A Novel Approach to Modelling of MOS Devices – Development of a Simulation Tool for MOSFET Analysis

Third Year Individual Project – Final Report

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Abstract

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# Introduction

## Background and motivation

The utilization of semiconductor materials has undeniably paved the way for the most significant milestones in the electronics industry. With the invention of first bipolar transistor in 1947, the modern electronics era started. At its core, these tri-terminal devices function as electrical switches, facilitating the execution of mathematical logic operations through the manipulation of voltage and current.

Preceding transistors, their predecessors, such as vacuum tubes also were used for digital computing, however, the use of semiconductor materials has allowed transistors to be much faster, use less power and become progressively smaller, down to nanometric scales today. By 1954, transistors had already replaced forty-eight years old vacuum tubes in many applications [1].

Building upon the success of transistors, further advancements emerged with the invention of Metal-Oxide-Semiconductor Field Effect Transistors (MOSFETs) in 1960. Unlike the bipolar transistors which are controlled by current, MOSFETs are controlled by electric field, thus allowing faster switching speeds. This attribute has propelled MOSFETs to the forefront of contemporary electronic applications. Today, they are predominantly used in all advanced integrated circuits, microprocessors, memory units and more [2].

In 1965, Gordon E. Moore observed that the density of transistors in integrated components was doubling every two years, indicating an exponential decrease in size [3]. This phenomenon owes its validity to the inherent semiconductor nature of these devices. Notably, Moore's Law, encapsulating this doubling trend, persists even 59 years later. As manufacturing complexities and rising fabrication costs pose challenges and signals Moore’s Law’s end, ongoing innovations, such as leveraging analog behaviours of semiconductor devices, sustains improvements in integrated circuit performance [4]. One constant in this evolving landscape is that technological advancement consistently results in an increase in the complexity of device physics.

The initial fabrication cost for transistors of this size is notably high. As a result, the study of transistor modelling becomes crucial. In this project, various MOSFET models will be explored, simulations will be implemented for transistors of diverse sizes, and simulation results will be compared with actual measurements to emphasize the need for enhanced models in transistor simulation.

## Aims and Objectives

The primary objective of this project is building a MOSFET model and developing a tool capable of accurately simulating models for predicting the transfer and output characteristics of a commercial MOSFET. To achieve this overarching aim, the following specific objectives will be pursued:

1. MOSFET Modelling – Theoretical Background

* Build a MOSFET model by introducing underlying semiconductor device behaviours.
* Identify publicly available MOSFET models with varying levels of complexity.

1. Software Development

* Develop a standalone software capable of simulation based on user-provided parameters.
* Enable a plug-and-simulate functionality to incorporate user-defined models seamlessly.

1. Performance Evaluation

* Conduct real-data measurements using an actual MOSFET device.
* Compare the results obtained from the real-data measurements with those generated by the simulation software.

## Project Overview

A diagram of a process

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Figure 1 illustrates the project plan, categorized into three key sections: (top) Theoretical Research focusing on publicly available models for MOSFETs up to 20nm, (middle) Main Software Development Plan, and (bottom) GUI Development Plan.

# Literature review

## Introduction

This chapter employs a bottom-up approach to delve into MOSFET modelling. MOSFETs are semiconductor devices, so modelling them requires a solid grasp of underlying semiconductor physics. This chapter will explain underlying properties of semiconductors and their resulting electrical characteristics. Starting with quantum mechanics and statistical mechanics, semiconductor’s carrier concentration in equilibrium will be established. Then the movement mechanics of the carriers will be established, leading to characterization of voltage-current relationship. Finally, the junctions of different semiconductor materials will be analysed to create the complete picture of semiconductor devices and create a simple MOSFET model.

## Quantum Mechanics Principles & Schrödinger Equation

To analyse any semiconductor device, moreover any electricity conducting device, electrons are the most fundamental unit that should be examined. While classical laws of physics yield high accuracy for the analyses of large objects, they become inadequate to describe the subatomic particles. For these particles, quantum mechanical principles are the backbones of their analysis. The quantum mechanics in its core has three principles [5]:

Energy is quantized in discrete packets called quanta and its energy is related to the frequency:

(1)

Where is the energy, is the Planck’s constant and is the frequency.

