Important equations from Chapter 3. (These may not be complete, so please take notes yourself when you read the chapter.)

Equations of Kinematics – for 2 dimensions

PROJECTILES

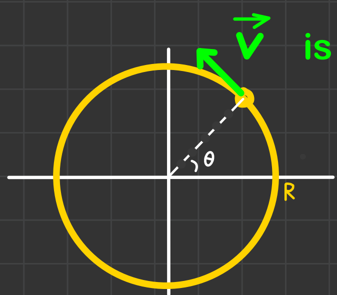
$= (v_0 \cos\theta,\ v_0 \sin\theta)$

$$x = x_0 + v_{0x} t$$
$$y = y_0 + v_{0y} t - \tfrac{1}{2} g t^2$$

$$\vec{v} = (v_{0x},\ v_{0y} - gt)$$
$$\vec{a} = (0, -g)$$

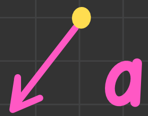
Look up the “range formula” problem 3.100.

UNIFORM CIRCULAR MOTION



$\vec{v}$ is tangential

$\vec{a}$ is centripetal



$$a_r = \frac{-v^2}{R}$$

$$\text{period} = \frac{2\pi R}{v}$$

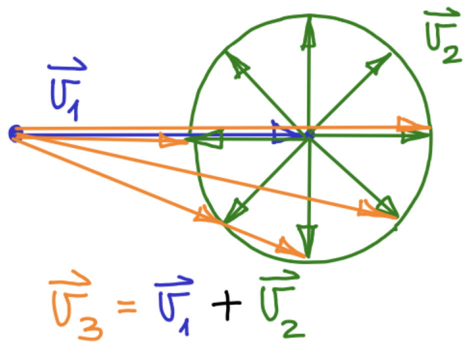
Problem Set 2 = Chapter 3 = Motion in 2D

PART 2 : Hints for Problem Set #2

1. RESCUE HELICOPTER

2. VECTOR ADDITION

Picture



Adding $\vec{v}_1$ and $\vec{v}_2$;

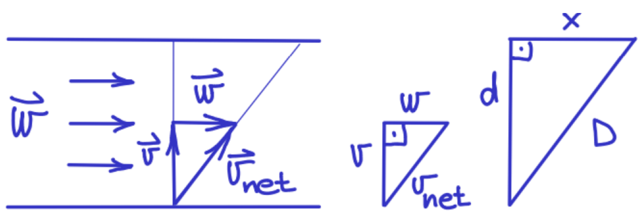
$$\vec{v}_3 = \vec{v}_1 + \vec{v}_2$$

Longest possible sum: $|\vec{v}_3| = |\vec{v}_1| + |\vec{v}_2|$

Shortest possible sum: $|\vec{v}_3| = |\vec{v}_1| - |\vec{v}_2|$

3. CROSSING A RIVER (numerical)

Draw a picture and label it.



d = 200m : width of the river
v = 4 m/s : speed of the boat
w = 3 m/s : speed of the river

Addition of velocities

$\vec{v}_{net} = \vec{v} + \vec{w}$ (velocity vector)

$v_{net} = \sqrt{v^2 + w^2}$ (speed)

D = v_{net} t = total distance traveled.

What is the time t to cross the river?

The width of the river is d, and the component of velocity in the direction across the river = v;

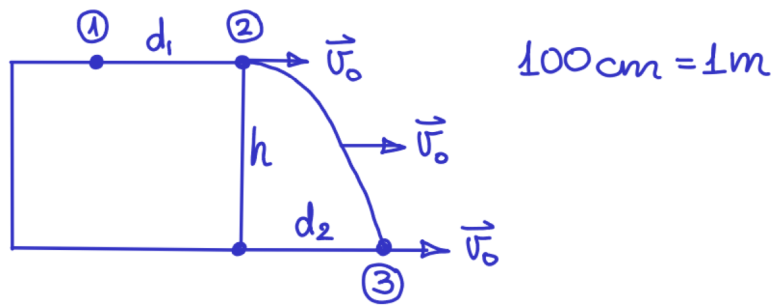
so t = d/v.

