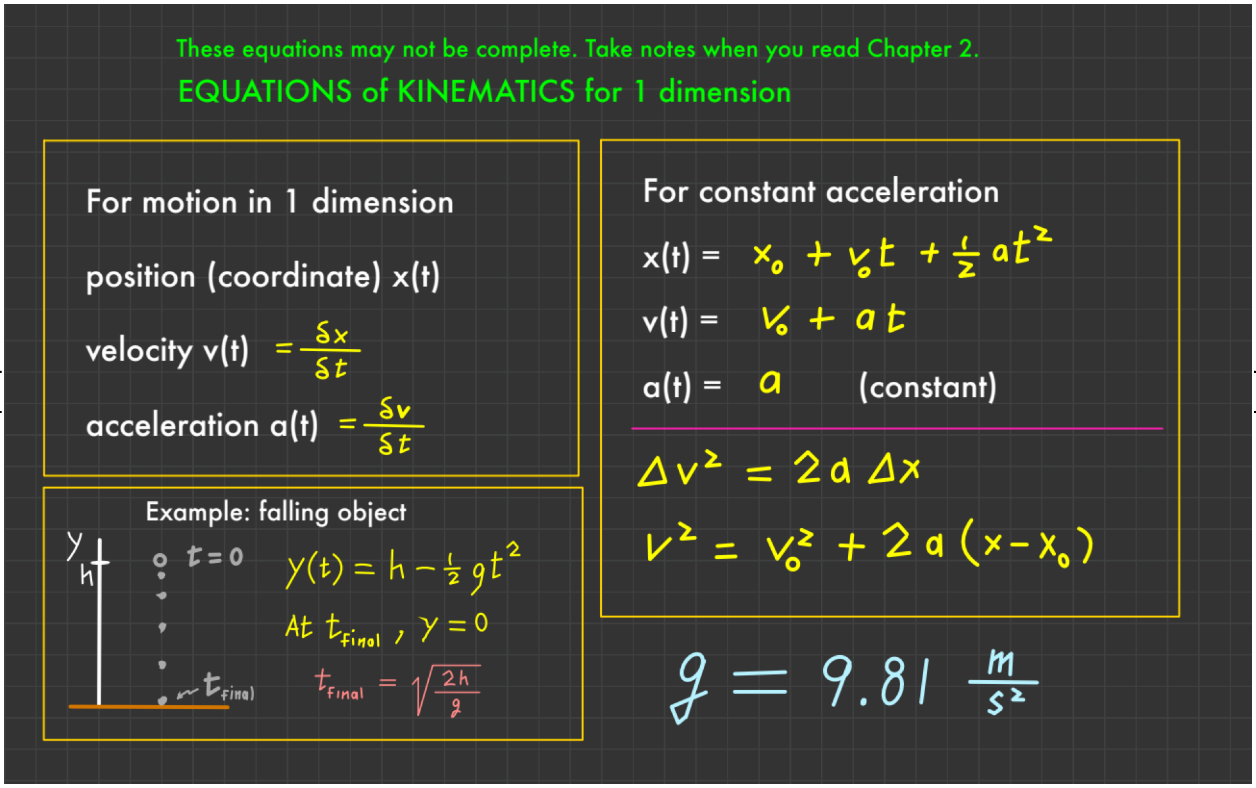
Problem Set 1 = Chapter 2 = Motion in 1 dimension

■ Part 1 ■ Equations from Chapter 2

This page provides a quick summary of Chapter 2, but is not sufficient to substitute for a complete reading of Chapter 2. Read the chapter (2 or 3 hours of concentration); take your own notes (about one 2-sided page in your notebook); refer to this page or your notes when you do the homework.

