FRM 二级公式表

整理人：齐齐子

市场风险

1. 收益率
   - ✓ 算术收益率(Holding Period Return)

$$r_t=\frac{P_t+D_t-P_{t-1}}{P_{t-1}}=\frac{P_t+D_t}{P_{t-1}}-1$$

   - ✓ 几何收益率:表示的是对数收益率。

$$R_t=\ln\left(\frac{P_t+D_t}{P_{t-1}}\right)=\ln(1+r_t)$$

Dt：分红

2. Normal VaR

$$VaR=|\mu-z_\alpha\sigma|$$
$$VaR=|\mu-z_\alpha\sigma|*P_{t-1}$$
$$VaR=-(\mu-z_\alpha\sigma)$$

μ：均值；σ：标准差；z_α：标准正态分布对应置信水平的关键值

3. Lognormal VaR

$$VaR=1-e^{\mu-z_\alpha\sigma}$$
$$VaR=(1-e^{\mu-z_\alpha\sigma})*P_{t-1}$$

4. standard error of a quantile estimator

$$SE(q)\approx\left(\frac{p\times(1-p)}{n\times f(q)^2}\right)^{0.5}$$

   - ✓ p =the proportion for the quantile
   - ✓ n =样本量
   - ✓ f(q) =the corresponding probability density function

5. Quantile estimators 的 confidence intervals

$$\left[q-z_{\alpha/2}\times SE(q),q+z_{\alpha/2}\times SE(q)\right]$$

6. Age-weighted Historical Simulation

$$\omega_{(i)}=\frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$$

   - ✧ $\omega_{(i)}$表示的是第 i 天前的权重。
   - ✧ λ，叫做衰减因子 the rate of decay or memory decay，取值范围：$0\leq\lambda\leq1$。λ越小，衰减的速度越大。λ越接近于大，衰减的速度越小。
   - ✧ 当$\lambda=1$时，所有的数据都是一个相等的权重。

7. Volatility-weighted historical simulation 波动率加权的历史模拟法

$$\frac{r_{t,i}^*}{r_{t,i}}=\frac{\sigma_{T,i}}{\sigma_{t,i}}\quad\Rightarrow r_{t,i}^*=\frac{\sigma_{T,i}}{\sigma_{t,i}}\times r_{t,i}$$

   - ✓ $r_{t,i}$ = actual return for asset i on day t
   - ✓ $\sigma_{t,i}$ = volatility forecast for asset i on day t (made at the end of day t – 1)
   - ✓ $\sigma_{T,i}$= current forecast of volatility for asset i
   - ✓ $r_{t,i}^*$：调整之后的收益率数据。

8. $VaR_{PT}=\sum_{i\in P}\frac{\partial VaR_P}{\partial V_{iT}}*V_{iT}=\sum_{i\in P}MVaR_{iT}*V_{iT}$

$$CVaR_i=VaR_P*\omega_i*\beta_{i,p}$$

   - ✓ i:投资组合中单个资产
   - ✓ P：投资组合
   - ✓ T：时间 T
   - ✓ ω_i:资产 i 在组合的权重
   - ✓ $\beta_{i,p}$：资产 i 头寸价值对整体投资组合价值的敏感程度
   - ✓ $\frac{\partial VaR_P}{\partial V_i}$=$MVaR_i$：marginal VaR 边际 VaR
   - ✓ $CVaR_i$=component VaR：成分 VaR

9. asymptotic standard error：

$$SE(VaR_{PT}^{1-c}) = \sqrt{\frac{c(1-c)}{T * f(VaR_{PT}^{1-c})^2}}$$

10. 广义极值理论(generalized extreme value theory,GEV)【齐齐子:这个公式直接考的概率不大，重要的是知道μ、σ和ξ的含义】

$$H_{\xi,\mu,\sigma} = \begin{cases} \exp[-(1+\xi\frac{x-u}{\sigma})^{-\frac{1}{\xi}}], \xi \neq 0 \\ \exp[-\exp\left(-\frac{x-\mu}{\sigma}\right)], \xi = 0 \end{cases}$$

- ✓ μ：the location parameter of the limiting distribution, which is a measure of the central tendency of Mn。【u 是极端值的平均数，反映的是集中趋势】
- ✓ σ：the scale parameter of the limiting distribution, which is a measure of the dispersion of Mn. 【σ是极端值的标准差，反映的是离散程度】
- ✓ ξ：the tail index, gives an indication of the shape (or heaviness) of the tail of the limiting distribution. 【ξ是形状参数，反映的是尾部肥瘦情况】
  - ■ When ξ>0: Fréchet distribution, heavy tails, like t-dist, Pareto dist.【这类分布是金融学用到的最多的一种极值分布】
  - ■ When ξ=0: Gumbel distribution, light tails, like normal or lognormal dist.
  - ■ When ξ<0: Weibull distribution, very light tails, not useful for modelling financial returns.

11. The Peaks-over-threshold Approach(POT) 【齐齐子:这个公式直接考的概率不大，重要的是能知道β和ξ的含义】

$$G_{\xi,\beta}(x) = \begin{cases} 1-(1+\frac{\xi x}{\beta})^{-\frac{1}{\xi}}, \xi \neq 0 \\ 1-exp(-\frac{x}{\beta}), \xi = 0 \end{cases}$$

β: a positive scale parameter

ξ: a tail index parameter

12. POT risk measures：

$$VaR = u + \frac{\beta}{\xi}\left\{\left[\frac{n}{N_u}(1-\alpha)\right]^{-\xi} - 1\right\}$$

$$ES = \frac{VaR}{1-\xi} + \frac{\beta - \xi u}{1-\xi}$$

- ✧ u：设定的阈值，去掉百分号（%）
- ✧ β和ξ：POT 分布的参数
- ✧ n 是观测的总值
- ✧ N_μ是超过阈值的
- ✧ α是计算 VaR 值的置信区间

13. Failure rate = N/T;
- ✧ N:the number of exceptions.
- ✧ T：样本空间

14. Type I Error vs Type II Error

【这里是一级学习的重点，二级做个了解。这个也在我们考纲范围内】
- ■ Type I Error 一类错误：P(type I error) = P(reject null hypothesis | the null is true)= level of significance α
- ■ Type II Error 二类错误：P(type II error) = P(not reject the null | the null is false) =β
- ■ 假设检验的势 power of test =1-β

Decision	True Situation	
	H_0 True	H_0 False
Do not reject H_0	Correct Decision	Type II Error 二类错误 (probability = β)
Reject H_0 (“accept” H_1)	Type I Error 一类错误 (probability = α)	Correct Decision (Power of Test=1-β)

15. Unconditional Coverage【齐齐子:这个公式直接考的概率不大，重要的是拒绝

原假设的判断】

- These regions are defined by the tail points of the log-likelihood ratio.
  - ✓ H_0: the model is correctly calibrated.【原假设：VaR 模型是正确的】

$$LR_{uc}=-2\ln[(1-p)^{T-N}p^N]+2\ln\{[1-(N/T)^{T-N}](N/T)^N\}$$

  - ✓ LRuc is asymptotically (i.e., when T is large) distributed chi square with one degree of freedom.
  - ✓ Reject the Ho if the LRuc >3.84.