4. CROSSING A RIVER (conceptual)

5. HORIZONTAL BASEBALL

6. MARBLE OFF A TABLE

Draw a picture and label the parameters in the picture.



Motion from [2] to [3]: this is the projectile motion, i.e., free fall in the y direction,

$$\Delta y = -\frac{1}{2} g (\Delta t)^2 \;; \quad \text{so } (\Delta t)_{23} = \sqrt{\frac{2h}{g}} \;;$$

$$\Delta x = v_0 \, \Delta t \;; \quad \text{so } \; d_2 = v_0 \, (\Delta t)_{23} \quad \text{(calculate } v_0\text{)}$$

The time to move from [1] to [3] is

$$(\Delta t)_{13} = \frac{d_1 + d_2}{v_0}$$

7. GOLF BALL

See the description of projectile motion in Section 3.4. Draw a picture and label it.

For a golf ball with initial angle θ,

$$x = v_0 \cos\theta \cdot t \qquad ; \qquad v_x = v_0 \cos\theta$$

$$y = v_0 \sin\theta \cdot t - \frac{1}{2} g\, t^2$$

$$v_y = v_0 \sin\theta - g\, t$$

At the peak, $v_y = 0$; $\qquad t_{\text{peak}} = \frac{v_0 \sin\theta}{g}$;

At the ground, y = 0; $\qquad t_{\text{return to ground}} = \frac{2\, v_0 \sin\theta}{g}$;

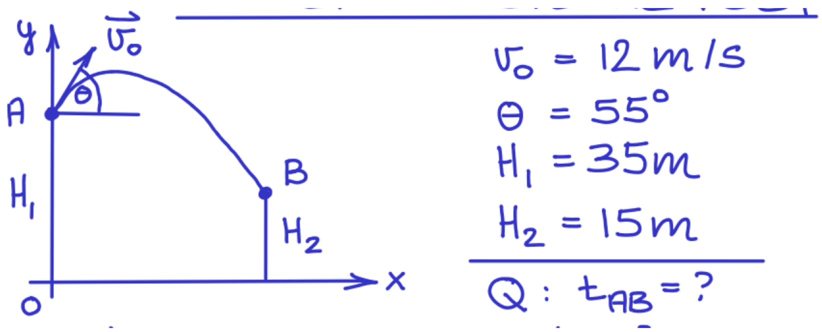
range = $x(t_{\text{ground}}) = R = \frac{v_0^2}{g} \sin(2\theta)$;

peak height = $y(t_{\text{peak}}) = H = \frac{v_0^2}{2\, g} \sin^2\theta$.

Use the proportionalities between these quantities and the initial speed v_0 to answer the questions.

8. SOCCER ON THE ROOF

Draw a picture and label the parameters on the picture.



Apply the equations for projectile motion to this problem;

$v_x(t) = v_0 \cos\theta$; $x(t) = v_0 \cos\theta \cdot t$

$v_y(t) = v_0 \sin\theta - gt$; $y(t) = H_1 + v_0 \sin\theta \cdot t - \frac{1}{2} g\, t^2$

Let t_2 = the time when the ball lands at B;

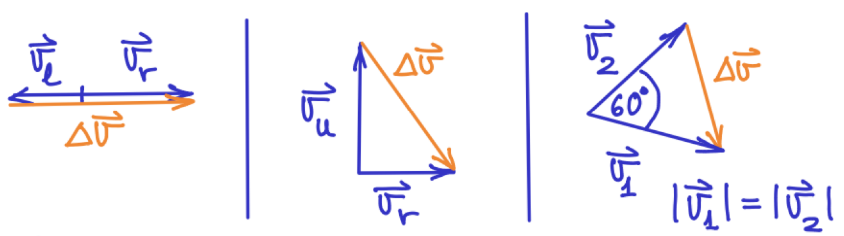
$y(t_2) = H_2$; then show that

$$\frac{1}{2}\, g\, t_2^2 - v_0 \sin\theta\, t_2 + (H_2 - H_1) = 0.$$

Solve this quadratic equation for t_2.