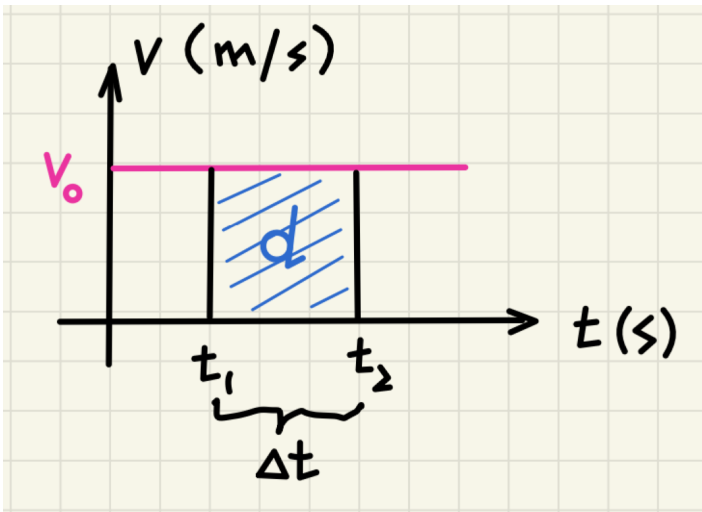


▪ Part 2 ▪ Homework hints on motion in 1 dimension

The next problems will give you practice applying the equations.
Try to do each problem by yourself! If you get stuck, peek at the hint.
The hints do not always show all the steps to the final answer.
Understand the hints and write out your solutions in your notebook.

1. Eighty miles per hour

2. Constant speed graph



The graph of v(t) is a horizontal line;

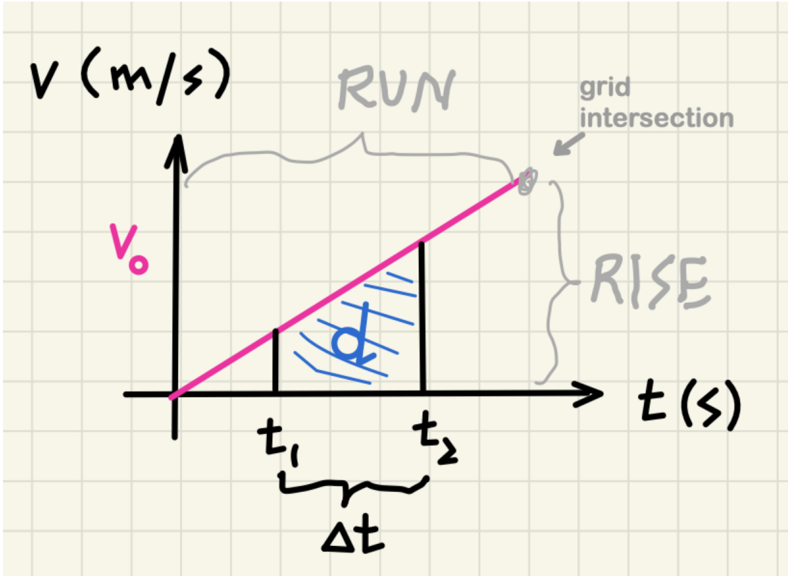
i.e., the speed is constant v_0 (read the value from the graph).

Distance $\quad d = v_0 \cdot \Delta t = v_0 \cdot (t_2 - t_1)$

Check the units: $\quad (m/s) \cdot s = m$

Note: the distance = the area under the graph of v(t).

3. Constant acceleration graph



The graph is a diagonal line, so the motion has constant acceleration.

Note: the slope of the graph of v versus t = acceleration;

$$a = \frac{\Delta v}{\Delta t} = \frac{\text{rise}}{\text{run}} = \text{slope}$$

<u>Check the units and graphical analysis for velocity:</u>

the unit of acceleration = $\frac{(m/s)}{s} = \frac{m}{s^2}$ = m/s/s

slope of the graph of v(t) = acceleration

<u>Check the units and graphical analysis for distance:</u>

distance (m/s)▫s = m

area under the graph of v(t) = distance

<u>Other equations:</u>

$d = v_{avg}$ ▫ $\Delta t = \frac{v_1 + v_2}{2}$ ▫ $(t_2 - t_1)$

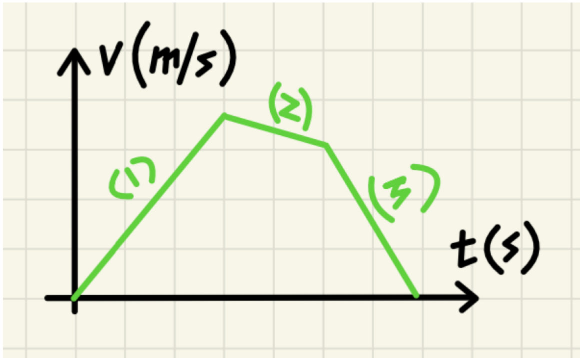
$d = \frac{1}{2} (a\, t_1 + a\, t_2) (t_2 - t_1)$