16. Conditional Coverage Models

- For conditional coverage, the overall test statistic (LRcc) is:

$$LR_{cc}=LR_{uc}+LR_{ind}$$

  - ✓ If $LR_{cc}>5.991$, H_0 can be rejected.
  - ✓ If $LR_{ind}>3.841$, independence alone can be rejected.

17. Basel Committee Rules for Backtesting

- 巴塞尔委员会，最多可接受四个异常值，也就是“绿灯”区域。如果异常值超过五个，银行就会进入“黄”或“红”区，并承担逐步增加的罚款，其中惩罚因子 k 从 3 增加到 4。

区域	例外值个数	Increase in k
绿	0-4	0
黄	5	0.4
	6	0.5
	7	0.65
	8	0.75
	9	0.85
红	10+	1

18. Mapping Options

$$c=Se^{-yt}N(d_1)-Ke^{-rt}N(d_2)$$

- ■ Long call option = long △ asset+ short (△S-c) bill
- ■ $N(d_1)=\triangle$
- ■ $Ke^{-rt}N(d_2)=SN(d_1)-c=\triangle S-c$

19. 平方根法则：$VaR_{n天}=\sqrt{n}\times VaR_{天}$

20. Correlation swap（the correlation fixed rate Payer）payoff：

$$\text{payoff}=NP\times(\text{realized }\rho-\text{fixed }\rho)$$

$$\rho_{realized}=\frac{2}{n^2-n}\sum_{i>j}\rho_{i,j}$$

  - ✓ NP：合约规模
  - ✓ $\rho_{i,j}$：Pearson correlation between asset i and j
  - ✓ n: number of assets

21. mean reversion(均值复归):

$$S_t-S_{t-1}=a(\mu_s-S_{t-1})$$

St: t 时刻的相关性

S_{t-1}: t-1 时刻的相关性

a: Degree of mean reversion, also called mean reversion rate or gravity, 取值范围：大于 0 小于 1

μ_s: 相关性的长期均值

22. $F_{hedge}=-F_{target}\times\frac{DV01_{target}}{DV01_{hedge}}\times\beta$

23. Jensen's Inequality 詹森不等式：

$$E\left[\frac{1}{(1+r)}\right]>\frac{1}{E(1+r)}$$

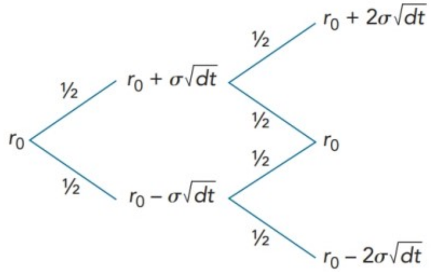
24. Model 1: normally distributed rates and no drift

$$dr=\sigma dw$$
$$dw=\varepsilon\sqrt{dt}$$

  - ✓ dr: change in interest rates over dt.
  - ✓ dt: a small time interval, measured in years.
  - ✓ σ: annual basis-point volatility of rate changes.
  - ✓ dw: normally distributed random variable with $N(0,\sqrt{dt})$.
  - ✓ ε是随机数；【ε是随机数。二叉树认为未来利率只有两种可能，即ε只有两种可能。大多数情况下简化，ε取值为“-1”或“+1”】

- Expected change of the rate: E(dr) = 0.

- Standard deviation of the rate: Std.(r) $=\sigma\sqrt{dt}$.
- 第一期利率是r_0，第二期利率是r_0+dr，即$r_0+\sigma\sqrt{dt}\times\varepsilon$(ε取值为“-1”或“+1”)，未来利率是$r_0+\sigma\sqrt{dt}$或者$r_0-\sigma\sqrt{dt}$，可得利率二叉树：



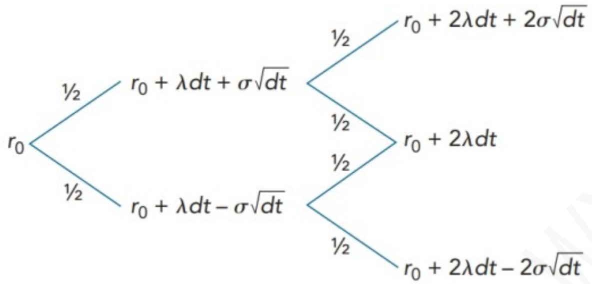
25. Model 2: Drift and risk premium

$$dr=\lambda dt+\sigma dw$$

λ:annual drift in interest rates.

- ✓ Expected change of the rate: E(dr) = 0.
- ✓ Standard deviation of the rate: Std.(r) $=\sigma\sqrt{dt}$.

- 利率二叉树：

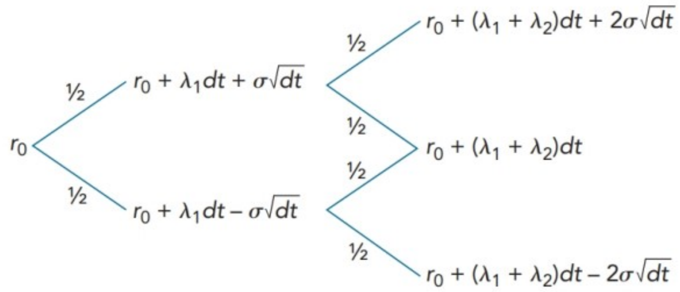


26. Ho-Lee Model:Time-dependent drift

$$dr=\lambda_t dt+\sigma dw$$

λ_t: a time-dependent drift term.

- ✓ It is clear that if $\lambda_1=\lambda_2$ then the Ho-Lee model reduces to Model 2.



27. Vasicek Model:Mean reversion

- Vasicek model captures mean reversion:

$$dr=\kappa(\theta-r)dt+\sigma dw$$

  - ✓ k:the speed of mean reversion.
  - ✓ θ :the long-run mean reversion level.
  - ✓ r:current interest rate level.
  - ✓ Drift term（趋势项）:$\kappa(\theta-r)dt$。The drift combines both interest rate expectations and risk premium
  - ✓ Stochastic term（随机项）:σdz

- Vasicek model: half time 半衰期
  - ✓ The expectation of the rate in the Vasicek model after T years is:

$$r_0e^{-\kappa T}+\theta(1-e^{-\kappa T})$$

  - ✓ factor's half-life (t)

$$t=\frac{\ln 2}{\kappa}$$

28. Model 3

$$dr=\lambda(t)dt+\sigma(t)dw$$

$$dr=\lambda(t)dt+\sigma e^{-\alpha t}dw$$

α是大于零的常数。意味着长期的波动率小于短期的波动率。

29. Cox-Ingersoll-Ross (CIR) Model

$$dr=k(\theta-r)dt+\sigma\sqrt{r}dw$$

- ✓ σ:yield volatility, which is constant.
- ✓ $\sigma\sqrt{r}$:annualized basis-point volatility
  - ■ $\sqrt{r_t}$ in the stochastic term forces interest rate (r) to be non-negative,

and higher interest rate will lead to higher volatility.

