Important equations from Chapter 4
(may not be complete—make your own notes)

{SEC.1} A **Force** is an interaction between two objects; a push or a pull; or action at a distance (gravity); or a force field (electricity&magn.)
Mass = inertia = resistance to acceleration
Net Force = the sum of all actions

{SEC.2} Newton's Laws of Motion

1st Law : $\vec{F}_{net} = 0 \rightleftarrows \vec{v} = 0$ inertia

2nd Law : $m\vec{a} = \vec{F}_{net}$ eqn. of motion

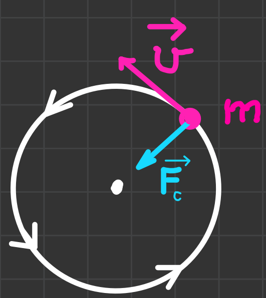
3rd Law : $\vec{F}_{BA} = -\vec{F}_{AB}$ action & reaction

$$ma_x = \sum F_x$$
$$ma_y = \sum F_y$$

{SEC.4} Friction and drag
Kinetic friction $F_K = \mu_K N$
Static friction $F_s \leq \mu_s N$

{SEC.5} Uniform circular motion

$$F_c = \frac{mv^2}{R}$$



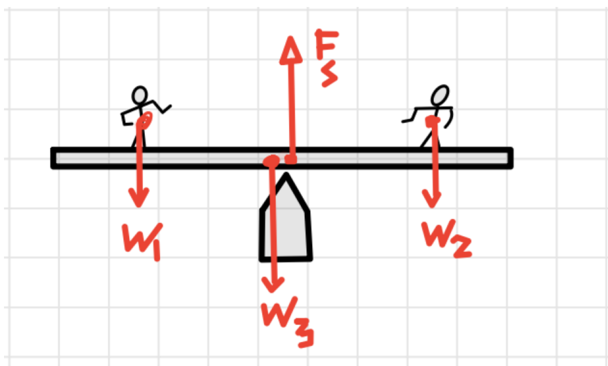
Hints for Problem Set #3

Problem Set#3 = Chapter 4 = Forces

1. TWO CHILDREN ON A BOARD

The net force on the board must be zero.

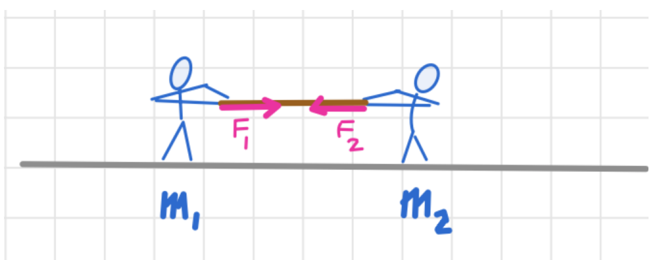
$F_s - W_1 - W_1 - W_1 = 0$



2. TWO CHILDREN ON A FROZEN LAKE

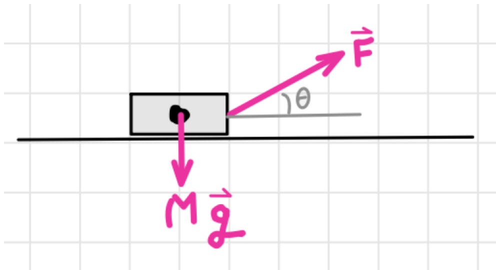
Newton's 3rd law ■■ $|F_1| = |F_2|$ = tension in the rope

Newton's 2nd law ■■ $a_1 = F/m_1$ and $a_2 = F/m_2$



3. BLOCK PULLED AT AN ANGLE

Draw a picture and label it.



$$\vec{F}_{net} = M\vec{g} + \vec{F}$$

Resolve the force vectors into components,

ans apply Newton's second law for each direction,

$$F_{net,x} = F\cos\theta = M\,a_x = M\,a$$

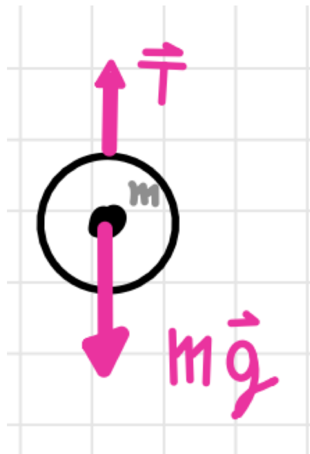
$$F_{net,y} = F\sin\theta - Mg = M\,ay = 0$$

Recall for constant acceleration

$$v = v_0 + a\,\Delta t = a\,\Delta t$$

4. LIFTING A MASS

Draw a force diagram



Apply Newton's second law,

$$m\,\vec{a} = \vec{F}_{net}$$

$$m\,a_y = T - m\,g$$

and solve for T.