Particles exhibit wavelike behaviour. The wavelength of a particle is related to its momentum.

(2)

Where is the de Broglie wavelength of a particle and is the momentum.

The momentum and the position of a particle cannot be described with absolute accuracy; the energy and the time that the particle has that energy cannot be described with absolute accuracy.

(3)

Where p, x, E, t are the uncertainties in momentum, position, energy, and time respectively. is the reduced Planck’s constant.

(4)

Although these three principles govern quantum mechanics, to apply them and characterize a system’s quantum-mechanical state, Schrödinger’s time-independent equation should be solved.

(5)

Where is the wavefunction that describes the quantum state of the system, is the total energy of the particle, is the potential experienced by the particle and is the mass of the particle. In this paper, Schrödinger’s equation will not be mathematically solved but rather the solutions will be examined to understand how electrons behave.

## Electron Models

### Classical Free Electron Model

This model relies on classical laws of physics and electrons are assumed to behave like a classical gas, adhering to the laws of kinetic theory and the electron energies are not quantized; they can take any value without quantum restrictions [6]. The classical free electron theory successfully verified Ohm’s law, explained thermal and electrical conductivities. However, its reliance on classical mechanics hindered its ability to explain crucial properties in semiconductors and insulators.

### Quantum Mechanical Electron Model – Potential Well

The Schrödinger’s equation’s solution for the potential well, illustrated in figure 2, has the following form.

(6)

A diagram of a region

Description automatically generated

Figure 2 Diagram of the infinite potential well [5]

Where is the wavenumber and represents the spatial frequency. To describe the energy of the system, can be written in terms of energy using wave-particle duality.

(7)

At the boundaries and of the potential well, the wavefunction is evaluated:

(8)

(9)

For any non-zero

(10)

(11)

The solution suggests that there are allowed energy levels for the electron to occupy. Allowed energy levels are illustrated in figure 3 for an infinite well with width .

A diagram of a rectangular object with numbers and equations

Description automatically generated with medium confidence

Figure 3 Distribution of allowed quantum levels in the potential well [7]

### Band Theory of Solids

Moving from the isolated particle to crystalline solids is necessary to understand how electrons behave. When atoms come together in an arranged way to create a crystal, there exists a potential that is different to the potential well previously assumed. The most basic repeating pattern in a crystal is called the primitive cell. When the primitive cell is replicated in three dimensions the crystal lattice is formed [8]. This periodic structure creates a periodic potential across the volume of the solid. Even without solving the Schrödinger’s equation for this potential, the conclusion of bands and energy gaps could be achieved by applying the quantum mechanical electron model to a crystal lattice. Due to wave particle duality, electrons can be treated as waves, and so the electron waves interacting with the periodic potential creates destructive and constructive interferences.

A linear solid of lattice constant , as illustrated in figure 4, can be examined to demonstrate the band formation. Bragg condition, which describes the reflections of a wave in a crystal lattice [9], is met at locations . At these locations the traveling electron wave is reflected continuously, thus becoming a standing wave. At and , so called first Brillouin zone of this lattice.

A diagram of a waveform

Description automatically generated

Figure 4 (a) The change in potential energy experienced by a conduction electron due to the ion cores in a linear lattice, (b) the energy bands due to first Brillouin zone edited [10]

For a quantitative description, the Schrödinger’s equation is solved for a specific crystal’s 3D potential and the solution yields to the specific E-k diagram of that crystal.

## Electrical Conduction in Semiconductor Crystals

Electrical conduction is simply the existence of a net movement of charge carriers. An intrinsic semiconductor at 0 K has all valence electrons bonded to the neighbouring nuclei. As the temperature increases, the bonds break, i.e. the electron gains enough energy to jump to the higher energy level, conduction band, and thus can freely move through the crystal. When the electron jumps to the conduction band, generating a negative charge carrier, it also generates a hole, a positive charge carrier, in the valence band. Thus, in the semiconductor, electrical conduction can be due to the movement of a single negative charge carrier at the conduction band or due to the collective recombination/generation of the electrons with holes. To simplify the calculations the latter is assumed to be the movement of a single hole. In figure 5, E-k diagrams of GaAs and Si are illustrated and is defined as the energy gap between the lowest level of conduction band and the highest level of the valence band.

