Solving Logarithmic and Exponential Equations

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Success in College Math

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Inverse Properties of Logarithms and Exponential Functions

For all x

$$\log_b b^x = x.$$

For $x > 0$

$$b^{\log_b x} = x.$$

How To - Solve Exponential Equations

To solve an equation containing an exponential expression:

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1. Isolate the exponential expression.
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3. Cancel the logarithm and exponential expression using the Inverse Property.

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To solve an equation containing an exponential expression:

1. Isolate the exponential expression.
2. Take the logarithm of both sides. Use the same base for the logarithm as the exponential expression.
3. Cancel the logarithm and exponential expression using the Inverse Property.
4. Solve the resulting equation.

Example 1

Solve the equation. Round your answer to three decimal places where appropriate.

$$4 \cdot 9^{2x} = 14$$

Example 1

Solve for the exponential expression.

$$4 \cdot 9^{2x} = 14$$

$$\frac{4 \cdot 9^{2x}}{4} = \frac{14}{4}$$

$$9^{2x} = 3.5$$

Example 1

- Take the logarithm base 9 of both sides.
- Cancel the logarithm and exponent.

$$9^{2x} = 3.5$$

$$\log_9 9^{2x} = \log_9 3.5$$

$$2x = \log_9 3.5$$

Example 1

- Solve for x.
- Round your answer to three decimal places.

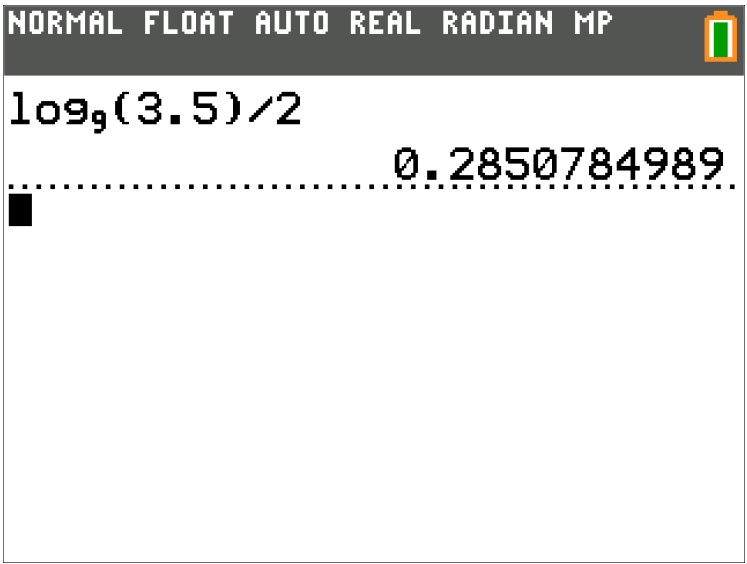
$$2x = \log_9 3.5$$

$$\frac{2x}{2} = \frac{\log_9 3.5}{2}$$

$$x = \frac{\log_9 3.5}{2} = 0.285$$

Example 1

- Solve for x.
- Round your answer to three decimal places.



Example 2

Solve the equation. Round your answer to three decimal places where appropriate.

$$10^{2x-18} + 12 = -3$$

Example 2

Solve for the exponential expression.

$$10^{2x-18} + 12 = -3$$

$$10^{2x-18} + 12 = -3$$

$$-12 \quad -12$$

$$10^{2x-18} = -15$$

Example 2

- Take the common logarithm of both sides.
- The common logarithm isn't defined for -15, so the equation does not have (real number) solutions.

$$10^{2x-18} = -15$$

$$\log 10^{2x-18} = \log(-15)$$

$\log(-15)$ is not defined for real numbers.

How To - Solve Logarithmic Equations

To solve an equation containing a logarithm:

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To solve an equation containing a logarithm:

1. Isolate the logarithmic expression.
2. Use both sides as an exponent in an exponential expression. Use the same base for the exponential expression as the logarithm.