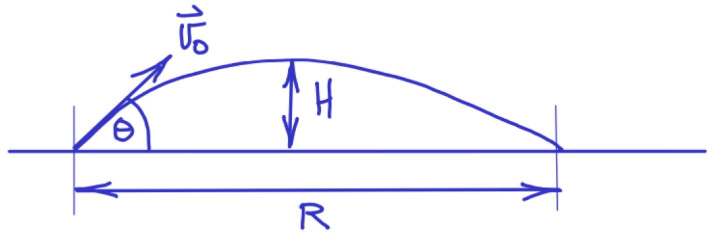
(There will be two solutions; you want the one where t_2 is positive.)

9. TWO ROCKS A, B, C



10. RATIO OF RANGE AND HEIGHT

Draw a picture and label it.



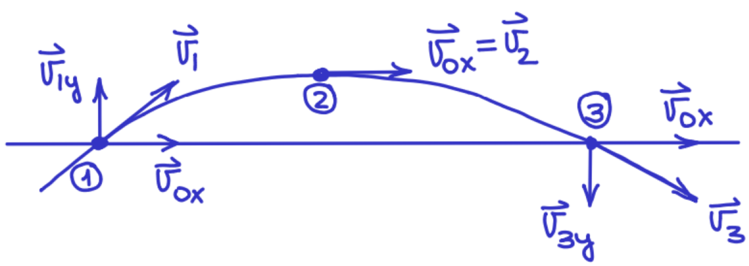
We have these equations for range and height; see the earlier problem on a golf ball,

$$R = \frac{v_0^2}{g}\sin(2\theta) \qquad ; \qquad H = \frac{v_0^2}{2g}\sin^2\theta\,.$$

Let $q = \frac{R}{H}$.

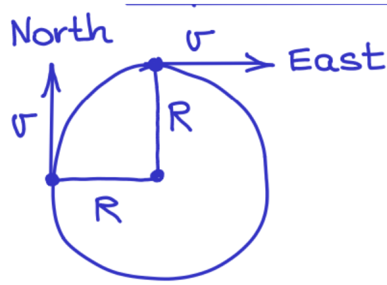
Simplify the equation to express q in terms of θ.

11. BASKETBALL PLAYER CONCEPTS



12. SUPERSONIC AIRPLANE

Draw a picture and label it.



$R = 50\,km = 5\times10^4\,m$

$v = 2750\,\frac{km}{h} = 764\,\frac{m}{s}$

(2.25 Mach)

Unit conversions: convert all parameters to SI base units; 1 km = 1000 m; 1 h = 3600 s.

Equations:

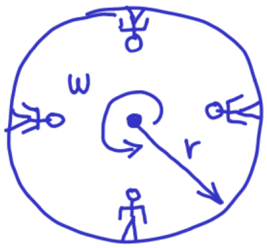
the length of the turn = d = $\frac{1}{4} \cdot 2\pi R$;

the time of the turn = t = $\frac{d}{v}$;

the centripetal acceleration, $a_r = -\frac{v^2}{R}$.

13. ARTICIFIAL GRAVITY

Draw a picture, and label the variables.



r : radius
d : diameter (d=2r)
ω : angular speed
(2001 Space Odyssey, Interstellar, Elysium)

To create artificial gravity equal to g,

$$|a_r| = \frac{v^2}{r} = r\omega^2 = g.$$

The acceleration vector is toward the center;
the floor of the cylinder is pushing the astronaut toward the center; just like on the Earth, the ground is pushing us vertically upward as Earth's gravity is pulling us downward. The direction of "artificial gravity" is inward.

The next problems are taken from the textbook, and will be discussed at a Zoom meeting.