Now use your calculator.

5. SPACECRAFT IN INTERSTELLAR SPACE

6. ELEVATOR CONCEPTS

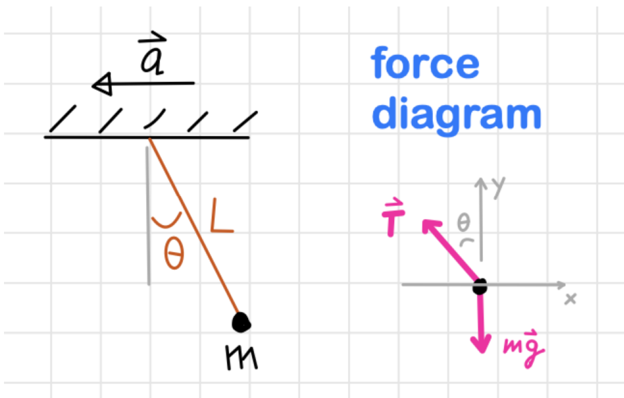
The net force (upward) is

F_{net} = T - Mg = Ma;

T = tension force of the cable, M = total mass

7. WHEN A JET TAKES OFF (pendulum in a jet plane)

Draw pictures, and label them.



We'll apply Newton's second law $m\,\vec{a} = \vec{F}_{net}$

Resolve the vectors into components ...

$m\,a_x = -T\sin\theta$ and $m\,a_y = T\cos\theta - mg$

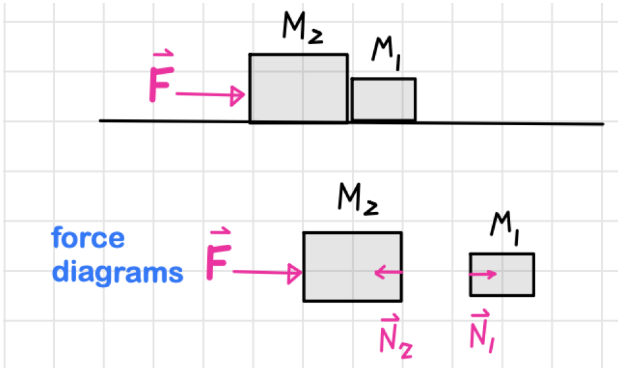
From the diagrams, $a_x = -a$ and $a_y = 0$.

Use algebra to write the equation for a, and only then pick up your calculator.

2 equations for 2 unknowns

8. TWO BLOCKS IN CONTACT

Draw separate force diagrams for the blacks.



Newton' s second law, TWICE,

$$M_2\ a_2 = F - N_2 \quad \text{and} \quad M_1\ a_1 = N_1.$$

The accelerations are equal $a_2 = a_1 \equiv a$ [Why?]

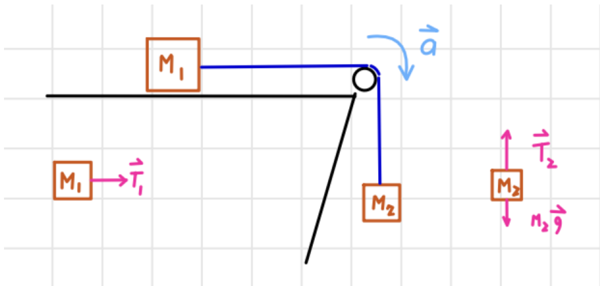
Newton's third law: $N_2 = N_1$ [Why?]

$\Longrightarrow$ Algebra - 2 equations for 2 unknowns.

9. FALLING AND PULLING

1 - Draw pictures and label them.

2 - Draw [redacted] diagrams for M_1 and M_2.



Equations:

$M_1\ a = T_1$ and $M_2\ a = M_2\ g - T_2$

$T_1 = T_2$ [Why?]