30. Lognormal model (model 4)

- The risk-neutral dynamics of the lognormal model are:

$$dr = ardt + \sigma rdw$$

- The Salomon Brothers Model (Lognormal model with deterministic drift)

$$d[\ln(r)] = a(t)dt + \sigma dw$$

- The Black-Karasinski Model (Lognormal model with mean reversion)

$$d[\ln(r)] = k(t)[\ln\theta(t) - \ln(r)]dt + \sigma(t)dw$$

31. Gauss+ model

- 短期利率因子 r 的变化:$d_{r_t} = -\alpha_r(m_t - r_t)dt$
- 中期利率因子 m 的变化:$d_{m_t} = -\alpha_m(l_t - m_t)dt + \sigma_m(\rho dW_t^1 + \sqrt{1-\rho^2}dW_t^2)$
- 长期利率因子 l 的变化: $d_{l_t} = -\alpha_l(u - l_t)dt + \sigma_l dW_t^1$
- $E(dW_t^1 dW_t^2) = 0$

信用风险

32. Altman's Z-Score

$$Z = 1.21X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 0.999X_5$$

  - X_1: Working capital to total assets,
  - X_2: Retained earnings to total assets,
  - X_3: Earnings before interest and taxes to total assets,
  - X_4: Market value of equity to book value of total liabilities, and
  - X_5: Sales to total assets.

- 判断：
  - Z-score>3.0，不太可能违约
  - 2.7＜Z-score＜3.0, there was reason to be "on alert 警惕."
  - 1.8＜Z-score＜2.7, there was a good chance of default 有很大的违约可能性
  - Z-score＜1.8，违约概率很高

33. Unconditional and Conditional default probability 无条件和条件违约概率
   - ✓ Cumulative PD：在一定时期内（例如，一年内、两年内、三年内）违约的概率

   Cumulative survival rate 累计生存概率（$SR_t^{cumulated}$）=1-cumulative PD

34. Unconditional default probability 无条件违约概率
   - ✓ 无条件违约概率是指在某一特定时间段内，某一实体（如公司或债券发行人）发生违约的概率，而不考虑其他任何条件或前提。

$$PD_k^{Uncond} = PD_{t+k}^{cumulated} - PD_t^{cumulated}$$

35. Conditional default probability 条件违约概率
   - ✓ 例如，存活（不违约）到第 n 年，其在第 n+1 年违约的概率。

$$PD_{t,t+k}^{cond} = \frac{PD_k^{Uncond}}{SR_t^{cumulated}}$$

36. Default intensity model 违约强度模型

$$\text{Cumulative PD} = 1 - e^{-\bar{\lambda}t}$$

$$\text{Cumulative survival rate} = e^{-\bar{\lambda}t}$$

$$\text{Unconditional PD} = e^{-\lambda_1 \times t_1} - e^{-\lambda_2 \times t_2}$$

   - ■ $\bar{\lambda}$ is the average hazard rate between time zero and time t.

37. credit spread

$$\text{credit spread} \approx EL(\%) = PD*LGD = \text{Hazard Rate}*(1-RR)$$

$$\rightarrow \text{Hazard Rate}(\lambda) = \text{credit spread}/(1-RR)$$

   - ✓ RR：回收率

38. CDS-bond Basis
   - ■ CDS-Bond Basis= CDS Spread - Bond Yield Spread

39. Merton model 莫顿模型

$$\text{股东权益}(E_0) = VN(d_1) - Ke^{-rT}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{V}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \quad d_2 = \frac{\ln\left(\frac{V}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

   - ✧ σ : Volatility of assets (assumed constant)

✓ 对于债权人来说【千万记得，债务价值不是 put 价值】

①$B_1 = \mathrm{Min}(K, V_1) = K - \mathrm{Max}(K - V_1, 0) = K - \text{put option}$

- ❖ Value of risky debt = value of risk-free debt - value of a put option
- ❖ 持有公司的债务，等同于持有一种无风险债券（面值为 K），并同时卖出一个欧式看跌期权（标的资产是公司资产，执行价格是 K）

②$B_1=V_1-E_1=V_1-\mathrm{Max}(V_1-K,0)=V_1\text{-call}$

- ❖ 持有公司的债务，等同于持有公司资产，同时卖出一个欧式看涨期权

✓ Risk-neutral probability of default：$1-N(d_2)$ 【违约概率=看涨期权不行权的概率=$1-N(d_2)$】

40. Distance to default (DD)违约距离

$$DD = d_2 = \frac{\ln\left(\frac{V}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \approx \frac{\ln(V) - \ln(K)}{\sigma}$$

- ■ As the distance to default declines, the company becomes more likely to default. 【DD 越小，越有可能违约】

41. Loan-to-value (LTV) ratio 贷款价值比: LTV=贷款金额/房屋价值
42. Debt-to-income (DTI) ratio 债务收入比: =月供/月总收入
43. accuracy ratio (AR) $= \frac{A_R}{A_P}$

    ✓ AR 越接近 1，说明这个评分模型越准确。

44. EL、UL、ULC(重要的不能再重要了)

(1) 预期损失 (Expected Loss, EL):指在未来一段时间（通常是一年）可能发生的平均损失。也就是损失的期望值。	
单个资产: EL(%) = PD * LGD EL($)= PD * LGD*EAD【主要用这个公式】	PD→信用评级、CS、莫顿模型计算 PD LGD=loss rate=1-回收率 EAD，也写作 exposure amount (EA)
组合: $EL_P = \sum_{i=1}^{n} PD_i \times LGD_i \times EAD_i$ 两资产组合: $EL_P=PD_1 \times LGD_1 \times EAD_1 + PD_2 \times LGD_2 \times EAD_2$	和资产之间的相关性 ρ 无关

(2) 非预期损失 (Unexpected Loss, UL)： UL is the standard deviation of credit losses, that is, the standard deviation of actual credit losses around the expected loss average (EL). 切记：在这个科目中，UL 的概念，与一级以及二级其他科目中的 Unexpected Loss 概念不同。	
单个资产： $UL = EAD \times \sqrt{PD_i \times \sigma_{LGD}^2 + LGD^2 \times \sigma_{PD}^2}$ Bernoulli 伯努利分布→$\sigma_{PD}^2 = PD \times (1 - PD)$	σ_{LGD}^2 ： LGD 的方差 σ_{PD}^2 ： PD 的方差
组合：$UL_p = \sum_i \sum_j \rho_{i,j} \times UL_i \times UL_j$ 两资产组合：$UL_p = \sqrt{UL_1^2 + UL_2^2 + 2 \times \rho_{1,2} \times UL_1 \times UL_2}$ 若单个资产的 UL 相等，相关系数相等→$UL_p = UL \times \sqrt{n + n \times (n-1) \times \rho}$	$\rho_{i,j}$:资产 i 和 j 的违约相关性
(3) Unexpected Loss Contribution (ULC)： ULC 是指单个资产（如 loan）对整个 UL 的边际贡献。即衡量一个特定资产在整个组合可能出现的损失中占了多大的份额。【重点掌握两资产】	
$ULC_i = \frac{UL_i \times \sum_j \rho_{i,j} \times UL_j}{UL_p}$	
两资产：$ULC_1 = \frac{UL_1^2+\rho_{i,j}\times UL_1 \times UL_2}{UL_p}$ $ULC_2 = \frac{UL_2^2+\rho_{i,j}\times UL_1 \times UL_2}{UL_p}$ $ULC_1 + ULC_2 = UL_p$	
前提：n 笔 loan；每笔 loan 规模相同、风险特质相同 $ULC_i = \frac{UL_p}{n} = UL_i \times \sqrt{\frac{1}{n} + \rho \times (1 - \frac{1}{n})}$	the correlation between assets increases, the bank suffers from concentration risk 集中度风险。
前提：n 比较大时 $ULC_i = UL_i \times \sqrt{\rho}$	