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To solve an equation containing a logarithm:

1. Isolate the logarithmic expression.
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3. Cancel the exponential expression and logarithm using the Inverse Property.

How To - Solve Logarithmic Equations

To solve an equation containing a logarithm:

1. Isolate the logarithmic expression.
2. Use both sides as an exponent in an exponential expression. Use the same base for the exponential expression as the logarithm.
3. Cancel the exponential expression and logarithm using the Inverse Property.
4. Solve the resulting equation.

Example 3

Solve the equation. Round your answer to three decimal places where appropriate.

$$2 \cdot \log(6x) + 16 = 14$$

Example 3

Solve for the logarithm.

$$2 \cdot \log(6x) + 16 = 14$$

$$2 \cdot \log(6x) + 16 = 14$$

$$-16 \quad -16$$

$$2 \cdot \log(6x) = -2$$

$$\frac{2 \cdot \log(6x)}{2} = \frac{-2}{2}$$

$$\log(6x) = -1$$

Example 3

- Use both sides as an exponent with base 10.
- Cancel the exponent and logarithm.
- The negative exponent is defined, so we know there are solutions.

$$\log(6x) = -1$$

$$10^{\log(6x)} = 10^{-1}$$

$$6x = 10^{-1}$$

Example 3

- Solve the equation for x.
- Round the answer to three decimal places.

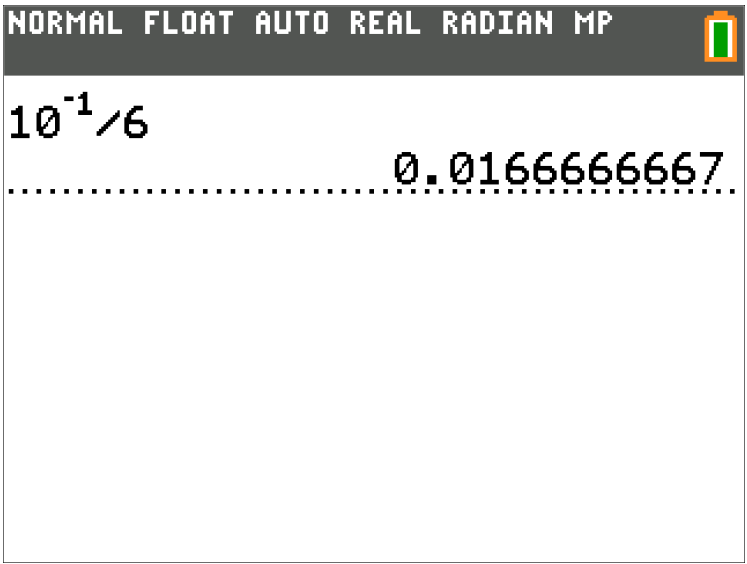
$$6x = 10^{-1}$$

$$\frac{6x}{6} = \frac{10^{-1}}{6}$$

$$x = \frac{10^{-1}}{6} = 0.017$$

Example 3

- Solve the equation for x.
- Round the answer to three decimal places.



Example 4

Solve the equation. Round your answer to three decimal places where appropriate.

$$\log_8 (3x - 18) = 3$$

Example 4

- Use both sides of the equation as an exponent with base 8.
- Cancel the exponent and logarithm.

$$\log_8(3x-18)=3$$

$$8^{\log_8(3x-18)}=8^3$$

$$3x-18=8^3$$

Example 4

- Solve the equation for x.
- Round the answer to three decimal places.

$$3x - 18 = 8^3$$

$$3x - 18 = 8^3$$

$$+18 \quad +18$$

$$3x = 8^3 + 18$$

$$\frac{3x}{3} = \frac{8^3 + 18}{3}$$

$$x = (8^3 + 18)/3 = 176.667$$

Example 4

- Solve the equation for x.
- Round the answer to three decimal places.