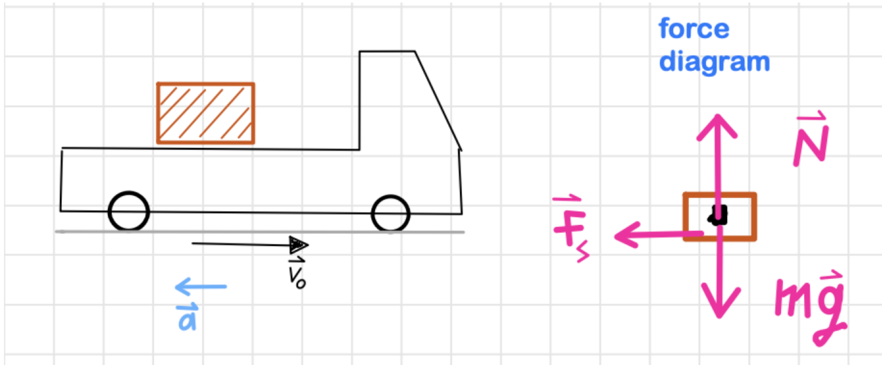
The rest is algebra.

10. BOX ON A TRUCK

Draw a picture, and draw a force diagram.

Be sure to identify all the forces.

$\vec{F}_S$ = force of "static friction".



Equations (which you know from reading Chapter 4)

$F_{s,max} = \mu_S$ N ; in other words, $F_s < \mu_S$ N;

$\vec{F}_{net}$ = m $\vec{a}$;

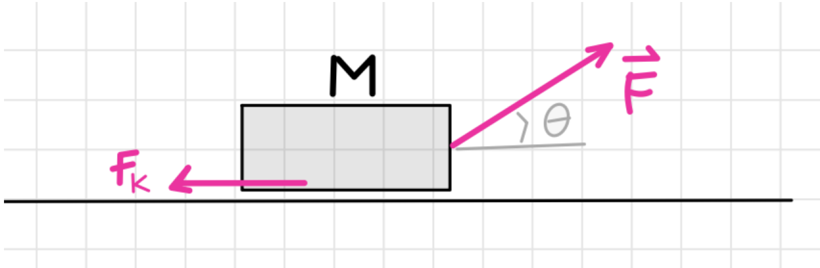
a_x = - m a ; a_y = 0 ;

$F_{net,x}$ = Fs ; $F_{net,y}$ = N - m g = 0 ;

The rest is algebra.

Also, recall for constant acceleration, v^2 = 2 a d.

11. BLOCK PULLED AT AN ANGLE WITH FRICTION



Draw a force diagram showing all three forces.

Resolve the force vectors into xy-components,

and apply Newton's second law for each direction.

$$M\ a_x = F\cos\theta - F_{\text{kinetic friction}}$$

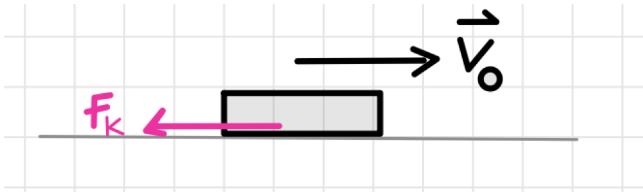
$$M\ a_y = N + F\sin\theta - Mg = 0$$

Also, $F_k = \mu_k N$.

The rest is algebra.

Then, $v_x = a_x\ t$.

12. HOCKEY PUCK WITH FRICTION



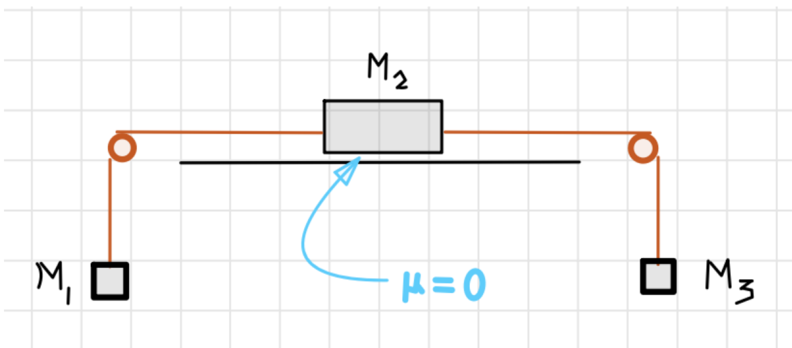
Draw a force diagram with all the forces.