45. Default correlation 违约相关性

- ● 联合违约概率 joint default probability(π_{12}):在时间范围 T 内两者都违约的概率

$$\rho=\frac{Cov(X_1X_2)}{\sigma_{X_1}\sigma_{X_2}}=\frac{\pi_{12}-\pi_1\pi_2}{\sqrt{\pi_1(1-\pi_1)}\sqrt{\pi_2(1-\pi_2)}}$$

- ✓ π_1：资产 1 的违约概率；π_2：资产 2 的违约概率

46. Credit VaR

Credit VaR= WCL-EL

- ✓ Worst case loss (WCL): Loss at the confidence level

47. Single factor model

$$\alpha_i=\beta_i\times m+\sqrt{1-\beta_i{}^2}\times\varepsilon_i$$

$$\alpha\sim N(0,1)$$

$$E(\alpha)=\beta E(m)+\sqrt{1-\beta^2}E(\varepsilon)=0$$

$$\sigma^2(\alpha)=\beta^2\sigma^2(m)+(1-\beta^2)\sigma^2(\varepsilon)+2\beta\sqrt{1-\beta^2}*cov(m,\varepsilon)=1$$

- ✧ β_i:own correlation between market value.

48. Asset return correlation of the two firms =$\beta_i\beta_j$
49. unconditional PD 与 conditional PD

- ● 当 m 未知时，通过单因素模型得出的是非条件违约概率（unconditional PD）。
- ● 当 m 已知时，即市场因子 m 设定为常数（$\overline{m}$，对市场情况有一定预期），通过单因素模型得出的是条件违约概率（conditional PD）。

$$\alpha=\beta\overline{m}+\sqrt{1-\beta^2}\varepsilon$$

- ✓ 其中 $E(\alpha)=\beta\overline{m}$，$\sigma^2(\alpha)=1-\beta^2\rightarrow\alpha$服从均值为$\beta\overline{m}$，方差为 $1-\beta^2$的正态分布。

50. Vasicek model

- ● Capital requirement(Credit VaR)=(WCDR- PD)×EAD×LGD

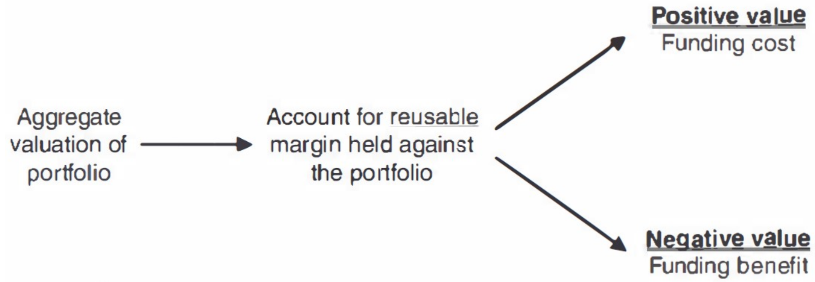
$$WCDR(T,X)=N(\frac{N^{-1}(PD)-\sqrt{\rho}N^{-1}(X)}{\sqrt{1-\rho}})N(\frac{N^{-1}(PD)-\sqrt{\rho}N^{-1}(0.001)}{\sqrt{1-\rho}})$$

- ✓ WCDR * LGD * EAD is the loss at this 99.9 percentile point.
- ✓ PD * LGD * EAD is the expected loss.

51. Credit exposure 信用敞口

exposure=max(Value,0)

52. Funding Costs and Benefits



- ● funding = value-margin
  - ✓ funding＞0→funding cost(理解：cost 理解被占便宜)
  - ✓ funding＜0→funding benefit(理解：benefit 占别人便宜)
- ● 收到的保证金应从敞口中扣除，而支付的保证金应加到敞口中。

Positive exposure=max(value−VM−IM^R,0)

Negative exposure = min(value -VM + IM^P, 0)

funding=value−VM+IM^P

- ✓ VM：变动保证金
- ✓ IM^R：收到的初始保证金。
- ✓ IM^P ：支付的初始保证金。

53. Credit value adjustment (CVA)信用价值调整

risky value = risk-free value + CVA

【这里的 CVA 是负值。有风险的资产价值通常小于无风险资产】

54. Unilateral CVA(UCVA)单边 CVA:

- ● 站在自己的角度(自己不会违约)，讨论交易对手的违约风险

$$UCVA=-\sum_{i=1}^{m}PV(EL_i)=LGD\times\sum_{i=1}^{m}EPE(t_{i-1},t_i)\times PD(t_{i-1},t_i)$$

- ✓ LGD:The loss given default.
- ✓ $EPE\big((t_{i-1},t_i)\big)$：The discounted EPE for the relevant dates in the future given by t_i. 【注意这里的 EPE 已经是现值形式，所以无需再折现；做题注意，exposure 是否已经是现值形式。如果是，直接使用。如果不是，还需折现】
- ✓ $PD(t_{i-1},t_i)$：边际违约概率

55. CVA as a Spread：UCVA is expressed in the same units as the credit spread

$$UCVA = -\text{average EPE} \times \text{spread}$$

✓ Credit component: the credit spread of the counterparty

■ spread=LGD*λ，体现违约风险/信用风险

56. Netting CVA：多笔交易的 CVA 净额结算

$$CVA^{NS} \le \sum_{i=1}^{n} CVA_i$$

✓ CVA^{NS}：净额结算的 CVA

✓ 净额结算的 CVA 小于等于单个资产 CVA 加总。【类似组合 VaR 和单个资产 VaR 加总概念】

57. Bilateral credit value adjustment(BCVA)双边信用价值调整

● 综合考虑了 CVA 和 DVA 的共同影响

✓ 站在 A 方角度：$CVA = -LGD_B \sum_{i=1}^{m} EPE(t_i) \times PD_B(t_{i-1}, t_i) \times (1 - PD_A(0, t_{i-1}))$

✓ 站在 B 方角度：$DVA = -LGD_A \sum_{i=1}^{m} ENE(t_i) \times PD_A(t_{i-1}, t_i) \times (1 - PD_B(0, t_{i-1}))$

$$BVCA = -|CVA| + |DVA|$$

✧ $1 - PD_A(0, t_{i-1})$:A 方存活到t_{i-1}的概率，存活率

✧ $1 - PD_B(0, t_{i-1})$:B 方存活到t_{i-1}的概率，存活率

✧ 【在双边 BCVA 中，A 被 B 伤害的前提是 A 还活着；B 被 A 伤害的前提是 B 还活着】

$$BCVA = -\mathit{average}\ \text{EPE} \times \text{Spread}_C - \text{average ENE} \times \text{Spread}_P$$

58. Incremental CVA

● 增量 CVA:是指加入一笔新交易对 CVA 的影响

● 类似《投资》科目 incremental VaR 概念

$$CVA^{新-旧} = CVA^{新} - CVA^{旧}$$

59.

$$Bond_{risky} + CDS = Bond_{risk\ free}$$

$$Bond_{risky} = Bond_{risk\ free} - CDS$$

60. Upfront premium 预付保费(%)≈(Credit spread - Fixed coupon) × Duration

61. Default'01

✓ Default'01 用于衡量当 PD 变动一个基点时，对于证券化产品价值的影响。通常以违约概率每上升或下降 10 个基点为整体衡量对于证券化产品价值的影响：

Default'01

$=\frac{1}{20}[(\text{mean value/loss based on } \pi + 0.001) - (\text{mean value/loss based on } \pi - 0.001)]$

投资

62. CAPM：只有一个 factor-市场组合【一级知识，二级也要会】

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

■ $E(R_i)$: expected return on risky asset i.