$d = \frac{1}{2} a (t_2 + t_1)(t_2 - t_1)$

$d = \frac{1}{2} a (t_2^2 - t_1^2)$

$d = d_2 - d_1$

4. Car between two traffic lights



- There are three "stages".

  1st stage: a_1 = slope = $\frac{\Delta v}{\Delta t}$ (positive)

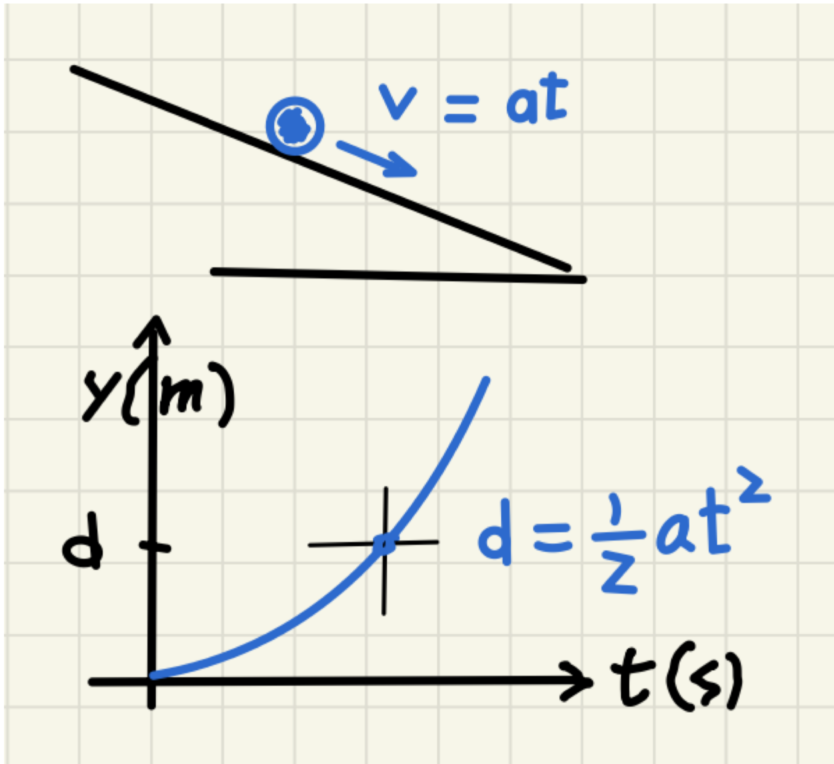
  2nd stage: a_2 = slope = $\frac{\Delta v}{\Delta t}$ (negative)

  3rd stage: a_3 = slope = $\frac{\Delta v}{\Delta t}$ (negative)

(negative acceleration is deceleration; i.e. velocity is decreasing)

- DISTANCE (or, displacement) = the area under the graph of v(t); calculate the three areas and add them for the total distance