$F_k = \mu_k N$.

Recall from kinematics, $\Delta(v_x^2) = 2\ a\ \Delta x$

13. PULLING AND SLIDING

14. THREE BLOCKS, NO FRICTION

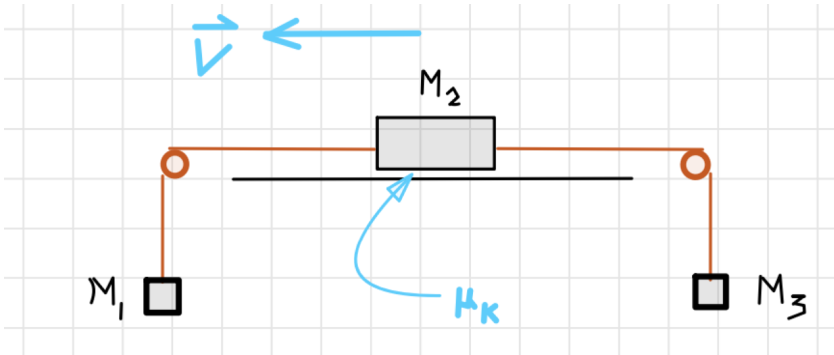


Draw three force diagrams, and write the corresponding equations.

Use algebra to derive an expression for a.

Now use your calculator.

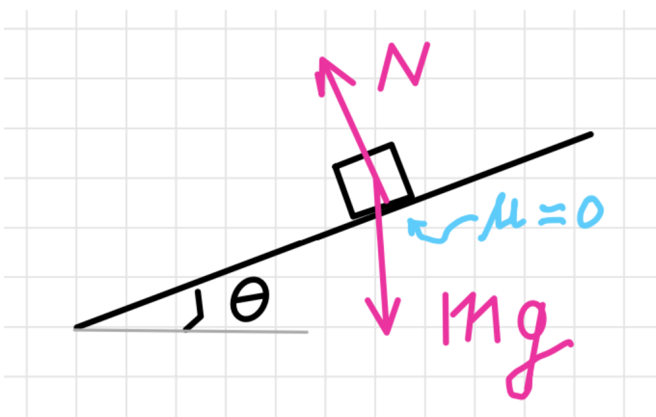
15. THREE BLOCKS WITH FRICTION



Draw three force diagrams, and write the corresponding equations.

The velocity is constant so a = 0.

16. BLOCK ON A FRICTIONLESS INCLINE

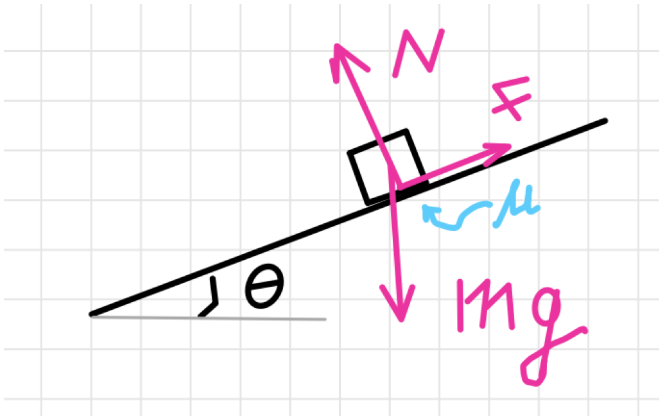


Newton’s second law $\quad m\ a = F_{\text{tangential}}$

$= m\ g\ \sin\theta$

“Tangential” means down parallel to the surface.

17. INCREASING THE ANGLE



Resolve the forces into normal (perpendicular upward) and tangential (parallel downward) components.

■When it starts to slide, $F_{fric.} = \mu_s N$ and the net force is zero.

■When it is sliding, $F_{fric.} = \mu_k N$ and $d = \frac{1}{2} a t^2$.

18. TWO MASSES ONE INCLINE NO FRICTION

Draw a picture.