A graph of energy and energy

Description automatically generated

Figure 5 (a) E-k diagram of GaAs, (b) E-k diagram of Si [5]

The energy of a carrier is related to wavenumber, in 3D case wavevector, of that carrier (eq. 7). However, the E-k graphs found by solving to the Schrödinger’s equation suggests that the mass of the particle changes as it moves through the lattice. This phenomenon is defined as the effective mass, , and can be approximated using the curvature of the E-k diagram [5].

(12)

## Intrinsic Charge Carrier Concentration

Perhaps the most important parameter for a semiconductor is the charge carrier concentration. To find the charge carrier concentration, density of states,, and probability of state, , functions are simply integrated. Former giving the number of available quantum states and latter giving the probability of a state being occupied using Fermi-Dirac distribution.

For electron charge carrier concentration in thermal equilibrium :

& (13a, 13b)

(14)

For hole charge carrier concentration in thermal equilibrium :

& (15a, 15b)

(16)

Where is the Boltzmann’s constant, and are effective density of states for conduction and valence bands respectively, is the fermi energy level. Fermi energy level is the energy in which probability of state function is 0.5. For intrinsic semiconductors, intrinsic fermi level, , can be calculated by equalizing (eq 14 & 16):

(17)

The hole and free electron concentration is equal in intrinsic semiconductors, it is defined as the intrinsic carrier concentration, [11]. Intrinsic carrier concentration at thermal equilibrium is constant for a semiconductor:

(18)

## Semiconductor Doping and Change in Fermi Level

Doping is the addition of impurities to semiconductor crystals to change their properties in a controlled matter. The addition of extra electrons from donor atoms or holes from acceptor atoms change the carrier concentrations of intrinsic semiconductors. Electron doped and hole doped semiconductors are called N-type and p-type respectively. Since the Fermi level is directly related to the probability of an energy state being occupied, the addition of charge carriers changes its position from the intrinsic fermi level [11].

& (19a, 19b)

Where eq. 19a is for the case of hole doping, eq. 19b is for the case of electron doping, and are the concentrations of donor and acceptor atoms. Here the importance of Fermi level should be emphasized. It is, as defined, a statistical value, and is extensively used to analyze semiconductor devices since it is a result of all particle behaviors mentioned so far in this paper. Since intrinsic carrier concentration is not related to Fermi level, eq. 18, regardless of doping, it stays the same, thus the product of hole and electron concentrations is a constant.

## Carrier Drift & Diffusion Current

Drift is the movement of carriers due to the electrostatics force acting on them in the presence of an external field and it is the first movement mechanism that will be examined [12]. The current density due to drift is given in terms of carrier concentrations & , and charge velocities due to drift & :

& (20a, 20b)

The velocities of the carriers can be given in terms of carrier mobilities & :

& (21a, 21b)

The mobility is a proportionality constant and abstracts scattering mechanisms. The charge carrier’s interaction with the vibrating atoms of the lattice, phonon scattering, and the charge carrier’s interactions with the doped impurities, ionized impurity scattering, are abstracted. They are derived in terms of lattice properties, temperature, and impurity concentrations [8], however, for the purposes of this paper, since the resulting mobilities can be measured are not important.

The other carrier movement mechanism is due to the concentration gradient of the charges, the diffusion current. It is derived using Fick’s law of diffusion and given in terms of charge concentration gradients in 3D & :

& (22a, 22b)

Where & electron and hole diffusion coefficients respectively and related to the mobilities by the Einstein relation:

(23)

As a result, the total current density, , is given as:

(24)

## Recombination & Generation of Electron-Hole Pairs

In nonequilibrium, i.e. when external voltage is applied, heat is changed or light is emitted, carrier concentrations discussed in chapters 2.5 & 2.6 changes due to the generation and recombination of carriers and eq. 18 does not hold true. In equilibrium the rate of generation of electrons & holes and the rate of recombination of electrons & holes, & , & respectively, are equal:

