Definition 0.1. Let A be an object set.We define a total order on A by the one-to-one mapping

$$\tau : A \to \{1, 2, ..., |A|\}$$

that is,$\forall x \neq y \in A, x < y \Leftrightarrow \tau(x) < \tau(y)$.

Let L be the set that contains all of the τ .

Definition 0.2. A function $f : L^n \to A$ is called a choice function.

Definition 0.3. A choice function f can be *strategically manipulated* by voter i if for some $\tau_1, ..., \tau_n \in L$ and some $\tau_i' \in L$ we have that $\tau_i(a) < \tau_i(a')$ where $a = f(\tau_1, ..., \tau_n)$ and $a' = f(\tau_1, ...\tau_i', ..., \tau_n)$. That is, voter i that prefers a' to a can ensure that a' gets chosen rather than a by strategically misrepresenting his preferences to be τ_i' rather than τ_i . f is called *incentive compatible* if it cannot be manipulated.

Definition 0.4. A choice function f is monotone if $f(\tau_1, ..., \tau_i, ..., \tau_n) = a \neq a' = f(\tau_1, ..., \tau_i', ..., \tau_n)$ implies($\Rightarrow$) that $\tau_i(a') < \tau_i(a)$ and $\tau_i'(a) < \tau_i'(a')$. That is if the choice changed from a to a' when a single voter i changed his vote from τ_i to τ_i' then it must be because he switched his preference between a and a' .

Definition 0.5. Let $\vec{v} = (v_1, ..., v_n)$ be an n-dimentional vector. We will denote the $(n-1)$-dimentional vector in which the i-th coordinate is removed by $\vec{v_{-i}} = (v_1, ..., v_{i-1}, v_{i+1}, ..., v_n)$. Thus we have three equivalent notations:$\vec{v} = (v_1, ..., v_n) = (v_i, \vec{v_{-i}})$.

Definition 0.6. A (direct revelation) mechanism is a choice function f and a vector of utility functions $u_1^f, ..., u_n^f$

$$f : V_1 \times ... \times V_n \to A \tag{0.1}$$

$$u_i^f : V_1 \times ... \times V_n \to \mathbb{R} \tag{0.2}$$

Definition 0.7. A mechanism $(f, u_1^f, ..., u_n^f)$ is called incentive compatible if $\forall i, \exists v_i \in V_i, \forall v_i' \in V_i$,

$$u_i^f(v_i, v_{-i}) \geq u_i^f(v_i', v_{-i}) \tag{0.3}$$

Definition 0.8. A game with(independent private values and) strict incomplete information for a set of n players is given by the following ingredients:

(i)For every player i , a set of *actions* X_i.

(ii)For every player i , a set of *types* T_i . A value $t_i \in T_i$ is the private information that i has.

(iii)For every player i , a *utility function* $u_i : T_i \times X_1 \times ... \times X_n \to \mathbb{R}$, where $u_i(t_i, x_1, ..., x_n)$ is the utility achieved by player i , if his type is t_i, and the profile of actions taken by all players is $x_1, ..., x_n$.

Definition 0.9. (i)A strategy of a player i is a function $s_i : T_i \to X_i$.

(ii)A strategy s_i is a (weakly) dominant strategy if for every t_i we have that the action $s_i(t_i)$ is a dominant strategy in the full information game defined by t_i . Formally:For all t_i, allx_{-i} and all x_i' we have that

$$u_i(t_i, s_i(t_i), x_{-i}) \geq u_i(t_i, x_i', x_{-i}) \tag{0.4}$$

A profile $s_1, ..., s_n$ is called a dominant strategy equilibrium if each s_i is a dominant strategy.

Definition 0.10. (i)A mechanism for n players is given by

(a)players' type spaces $T_1, ..., T_n$,

(b)players' action spaces $X_1, ..., X_n$,

(c)an alternative set A ,

(d)an outcome function $a : X_1 \times ... \times X_n \to A$ and ,

The game with strict incomplete information induced by the mechanism is given by using the types spaces T_i , the action spaces X_i , and the utilities $u_i(t_i, x_1, ..., x_n)$.

(ii)The mechanism implements a choice function $f : T_1 \times ... \times T_n \to A$ in dominant strategies if for some dominant strategy equilibrium $s_1, ..., s_n$ of the induced game , where $s_i : T_i \to X_i$, we have that for all $t_1, ..., t_n$, $f(t_1, ..., t_n) = a(s_1(t_1), ..., s_n(t_n))$.

Proposition 0.1 (Revelation Principle)**.** *If there exists an arbitrary mechanism that implements f in dominant strategies , then there exists an incentive compatible mechanism that implements f. The payments of the players in the incentive compatible mechanism are identical to those , obtained at equilibrium , of the original mechanism.*

Proof. The new mechanism will simply simulate the equilibrium strategies of the players . That is , let $s_1, ..., s_n$ be a dominant strategy equilibrium of the original mechanism , we define a new direct revelation mechanism :

$$f(t_1, ..., t_n) := a(s_1(t_1), ..., s_n(t_n)) \tag{0.5}$$

$$u_i^f(t_1, ..., t_n) := u_i(t(i), s_1(t_1), ..., s_n(t_n)), \text{ where } t(i) := t_i \tag{0.6}$$

. Now since each s_i is a dominant strategy for player i , then for every t_i, x_{-i}, x_i' we have that

$$u_i(t(i), s_i(t_i), x_{-i}) \geq u_i(t(i), x_i', x_{-i}) \tag{0.7}$$

$$u_i\left(t(i), s_i(t_i), s_{-i}(t_{-i})\right) \geq u_i\left(t(i), s_i(t_i'), s_{-i}(t_{-i})\right) \tag{0.8}$$

$$u_i^f(t_i, t_{-i}) \geq u_i^f(t_i', t_{-i}) \tag{0.9}$$

, which gives the definition of incentive compatibility of the mechanism $(f, u_1^f, ..., u_n^f)$.

□