■ $E(R_m) - R_f$: market portfolio risk premium.

■ β_i : systematical risk of asset i.

■ $\beta_i[E(R_m) - R_f]$: the expected return premium above the risk-free rate (as required by investors according to the CAPM)

63. APT 模型【一级知识，二级也要会】

$$E(r_i) = r_f + \beta_{i,1}E(f_1) + \beta_{i,2}E(f_2) + \cdots + \beta_{i,k}E(f_k)$$

✧ βi,k is the beta of asset i with respect to factor k and E(fk) is the risk premium of factor k.

64. Three-factor model

$$E(r_i) = r_f + \beta_{i,MKT}E(r_m - r_f) + \beta_{i,SMB}E(SMB) + \beta_{i,HML}E(HML)$$

65. Four-factor model(Carhart Model)

$$E(R_i) = R_f + \beta_{i,MKT}E(R_m - R_f) + \beta_{i,SMB}E(SMB) + \beta_{i,HML}E(HML) + \beta_{i,WML}E(WML)$$

66. Excess returns(r_t^{ex},超额收益）

✓ 也叫 active returns 积极收益

$$r_t^{ex} = r_t - r_t^{bmk}$$

◆ r_t:the return of an asset or strategy

◆ r_t^{bmk}: the benchmark return.

67. Alpha(α)=超额收益的均值

$$\alpha = \frac{1}{T}\sum_{t=1}^{T} r_t^{ex}$$

68. Tracking error($\bar{\sigma}$,TE,追踪误差)=超额收益的标准差(standard deviation)

69. Information ratio(IR,信息比率) is the ratio of alpha to tracking error($\bar{\sigma}$):
   - ✓ 表示 the average excess return per unit of risk

$$IR = \frac{\alpha}{\bar{\sigma}} = \frac{\alpha}{TE}$$

$$IR \approx IC \times \sqrt{BR}$$

   - ◆ IR is the information ratio
   - ◆ IC is the information coefficient 信息系数, which is the correlation of the manager's forecast with the actual;
     - ✧ IC 是经理的预测与实际回报的相关性【侧重预测的准确性】
   - ◆ BR is the breadth of the strategy (how many bets are taken).
     - ✧ BR 是策略的广度【进行了多少个预测，侧重数量】

70. Scale the alphas

Alpha= volatility*IC*score

   - ✧ Information coefficient (IC) and residual risk (volatility)是常数.
   - ✧ Score 服从 standard normal distribution
   - ✧ alphas 服从正态分布，其均值为 0，标准差为 IC × residual risk
     - ■ scale of the alphas 将取决于管理者的信息系数

71. Determination of Risk Aversion

✓ 信息比率、风险厌恶（Risk Aversion）和最佳主动风险之间的最优关系：

$$(\text{Risk Aversion})\lambda = \frac{IR}{2 \times \varphi}$$

   - ■ φ= tracking error=active risk

72. Marginal Contribution to Value Added（MCVA）
   - ✓ MCVA，表示当增加或减少投资组合中某个资产的持有量时，该资产对整个投资组合价值增加的贡献。

$$MCVA_n = \alpha_n - 2 \times \lambda_A \times \varphi \times MCAR_n$$

   - ✧ MCVAn : Marginal contribution to value added
   - ✧ MCARn : Marginal contribution to active risk
   - ✧ λ_A: active risk aversion
   - ✧ φ: active risk

73. N 资产组合的标准差(满足 3 个假设)：

$$\sigma_p = \sigma \times \sqrt{\frac{1}{N} + \left(1 - \frac{1}{N}\right) \times \rho}$$

   - ■ three assumptions
     - ✧ Each asset of the portfolio has equally weighted(σ)
     - ✧ Each individual position has the same standard deviation of return
     - ✧ Each pair of returns has only one correlation(ρ)

74. Undiversified VaR 未分散化 VaR

● 隐含：资产之间的相关系数ρ=1

$$VaR_p = VaR_1 + VaR_2$$

75. Marginal VaR 边际 VaR (MVaR)
   - ✓ 含义：在原有组合中，A 资产多投资$1,组合 VaR 的变化额。

$$MVaR_A = \frac{\partial VaR_P}{\partial V_A} = z_\alpha \times \frac{Cov(R_A, R_P)}{\sigma_P}$$
$$= z_\alpha \times \rho_{A,P} \times \sigma_A$$
$$= z_\alpha \times \beta_{A,P} \times \sigma_P$$
$$= \frac{VaR_P}{V_P} \times \beta_{A,P}$$

小结论：对于同一个组合 p 而言，资产 A 的 β 越大，其 MVaR 越大

76. Incremental VaR 增量 VaR (IVAR)

● 精确式：考试可主要用这个公式

$$\text{Incremental } VaR_A = VaR_{P+A} - VaR_P$$

● 近似式：慎用,只有当V_A足够小，才会比较准确

$$\text{Incremental } VaR_A \approx MVaR_A \times V_A (\text{any amount})$$

77. Component VaR 成分 VaR (CVaR)

$$\mathrm{CVaR_A = MVaR_A \times V_A = VaR_A \times \rho_{A,p}}$$

$$\mathrm{VaR_P = CVaR_1 + CVaR_2 + \cdots + CVaR_n = VaR_P(\sum_{i=1}^{N} \omega_i\beta_{i,p})}$$

- VaR Contribution to Asset A $= \frac{\mathrm{CVaR_A}}{\mathrm{VaR_P}} = \frac{\mathrm{MVaR_A \times V_A}}{\mathrm{VaR_P}} = \frac{\frac{\mathrm{VaR_P}}{\mathrm{V_P}} \times \beta_{A,P} \times V_A}{\mathrm{VaR_P}} = \frac{\beta_{A,P} \times V_A}{V_P} = \omega_A \times \beta_{A,P}$
- 所有贡献率求和为 100%：$\sum_{i=1}^{N} \omega_i\beta_{i,p} = 100\%$

78. 只考虑风险，确定最优组合
- 目标是降低组合风险。缺点，没有考虑 return 情况
- 方法：
  - ✓ 1.Positions should be cut first where the marginal VaR is the greatest, keeping portfolio constraints satisfied. 【找到 MVaR 最大的头寸，减少该资产的投资，增加到其他资产的投资上】
  - ✓ 2.This process can be repeated up to the point where the portfolio risk has reached a global minimum.【不断重复，直到组合风险达到最小。】
  - ✓ At this point, all the marginal VaRs, or the portfolio betas, must be equal。【此时，所有的资产的 MVaR 相等，同时$\beta_{i,p}$或者组合的 β =1】

$$\mathrm{MVaR_1 = MVaR_2 = \cdots = MVaR_n}$$
$$\beta_{1,p} = \beta_{2,p} = \cdots = \beta_{n,p}$$
$$\beta_p = 1$$

79. 考虑风险和收益，确定最优组合

$$\frac{\mathrm{E(R_A) - R_f}}{\mathrm{MVaR_A}} = \frac{\mathrm{E(R_B) - R_f}}{\mathrm{MVaR_B}}$$