5. Galileo's marble experiment

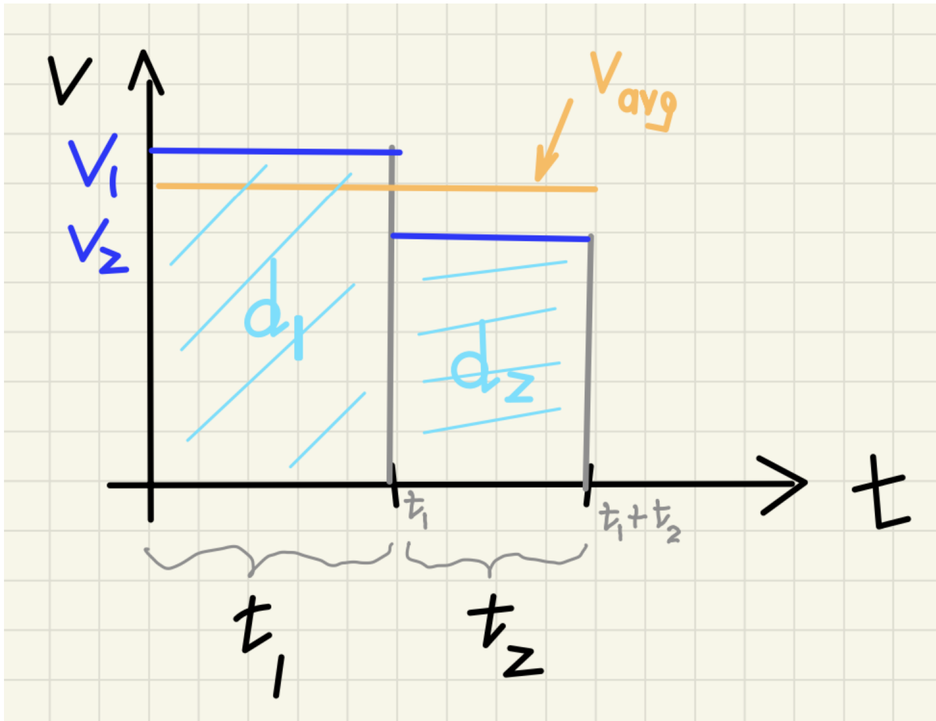


For constant acceleration with v_0 = 0,

$$d = \frac{1}{2}\, a\, t^2 \quad \text{which implies} \quad a = \frac{2\,d}{t^2}$$

Any point of the curve can be applied to this equation; to get an accurate value use a grid intersection point for (t,d).

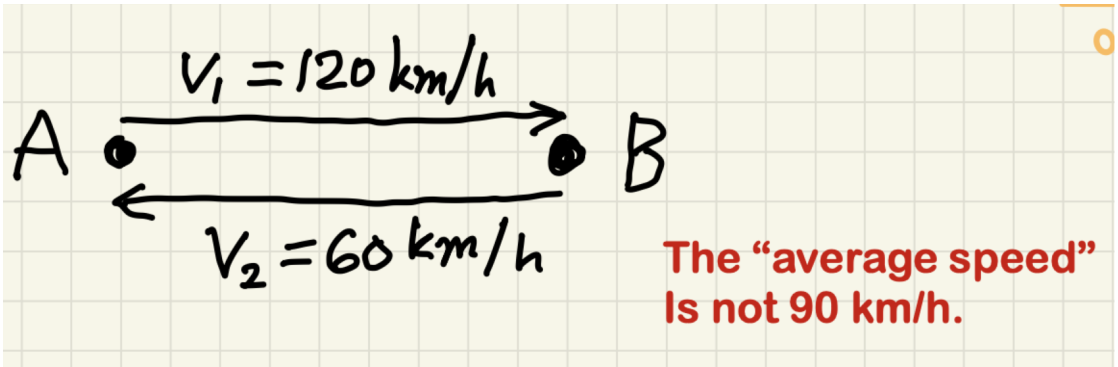
6. Average speed of a truck on a trip



$$\text{Average speed} = \frac{d_1 + d_2}{t_1 + t_2} = \frac{v_1\,t_1 + v_2\,t_2}{t_1 + t_2}$$

The <u>average speed</u> is the total distance divided by the total time; i.e., it is the "weighted average" of speeds (v_1 and v_2), weighted by the times (t_1 and t_2).

7. Average speed on a round trip



$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\overline{V} = \frac{d+d}{t_1+t_2} = \frac{2\,d}{\frac{d}{v1}+\frac{d}{v2}}$$

$$\overline{V} = \frac{2\,v_1\,v_2}{v_1+v_2}$$ = a kind of "geometrical mean"

8. Speed as a function of time

We have the graph of $v(t)$ ⟵ velocity as a function of time.

Recall from earlier problems:

The distance is the area under the graph of $v(t)$.