(25)

In nonequilibrium, however, there are few mechanisms that change these rates causing imbalance in carrier concentrations. First of which will be discovered is direct band-to-band transition. In direct band-to-band recombination an electron in conduction band simply loses energy, by emitting a photon with energy equal to the bandgap energy and combines with a hole in in valence band. This phenomenon happens more frequently in direct bandgap semiconductors and less frequently for in-direct bandgap materials. Direct bandgap materials have their valence band's maximum energy and conduction band's minimum energy occurring at the same momentum, meaning they share the same "k" value in the E-k plot [13]. The recombination rate for band-to-band recombination with low level-injection assumption (i.e. assuming the concentration of excess charges, & , being significantly smaller than the majority carrier concentrations in thermal equilibrium) is given for p-type and n-type materials respectively:

& (26)

Where represents excess minority carrier lifetime and is a measurable quantity.

The other mechanism of transition, indirect transition, is the dominant mechanism for direct semiconductors. Indirect transition occurs due to the crystal defects. Since now, the energy gap was considered to have zero probability of state, however, due to the defects there exists allowed energy states in the bandgap called traps. In indirect transition electron does not move directly between conduction and valence bands rather moves to the trap. This transition changes the excess minority carrier lifetime defined in eq. 26, however, since it is a measurable quantity, the eq. 26 encapsulates this transition mechanism and hold true.

## Continuity Equation

Using all the mechanics discussed so far, an equation can be derived and later be used as the main tool to analyse and model all semiconductor devices [14]. The charge distribution inside the semiconductor as a function of space and time is given in the continuity equation for electrons and holes respectively:

(27a)

(27b)

## Summary

# Methods

## Introduction

This chapter is divided into 2 parts: MOSFET Model and Simulation software development. In the first part, a basic MOSFET model will be built using the semiconductor physics discussed in chapter 2. MOSFETs are complicated devices utilizing several interfaces, so, to develop a model PN junction and MOS interfaces will be analysed. Analysis of these interfaces will yield key relationships of device and material parameters. With these relationships, famously known MOSFET model for long channel devices will be derived using gradual channel approximation. In the second part, a standalone simulation tool will be introduced and its implementation will be explored.

## MOSFET Model Development

### PN Junction

##### PN Junction - Conceptual Introduction

When a p-type and an n-type semiconductor are put in contact with each other, PN junction will form. Assuming equilibrium, before contact, both types of semiconductors will be neutral.

During contact, the equilibrium will be disturbed due to minority carriers’ diffusion to opposite side. During the diffusion, p-type semiconductor will gain a net negative charge while the n-type semiconductor will gain a net positive charge. Diffusing minority carriers will recombine with the majority carriers on the other side leaving behind the ionized dopant atoms in the junction. A region of no mobile carriers will be created. In this region there will only be these ions creating a charge difference, thus an electric field will be created [15]. The equilibrium will be reestablished when the force of electric field balances out the force of diffusion.

The resultant electric field will act as a potential barrier, , pushing the majority carriers away from the junction and creating a region of no mobile carriers. This depletion region will not allow any charge movement from one side to another, making the PN junction in equilibrium act like an insulator. The junction and the energy diagrams before and after contact are illustrated in figure x.

A diagram of a diagram of a number

Description automatically generated with medium confidence

Figure x PN junction before contact (a), transient state (b), and after equilibrium is established (c). Simplified view before contact (a1) and after equilibrium (c1). - and + represents charges; circled charges represent ionized doped charges and non-circled charges represent mobile charges. Band diagram before contact (a2) and after equilibrium (c2); and represents mobile charge carriers.

##### PN Junction – No Bias

Three important PN junction parameters are conceptually explained in the previous section: electric field, built-in potential and space charge region. In this section, they will be mathematically evaluated.

The occurrence of a built-in potential is due to the fermi-level mismatch and since fermi level is a statistical value, in equilibrium it will be constant throughout the whole material. This results in energy bands bending towards the junction. Its value can be derived from (figure x c2) using (19a & 19b):

(28)

Where is thermal voltage.