- ✓ 具体方法：
  - ✧ 若 $\frac{\mathrm{E(R_A)-R_f}}{\mathrm{MVaR_A}} > \frac{\mathrm{E(R_B)-R_f}}{\mathrm{MVaR_B}}$，应该卖出 B 资产买入 A 资产；
  - ✧ 若 $\frac{\mathrm{E(R_A)-R_f}}{\mathrm{MVaR_A}} < \frac{\mathrm{E(R_B)-R_f}}{\mathrm{MVaR_B}}$，应该卖出 A 资产买入 B 资产；
  - ✧ 若 $\frac{\mathrm{E(R_A)-R_f}}{\mathrm{MVaR_A}} = \frac{\mathrm{E(R_B)-R_f}}{\mathrm{MVaR_B}}$，此时投资组合不需要调仓。

- 小结
  - ✓ 如果不考虑资产的收益率，再平衡的最终状态为 $\mathrm{MVaR_i = MVaR_j}$。
  - ✓ 如果考虑资产的收益率，再平衡的最终状态为$\frac{\mathrm{E(R_A)-R_f}}{\mathrm{MVaR_i}} = \frac{\mathrm{E(R_B)-R_f}}{\mathrm{MVaR_j}}$。

80. Funding Risk 资金风险

$$\Delta\text{Surplus} = \Delta\text{Assets} - \Delta\text{Liabilities}$$

$$R_S = \frac{\Delta S}{A} = \frac{\Delta A}{A} - \frac{\Delta L}{L} \times \frac{L}{A} = R_{asset} - R_{\text{liabilities}} \times \frac{L}{A}$$

$$\text{Expected surplus} = A \times (1 + R_A) - L \times (1 + R_L)$$

$$\text{Expected surplus growth} = A \times R_A - L \times R_L$$

$$\sigma_{Surplus} = \sqrt{A^2\sigma_A{}^2 + L^2\sigma_L{}^2 - 2\mathrm{A}\sigma_A\mathrm{L}\sigma_L\rho}$$

$$\text{Surplus at risk} = |\text{Expected surplus growth} - z_\alpha \times \sigma_{Surplus}|$$

81. The optimal allocation across managers is:

$$\text{weight of portfolio managed by manager i} = \frac{\mathrm{IR_i} \times (\text{Portfolio's tracking error volatility})}{\mathrm{IR_P} \times (\text{manager's tracking error volatility})}$$

82. liquidity duration 流动性久期

$$\mathrm{LD_i = \frac{Q_i}{0.15 \times V_i}}$$

- ❖ Qi= Number of shares held in security i
- ❖ Vi= Daily volume of security i

- ❖ 15%：指的是在清算某个证券时，每天交易的最大允许比例 (Maximum daily volume allowed for liquidation)【15%这个是原版书给出】
  - ■ 注意，做题时这个比例不一定是 15%，按照题目给的数据计算。

83. Holding Period Return 持有期收益率

$$HPR = \frac{\text{End Value} - \text{Beginning Value}}{\text{Beginning Value}} = \frac{P_1 - P_0 + D_1}{P_0}$$

- ✓ D_1：1 时刻的收益，比如分红等
- ✓ p_0：0 时刻资产价格
- ✓ p_1：1 时刻资产价格

84. Dollar-Weighted rates of return 货币加权收益率
- ✓ 也叫 Internal rate of Return (IRR)，也叫 Money-Weighted rates of return

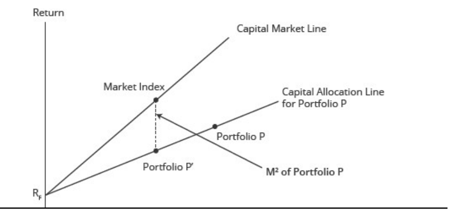
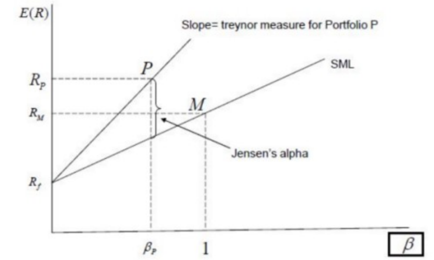
$$CF_0 + \frac{CF_1}{1+MWR} + \ldots + \frac{CF_N}{(1+MWR)^N} = 0$$

85. Time-weighted rates of return 时间加权收益率
- ✓ 也叫 Geometric Mean Return 几何平均收益率
- ✓ 考虑“利滚利”的情况，即期初的投资按照几何平均收益率进行复利增长直至期末
- ✓ 也体现在整体期间的平均收益表现

$$(1+TWR)^N = (1+HPR_1)(1+HPR_2)(1+HPR_3)\ldots\ldots(1+HPR_n)$$

$$\rightarrow TWR = [(1+HPR_1)\times(1+HPR_2)\times\ldots\times(1+HPR_n)]^{\frac{1}{N}} - 1$$

86. Risk-Adjusted Performance Measures 风险调整后收

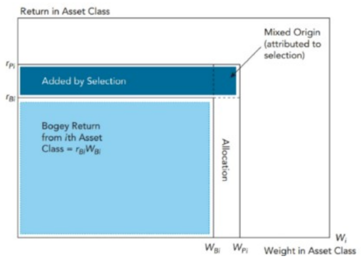
Sharpe ratio 夏普比率	Sharpe Ratio measures the excess return per unit of total risk. 也叫 reward-to-variability ratio $SR = \frac{R_p - R_f}{\sigma_p}$	
M-Squared M 平方 ：与夏普比率相同的排序	$M^2 = \frac{\sigma_M}{\sigma_P}(R_p - R_f) - (R_M - R_f)$ $= \sigma_M \times (SR_P - SR_M)$	
注意：夏普比率与 M 方适用于没有充分分散化的组合 M2 越大越好，排序结果与夏普比率一致（identical）		
Treynor ratio 特雷诺比率	$TR = \frac{R_p - R_f}{\beta_p}$	特雷诺比率越大越好，但不适用于贝塔为负的资产（negative-beta assets）
Jensen's Alpha 詹森阿尔法	α_p $= R_p - \{R_f + \beta_p * [E(R_m) - R_f]\}$ $= R_p - CAPM$	
注意：特雷诺指数和詹森阿尔法适用于充分分散化的组合		
信息比率IR = $\frac{\alpha}{\sigma_\alpha}$		

87. Performance attribution 业绩归因

(1) 资产配置（Asset Allocation）
- ✓ Contribution from Asset Allocation= Excess weight in asset class × Benchmark return = $(W_P-W_B) \times R_B$

(2) 证券选择（Security Selection）
- ✓
- ✓ Contribution from Security Selection= Weight in asset class × Excess return = $W_P\times(R_P - R_B)$

- ● Total Contribution= $W_P \times R_P - W_B \times R_B$



流动性风险

88. 交易流动性风险

Bid-ask spread (dollar 形式): p = offer price − bid price【高减低】

$$\text{Bid-ask spread(\%形式)}: s = \frac{\text{offer price} - \text{bid price}}{\text{mid market price}}$$

$$\alpha = \text{mid market price} = \frac{\text{offer price} + \text{bid price}}{2}$$

✓ In normal market:

$$\text{Cost of liquidation} = \sum_{i=1}^{n} \frac{1}{2} \times (s_i \alpha_i)$$

✓ In stressed market:

$$\text{Cost of liquidation} = \sum_{i=1}^{n} \frac{1}{2} \times (\mu_i + \lambda\sigma_i)\alpha_i$$