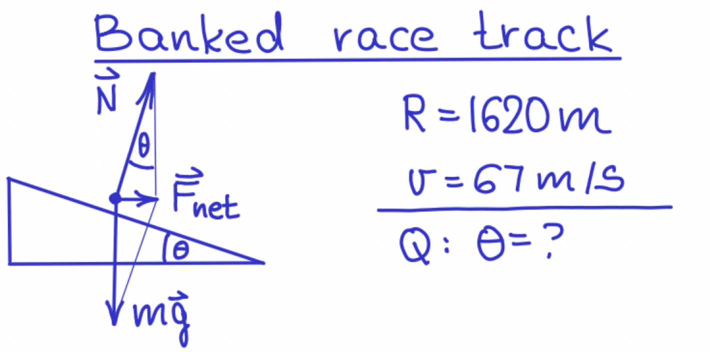
Draw the force diagrams, and write the equations.

$T_1 = T_2$ because the pulley is massless.

$a_1 = a_2$ because the cord has fixed length.

19. TWO MASSES ONE INCLINE WITH FRICTION

20. BANKED TRACK



The uniform circular motion is ***horizontal;***

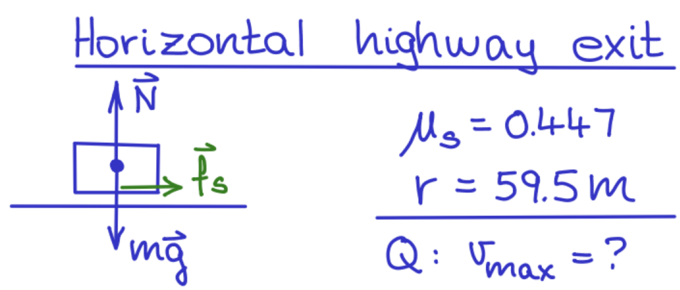
write the equations

$$a = \frac{F_{net}}{m} \quad ; \quad a = \frac{m\,v^2}{R} \quad ;$$

$$F_{net} = F_{horizontal} = \text{N} \sin\theta$$

The net vertical force must be 0.

21. HIGHWAY EXIT WITH FRICTION

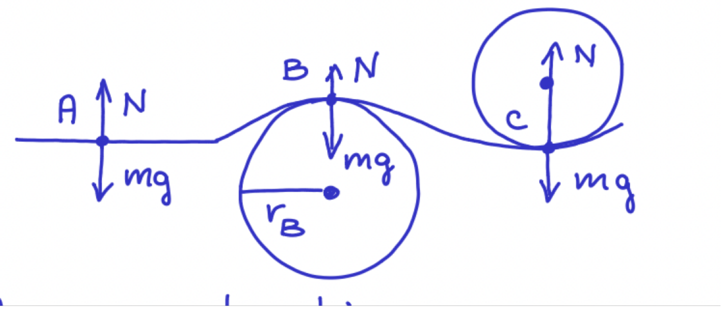


$$f_{s,max} = \mu_s \cdot N = \mu_s \cdot mg$$

22. CAR ON A HILLY ROAD

Draw a picture, and force diagrams.

Analyze Newton's second law for each force diagram.



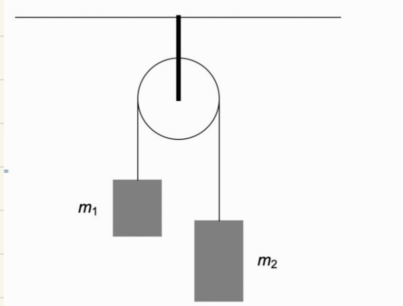
Note: the next problems are taken from the textbook;

- try to do them without the hints;
- if you get stuck, peek at the hints;
- they will also be discussed at the Zoom meeting

Problem 4-62

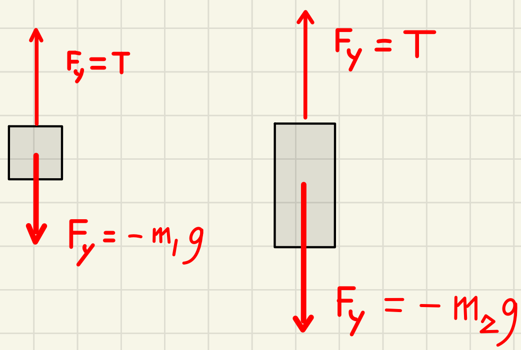
See the figure. Calculate (A) the acceleration and (B) the string tension. {Hints: Draw force diagrams for both masses. The pulley is massless and frictionless.}

Atwood's Machine



Data : m_1 = 0.33 kg; m_2 = 0.48 kg]

1/ Draw picture(s)



The positive direction is upward.