To quantify the electric field, charge densities across the device should be examined. For an easier analysis, so far, the junction is assumed to be linear; for the derivation of electric field, the depletion layer is assumed to be abrupt [16] illustrated in (figure x).

A diagram of a graph

Description automatically generatedA diagram of a triangle

Description automatically generatedA diagram of a graph

Description automatically generated

Figure x

Using Poisson’s equation and Gauss’s law gives the electric field in terms of charge density and potential:

(29)

Where is the electrical potential at position x, is the net charge density, is the permittivity of the semiconductor. For one dimensional analysis the electric field in p and n sides respectively:

, (30a)

, (30b)

The integration constants are in the form that gives results in no electric field outside the depletion layer, i.e. they are zero. The potential for p and n sides respectively:

, (31a)

, (31b)

Since the built-in potential is the potential difference between and , the net potential at is 0, thus, can be calculated. The net potential at will then be equal to and .

(32)

So, the built-in potential in terms of depletion region width can be written as:

(33)

The electric field is continuous, so at eq. 30a & 30b can be used to express in terms of :

, , (34)

By substituting eq. 34 into eq. 33, depletion regions widths and can be found:

& (35a, 35b)

Finally, the total depletion width:

(36)

It is important to note this analysis is done for the doping profile given at in (figure x), however, for any doping profile, the same analysis can be done by changing the net charge density function, .

### MOS Capacitor

##### MOS Capacitor – Conceptual Introduction

Metal-oxide-semiconductor structure is a specific type of metal-insulator-semiconductor structure and is at the core of MOSFET devices. In MOSFETs, the output current is regulated through the voltage applied to this structure. The structure is like a parallel plate capacitor as illustrated in figure x.

A diagram of electrical circuits

Description automatically generated

Figure x

The figure illustrates a p-type semiconductor; however, an n-type semiconductor could also be used. To simplify, the semiconductor will be assumed to always be p-type in this section unless explicitly stated and the jargon will be used accordingly. When an external voltage is applied to the metal, depending on the polarity of the voltage, the holes, majority charge carriers, will either be attracted to or pushed away from the oxide interface. These events are called accumulation of holes and depletion respectively. During the depletion, a region of no carriers will be created in the interface like the case of PN junction. This region will be called space charge region or depletion region. When further voltage is applied, holes will be further pushed and electrons will have a high concentration, effectively creating an n-type channel in the oxide-semiconductor interface, and this is called the inversion of electrons. MOS capacitor will operate in these three regions.

##### MOS Capacitor – Threshold Inversion Point

In this section, an in-depth analysis of MOS structures will be undertaken starting with an isolated semiconductor. The threshold voltage, a crucial MOSFET parameter, will be derived from device and interface parameters.

To quantitatively analyse the depletion region, the energy-band diagram of a p-type semiconductor is illustrated in figure x where bending of bands is present for any reason.

A diagram of a power line

Description automatically generated

Figure x

Surface potential, , is defined to quantify the bending of the bands. The thickness of the depletion region, , could be derived using the depletion width formula derived for PN junctions (eq. 36). The formula simplifies due to in depletion region:

, (36a, 36b)

If the surface potential is increased further, the bends will bend further, and the depletion region thickness will increase. The threshold inversion point will be reached when the concentration of majority carriers in the bulk semiconductor is equal to the concentration of minority carriers on the surface. The voltage in which this point is reached is defined as the threshold voltage, , and it occurs when [17]. After the threshold inversion point is reached, further increase in surface potential will only slightly change the space charge region’s thickness [18], thus, its maximum is defined:

, (37a, 37b)

So far, the metal and oxide are not considered. Now, a complete MOS structure will be analysed, thus the specific material choices for the MOS structure will be important. For this analysis, silicon dioxide () is assumed to be the oxide layer, p-type silicon (-) is assumed to be the semiconductor substrate. With different material choices the interface could be made so in equilibrium it can in inversion. For this case the energy band diagram is illustrated in figure x.