✧ α_i: the dollar value of the position in the instrument

✧ λ：标准正态分布对应的关键值（单尾）；99%置信水平，λ=2.326；95%置信水平，λ=1.645

✧ μ_i：Bid-Offer Spread 的均值

✧ σ_i：Bid-Offer Spread 的标准差

89. Liquidity-adjusted VaR

✓ In normal market:

$$\text{Liquidity} - \text{Adjusted VaR} = \text{VaR} + \sum_{i=1}^{n} \frac{1}{2} \times (s_i \alpha_i)$$

✓ In stressed market:

$$\text{Liquidity} - \text{Adjusted VaR} = \text{VaR} + \sum_{i=1}^{n} \frac{1}{2} \times (\mu_i + \lambda\sigma_i)\alpha_i$$

✧ Spread risk factor： $\frac{1}{2} \times (\mu_i + \lambda\sigma_i)$

90. 关于 VaR→Adjusting VaR for liquidating

■ 潜在假设：σ 不变；VaR 独立；均匀清仓

$$\text{VaR} \times \sqrt{\frac{(1+T) \times (1+2T)}{6T}}$$

■ T is the required for the orderly liquidation of a position.【T 是清仓所用的时间】

■ Square root rule is overstating VaR for different time horizons

91. Liquidity Coverage Ratio 流动性覆盖率（LCR）

✓ LCR：优质性流动资产储备与未来 30 日的资金净流出量的比值，且该比值应大于等于 100%

$$\text{LCR} = \frac{\text{High} - \text{quality liquid assets}}{\text{Net cash outflows in a 30} - \text{day period}} \geq 100\%$$

92. 净稳定融资比率（net stable funding ratio,NFSR）

✓ 关注长期流动性问题

$$\text{NFSR} = \frac{\text{可用的稳定资金(Amount of stable funding, ASF)}}{\text{所需的稳定资金(Required Amount of stable funding, RSF)}} \geq 100\%$$

93. Leverage ratio= $\frac{A}{E} = \frac{D+E}{E} = 1 + \frac{D}{E}$

✓ A：asset；D：date 借来的钱；E：equity 自己的钱

✓ A=D+E

94. Leverage effect 杠杆效应:

$$r_E = L * r_A - (L-1)r_D = r_A + \frac{D}{E}(r_A - r_D) = r_D + L * (r_A - r_D)$$

- ✓ Effect of increasing leverage: $r_A - r_D$

  r_A= return on assets

  r_E: return on equity(ROE)【本质上用自己的钱赚的收益率】

  r_D: cost of debt

  L = leverage ratio

95. 回购

$$\text{Repurchase Price} = \text{Purchase Price}_0 \times \left(1 + \frac{T}{360} \times \text{Repo rate}\right)$$

96. Haircut(h)：自己的钱（保证金），通常带%

- ✓ 1-h：借钱比例
- ✓ $\text{Haircut} = \frac{\text{Security Price}_0 - \text{Purchase Price}_0}{\text{Security Price}_0}$

$$\text{leverage} = \frac{1}{h}$$

97. Leveraged returns to housing(Assuming 10% price appreciation) 【房贷】

Leverage=1/down payment(%)

- ✓ down payment 首付比例

98. Gross leverage 和 Net leverage

Gross leverage=(多头头寸价值+空头头寸价值)/资本

Net leverage=(多头头寸价值-空头头寸价值)/资本

99. TEY

$$\text{tax} - \text{equivalent yield (TEY)} = \frac{\text{市政债券收益率}}{1 - \text{税率}}$$

$$= \frac{\text{After tax return on a tax} - \text{exempt investment}}{(1 - \text{ firm's marginal tax rate})}$$

100. net liquidity position (L)净流动性

net liquidity position (L)净流动性= 流动性供给 supply for liquidity- 流动性需求 demand for liquidity

101. Esitmated liquidity deficit or surplus=Δdeposits-Δloans

- ✓ 当存款增加（Δdeposits＞0）而贷款减少（Δloans＜0）时，流动性增加;
- ✓ 当存款减少而贷款增加时，流动性减少。

102. term structure of expected liquidity (TSLe)

=Causes of liquidity+Sources of liquidity

=TSECCF+TSCLGC

103. Cash flows at risk

- ✧ positive cash-flow-at-risk: $CFaR_\alpha^+ = CF_\alpha^+ - CF_e$
- ✧ negative cash-flow-at-risk: $CFaR_\alpha^- = CF_\alpha^- - CF_e$

104. Interest-sensitive Gap Management

★	NIM = NII / earning assets
★ ★	NII = interest income – interest expense 简化公式→NII = interest income – interest expense $=ISA*i_{ISA}-ISL*i_{ISL}$
★	$\Delta NII=ISA*\Delta i_{ISA}-ISL*\Delta i_{ISL}=(ISA-ISL)*\Delta i$ 【假设 $\Delta i_{ISA}=\Delta i_{ISL}=\Delta i$，i：利率】
★	Interest-sensitive gap(IS gap) =ISA - ISL
★	cumulative gap=summing the IS gaps
★	Relative IS gap = IS gap/Size of financial institution
★	Interest Sensitivity Ratio (ISR) = ISA/ISL

105. Duration Gap Management

NW = assets - liabilities=A-L

$$\Delta NW = \Delta A - \Delta L = \left(-D_A \times \frac{\Delta i}{1+i} \times A\right) - \left(-D_L \times \frac{\Delta i}{1+i} \times L\right)$$

$$= -\left(D_A - D_L \times \frac{L}{A}\right) \times \frac{\Delta i}{1+i} \times A$$

- ✧ D_A：资产的麦考林久期; D_L：负债的麦考林久期
- ✧ $D_A - D_L$： Duration gap
- ✧ $D_A - D_L \times \frac{L}{A}$: Leverage-adjusted duration gap
- ✧ $\frac{L}{A}$: leverage adjusted indicator

106. Duration gap management

① Defensive gap management

Portfolio immunization strategy：$D_A - D_L \times \frac{L}{A}=0$

② Aggressive gap management

Expected Change in Interest Rates	Aggressive Management’s Most Likely Action	Best Interest-Sensitive GAP Position to Be in
Rates ⬆	$D_A - D_L \times \frac{L}{A}<0$：Reduce DA and increase DL	Net worth increase
Rates ⬇	$D_A - D_L \times \frac{L}{A}>0$：Increase DA and reduce DL	Net worth increase

操作风险

107. RAROC 计算

✓ RAROC 表示每投入一块钱的资源(经济资本)所获得回报；越高越好

$$RAROC = \frac{\text{经过风险调整的收益}}{\text{经济资本}} = \frac{RAR}{\text{Economic Capital}}$$

✓ After-tax expected risk-adjusted return(RAR) 【RAR 现在的原版书也叫 after-tax expected risk-adjusted net income】

■ RAR = Expected Revenues –Costs – Expected Losses – Taxes +Return on EC ±Transfer

✧ “+”：融资项目；“-”：投资项目

✓ Economic capital(EC)

■ EC = Risk capital + Strategic risk capital

■ Strategic risk capital = goodwill 商誉 + burned-out capital

108. Hurdle Rate

$$h_{AT} = \frac{CE \times r_{CE} + PE \times r_{PE}}{CE + PE}$$

✓ CE = market value of common equity

✓ PE = market value of preferred equity

✓ r_{CE}= cost of common equity (可以使用 CAPM 公式计算)

