The Multiplication Rule for Counting

Detailed Examples

Introduction

Using the Multiplication Rule for Counting can look difficult at first. The key is to follow the following steps.

1. Determine how many decisions must be made for each step of the problem. Write a slot for each decision.
2. Fill in the number of possibilities for each decision in the corresponding slot. Work on each decision one at a time.
3. Multiply the numbers in each slot. The product is your answer.

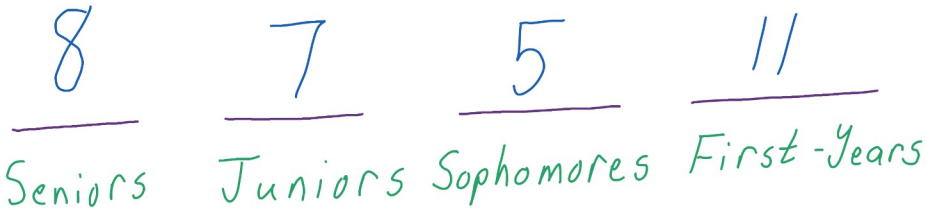
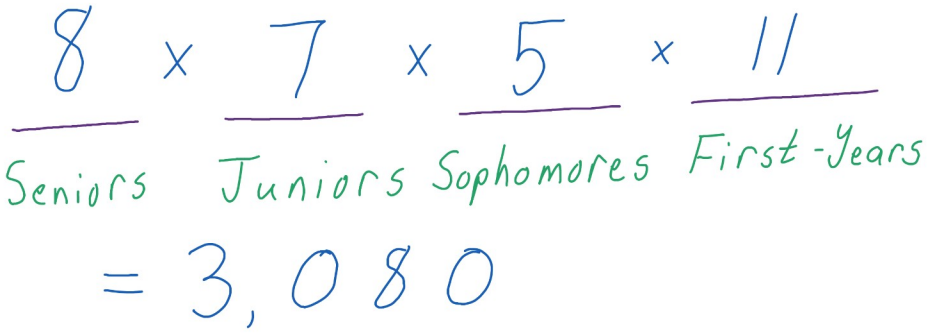
These problems can have large numbers for answers.

Examples

Example 7.2

The University Combinatorics Club has 31 members: 8 seniors, 7 juniors, 5 sophomores, and 11 first-years. How many possible 4-person committees can be formed by selecting 1 member from each class?

Step	Work
Write four slots, one each for: seniors, juniors, sophomores, and first-years.	

Step	Work
Fill in the slots: 8 for the seniors, 7 for the juniors, 5 for the sophomores, and 11 for the first-years.	
Multiply the numbers in each slot.	

The answer is 3,080 ways to pick people for the committee.

Example 7.3

The standard license plates for vehicles in a certain state consist of 6 characters: 3 letters followed by 3 digits. There are 26 letters in the alphabet and 10 digits (0 through 9) to choose from. How many license plates can be made using this format?

Step	Work
Write six slots: three for the three letters and three for the numbers.	$\underset{\text{letter}}{\underset{1^{st}}{\square}} \quad \underset{\text{letter}}{\underset{2^{nd}}{\square}} \quad \underset{\text{letter}}{\underset{3^{rd}}{\square}} \quad \underset{\text{number}}{\underset{1^{st}}{\square}} \quad \underset{\text{number}}{\underset{2^{nd}}{\square}} \quad \underset{\text{number}}{\underset{3^{rd}}{\square}}$
Fill in the slots. The first three slots are 26, which is the number of letters. The last three slots are 10, which is the number of numbers.	$\underset{\text{letter}}{\underset{1^{st}}{26}} \quad \underset{\text{letter}}{\underset{2^{nd}}{26}} \quad \underset{\text{letter}}{\underset{3^{rd}}{26}} \quad \underset{\text{number}}{\underset{1^{st}}{10}} \quad \underset{\text{number}}{\underset{2^{nd}}{10}} \quad \underset{\text{number}}{\underset{3^{rd}}{10}}$
Multiply the numbers in each slot.	$\underset{\text{letter}}{\underset{1^{st}}{26}} \times \underset{\text{letter}}{\underset{2^{nd}}{26}} \times \underset{\text{letter}}{\underset{3^{rd}}{26}} \times \underset{\text{number}}{\underset{1^{st}}{10}} \times \underset{\text{number}}{\underset{2^{nd}}{10}} \times \underset{\text{number}}{\underset{3^{rd}}{10}}$ $= 17{,}576{,}000$

The number of possible license plates is 17,576,000. For a large state, having this many possibilities is good so you will not run out of plates.