9. Acceleration histogram

We have the graph of $a(t)$ ⟵ acceleration as a function of time.

Graphical analysis:

The change of velocity is the area under the graph of $a(t)$;

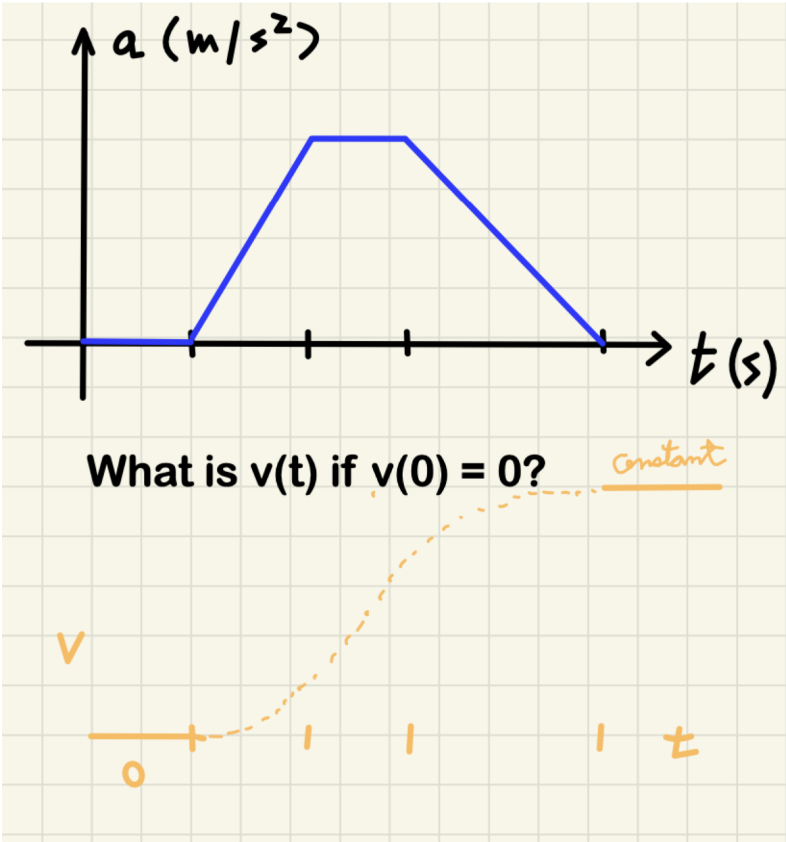
$\delta v = a\,\delta t$ in each time interval where a is constant

$\Delta v = \Sigma\, a\,\delta t$ sum over the time intervals.

The distance traveled during a time interval where a is constant

$\delta x = v_{average}\,\delta t$; Total distance = $\Sigma\,\delta x$

10. Acceleration profile



The area under the graph of acceleration is the change in velocity;

theorem: Δv = area under the graph of a(t)

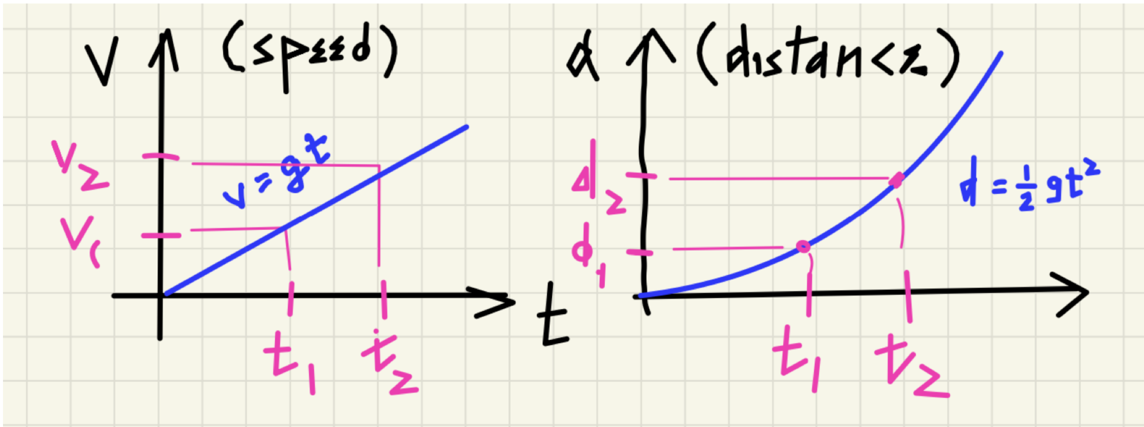
proof: For one small time interval, $\delta v = a(t)\ \delta t$.