✓ r_{PE}=cost of preferred equity (yield on preferred shares)

● Decision Rule【决定做不做某个项目】

✓ RAROC > hurdle rate，意味着收益>成本→做这个项目

✓ RAROC < hurdle rate，意味着收益<成本→不做这个项目

109. Adjusted RAROC

$$ARAROC = RAROC - \beta_E(R_M - r_f)$$

✓ r_f = risk-free rate = hurdle rate

✓ R_M = expected return on market portfolio

✓ β_E =firm's equity beta

✓ $R_M - r_f$ =excess return over risk-free rate to account for the systematic risk of the project

● decision rules

✓ adjusted RAROC > r_f→做这个项目

✓ adjusted RAROC < r_f→不做这个项目

110. Basel I

$$\text{理念式：capital adequacy ratio} = \frac{\text{资本 capital}}{\text{有风险的资产}}$$

111. The 1995 and 1996 Amendments 95/96 年修正案-market risk

✓ Internal Model-Based Approach 内部模型法

$$\text{Market Risk}=\text{Max}(VaR_{t-1},\ m \times VaR_{avg})$$

■ VaR_{t-1}=previous day's VaR

■ VaR_{avg} = the average VaR over the past 60 trading days

■ m = multiplicative factor 【取值范围：3 到 4，监管机构控制】

■ 10-trading-day time horizon and a 99% confidence level

■ Considers correlations between the instruments

112. Basel II

Credit risk capital requirement 【修正和新增】	① Standardized approach（计算 RWA） →原巴塞尔 1 ② Foundation internal ratings-based (IRB) approach（计算 capital） →新增 ③ Advanced IRB approach（找 capital） →新增
Operational risk capital requirement 【新增】	④ Basic indicator approach（计算 capital） ⑤ Standardized approach（计算 capital） ⑥ Advanced measurement approach（计算 capital）
market risk capital requirement 【无变化】	巴塞尔 2 中，并没有关于市场风险资本金要求的更改，在 95/96 年修正案中已经考虑。为了知识的完整性，补充到这里： ⑦ Standardized approach 标准法（计算 capital） ⑧ Internal Model-Based Approach 内部模型法（计算 capital）

113. Credit risk capital requirement

① The standardized approach→计算 RWA

② The IRB approach→计算 capital

- Advanced IRB approach：【银行有更大的自由度】
  - ✓ 银行自己估计：probability of default (PD), loss given default (LGD), exposure at default (EAD), and the maturity adjustment (MA)
- Foundation IRB approach
  - ✓ 银行自己估计：PD
  - ✓ 监管机构：LGD, EAD, and MA
    - ■ The EAD 类似巴塞尔 1 的 credit equivalent amount (CEA),考虑 netting.
    - ■ MA is set to 2.5 in most cases.

$$\text{Credit capital} = \sum_i \mathrm{EAD}_i \times \mathrm{LGD}_i \times (\mathrm{WCDR}_i - \mathrm{PD}_i)$$

$$= \sum_i \mathrm{EAD}_i \times \mathrm{LGD}_i \times \mathrm{WCDR}_i - \mathrm{EL}$$

【关于上面这个公式，可以和《信用风险》科目的“Chapter 11 Regulatory Capital”一起理解】

$$\text{Credit RWA} = 12.5 \times \text{Credit capital} \times \text{MA}$$

114. Operational risk capital requirement

① The basic indicator approach(BIA)基本指标法→计算 capital

$$K_{BIA} = \frac{\sum_{i=1}^{n} \mathrm{GI}_i \times 15\%}{n}$$

- ✓ GI_i:某一年的 gross income

- 当过去三年中有某年收入负数时，将负数按 0 来计，分母 n 变为 n-1。

② The standardized approach(SA)标准法

Business Line	Capital (% of Gross Income)
Corporate finance	18%
Trading and sales	18%
Retail banking	12%
Commercial banking	15%
Payment and settlement	18%
Agency services	15%
Asset management	12%
Retail brokerage	12%

$$K_{SA} = \frac{\sum_{\text{year } 1-3} \max[\sum (\mathrm{GI}_{1-8} \times \beta_{1-8}), 0]}{3}$$

- 当过去三年中有某年收入负数时，将负数按 0 来计，分母依然为 3

③ The advanced measurement approach (AMA)高级计量法

- AMA 比其他两种方法更复杂
- Banks were required to treat operational risk like credit risk and set capital equal to the 99.9 percentile of the loss distribution minus the expected operational loss.
- 核心思想：银行需要考虑八大业务条线和七类损失事件的每一种组合。对于 56 个（=7*8）组合中的每一个，估计一年损失。然后对这些估计进行汇总，得出 loss distribution，确定总资本要求→LDA

115. Solvency II 计算 SCR

- ✓ Standardized approach
- ✓ Internal models approach
  - ■ SCR 基于一年期的在险价值（Value-at-Risk，VaR）概念，置信水平为 99.5%。
  - ■ Internal models must satisfy three criteria.
    - ❖ First, the data and methodology must be sound.

- ❖ Second, risk assessments must be calibrated to be in accordance with target criteria set by the regulator.
- ❖ Finally, the model must be used in actual business decision-making.

116. Basel 2.5→市场风险 Market Risk Capital

$$MRC_t^{IMA} = Max\left(M_r * \frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}, VaR_{t-1}\right) + Max\left(M_s \frac{1}{60}\sum_{i=1}^{60} SVaR_{t-i}, SVaR_{t-1}\right) + SRC_t + IRC_t$$

- ✓ SRC_t ： Specific Risk Charge, capture default risk
- ✓ IRC_t ： Incremental Risk Charge, estimate losses associated with rating downgrades.

117. Capital ratio requirements under Basel III

		Tier 1 Equity Capital	Total Tier 1 capital	Total capital
所有银行的普适性要求	Minimum capital requirements (MCR)	4.5%	6%	8%
	CCB	2.5%		
	MCR+CCB	7.0%	8.5%	10.5%
	CCyB	0%~2.5%		
只对 G-SIB 银行	G-SIB buffer	1%~3.5%		

118. Leverage ratio

Leverage Ratio = Tier 1 capital /Total Exposure ≥ 3%

119. SMA 计算-操作风险资本金

✧ Step 1: Find the business indicator (BI) 【统计收入】

BI = ILDC + SC + FC

- ❖ I：利息（interest）收益；
- ❖ L：经营租赁（lease）收益；
- ❖ DC：股利分红（dividend component）收益；
- ❖ SC：服务费用（service component）收益，比如托管业务、中间业务等；
- ❖ FC：银行的金融收益，例如股权投资等。

✧ Step 2: Calculate the business indicator component (BIC)

BIC = BI × marginal coefficients.

Bl Bucket	Bl Range (bn)	Marginal Bl Coefficients α
1	<= 1	12%
2	1 < BI <= 30	15%
3	> 30	18%

✧ Step 3: Find the Internal Loss Multiplier (ILM)计算内部损失乘数 ILM

$$ILM = Ln[\exp(1) - 1 + (\frac{LC}{BIC})^{0.8}]$$

- ❖ 损失成分 LC（Loss Component）等于银行过去 10 年平均历史损失的 15 倍

✧ Step 4: Calculate Risk Capital Requirement 计算操作风险的资本金要求

ORC = BIC × ILM