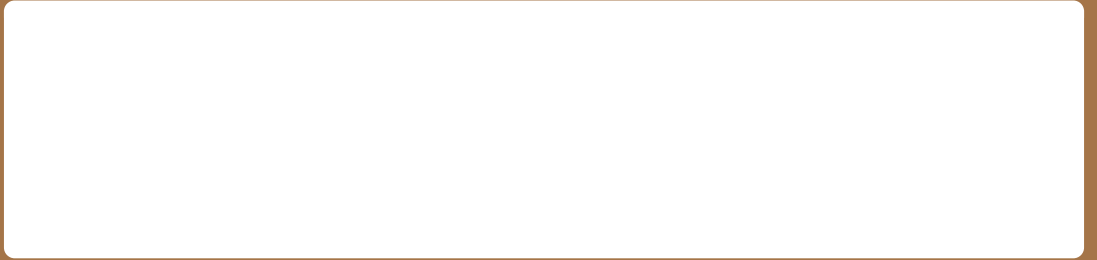# Solving Logarithmic and Exponential Equations

Quick Start

Success in College Math

Overview

Overview

1. Inverse Properties of Logarithms and Exponential Functions

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1. Inverse Properties of Logarithms and Exponential Functions
2. Solving Exponential Equations

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2. Solving Exponential Equations
3. Solving Logarithmic Equations

Inverse Properties of Logarithms and Exponential Functions

For all x

$$\log_b b^x = x.$$

For $x > 0$

$$b^{\log_b x} = x.$$

How To - Solve Exponential Equations

To solve an equation containing an exponential expression:

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How To - Solve Exponential Equations

To solve an equation containing an exponential expression:

1. Isolate the exponential expression.
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3. Cancel the logarithm and exponential expression using the Inverse Property.
4. Solve the resulting equation.

Example 1

Solve the equation. Round your answer to three decimal places where appropriate.

$$4 \cdot 9^{2x} = 14$$

Example 1

Solve for the exponential expression.

$$4 \cdot 9^{2x} = 14$$

$$\frac{4 \cdot 9^{2x}}{4} = \frac{14}{4}$$

$$9^{2x} = 3.5$$

Example 1

- Take the logarithm base 9 of both sides.
- Cancel the logarithm and exponent.

$$9^{2x} = 3.5$$

$$\log_9 9^{2x} = \log_9 3.5$$

$$2x = \log_9 3.5$$

Example 1

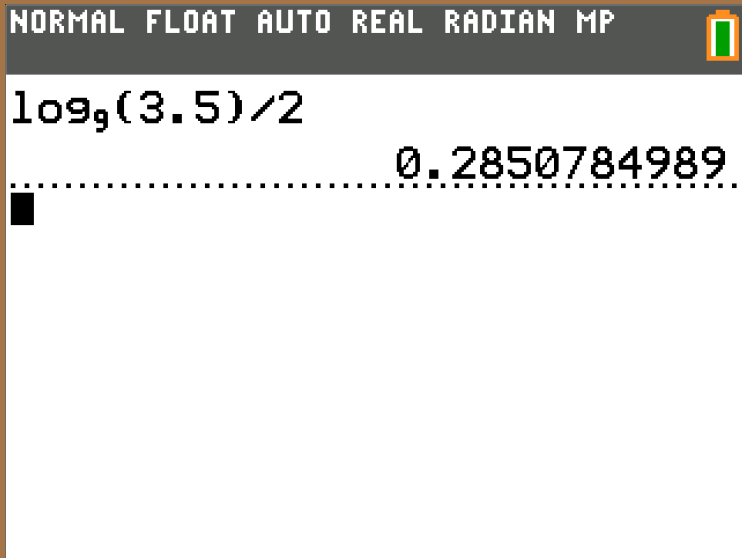- Solve for *x*.
- Round your answer to three decimal places.

$$2x = \log_9 3.5$$

$$\frac{2x}{2} = \frac{\log_9 3.5}{2}$$

$$x = \frac{\log_9 3.5}{2} = 0.285$$

Example 1

- Solve for *x*.
- Round your answer to three decimal places.

$\log_9(3.5)/2$

0.2850784989

Example 2

Solve the equation. Round your answer to three decimal places where appropriate.

$$10^{2x-18} + 12 = -3$$

Example 2

Solve for the exponential expression.

$$10^{2x-18} + 12 = -3$$

$$10^{2x-18} + 12 = -3$$
$$\underline{\phantom{10^{2x-18}} - 12 \quad - 12}$$
$$10^{2x-18} = -15$$

Example 2

- Take the common logarithm of both sides.
- The common logarithm isn't defined for -15, so the equation does not have (real number) solutions.

$$10^{2x-18} = -15$$

$$\log 10^{2x-18} = \log(-15)$$

$\log(-15)$ is not defined for real numbers.