For the entire interval Δt, $\Delta v = \Sigma\ \delta v$

$v(t_2) - v(t_1) = \Delta v = \Sigma\ a(t)\ \delta t$

$\Sigma\ a(t)\ \delta t$ = area under the graph.

11. Distance covered by a falling object



General equations for free fall,

$$v(t) = v_0 - g\,t$$
$$y(t) = y_0 + v_0\,t - \frac{1}{2}\,g\,t^2$$

Between times t_1 and t_2,

$$\Delta y = y_2 - y_1 = [y_0 + v_0 t_2 - \frac{1}{2} g t_2^2] - [y_0 + v_0 t_1 - \frac{1}{2} g t_1^2]$$
$$\Delta y = v_0(\,t_2 - t_1) - \frac{1}{2}\,g\,(\,t_2^2 - t_1^2\,)$$

In this problem, $v_0 = 0$.

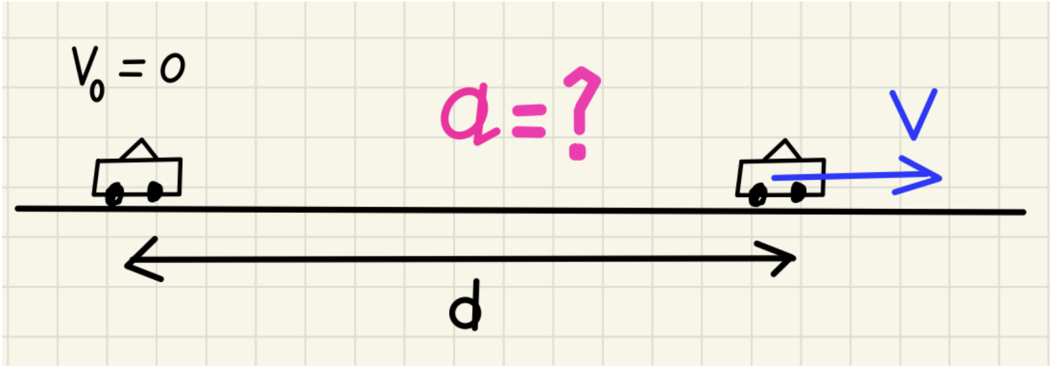
The displacement = Δy (a negative value means downward)

The distance = $|\Delta y|$ (distance is always positive)

$$= \frac{1}{2} g\,(\,t_2^2 - t_1^2\,)$$

Comment: note that $|\Delta y| = \dfrac{V_1 + V_2}{2} \cdot (\,t_2 - t_1\,)$.

12. Acceleration of a passenger car



General equations for constant acceleration,

$$v(t) = v_0 + at\ ,$$
$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2.$$

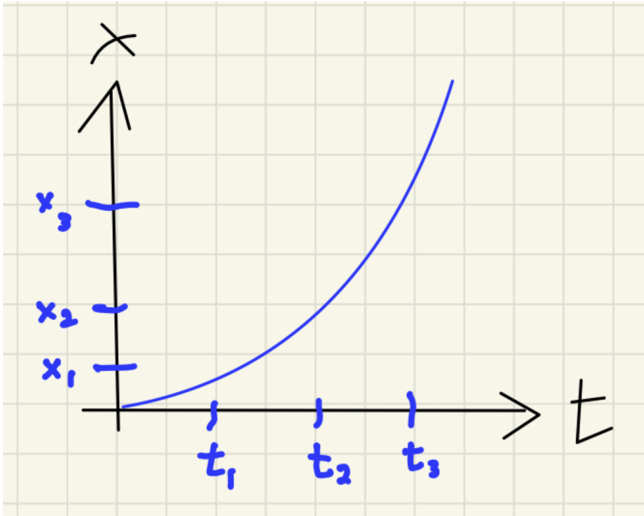
For this problem, $x_0 = 0$ and $v_0 = 0$.

