying or dividing by a negative number, reverse the direction of the inequality.
- Three-part inequalities use the same rules.

Quadratics (Part 1)

- Multiply binomials with FOIL:

  $(x + a)(x + b)$

  $= x^2 + (a + b)x + a \cdot c$

- Can factor with FOIL by reversing the process above.

- To solve a quadratic equation:
  1. Get 0 on one side.
  2. Factor the polynomial.
  3. Set the factors equal to 0.
  4. Solve the resulting equation.

Quadratics (Part 2)

- Quadratic formula to solve
$$ax^2 + bx + c = 0$$
is
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$
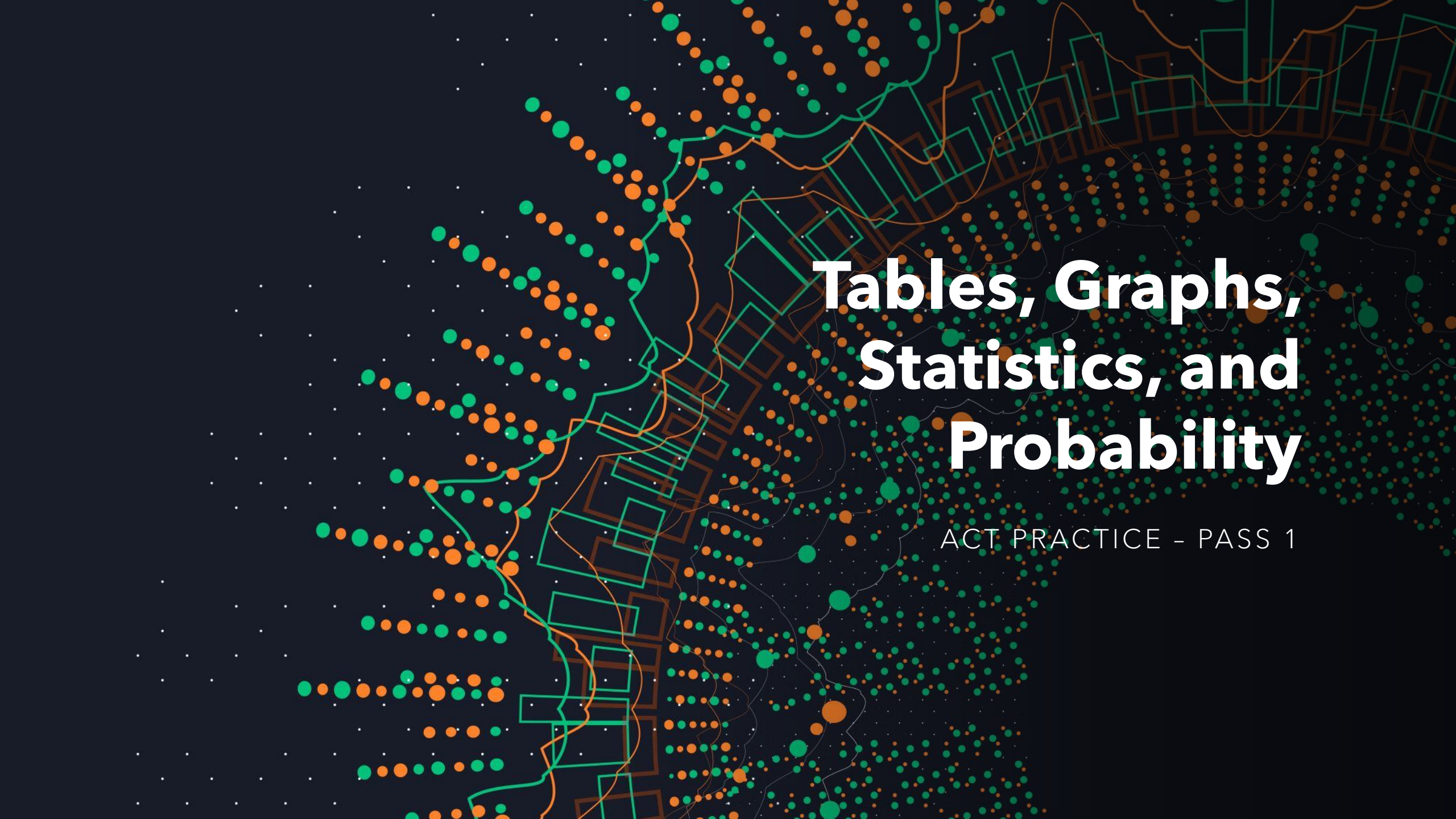
- The quadratic formula always works.

- Factoring is faster when it works.

Special Factoring Formulas

- $a^2 + 2ab + b^2 = (a + b)^2$

- $a^2 - 2ab + b^2 = (a - b)^2$

- $a^2 - b^2 = (a - b)(a + b)$

Word Problems

- Translate the problems into algebra.