A diagram of a vacuum level

Description automatically generated

Figure x

The work function, , is defined as the energy needed to remove an electron from the fermi level to the vacuum level. For metals this value is dependent on temperature, however it is fairly constant for a long range of temperatures [19] and can be treated as a material constant, while for semiconductors, the fermi level should be calculated using (eq. 19a 19b).

The electron affinity, , is defined as the energy needed to remove an electron from the bottom of the conduction band to the vacuum level, thus it is a material constant for both metals, insulators and semiconductors [20].

When the materials are in contact and thermal equilibrium is reached, the resulting energy-band diagram is illustrated in (figure x.b). There exist charge traps in the oxide-semiconductor interface, similar to those mentioned in section 2.8, due to oxidation process during device fabrication. These charge traps result in a potential difference across the oxide layer,. As a result, the bending of the bands depends on this potential as well as the fermi level differences of metal and semiconductor. The fermi level difference before contact can be quantified by defining metal-semiconductor work function difference, , that will be constant for a given gate-semiconductor material choice:

(38)

After contact, the fermi level on the semiconductor-oxide interface and metal-oxide interface should be equal in equilibrium:

(39)

Eq. 39 can be simplified and rearranged using eq. 38:

(40)

So far, the analysis assumed no potential bias. If there exists an external potential, the gate voltage, , surface potential and the potential across the oxide will change, thus eq. 40 will no longer be equal to zero:

(41)

Since the oxide is a dielectric, voltage across it can be derived using Gauss’s law [17] and depletion region thickness formula derived in (eq. 36a, 36b). The voltage across the oxide during inversion:

(42)

Where , is the charge density ( is in metal side of metal-semiconductor interface, is in semiconductor side of oxide-semiconductor interface, is the trapped charge in the semiconductor side of the oxide semiconductor interface), , is the oxide capacitance per unit area and is the thickness of the oxide. It is important to note that charge in the channel created due to inversion is ignored in this derivation.

Finally, a formula for threshold voltage can be derived assuming the condition :

(43)

(44)

The required voltage to create an inversion, i.e. to make the minority carrier concentration in the oxide-semiconductor interface equal to the majority carrier concentration in the bulk semiconductor, can now be calculated from manufacturing parameters.

### MOSFET Model – Long Channel Model

In this section, a basic and well known MOSFET model, long channel model, will be derived [18] [21] [22]. The model will not consider many quantum-mechanical effects which will ultimately result in low accuracy prediction of device behaviour. However, the model is beneficial to understand the MOSFET device.

To create a MOSFET, in addition to the previously analysed MOS capacitor, two heavily doped semiconductors are added to the substrate. These semiconductors are called source and drain. The metal of the MOS capacitor is called the gate. These are the three terminals of a MOSFET. The device is called NMOS if the source and drain are n-type and PMOS if they are p-type. The source and drain are always the same type while the substrate is the opposite type. The device is illustrated in figure x.

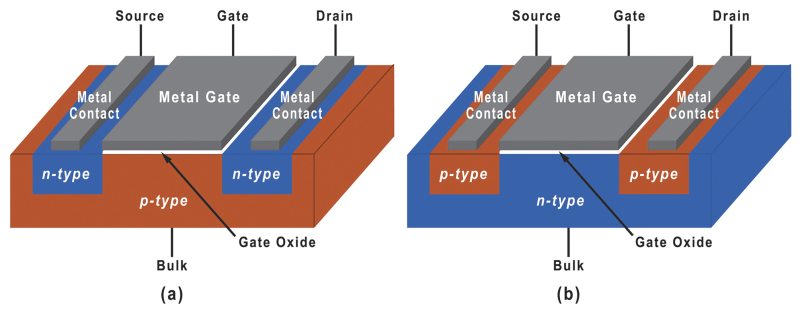


Figure x

Under no potential, the structure has two PN junctions that would not allow current flow from source to drain. A potential applied to the gate (defined as in the previous section and is applied from gate to bulk, here will be labelled as for convention and will assume the bulk is connected to the source) will control the conductivity of these PN junctions. In accumulation region, the conductivity will be decreased, ideally will be zero; in depletion region the conductivity will still be ideally zero; in inversion region the conductivity will be nonzero, and a conductive channel will be formed. In the most ideal MOSFET model, the switch model, this behaviour is modelled such that once the threshold inversion point is reached, a channel with infinite conductivity will be present. Unlike this, in the long channel model, conductivity of the channel is defined using the previously discussed topics and the effect of the potential across source and drain,, on the channel conductivity is considered. The model assumes following [18]:

1. Only drift current occurs in the channel.
2. There is no leakage current due to MOS capacitor.
3. The potential in x-direction gradually decreases from source to drain and the potential in y-direction quickly decreases from gate to the bulk semiconductor. This assumption is called the gradual channel approximation.
4. All trapped charges in the oxide are at the semiconductor interface.
5. Carrier mobility is constant throughout the channel.
6. Generation, recombination and hole current are negligible.

Assuming an NMOS device, illustrated in figure x, the derivation is as follows:

Diagram of a diagram of a rectangular structure

Description automatically generated

Figure x

Due to assumption 1, the current density is equal to the drift current density that is derived in (eq. 20, 21):

(45)

Since the electron concentration is not constant it is given in as a function of position y. The total current in x-direction is:

(46)

Where is the charge per unit area in the channel and is the width of the channel. To find the charge in the channel, Gauss’s law is applied to the surface illustrated in figure x.a.

Diagram of a diagram of a structure

Description automatically generatedA diagram of a diagram

Description automatically generatedA diagram of a curve

Description automatically generated

Figure x

(47)

The electric field vectors to surfaces 1 and 2 cancel each other and no electric field is present at bulk semiconductor, so the integral is only solved for surface 4:

(48)

Electric field across the oxide at x-position x can be written in terms of the oxide potential eq. 43:

(49)

The total charge in the channel can be derived using eq. 48, 49 and written in terms of threshold voltage using eq. 42, 43:

(50)

The channel charge concentration can be substituted back to eq. 46:

(51)

Finally, the electric field can be written in terms of potential and the equation can be integrated through the channel length:

(52)

The equation can be rearranged:

(53)

This equation is valid only for and . The equation shows for a given gate-to-source voltage, increase in drain-to-source voltage will quadratically increase and quadratically decrease. When maximum current occurs at , the saturation voltage, , is defined.

This model assumes after saturation voltage, current stays constant and before threshold voltage there is no current. So, the complete long channel model for NMOS becomes:

, (54)

, (55)

, (56)

### Review of Current MOSFET Models

The long channel model is a very simple model. Although its accuracy is poor and gets even poorer as the channel size gets shorter, it is still used quite extensively. Mainly for educational purposes and as a hand-calculation. It is also important to note that this model does not consider any frequency effects and thus purely a DC model.

Currently the MOSFET sizes are down to few nanometres, so the quantum effects become crucial to consider. As more effects are considered, the models get more complicated and computationally expensive. The models are categorized into four categories [23]:

* Table lookup models: Either experimental or analytical model simulation data is stored for various conditions and used for further simulation. No underlying physical insights in the models.
* Empirical models: Real data measurements are modelled by curve-fitting. No underlying physical insights in the models.
* Analytical models: Models developed purely from device physics through rigorous mathematical calculations. They are the most computationally expensive models. Further categorized in two categories
* Semi-empirical analytical models: Combines of analytical and empirical elements. The model is analytically developed, and the accuracy is further improved by curve fitting with empirical values. Extensively used in current industry.

The choice of a model depends on the aim. Since the aim of this project is to simulate a MOSFET before production, no data measurement is possible unless the same, or similar if some inaccuracy is tolerable, technology is used. Therefore, analytical or semi-empirical analytical models should be developed. In this paper, a simple yet complete development process of an analytical model is undertaken to develop the long channel model. Device outputs can be simulated by choosing the geometry of the device and the material properties of the metal, oxide and semiconductors.

The most advanced simulators such as Silvaco TCAD or Synopsys Sentaurus are purely physical models that numerically solves the fundamental equations derived in section two such as continuity equation, transport equation, Poisson’s equation alongside other quantum mechanical equations for the specific 2D or 3D device structure for the most accurate output data.

