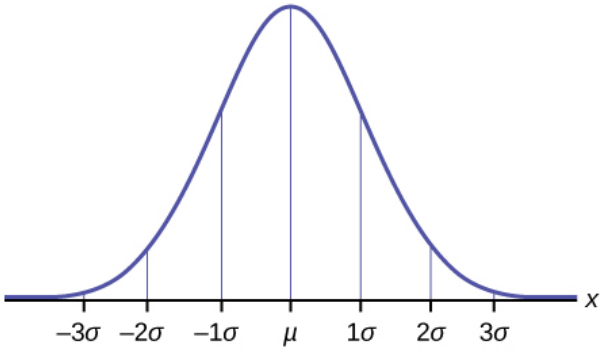
1 The Normal Distribution

Normal Distributions

- Many data sets follow a **normal distribution**.
- The picture to the right shows a distribution function for a normal distribution.
- The distribution of the data is determined by the area under the curve of the graph.



Standard Normal Distribution

- We write $X \sim N(\mu, \sigma)$ to say a random variable X is distributed with a normal distribution with mean μ and stand deviation σ.
- The **standard normal distribution** has mean 0 and standard deviation 1.
- We can change any normal distribution to a standard normal distribution using a **z-score**.

$$z = \frac{x - \mu}{\sigma}$$

- We can use z-scores to measure standard deviations from the mean

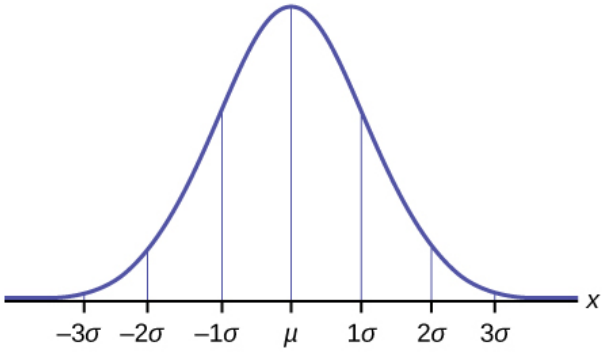
Example 1
Suppose $X \sim N(-1, 2)$.

- What is the z-score of $x = 2$?
- What is the z-score of $x = -13$?
- What value is 2 standard deviations to the right of the mean?
- What value has a z-score of -0.5?

2 The Empirical Rule

The Empirical Rule

- About 68% of the x values lie between -1σ and $+1\sigma$ of the mean μ.
- About 95% of the x values lie between -2σ and $+2\sigma$ of the mean μ.
- About 99.7% of the x values lie between -3σ and $+3\sigma$ of the mean μ.
- Notice that almost all the x values lie within three standard deviations of the mean.



Example 2
Suppose x has a normal distribution with mean 50 and standard deviation 6. What can we say about the distribution of the data?

Example 3
From 1984 to 1985, the mean height of 15 to 18-year-old males from Chile was 172.36 cm, and the standard deviation was 6.34 cm.

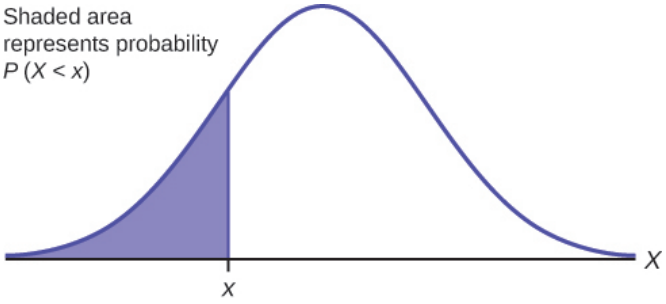
- About 68% of the values lie between what two values?
- About 95% of the values lie between what two values?
- About 99.7% of the values lie between what two values?

3 Using the Normal Distribution

Using Normal Distributions

If $X \sim N(\mu, \sigma)$:

- Calculate $P(X < x)$ by finding the area to the left of x under the bell curve.
- Calculate $P(X > x)$ by finding the area to the right of x under the bell curve.
- Note: $P(X > x) = 1 - P(X < x)$.



Calculator Directions

To calculate the probability that a normally distributed random variable is between two values:

1. Go to `Distr -> 2:normalcdf(`.
2. Enter: lower bound, upper bound, mean, standard deviation.
3. Type ")" and `Enter`.

Note, use "1 EE 99" for infinite bounds.

Example 4

The final exam scores in a class were normally distributed with a mean of 63 and a standard deviation of five.

- Find the probability that a randomly selected student scored more than 65 on the exam.
- Find the probability that a randomly selected student scored less than 85 on the exam.
- Find the probability that a randomly selected student scored between 58 and 68.

Inverse Normal Distribution Function

To find the value of x that puts a given area to the left of x, use the `invNorm` function in the `Distr` menu.

Example 5

The final exam scores in a statistics class were normally distributed with a mean of 63 and a standard deviation of five.

- Find the 70th percentile for the final exam.
- Find the 40th percentile for the final exam.

Example 6

Suppose that the average number of hours a personal computer is used for entertainment is two hours per day and the standard deviation is half an hour.

- Find the probability that a computer is used for entertainment between 1.8 and 2.75 hours per day.
- Find the maximum number of hours per day that the bottom quartile of households use a personal computer for entertainment.