- Read carefully.

- Solve the algebra next.

- Be sure you find what the problem is really looking for.

Tables, Graphs, Statistics, and Probability

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Common Graphs

- Tables – list data, sometimes with frequencies

- Bar graphs – shows frequency of different values.

- Histogram – look like bar charts. Puts data into bins (ranges) first.

- Circle graphs – pie charts.

- Piecewise graphs – equation for graph changes with x-values

Statistics

- Mode – most common data value.

- Mean – Average. Add the data values and divide by the number of values.

- Median – Value that divides the data into upper half and lower half.

- Range – Largest data value minus the smallest data value.

Probability

- Formula:

$$\text{probability} = \frac{\text{number of successful outcomes}}{\text{number of possible outcomes}}$$

- Probability measures likelihood of an event.

- Write probability as fractions or decimals.

- Probabilities are numbers between 0 and 1.

- Find the probability that an event doesn't happen by subtracting 1 and the probability the event happens.

Functions

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Functions and Function Notation

- Function gives unique output for a given input.

- Set of inputs to a function is the domain.

- Set of outputs to a function is the range.

- Function notation:

$$y = f(x)$$

  - x = input
  - y = output
  - f is the function name

Graphs of Functions

- Points of graph determine function values.

- x-coordinates are the inputs.

- y-coordinates are the outputs.

- Domain is all x-coordinates in the graph.

- Range is all y-coordinates in the graph.

Geometry

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Lines and Angles

- Adjacent angles – can add measures to find the total

- Complementary angles – add to **90°**.

- Supplementary angles – add to **180°**

- Parallel lines don't intersect

- Perpendicular lines intersect at right angles

- Vertical angles formed by intersecting lines.
  - Opposite angles equal
  - Adjacent angles supplementary

Parallel Lines and a Transversal

- Two parallel lines intersected by a third line called a transversal

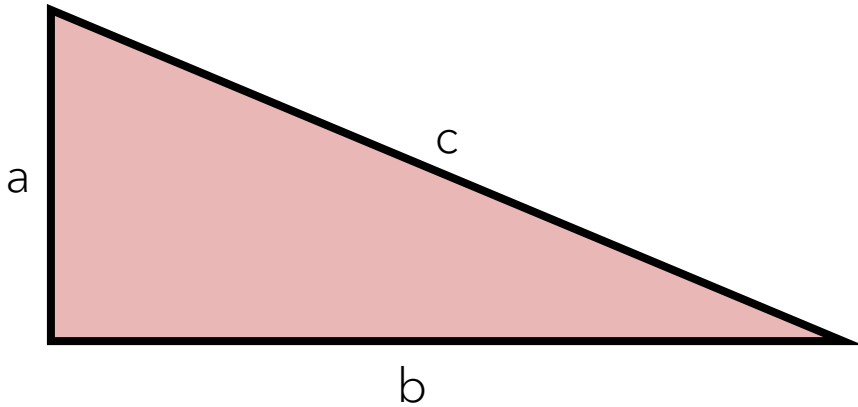- All angles are either equal measure or supplementary.
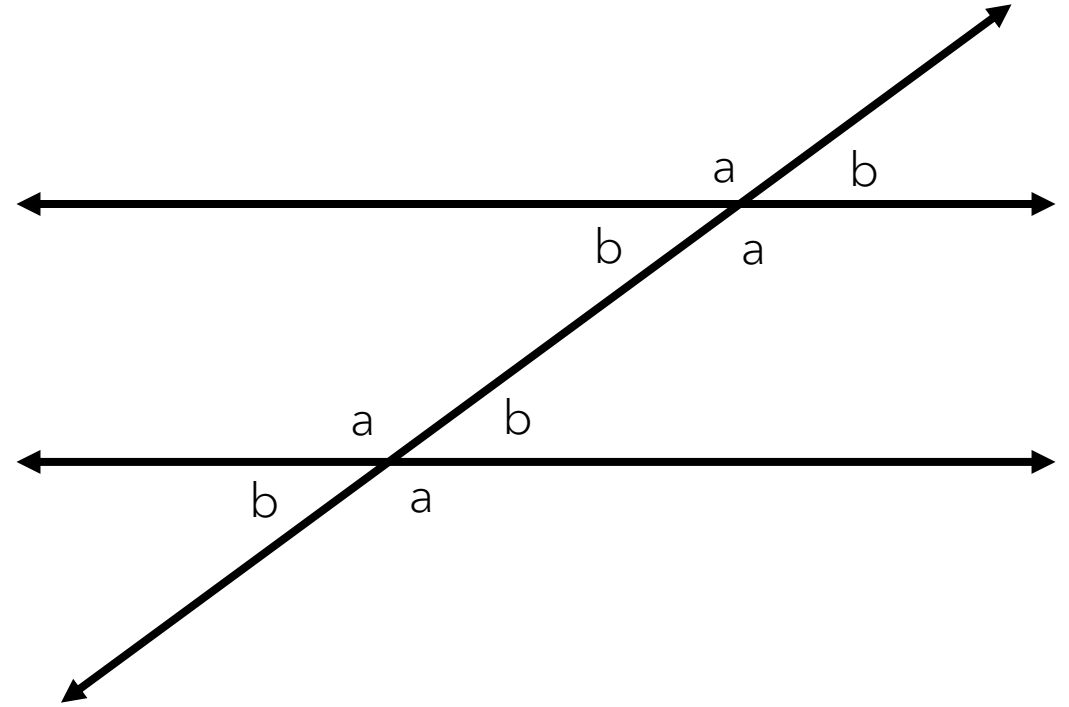
Triangle Properties