### Improvements on the Model

In this section the long channel model will be improved by adding the effects that were ignored. The mathematical derivation will not be included; however, the underlying quantum mechanical principle will be briefly introduced. For simplicity, these effects might be included with the addition of non-physical variables, ultimately categorizing the models semi-empirical analytical. The improvements on the model will be discussed in the results and discussion chapter.

Channel Length Modulation

Velocity saturation

Etc.

Etc.

## Simulation Software Development

A standalone simulation software is built as part of this project. In this section the program’s capabilities and limitations will be explained. The possible improvements of the program will be explained in future work section.

The whole program is written solely in C++ language.

Its frontend is designed using Qt framework. The user interface is designed to be easy-to-use, allowing user to save a MOSFET device with its parameters. Since the parameters depend on the model used, in MOSFET creation the model should be chosen. When a model is chosen the user interface will have a brief description or a pdf description of the model. Once the device is saved with its unique name it can be simulated. The simulation windows available are output simulation and transfer simulation. Output simulation makes a sweep of drain-to-source voltage for a given gate-to-source voltage while transfer simulation does vice versa.

The backend is implemented a modular approach. The program does not need any altering if a new model is to be added. Instructions to add a new model will be available with the release of the program. Simply the model needs to be added in the source code. The only part of code that is not under my ownership is the plotting libraries of “progsbase” company. The plotting libraries’ licence give me full control and rights to use/change/commercialize the code.

A screenshot of a computer

Description automatically generated A screenshot of a computer

Description automatically generated

A graph of a number of lines

Description automatically generated with medium confidence

## Summary

# Results and discussion

## Software Development

## Simulation vs Measurement

## Discussion

## Summary

# Conclusions and future work

## Conclusions

## Future work

* Light simulation -> optoelectronics
* Modular program so it can be expanded
* Plastic transistor modelling
* Using AI to create empirical models – Since cost of manufacturing is low it is viable to create empirical models.

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Appendices

A Project outline

Project outline as submitted at the start of the project is a required appendix. Put here.

B Risk assessment

Risk assessment is a required appendix. Put here.

C Tables of Symbols – etc.

Notes: Subscript represents the variable at equilibrium condition. Subscripts & are used to represent the variable for n and p type semiconductors respectively.

Table I - Table of Symbols - Chapter 2

|  |  |  |  |
| --- | --- | --- | --- |
| Symbol | Meaning | Symbol | Meaning |
|  | Energy |  | Planck’s constant |
|  | Frequency |  | Wavelength |
|  | Momentum |  | Uncertainty in momentum |
|  | Uncertainty in position |  | Uncertainty in energy |
|  | Uncertainty in time |  | Reduced Planck’s constant |
|  | Wavefunction |  | Mass |
|  | Electrical potential |  | Wavevector or wavenumber |
|  | Bandgap energy |  | Effective mass |
|  | Electron carrier concentration |  | Hole carrier concentration |
|  | Density of states function |  | Probability of state function |
|  | Fermi level energy |  | Conduction band energy |
|  | Valence band energy |  | Boltzmann’s constant |
|  | Temperature |  | Effective density states at conduction band |
|  | Effective density states at valence band |  | Intrinsic carrier concentration |
|  | Elementary charge |  | Intrinsic Fermi energy level |
|  | Fermi potential for p-type semiconductor |  | Fermi potential for n-type semiconductor |
|  | Acceptor atom concentrations |  | Donor atom concentration |
|  | Drift current density |  | Drift velocity |
|  | Electron mobility |  | Hole mobility |
|  | Electric field |  | Diffusion current density |
|  | Electron diffusion coefficient |  | Hole diffusion coefficient |
|  | Electron generation rate |  | Hole generation rate |
|  | Electron recombination rate |  | Hole recombination rate |
|  | Excess minority carrier lifetime |  |  |

Table 2 - Table of Symbols - Chapter 3

Includes Table 1. Some symbols can be used for different variables and some variables can have different symbols.

|  |  |  |  |
| --- | --- | --- | --- |
| Symbol | Meaning | Symbol | Meaning |
|  | Boltzmann’s constant |  |  |
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