2/ Write equations from the force diagrams

$$m_1 a_1 = T - m_1 g$$
$$m_2 a_2 = T - m_2 g$$

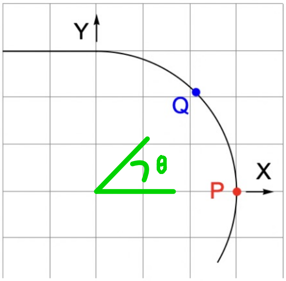
ALSO $a_2 = -a_1$ (why?)

$\therefore$ write $a_1 = +a$ and $a_2 = -a$

3/ Algebra. Solve 2 equations for 2 unknowns

Problem 4-104

A car rounds a circular curve with radius R. From P to Q the speed v_0 is constant. (A) Determine the acceleration vector at P. (B) Suppose the driver presses the accelerator pedal at Q, and the speed is then increasing at rate a_τ. Determine the acceleration vector just after Q.



[Data : R = 150 m; v_0 = 10.5 m/s; a_τ = 2.4 m/s^2]

$\theta = 45°$

Equations from P to Q

Uniform circular motion

$\vec{a} = a_r \hat{r}$; $a_r = -\frac{v^2}{R}$

At P, $\hat{r} = \hat{i}$

$\vec{a}_P = -\frac{v_0^2}{R}\hat{i} = \left(\underbrace{-\frac{v_0^2}{R}}_{a_{Px}}, \underbrace{0}_{a_{Py}}\right)$

Equations after Q

$\vec{a} = a_r \hat{r} + a_\theta \hat{\theta}$

$a_r = -\frac{v^2}{R}$ and $a_\theta = a_\tau$

At Q, $\vec{a}_Q = (a_{Qx}, a_{Qy})$

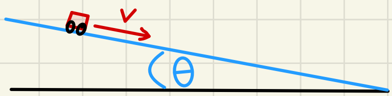
Trigonometry

Problem 4-76

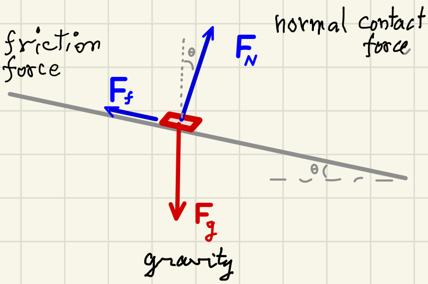
A car slides at constant speed down an icy road, whose slope is Θ. Find the coefficient of friction between the tires and the road.

[Data : Θ = 1.5 deg;]

Pictures



Force diagram

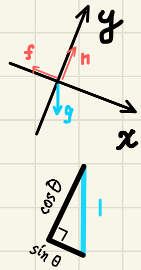


Coefficient of kinetic friction

$$F_f = \mu F_n$$

$$\therefore \mu = \frac{f}{n}$$

Convenient coordinates



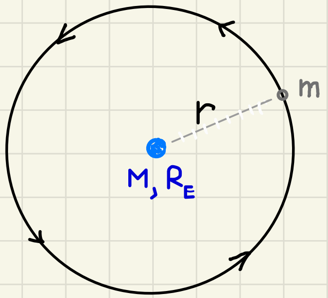
$$F_y = F_n - mg\cos\theta = 0$$

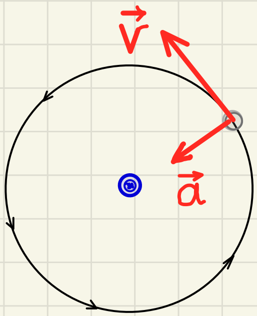
$$F_x = mg\sin\theta - F_f = 0$$

By Newton's first law, the net force is 0; $F = F_g + F_n + F_f = 0$.

The moon orbits Earth in approximately 27.3 days. The orbital radius r is approximately 384,000 km.
(A) Compute the centripetal acceleration.
(B) Based on his theory of gravity, Newton estimated $a \approx g\, R_{\text{Earth}}^2 / r^2$. Calculate Newton's estimate.

Newton's famous calculation





$$\boxed{\text{(A)}\quad a = \frac{v^2}{r} \quad \text{and} \quad v = \frac{2\pi r}{T}}$$

$r = 3.84 \times 10^8$ m

$T = 27.3 \times 24 \times 3600$ s

$R_E = 6371$ km