Given: $x(t) = d$ and $v(t) = v$;

then $\longrightarrow$ $v = a\,t$ and $d = \frac{1}{2}at^2$.

Algebra: Two equations with two unknowns (a and t); solve for a.

13. Three positions



We are given (t_1,x_1), (t_2,x_2), (t_3,x_3); these are three points on a parabola;

$$x(t) = x_0 + v_0\, t + \frac{1}{2} a\, t^2 \ .$$

Algebra

Determine x_0, v_0, and a from the values of (t_1,x_1), (t_2,x_2), (t_3,x_3).

$$x_2 - x_1 = v_0(t_2 - t_1) + \frac{1}{2} a\, (t_2^2 - t_1^2)$$

$$x_3 - x_2 = v_0(t_3 - t_2) + \frac{1}{2} a\, (t_3^2 - t_2^2);$$

the terms $\propto v_0$ are equal;

$$(x_3 - x_2) - (x_2 - x_1) = \frac{1}{2}\, a\, (t_3^2 - 2t_2^2 - t_1^2);$$

solve for a.

Now go back and calculate v_0 and x_0.

■ Part 3 ■ More Homework Hints

The next problems are taken from the textbook,
at the end of Chapter 2. Some are more advanced that the previous problems, involving more analysis than just a single equation.

To solve these problems-

1- start by drawing a picture or a diagram and labeling the variables;

2- next, write the relevant equations–with variables, not numbers!

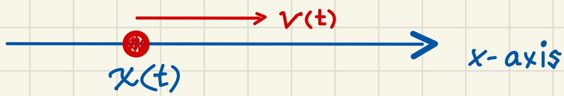
3- use algebra to analyze the equations and answer the question;

4- use your calculator to get the final numerical answer for LONCAPA.

The hints show how to get started. There will also be a Zoom meeting to review the solutions. (The Zoom meeting will be recorded in case you have a time conflict.)

Take notes in your Physics Notebook.

For example,
what are the equations for constant acceleration a?



(1) $v = v_0 + at$ where $v_0 = v(0)$

(2) $x = x_0 + v_0 t + \frac{1}{2} a t^2$ where $x_0 = x(0)$

(3) $\Delta(v^2) = 2a\,\Delta x$

$v^2 - v_0^2 = 2a(x - x_0)$

Δ NOTATION

ΔQ = a change of Q

$\Delta Q = Q_{FINAL} - Q_{INITIAL}$

Problem 2.36

(A) What is your average speed if you travel 11 m/s for 11 s and then 22 m/s for 11 s. (B) What is your average speed if you travel 11 m/s for 800 m and then 22 m/s for 800 m.

EQ: $\Delta x = v\,\Delta t$

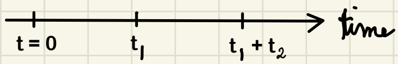
PICTURE

DELTA NOTATION

ΔQ = a change of the quantity Q

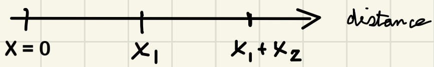
$\Delta Q = Q_{final} - Q_{Initial}$

(A)



$$V_{AV} = \frac{\Delta x}{\Delta t} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} = \frac{33 \times 11}{22}\ \frac{m}{s}$$