- Interior angles of a triangle add to **180°**

- Similar triangles have same angle measures and proportional sides.

- Congruent triangles have same angles and sides

- Sum of interior angles of polygon with n sides
$$(n - 2) \cdot 180°$$

Right Triangles

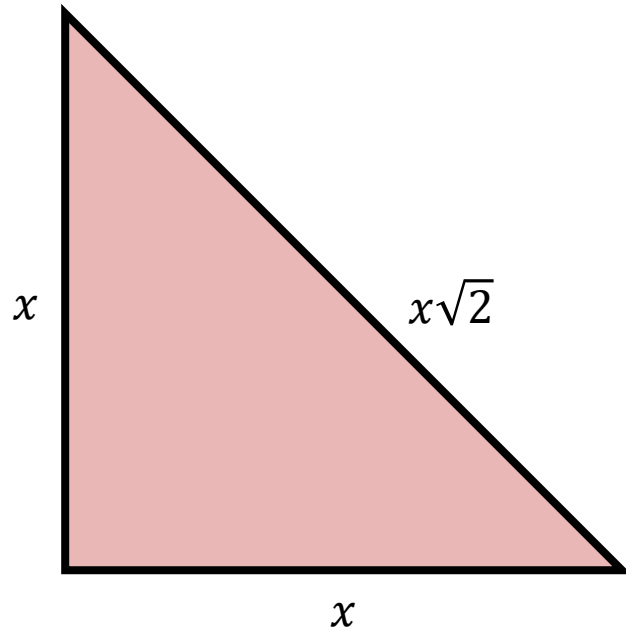- Pythagorean Theorem

$$a^2 + b^2 = c^2$$

- Common Pythagorean Tripples:
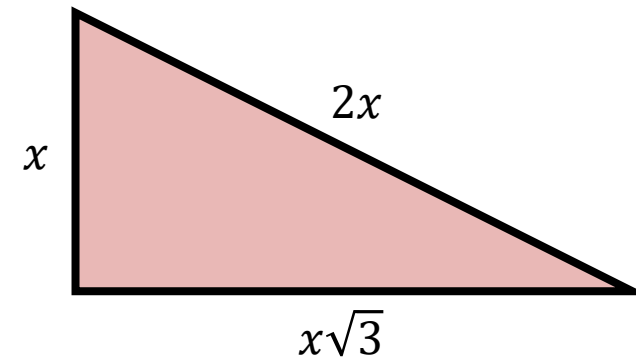 - 3, 4, 5
  - 5, 12, 13

Famous Right Triangles

- 45-45-90 Right Triangle

- 30-60-90 Right Triangle

Polygons

- Perimeter – distance around polygon

- Area – amount of space inside polygon

- Parallelogram – quadrilateral with two pairs of parallel sides

- Trapezoid – exactly one pair of parallel sides

- Area of parallelogram
$$A = b \cdot h$$

- Area of trapezoid
$$A = \frac{1}{2}h(B + b)$$

- Area of triangle
$$A = \frac{1}{2}b \cdot h$$

Circle

- Circumference – distance around circle

- Circumference of circle
$$C = 2\pi r$$
$$C = \pi d$$

- Area of circle
$$A = \pi r^2$$

- Arc – part of a circle circumference

- Central angle – angle with vertex at center of circle
$$\frac{\text{Central angle}}{360°} = \frac{\text{arc length}}{\text{circumference}}$$
$$= \frac{\text{sector area}}{\text{circle area}}$$

Remaining Topics

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Remaining Topics

- Absolute Value Equations
- Matrices
- Complex Numbers
- Sequences
- Graphs of Inequalities
- Systems of Equations

- Translations of Functions
- Trigonometric Identities
- Graphs of Trigonometric Functions
- Unit Conversions
- 3-Dimensional Shapes
- Trigonometric Ratios