Problem 3-36

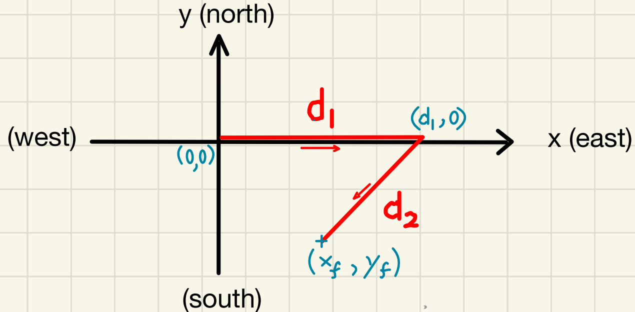
Walk distance d_1 east, then distance d_2 southwest, in time T. Compute (A) your final distance from the starting point; and (B) average velocity.

[Data : x direction = east and y direction = north; d_1 = 1233 m; d_2 = 886 m; T = 20.20 minutes.]

Velocity is a vector.

$$\vec{v}_{AVG} = \frac{\Delta \vec{x}}{\Delta t}$$

Start with a picture.
Then write equations.



(A) $\sqrt{x_f^2 + y_f^2}$

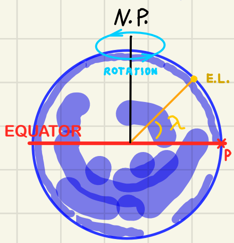
(B) $\vec{v} = \left(\frac{x_f}{T}, \frac{y_f}{T} \right)$

Problem 3-80

(A) Compute the centripetal acceleration of a point on Earth's equator. (Data is available in Appendix E.) (B) Express the result as a fraction of g. (C) Compute the centripetal acceleration of MSU.

Circular motion — centripetal acceleration

Start with a picture.



R = 6371 km
T = 24 hr

(convert to seconds)

Now the equations…

(A) At the point P,

$$a = \frac{v^2}{R} \quad \text{and} \quad v = \frac{2\pi R}{T} \longrightarrow a = \frac{4\pi^2}{T^2}$$

(C) At MSU,

$$a = \frac{v^2}{r} \quad \text{and} \quad v = \frac{2\pi r}{T} \quad \text{and} \quad r = R\cos\lambda$$

$$\lambda = \text{latitude}$$

(The latitude of MSU is a little smaller than 45°)
look up

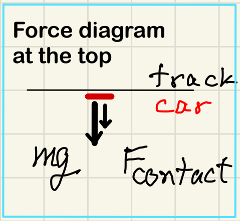
(B) a / g

Problem 3-83

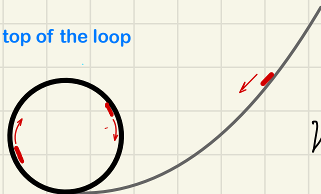
In a loop-the-loop roller coaster, the minimum centripetal acceleration at the top of the loop is g; less than g the car will fall off the track. If the radius of the loop is R, what is the minimum *speed* of the car?

[Data : R = 8.2 m]

picture



At the top of the loop



We need $a_c > g$

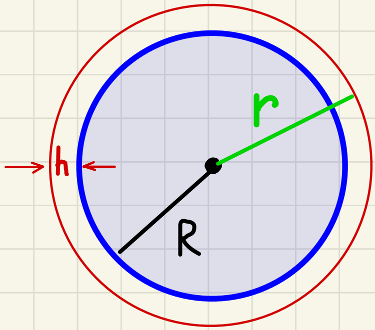
What is a_c?

Then what is minimum v?

Problem 3-90

A satellite in a circular orbit at altitude h above Earth's surface has period of revolution P. Compute (A) the speed, and (B) the centripetal acceleration, of the satellite.

[Data : h = 383 km; P = 92.11 minutes;]



$r = R + h$

$v = \dfrac{2\pi r}{P}$ ← P = period of revolution

$a_c = \ ?$

How does it compare to g ?