

**Definition 0.1.** Let  $A$  be an object set. We define a total order on  $A$  by the one-to-one mapping

$$\tau : A \rightarrow \{1, 2, \dots, |A|\}$$

that is,  $\forall x \neq y \in A, x < y \Leftrightarrow \tau(x) < \tau(y)$ .

Let  $L$  be the set that contains all of the  $\tau$ .

**Definition 0.2.** A function  $f : L^n \rightarrow A$  is called a choice function.

**Definition 0.3.** A choice function  $f$  can be *strategically manipulated* by voter  $i$  if for some  $\tau_1, \dots, \tau_n \in L$  and some  $\tau'_i \in L$  we have that  $\tau_i(a) < \tau_i(a')$  where  $a = f(\tau_1, \dots, \tau_n)$  and  $a' = f(\tau_1, \dots, \tau'_i, \dots, \tau_n)$ . That is, voter  $i$  that prefers  $a'$  to  $a$  can ensure that  $a'$  gets chosen rather than  $a$  by strategically misrepresenting his preferences to be  $\tau'_i$  rather than  $\tau_i$ .  $f$  is called *incentive compatible* if it cannot be manipulated.

**Definition 0.4.** A choice function  $f$  is monotone if  $f(\tau_1, \dots, \tau_i, \dots, \tau_n) = a \neq a' = f(\tau_1, \dots, \tau'_i, \dots, \tau_n)$  implies ( $\Rightarrow$ ) that  $\tau_i(a') < \tau_i(a)$  and  $\tau'_i(a) < \tau'_i(a')$ . That is if the choice changed from  $a$  to  $a'$  when a single voter  $i$  changed his vote from  $\tau_i$  to  $\tau'_i$  then it must be because he switched his preference between  $a$  and  $a'$ .

**Definition 0.5.** Let  $\vec{v} = (v_1, \dots, v_n)$  be an  $n$ -dimensional vector. We will denote the  $(n-1)$ -dimensional vector in which the  $i$ -th coordinate is removed by  $v_{-i} = (v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_n)$ . Thus we have three equivalent notations:  $\vec{v} = (v_1, \dots, v_n) = (v_i, v_{-i})$ .

**Definition 0.6.** A game with (independent private values and) strict incomplete information for a set of  $n$  players is given by the following ingredients:

- (i) For every player  $i$ , a set of *actions*  $X_i$ .
- (ii) For every player  $i$ , a set of *types*  $T_i$ . A value  $t_i \in T_i$  is the private information that  $i$  has.
- (iii) For every player  $i$ , a *utility function*  $u_i : T_i \times X_1 \times \dots \times X_n \rightarrow \mathbb{R}$ , where  $u_i(t_i, x_1, \dots, x_n)$  is the utility achieved by player  $i$ , if his type is  $t_i$ , and the profile of actions taken by all players is  $x_1, \dots, x_n$ .

**Definition 0.7.** (i) A strategy of a player  $i$  is a function  $s_i : T_i \rightarrow X_i$ .

(ii) A strategy  $s_i$  is a (weakly) dominant strategy if for every  $t_i$  we have that the action  $s_i(t_i)$  is a dominant strategy in the full information game defined by  $t_i$ . Formally: For all  $t_i$ , all  $x_{-i}$  and all  $x'_i$  we have that  $u_i(t_i, s_i(t_i), x_{-i}) \geq u_i(t_i, x'_i, x_{-i})$ . A profile  $s_1, \dots, s_n$  is called a dominant strategy equilibrium if each  $s_i$  is a dominant strategy.

**Definition 0.8.** (i) A mechanism for  $n$  players is given by

- (a) players' type spaces  $T_1, \dots, T_n$ ,
- (b) players' action spaces  $X_1, \dots, X_n$ ,
- (c) an alternative set  $A$ ,
- (d) an outcome function  $a : X_1 \times \dots \times X_n \rightarrow A$  and,

The game with strict incomplete information induced by the mechanism is given by using the types spaces  $T_i$ , the action spaces  $X_i$ , and the utilities  $u_i(t_i, x_1, \dots, x_n)$ .

(ii) The mechanism implements a choice function  $f : T_1 \times \dots \times T_n \rightarrow A$  in dominant strategies if for some dominant strategy equilibrium  $s_1, \dots, s_n$  of the induced game, where  $s_i : T_i \rightarrow X_i$ , we have that for all  $t_1, \dots, t_n$ ,  $f(t_1, \dots, t_n) = a(s_1(t_1), \dots, s_n(t_n))$ .

**Proposition 0.1** (Revelation Principle). *If there exists an arbitrary mechanism that implements  $f$  in dominant strategies, then there exists an incentive compatible mechanism that implements  $f$ . The payments of the players in the incentive compatible mechanism are identical to those, obtained at equilibrium, of the original mechanism.*

*Proof.* The new mechanism will simply simulate the equilibrium strategies of the players . That is , let  $s_1, \dots, s_n$  be a dominant strategy equilibrium of the original mechanism , we define a new direct revelation mechanism :  $f(t_1, \dots, t_n) = a(s_1(t_1), \dots, s_n(t_n))$  . Now since each  $s_i$  is a dominant strategy for player  $i$  , then for every  $t_i, x_{-i}, x'_i$  we have that  $u_i(t_i, a(s_i(t_i), x_{-i})) \geq u_i(t_i, a(x'_i, x_{-i}))$  . Thus in particular this is true for all  $x_{-i} = s_{-i}(t_{-i})$  and any  $x'_i = s_i(t'_i)$  , which gives the definition of incentive compatibility of the mechanism  $(f, u'_1, \dots, u'_n)$  .

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