(B)



$$V_{AV} = \frac{x_1 + x_2}{\frac{x_1}{V_1} + \frac{x_2}{V_2}}$$

$$= \frac{1600}{800\left[\frac{1}{11} + \frac{1}{22}\right]}\ \frac{m}{s}$$

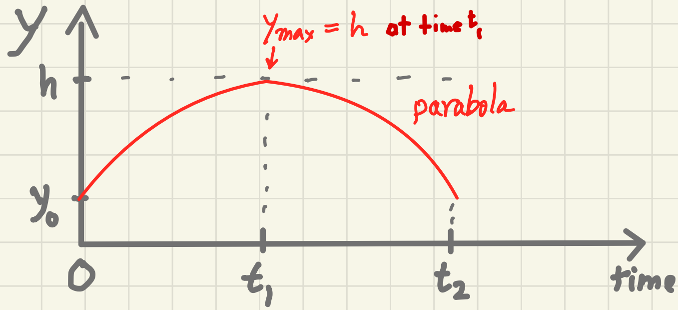
Problem 2-105

Attempting to run out the clock in a basketball game, a player throws the ball straight up from height y_0. An opposing player catches the ball coming down, at the same height. How much time was used up while the ball was in the air?

[Data : y_0 = 1.20 m; maximum height of the ball = 7.87 m]

Start with a picture.

Graphical Analysis



Equations and algebra

- $a_y = -g$
- $v_y = v_0 - gt$

at top $0 = v_0 - gt_1 \Rightarrow v_0 = gt_1$

- $y = y_0 + v_0 t - \frac{1}{2} g t^2$

at top $h = y_0 + \frac{1}{2} g t_1^2$

Use the equations; given y0 and h, solve for v0 and t1.

Answer $= t_2$

Problem 2-92

On a bicycle, a rider starts from rest, accelerates for time t_1, and then travels at constant speed for a time t_2. What is the total distance cycled?

[Data : initial acceleration = 2.09 $\mathrm{m/s^2}$; t_1 = 4.7 s; t_2 = 7.40 s;]

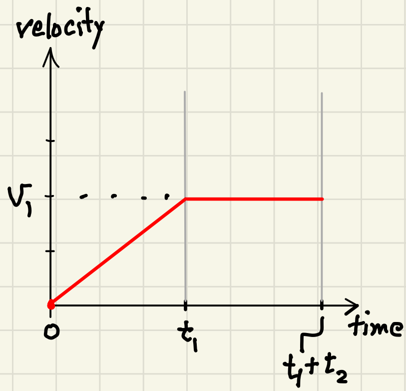
the right way

STEP 1
draw picture(s)

STEP 2
write equations

STEP 3
algebra

STEP 4
numbers



For $t < t_1$, $v = at$
$x = \frac{1}{2}at^2$

$\therefore v_1 = at_1$ and $x_1 = \frac{1}{2}at_1^2$

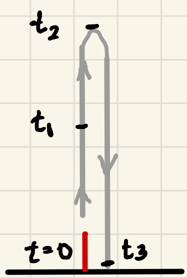
For $t > t_1$,
$v = v_1$
$x = x_1 + v_1(t - t_1)$
$x_2 = x_1 + v_1(t_2 - t_1)$

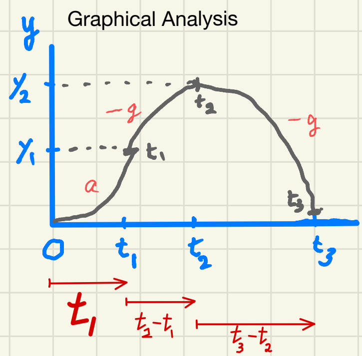
Problem 2-85

A rocket accelerates upward. Then the engine cuts off. Determine (a) the speed when the acceleration ends; (b) the maximum height; (c) the total time from launch to crash.

[Data : the upward acceleration is a = 12.1 m/s^2; the time of upwarc
s.]

STEP 1: draw 2 pictures





STEP 2: equations for $t \le t_1$

$v = at$ AND $y = \frac{1}{2}at^2$

$\therefore$ at t_1 | $v_1 = at_1$ AND $y_1 = \frac{1}{2}at_1^2$

STEP 3: equations for $t > t_1$

$v = v_1 - g(t - t_1)$

AND $y = y_1 + v_1(t-t_1) - \frac{1}{2}g(t-t_1)^2$

STEP 4: Algebra

Calculate t_3

$= t_1 + (t_2 - t_1) + (t_3 - t_2)$

STEP 5:

Use your calculator

(a) $V_1 =$

(b) $H =$

(c) $t_3 =$