

Composite and Inverse Functions

College Algebra

Definition – Function Composition

Definition

If f and g are two functions so that the range of f is the same as the domain of g , then the composite function $g \circ f$ is defined by

$$(g \circ f)(x) = g(f(x))$$

Example 1

Define

$$f(x) = x^2 + 5; \quad g(x) = 3x + 4$$

Find $(g \circ f)(2)$.

Find $(f \circ g)(2)$.

Example 2

Define

$$f(x) = x^2 + 5; \quad g(x) = 3x + 4$$

Find a formula for $(g \circ f)(x)$. Find a formula for $(f \circ g)(x)$.

Example 3

Define

$$f(x) = 3x - 6; \quad g(x) = \frac{1}{3}x + 2$$

Find a formula for $(g \circ f)(x)$. Find a formula for $(f \circ g)(x)$.

Definition – Inverse Functions

Two functions f and g are **inverse functions** if $(g \circ f)(x) = x$ and $(f \circ g)(x) = x$ for all x in the domains of f and g .

The inverse of the function f is denoted f^{-1} .

Procedure – Finding the Inverse of a Function

To find the inverse of the function $y = f(x)$:

1. Set up the equation $x = f(y)$.
2. Solve the equation for y .
3. Use the solution to define $y = f^{-1}(x)$.

Example 4

Find a formula for $f^{-1}(x)$.

$$f(x) = 2x - 9$$

Example 5

Find a formula for $f^{-1}(x)$.

$$f(x) = 2x^3 - 4$$