How To - Solve Logarithmic Equations

To solve an equation containing a logarithm:

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1. Isolate the logarithmic expression.

How To - Solve Logarithmic Equations

To solve an equation containing a logarithm:

1. Isolate the logarithmic expression.
2. Use both sides as an exponent in an exponential expression. Use the same base for the exponential expression as the logarithm.

How To - Solve Logarithmic Equations

To solve an equation containing a logarithm:

1. Isolate the logarithmic expression.
2. Use both sides as an exponent in an exponential expression. Use the same base for the exponential expression as the logarithm.
3. Cancel the exponential expression and logarithm using the Inverse Property.

How To - Solve Logarithmic Equations

To solve an equation containing a logarithm:

1. Isolate the logarithmic expression.
2. Use both sides as an exponent in an exponential expression. Use the same base for the exponential expression as the logarithm.
3. Cancel the exponential expression and logarithm using the Inverse Property.
4. Solve the resulting equation.

Example 3

Solve the equation. Round your answer to three decimal places where appropriate.

$$2 \cdot \log (6x) + 16 = 14$$

Example 3

Solve for the logarithm.

$$2 \cdot \log(6x) + 16 = 14$$

$$2 \cdot \log(6x) + 16 = 14$$
$$\underline{ -16 \quad -16}$$
$$2 \cdot \log(6x) = -2$$

$$\frac{2 \cdot \log(6x)}{2} = \frac{-2}{2}$$

$$\log(6x) = -1$$

Example 3

- Use both sides as an exponent with base 10.
- Cancel the exponent and logarithm.
- The negative exponent is defined, so we know there are solutions.

$$\log(6x) = -1$$

$$10^{\log(6x)} = 10^{-1}$$

$$6x = 10^{-1}$$

Example 3

- Solve the equation for x.
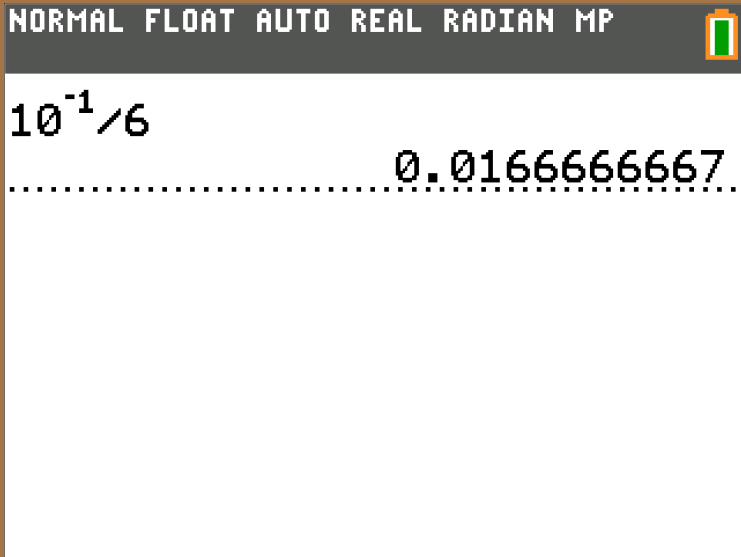- Round the answer to three decimal places.

$$6x = 10^{-1}$$

$$\frac{6x}{6} = \frac{10^{-1}}{6}$$

$$x = \frac{10^{-1}}{6} = 0.017$$

Example 3

- Solve the equation for *x*.
- Round the answer to three decimal places.

NORMAL FLOAT AUTO REAL RADIAN MP

10⁻¹/6

$$10^{-1}/6$$
$$\quad\quad\quad\quad\quad 0.0166666667$$

Example 4

Solve the equation. Round your answer to three decimal places where appropriate.

$$\log_8 (3x - 18) = 3$$

Example 4

- Use both sides of the equation as an exponent with base 8.
- Cancel the exponent and logarithm.

$$\log_8(3x - 18) = 3$$

$$8^{\log_8(3x - 18)} = 8^3$$

$$3x - 18 = 8^3$$

Example 4

- Solve the equation for x.
- Round the answer to three decimal places.

$$3x - 18 = 8^3$$

$$3x - 18 = 8^3$$
$$\underline{+18 \quad\quad +18}$$

$$3x = 8^3 + 18$$

$$\frac{3x}{3} = \frac{8^3 + 18}{3}$$

$$x = (8^3 + 18)/3 = 176.667$$

Example 4

- Solve the equation for *x*.
- Round the answer to three decimal places.

$(8^3+18)/3$

176.6666667